CONTOUREDUCATION

VCE Mathematical Methods ½
AOS 2 Revision [2.0]
Contour Check (Part 1)

Contour Check

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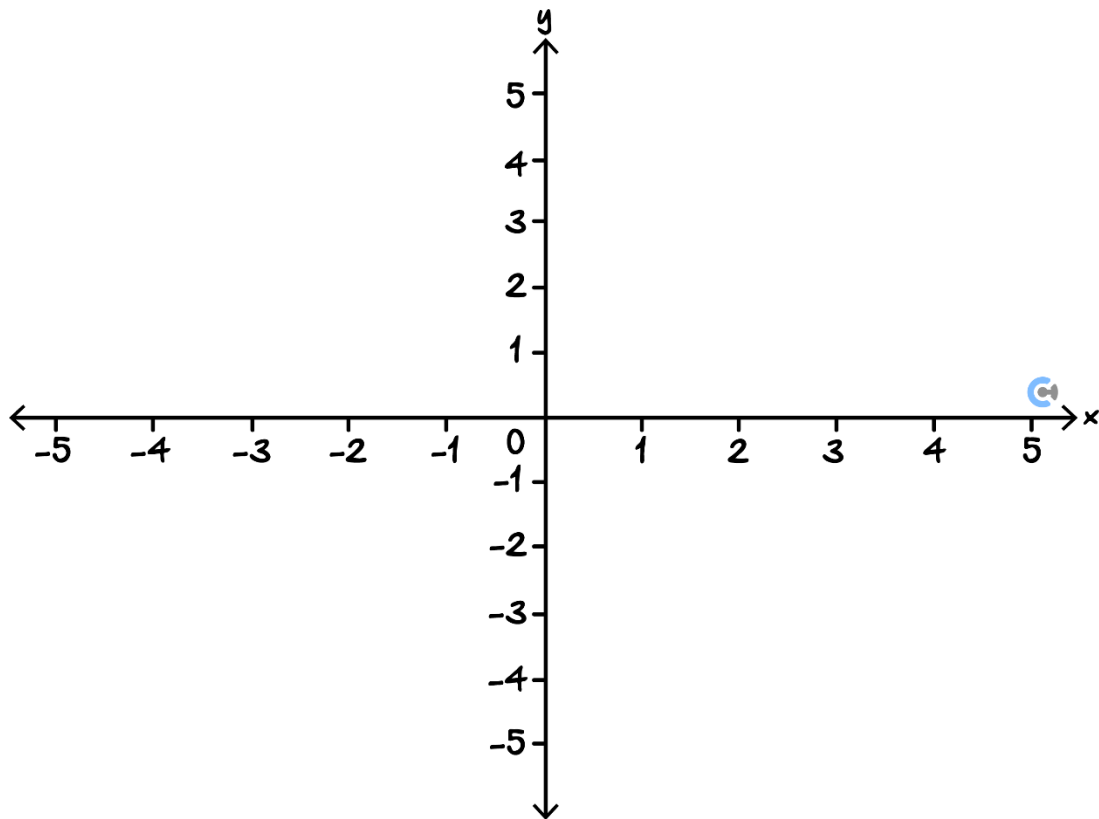## Section A: [2.1] – Functions & Relations I (Checkpoints)

Sub-Section [2.1.1]: Sketch and Find the Rule of Hyperbolas Functions

Question 1

Graph the following curve, labelling all intercepts and asymptotes with their equations.
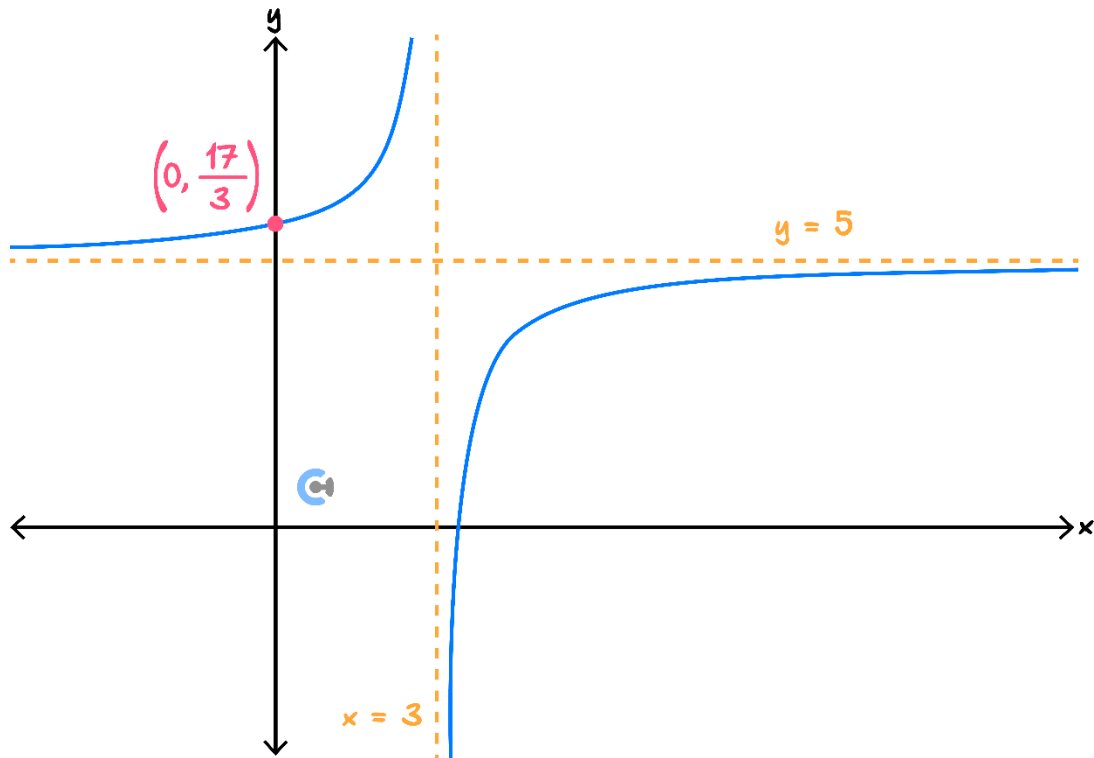
$$y = \frac{1}{x+1} - 2$$

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Question 2

Find the rule for the following graph, given it is of the form $y = \dfrac{a}{x - h} + k$.

$\left(0, \dfrac{17}{3}\right)$

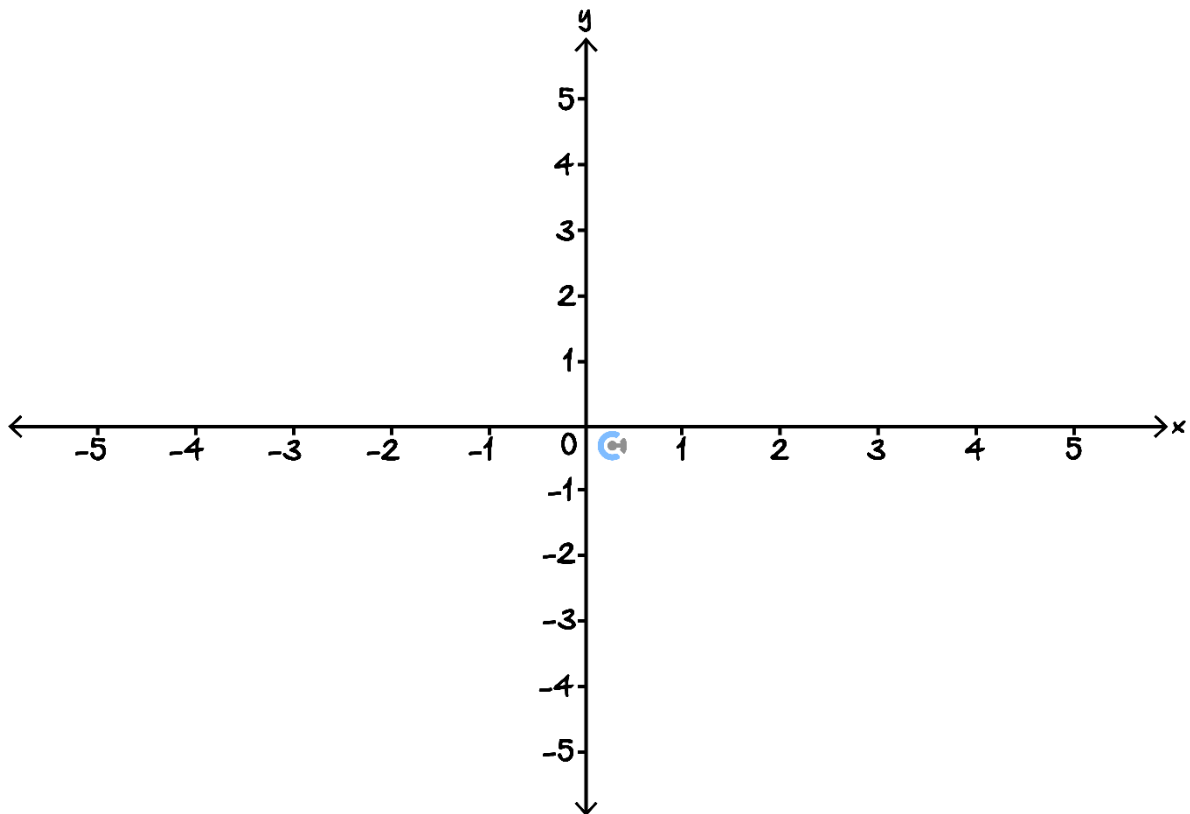$y = 5$

$x = 3$

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Question 3

Graph the following curve, labelling all intercepts and asymptotes with their equations.

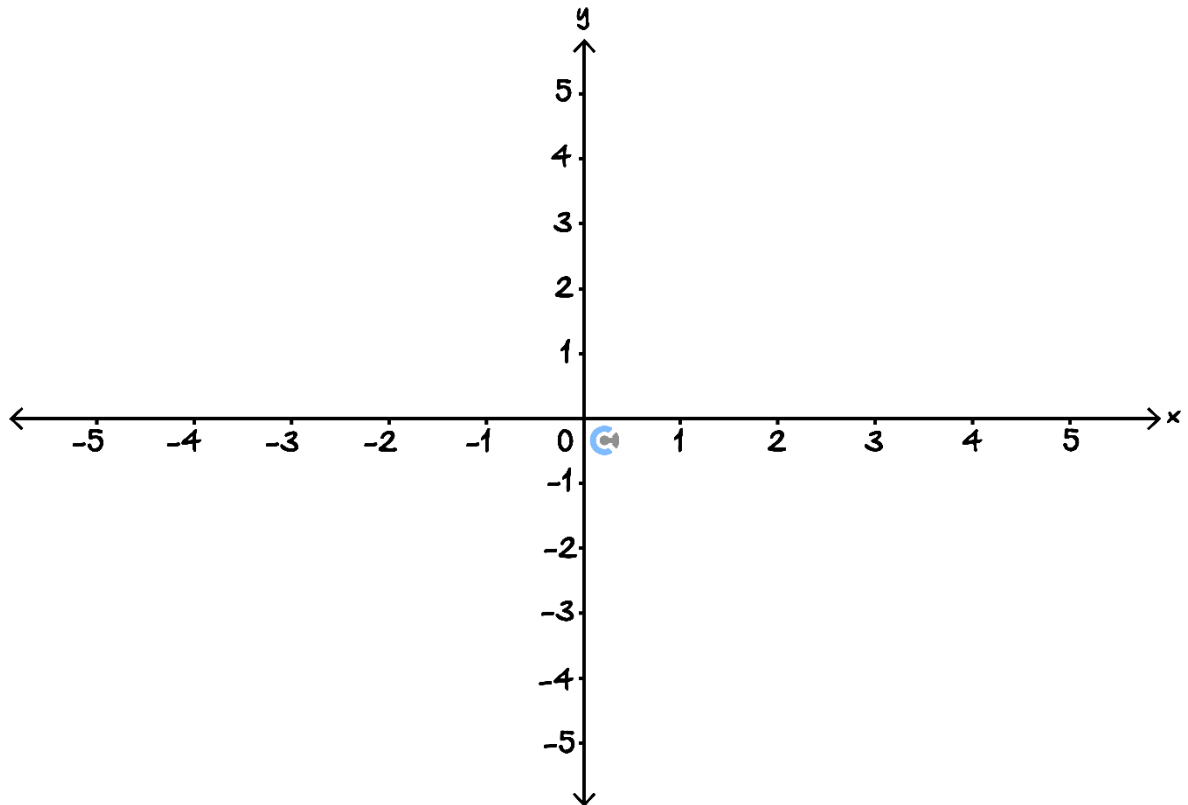$$y = 3 - \frac{2}{5 - 3x}$$
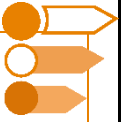

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Question 4

Graph the following curve, labelling all intercepts and asymptotes with their equations.

$$y = \frac{2 - 2x}{x - 2}$$

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Sub-Section [2.1.2]: Sketch and Find the Rule of Truncus Functions

Question 5

Graph the following curve, labelling all intercepts and asymptotes with their equations.

$$y = \frac{1}{(x-1)^2} + 2$$

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Question 6

Find the rule for the following graph, given it is of the form $y = \frac{a}{(x-h)^2} + k$.
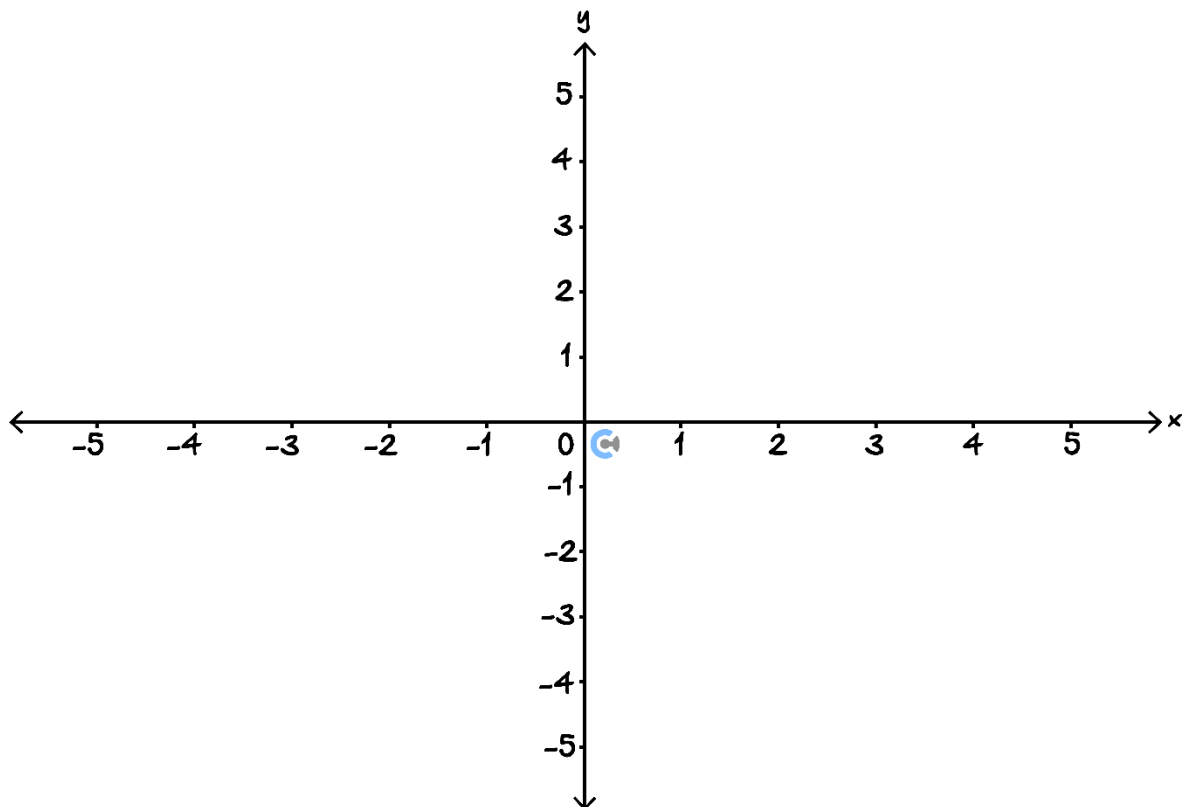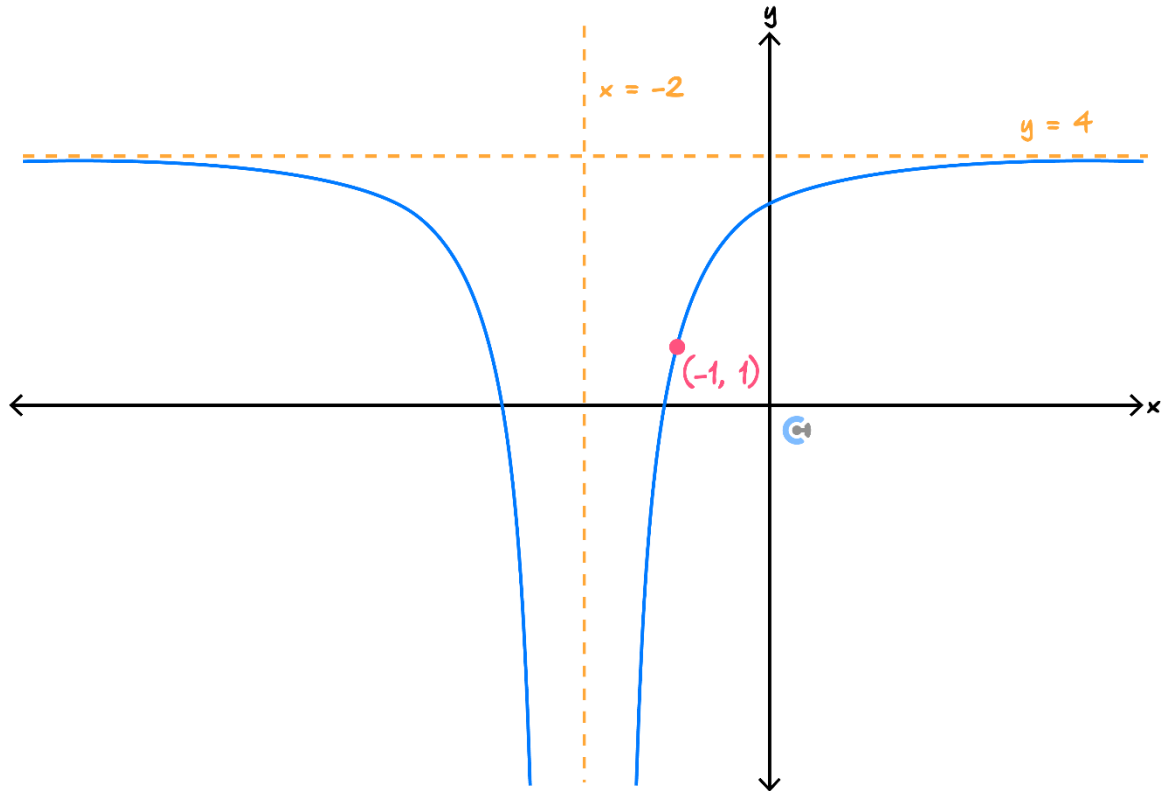

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Question 7

Graph the following curve, labelling all intercepts and asymptotes with their equations.
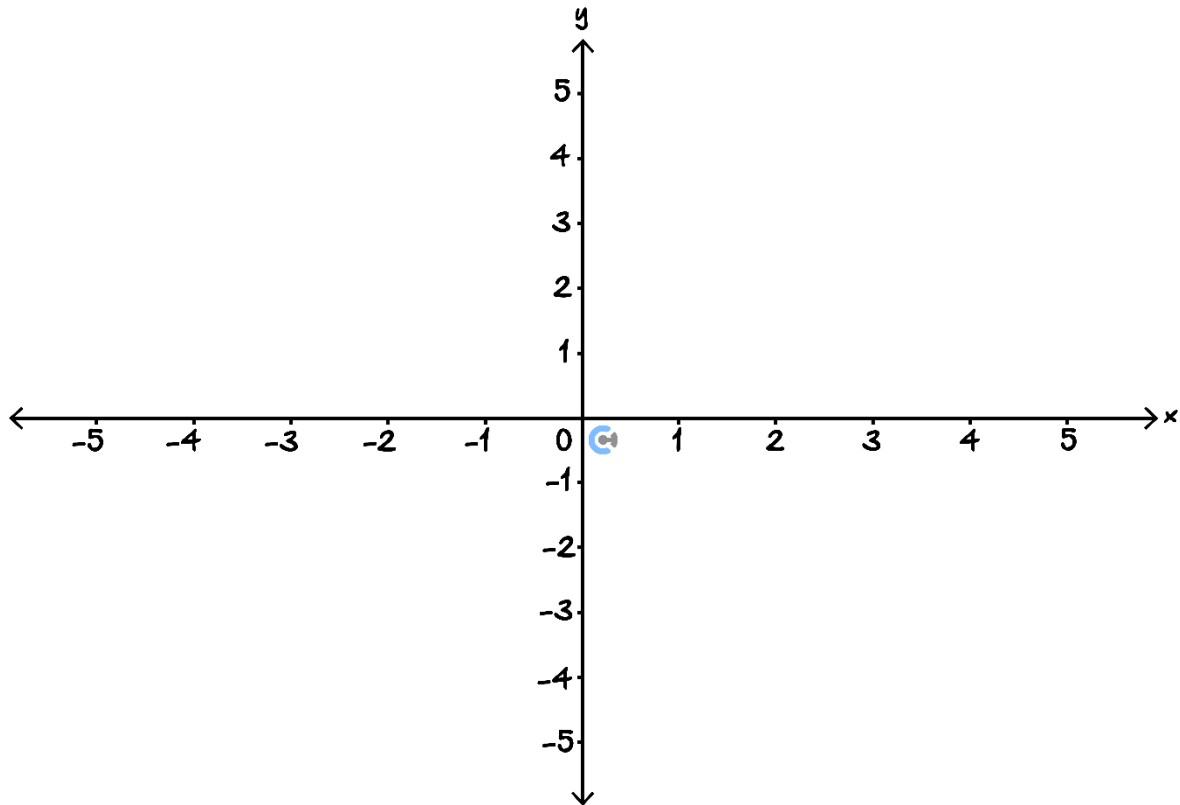
$$y = \frac{6}{(1-x)^2} - 3$$

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Question 8

Graph the following curve, labelling all intercepts and asymptotes with their equations.

$$y = \frac{x^2 - 4x - 1}{(x-2)^2}$$

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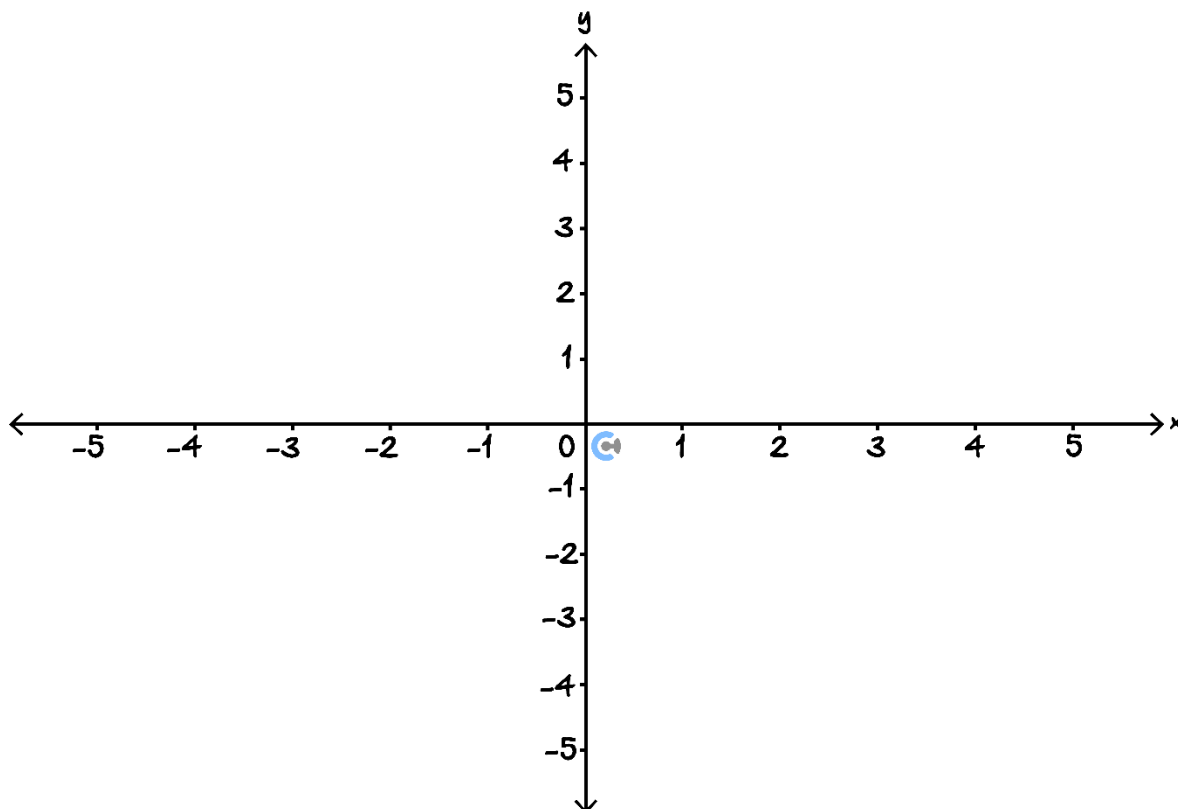
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Sub-Section [2.1.3]: Sketch and Find the Rule of Root Functions
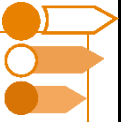
Question 9

Graph the following curve, labelling all intercepts and start points.

$$y = \sqrt{x - 2}$$

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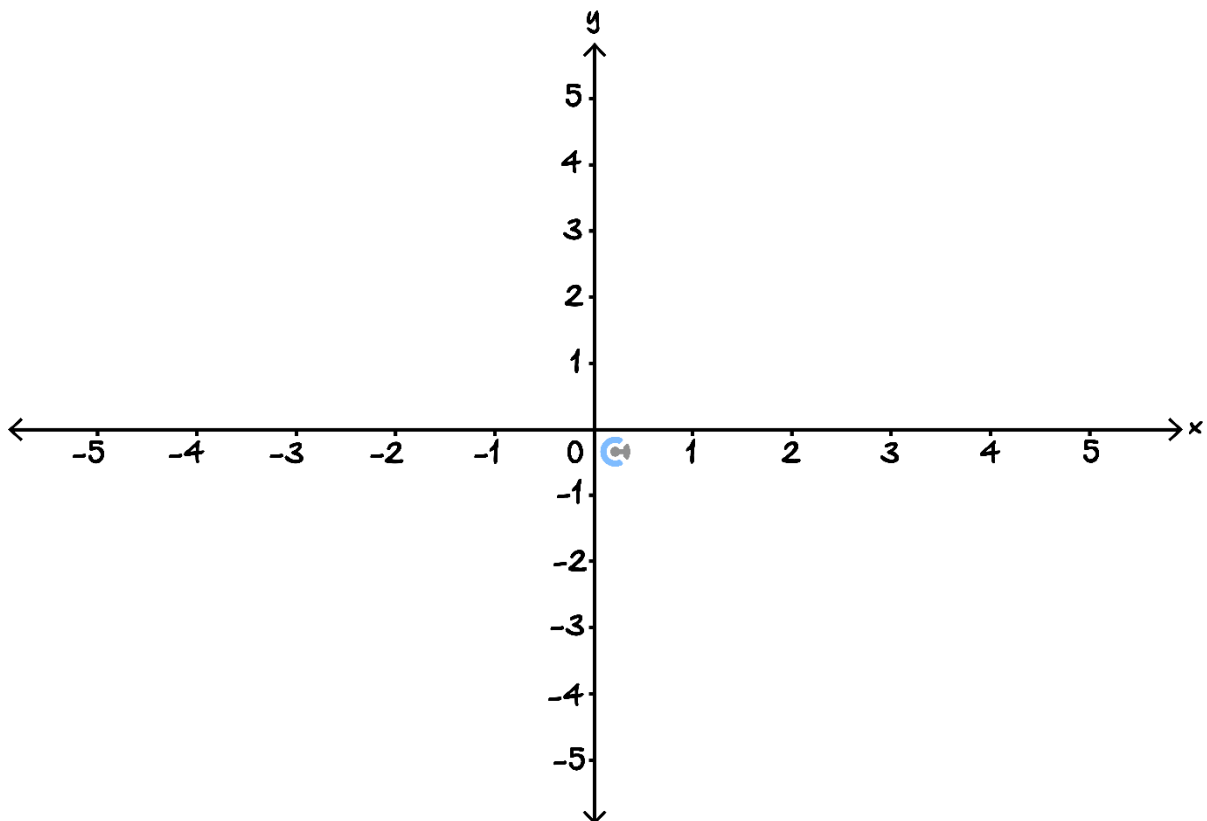
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Question 10

Find the rule for the following graph, given it is of the form $y = a\sqrt{h - x} + k$.

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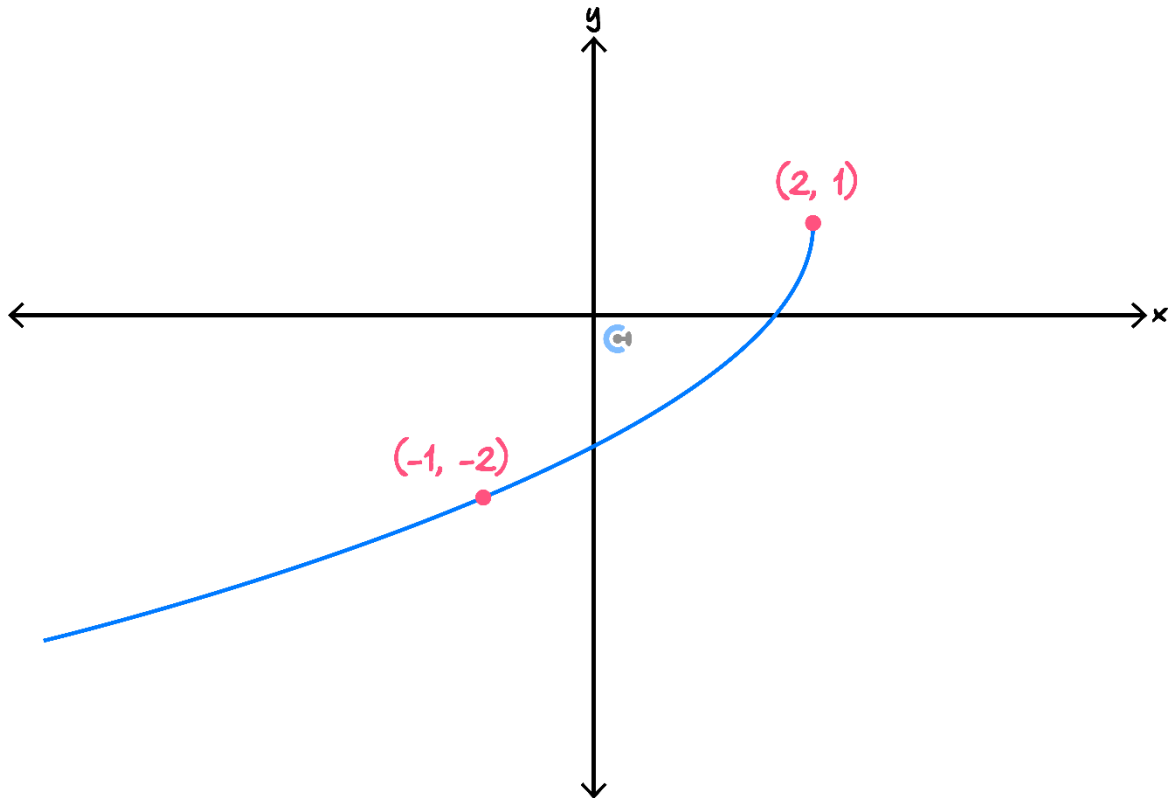
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Question 11

Graph the following curve, labelling all intercepts and start points.

$$y = 4 - \sqrt{5 - 2x}$$

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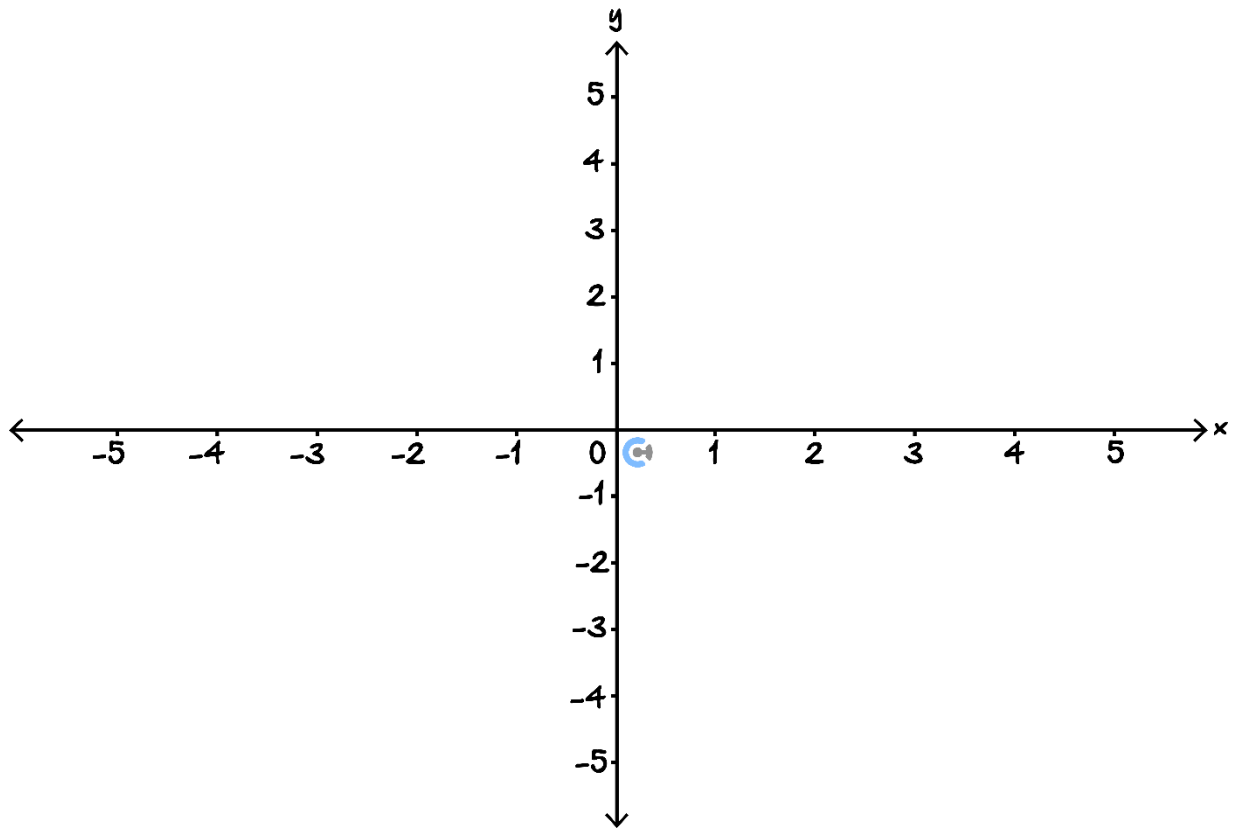
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Question 12

Graph the following curve, labelling all intercepts and turning points.

$$(y - 3)^2 = 5 - 2x$$

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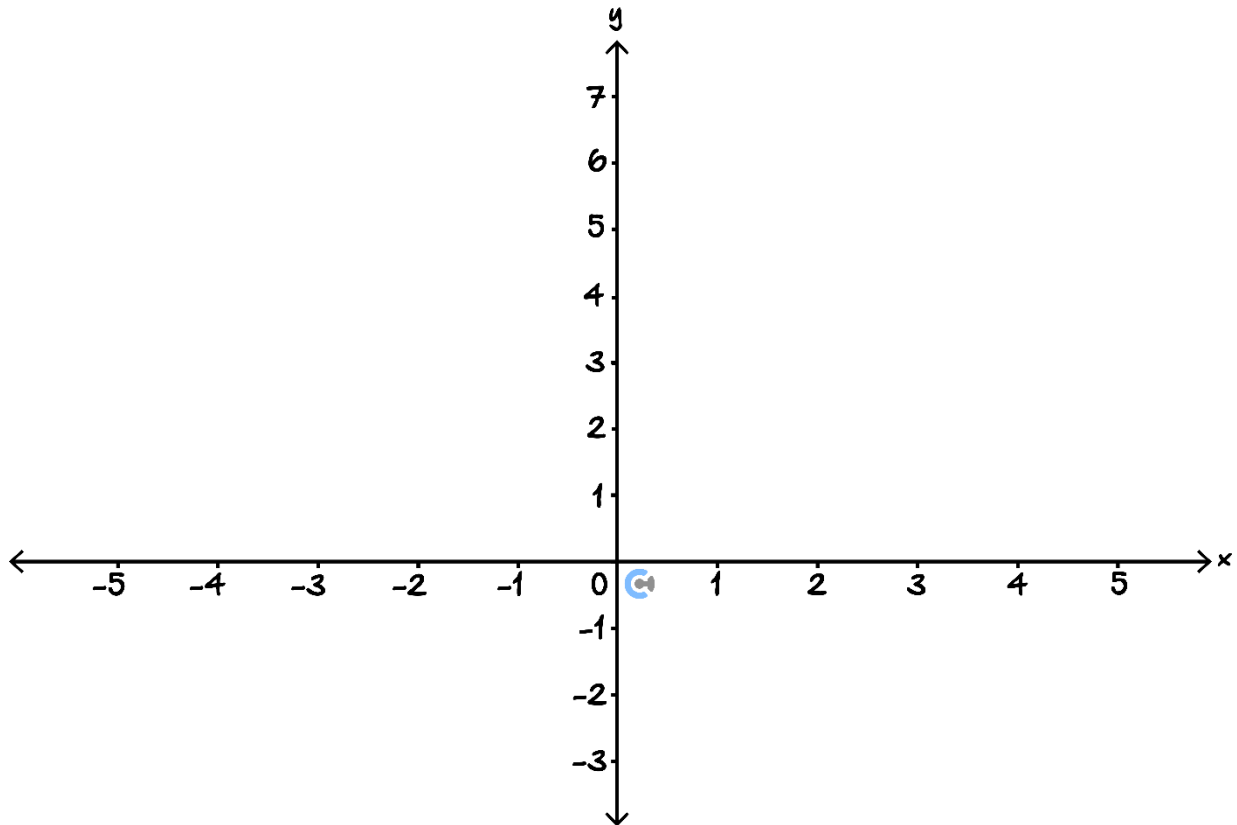
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Sub-Section [2.1.4]: Sketch and Find the Rule of Semicircles and Circles
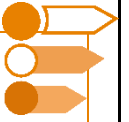
Question 13

Graph the following circle, label all intercepts.

$$(x-3)^2 + (y+4)^2 = 25$$

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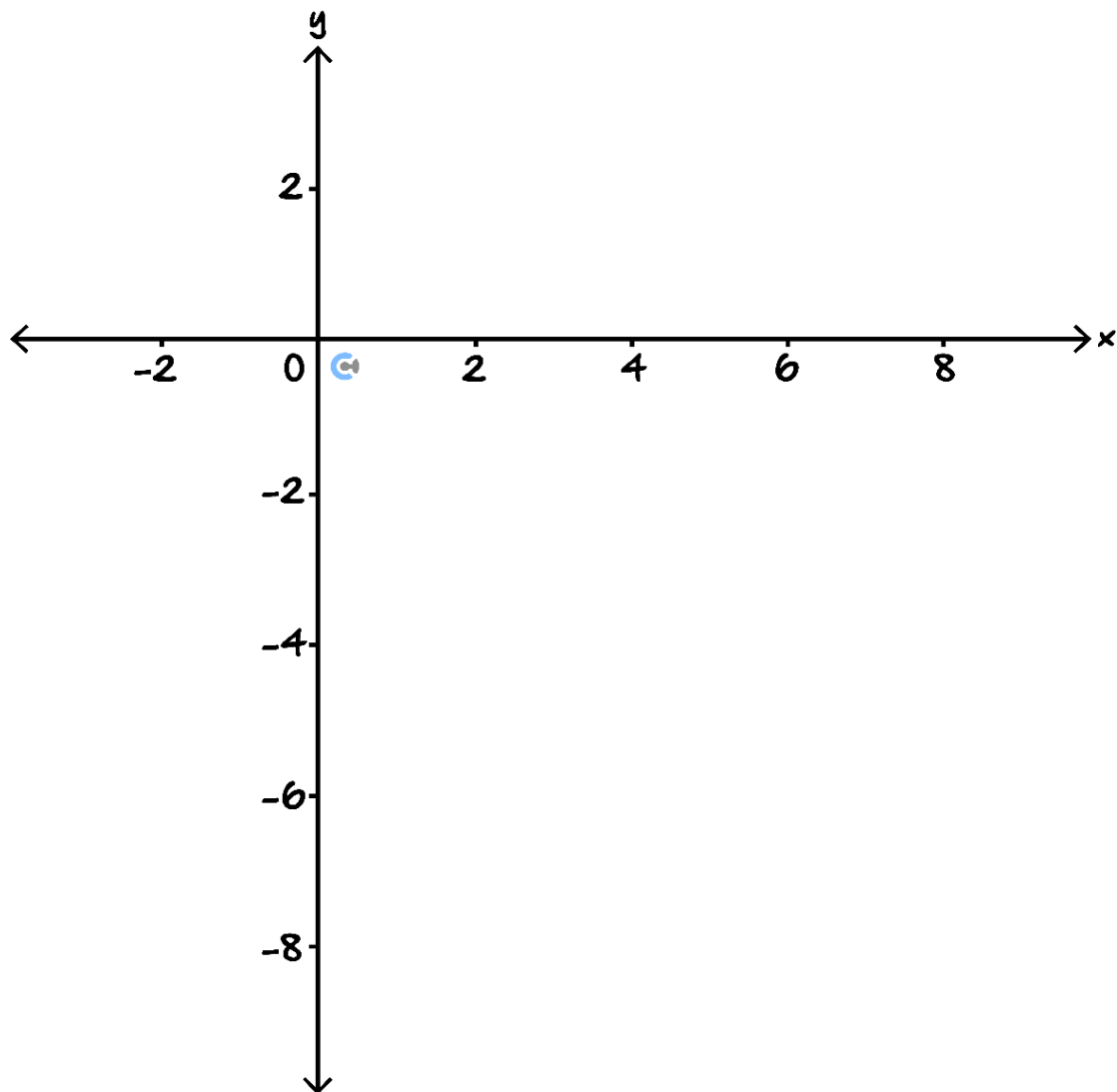
Question 14

Graph the following semi-circle, label all intercepts.

$$y = 3 - \sqrt{9 - (x + 2)^2}$$

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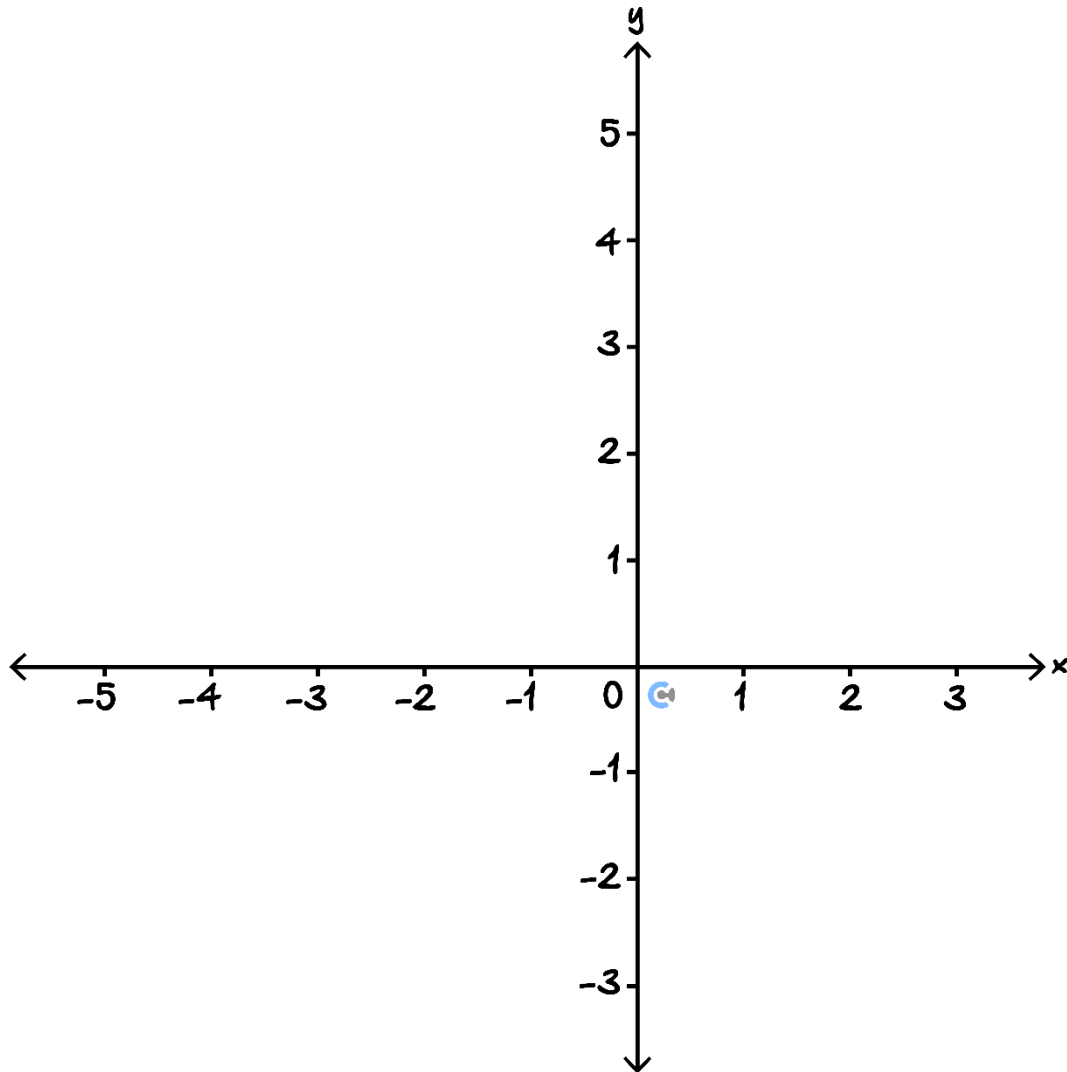
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Question 15

Graph the following circle, label all intercepts.

$$x^2 + y^2 + 2y - 4x = 4$$

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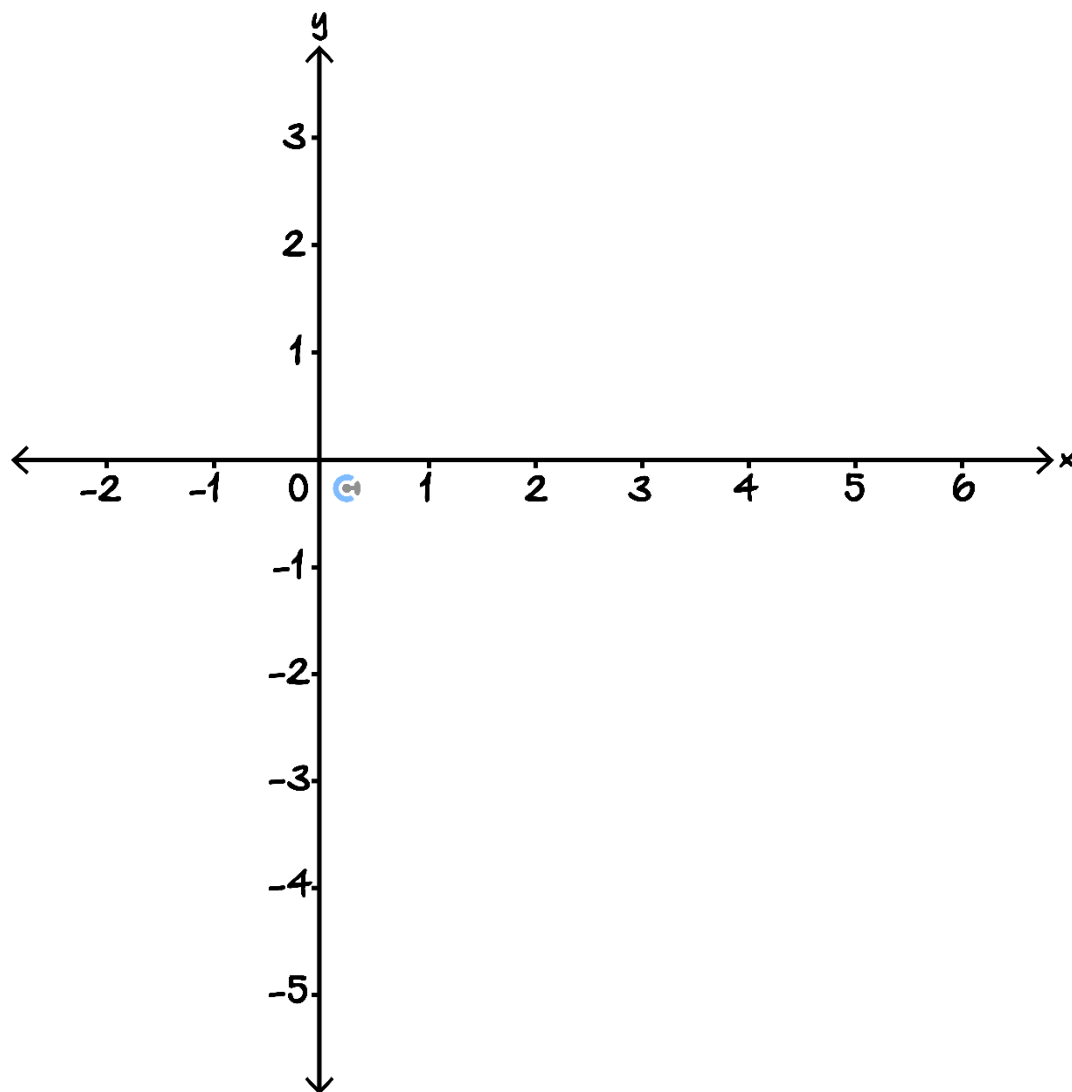
Question 16

Determine the equation of the semi-circle with radius 1, shown on the graph below.

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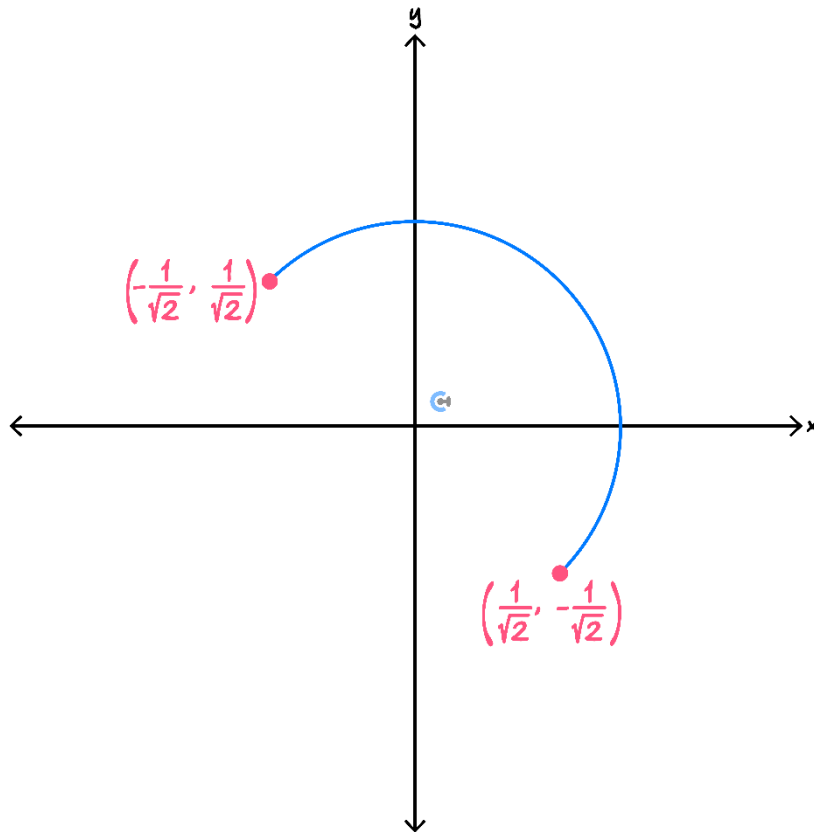
Sub-Section [2.1.5]: Identify the Type of Relations and Identify whether the Relation is a Function
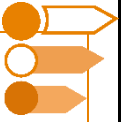
Question 17

For each of the following graphs, identify the type of relation depicted and whether the relation is a function.

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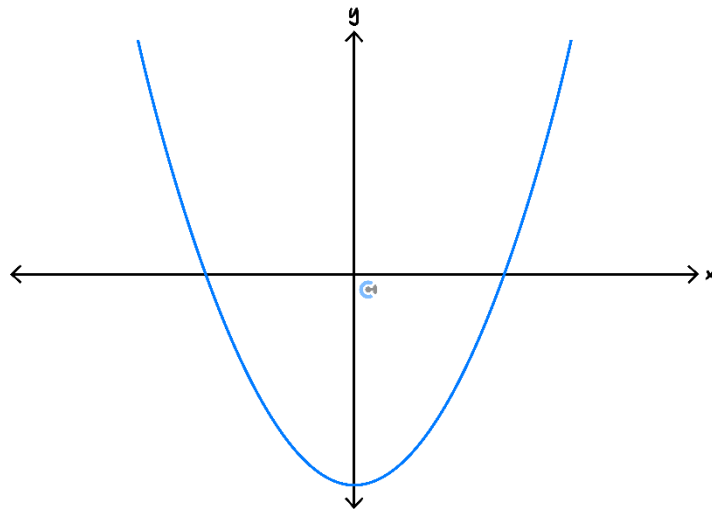
b.

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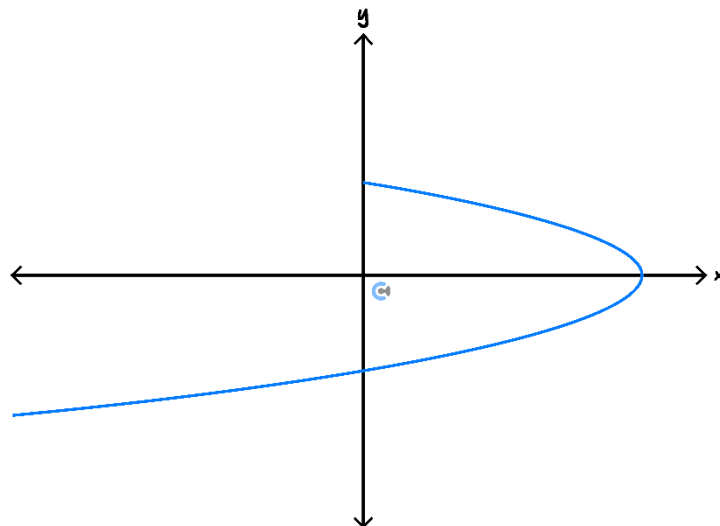
c.

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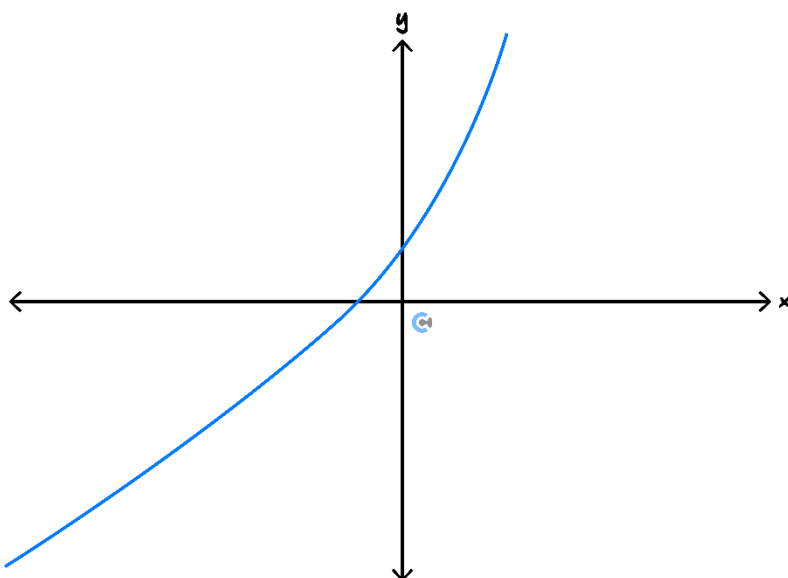
d.

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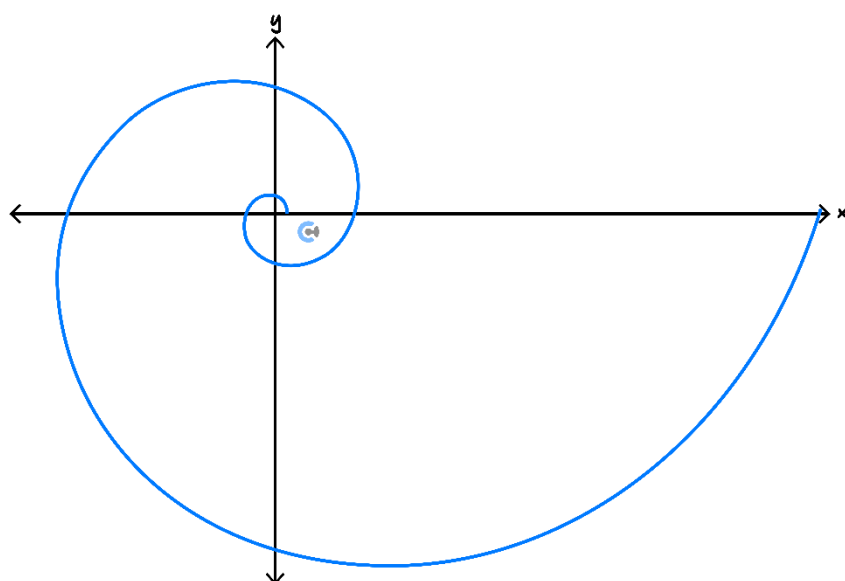
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Section B: [2.2] – Functions & Relations II (Checkpoints)

Sub-Section [2.2.1]: Find Domain and Range of Functions

Question 18

Find the domain of the following functions:

a. $y = \sqrt{5 - 2x}$

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b. $y = -\dfrac{3}{x^2 + 4x - 12}$

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c. $y = 2 \log_e(x + 1)$

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Question 19

Find the maximal domain of the following functions:

a. $y = \dfrac{\left(\sqrt{x^2+9x+18}\ \right)}{2}$

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b. $y = \dfrac{3}{\sqrt{6-5x-x^2}} - 4$

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c. $y = \log_5\left((1-x)(x+4)^2\right) + 1$

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Question 20

Express $f(x)$ in full function mapping notation.

a. $f(x) = \log_2(2x^2 + 7x + 6)$

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b. $f(x) = (3x^2 - 12x + 16)^{\frac{3}{2}}$

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c. $f(x) = 3\sqrt{-\frac{1}{4-2x} + 1}$

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Question 21

Find the maximal domain of the function $f(x) = x^2 + 4x + 12$ such that, the range of $f(x)$ is $[8,17]$.

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Sub-Section [2.2.2]: Sketch and Find the Domain and Range of Hybrid Functions

Question 22

Sketch the following graphs. Label all intercepts and endpoints.

a. $f(x) = \begin{cases} \frac{1}{3}(x-2)^2 - 3, & 2 \le x < 6 \\ -x - 1, & -1 < x < 2 \end{cases}$
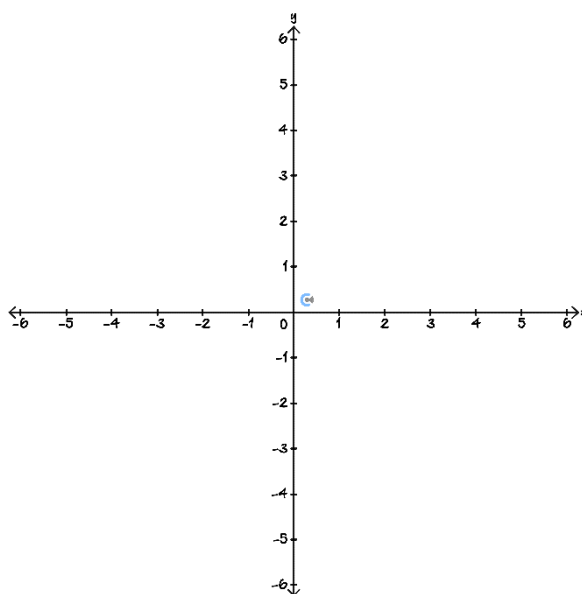
b. $f(x) = \begin{cases} 2x + 4, & -3 < x \le -2 \\ x^2 - x - 2, & -2 < x < 2 \end{cases}$
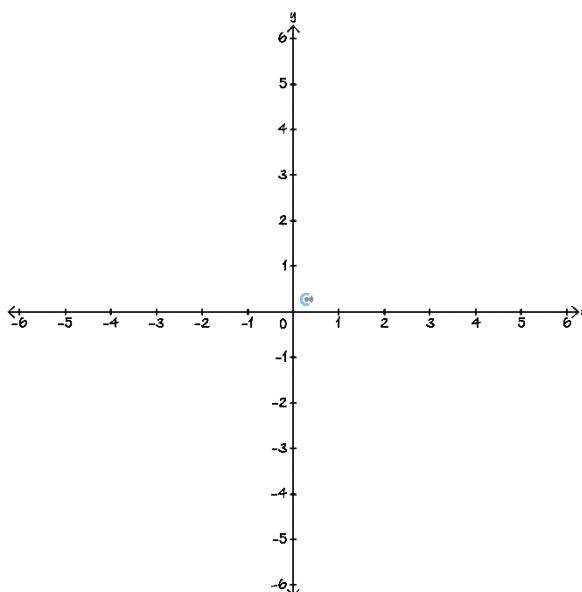
c. $f(x) = \begin{cases} 4 - x^2, & x \leq 0 \\ \dfrac{2}{x} - 1, & x \geq 1 \end{cases}$
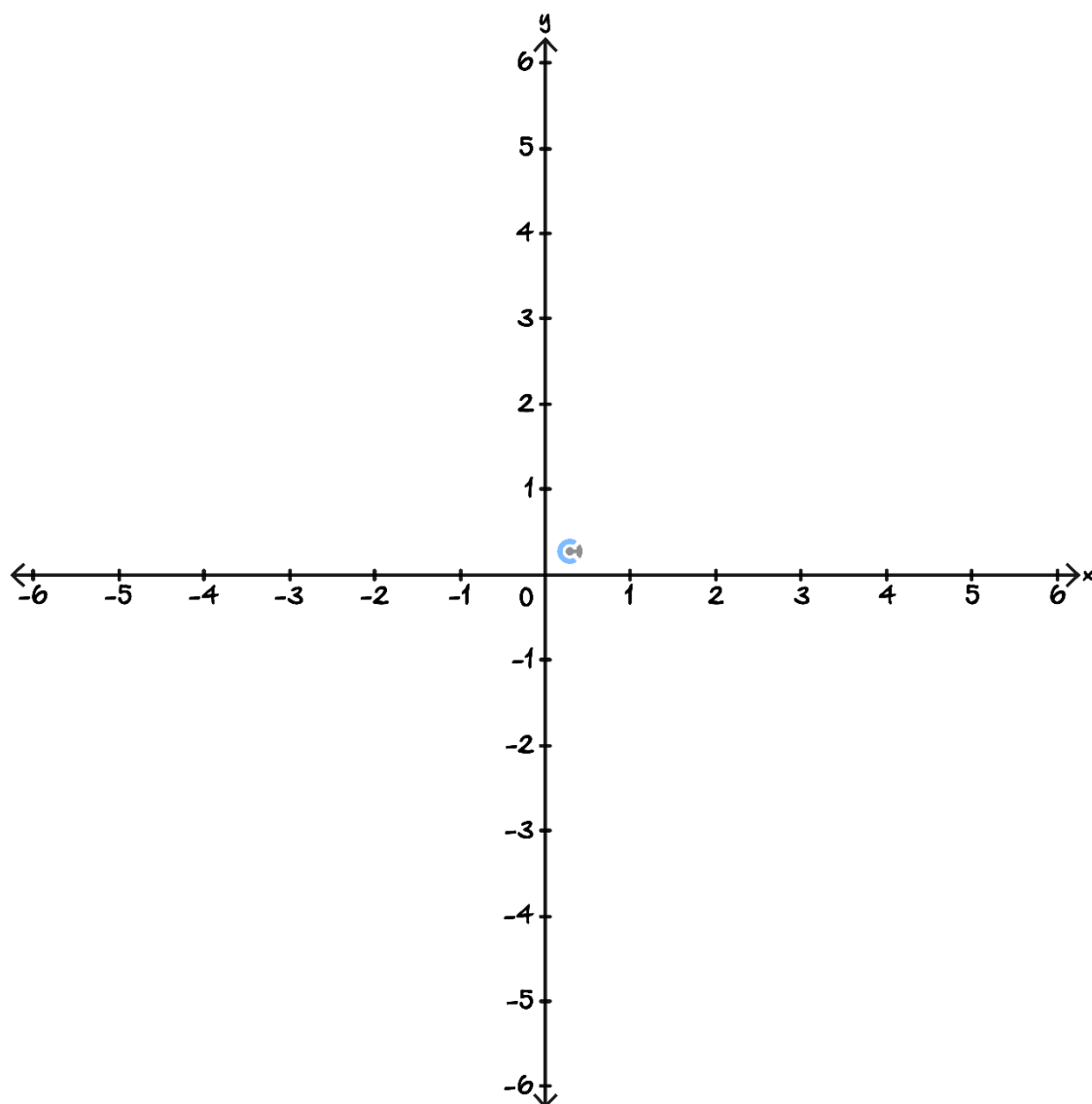
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Question 23

Find the range of the following piecewise functions.

a. $f(x) = \begin{cases} x - 2, & -4 \leq x < 1 \\ 2x - 2, & 1 \leq x \leq 3 \end{cases}$

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b. $f(x) = \begin{cases} x^2 - 4x + 6, & 0 < x < 5 \\ \frac{1}{2}x + 6, & -6 < x < 0 \end{cases}$

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c. $f(x) = \begin{cases} x + 4, & -3 < x < -1 \\ x^2, & -1 \le x \le 2 \\ x - 4, & 2 < x < 8 \end{cases}$

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Question 24

Find the maximal domain of the function: $f(x) = \begin{cases} \sqrt{8 - 2x} \\ \log_e(-x^2 + 5x + 6) \end{cases}$

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Sub-Section [2.2.3]: Find the Rule, Domain, Range, and Intersections between Inverse Functions

Question 25

The function $f(x)$ is defined as $f:[-5,1) \rightarrow \mathbb{R}, f(x) = \frac{2}{3-x} + 6$.

a. Find the equation of $f^{-1}(x)$.

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b. Determine the domain of $f^{-1}(x)$.

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c. State the range of $f^{-1}(x)$.

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Question 26

Consider the function, $g: (-\infty, 0] \to \mathbb{R}, \ g(x) = 2x^2 - 12x + 16$.

a. Find the equation of the inverse function.

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b. Find the domain of the inverse function.

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c. State the range of the inverse function.

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Question 27

Consider the function, $f(x) = 1 - \sqrt{7 - x}$.

a. Define the inverse function of $f(x)$, using full function mapping notation.

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b. Find the point of intersection between $f(x)$ and $f^{-1}(x)$.

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Question 28

Find the values of k such that, the graph $f: [0, \infty) \to \mathbb{R}, f(x) = x^2 + k$ and $f^{-1}(x)$ have 2 solutions.

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Section C: [2.3] – Functions & Relations Exam Skills (Checkpoints)

Sub-Section [2.3.1]: Restrict Domain such that the Inverse Function Exists

Question 29

For each of the following functions, a domain restriction is given with an endpoint a or b. Determine the minimum value of a or maximum value of b such that, the inverse function, f^{-1}, exists.

a. $f:(-\infty, b] \rightarrow \mathbb{R}, f(x) = (x + 1)^2 - 3$

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b. $f : [a, \infty) \rightarrow \mathbb{R}, f(x) = x^2 - 4x + 7$

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c. $f : [a, \infty) \rightarrow \mathbb{R}, f(x) = -x^2 + 8x - 11$

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Question 30

All the functions in this question are written in a non-standard form.

a. Consider the function:

$$f : (-\infty, a) \to \mathbb{R}, \quad f(x) = \frac{11 + 12x + 3x^2}{x^2 + 4x + 4}$$

Find the maximum value of a such that, $f(x)$ has an inverse.

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b. Consider the function:

$$g : (a, \infty) \to \mathbb{R}, \quad g(x) = \frac{x^2 + 8x + 18}{x^2 + 8x + 16}$$

Find the minimum value of a such that, $g(x)$ has an inverse.

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c. Consider the function:

$$h : (a, \infty) \to \mathbb{R}, \quad h(x) = \frac{3x^2 + 6x - 2}{x^2 + 2x + 1}$$

Find the minimum value of a such that, $h(x)$ has an inverse.

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Question 31

For each of the following semicircle functions, a domain restriction is given with an endpoint a.

Determine the minimum or maximum value of a such that, the inverse function exists.

a. Consider the semicircle function:

$$f : [-4, a] \to \mathbb{R}, f(x) = \sqrt{4 - (x + 2)^2}$$

Find the minimum value of a such that, $f(x)$ has an inverse.

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b. Consider the semicircle function:

$$g : [a, 4] \to \mathbb{R}, \qquad g(x) = 2 - \sqrt{8 + 2x - x^2}$$

Find the maximum value of a such that, $g(x)$ has an inverse.

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c. Consider the semicircle function:

$$h : [-5, a] \to \mathbb{R}, \qquad h(x) = \sqrt{20 - 16x - 4x^2} + 1$$

Find the maximum value of a such that, $h(x)$ has an inverse.

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Question 32

Consider the function:

$$f : [a, \infty) \to \mathbb{R}, \qquad f(x) = \frac{2x^2 + 8x + 11}{5 + 4x + x^2}$$

Find the maximum value of a such that, $f(x)$ has an inverse.

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Sub-Section [2.3.2]: Figure Out Possible Rule of a Graph
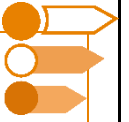
Question 33

Determine a possible rule for the following graphs:

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b.

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c.

$\left(-\frac{1}{2}, \frac{1}{4}\right)$

$(-1, 0)$

$(0, 0)$

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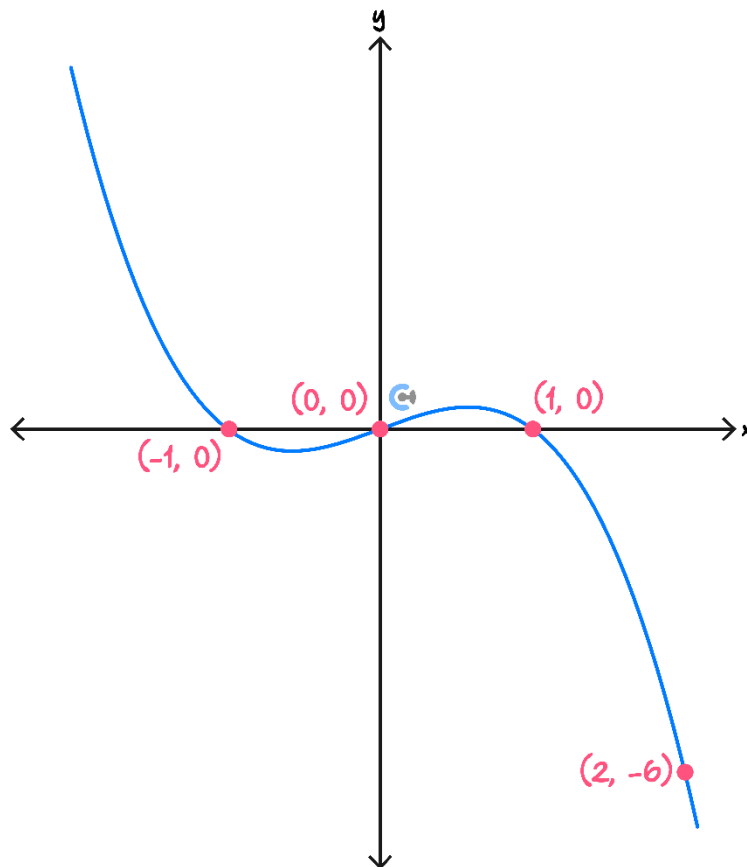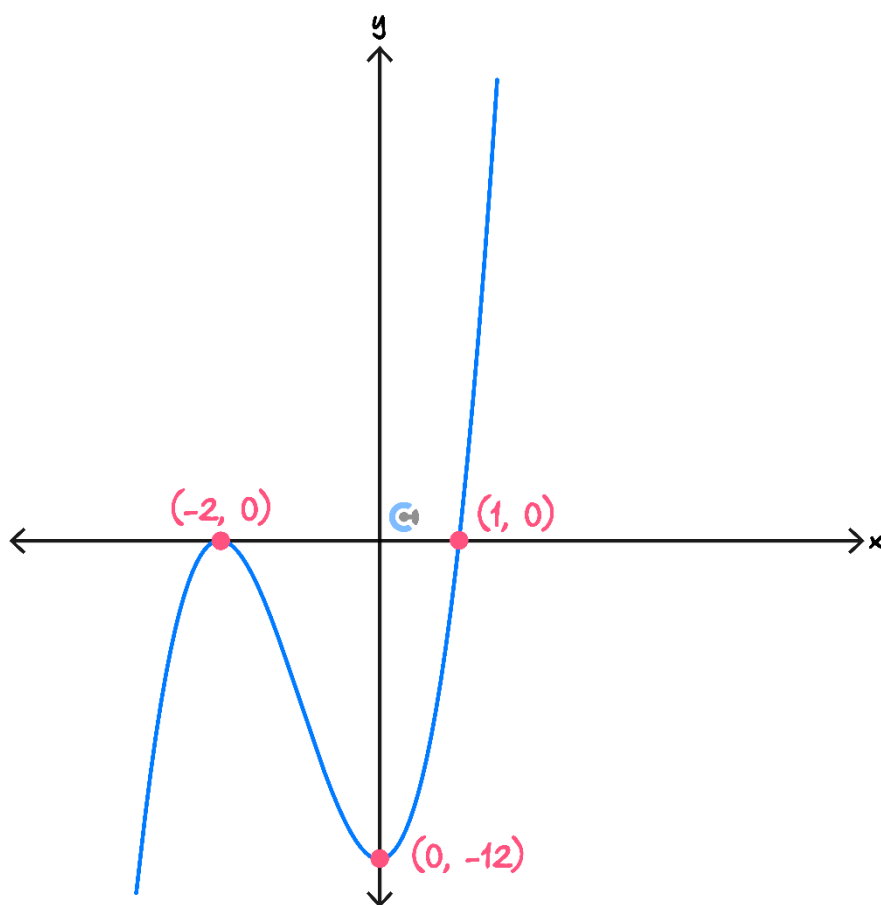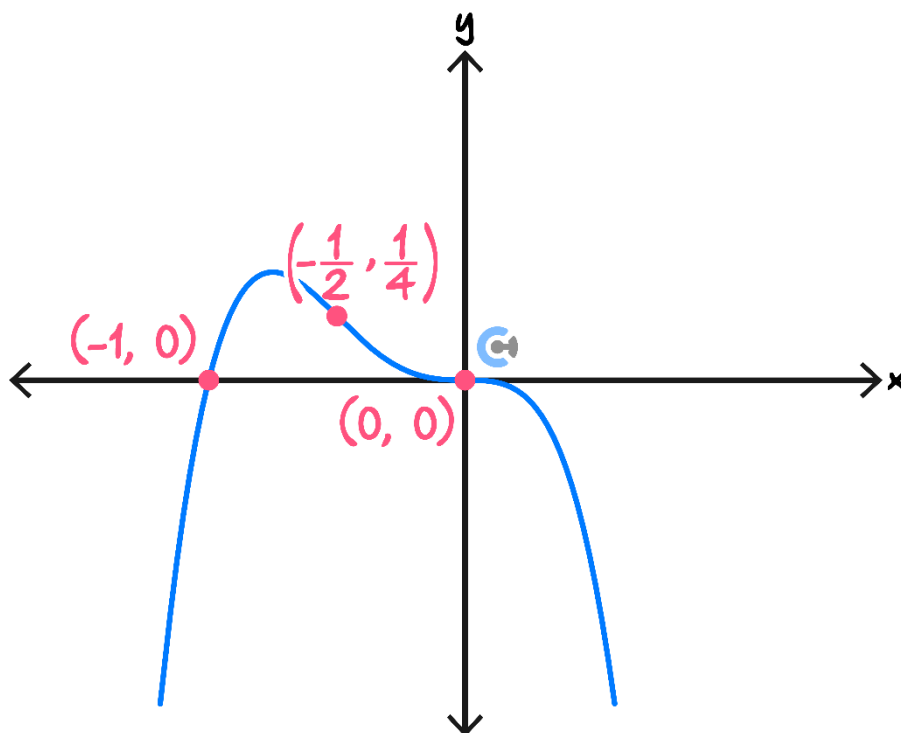
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Question 34

Determine a possible rule for the following graphs:

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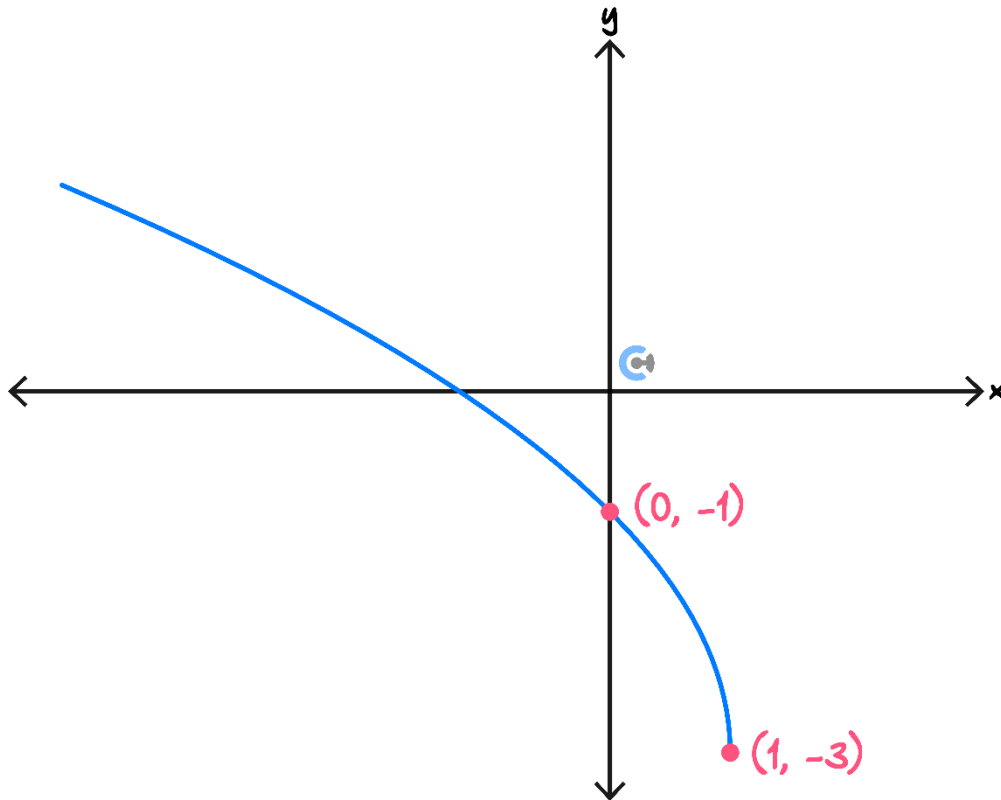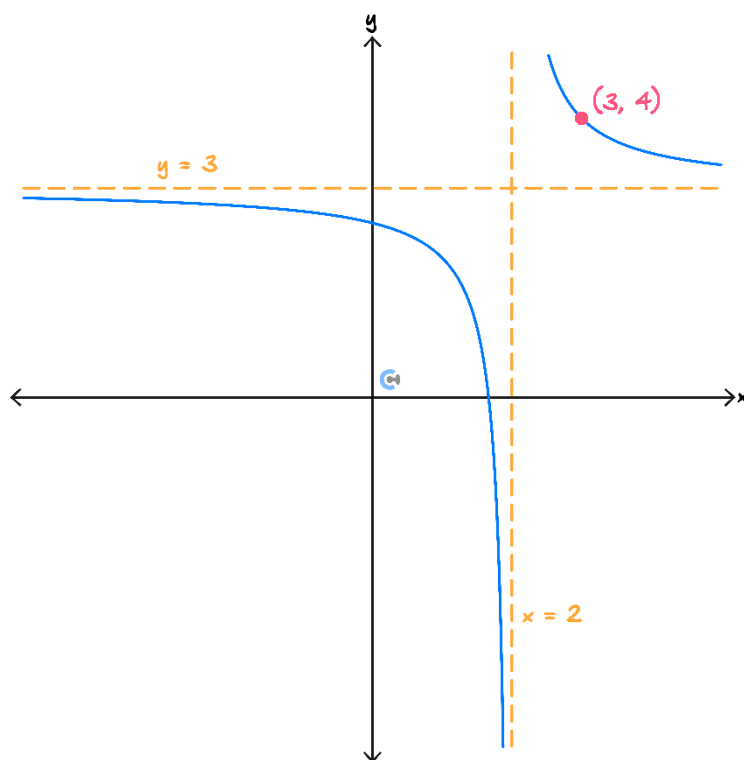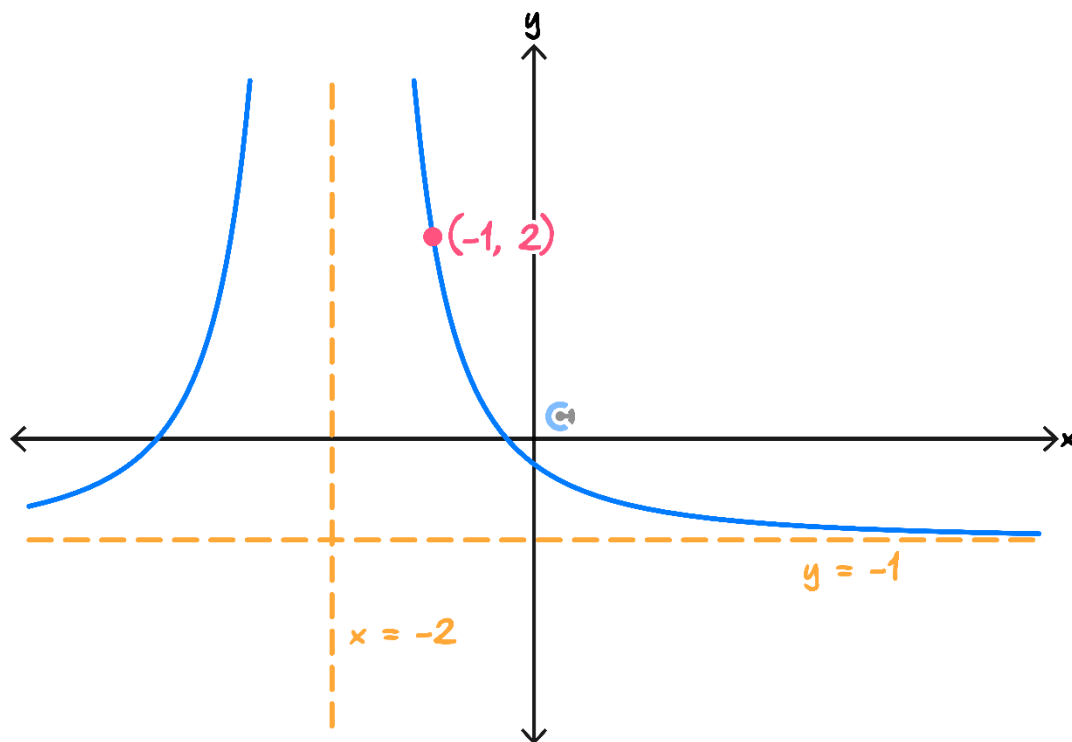
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Question 35

Determine a possible rule for the following graphs:
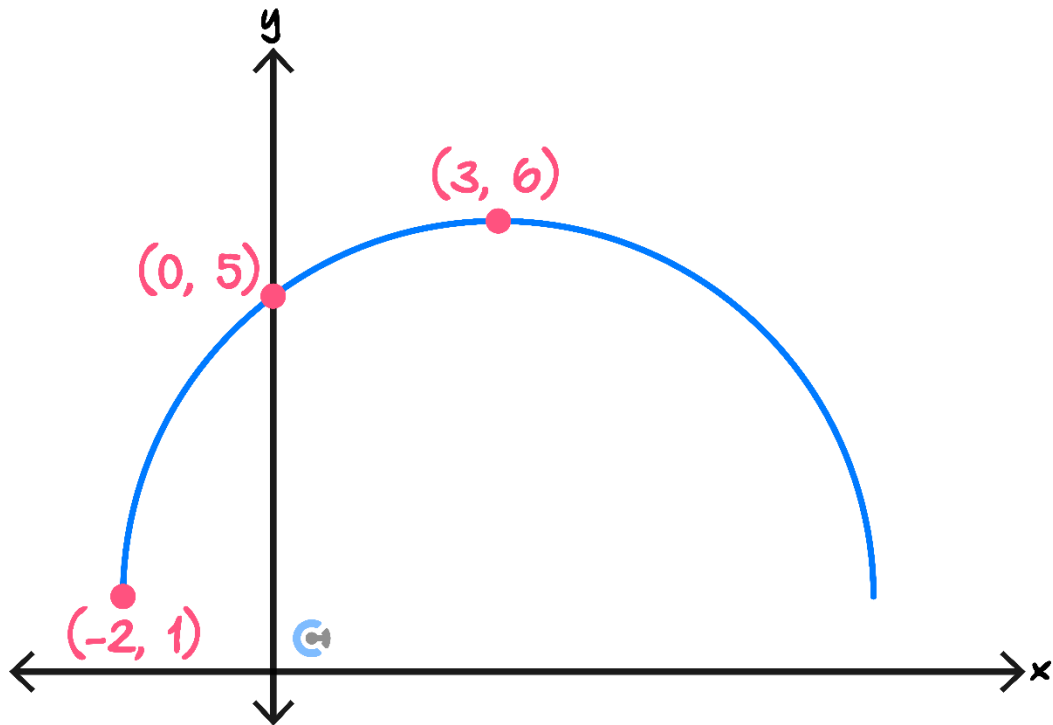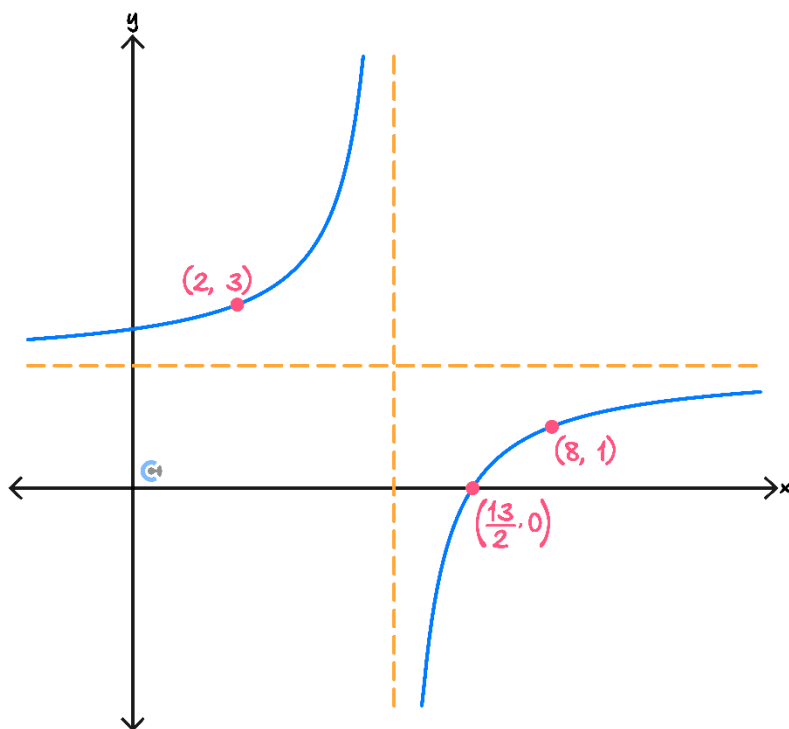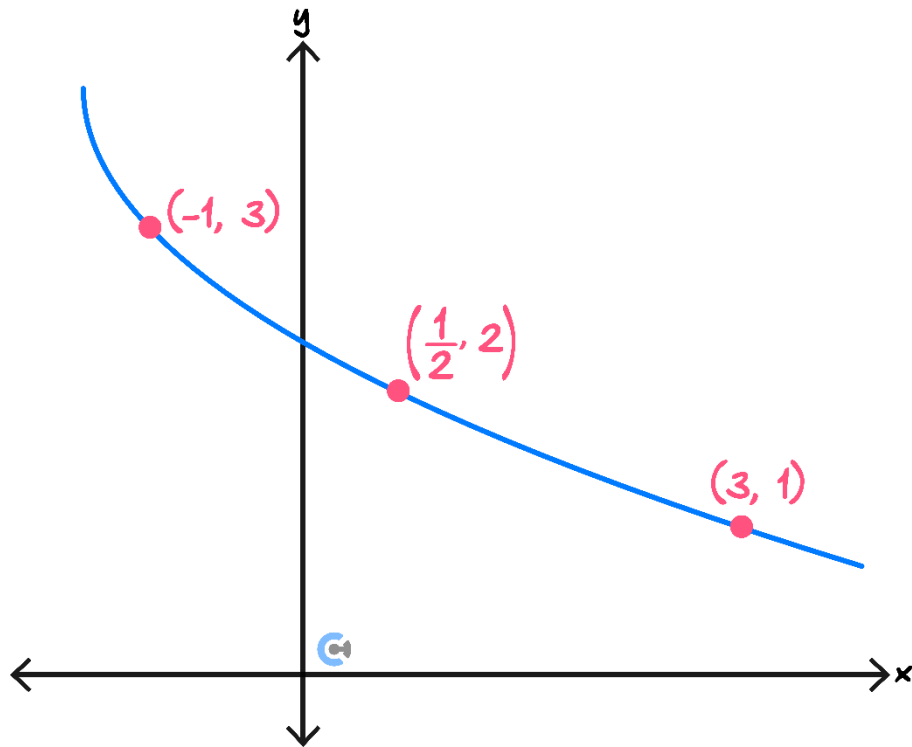
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b.

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Question 36

Determine a possible function for the following graph:

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Sub-Section [2.3.3]: Solve Number of Solution Problems Graphically

Question 37 Tech-Active.

Consider the function $f(x) = 4x^2 - 4x + 5$.

Determine the real values of k for which $f(x) = k$ has two solutions.

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Question 38 Tech-Active.

Consider the function $f(x) = x^3 + 3x^2 - 9x + 2$.

Determine the real values of k for which $f(x) = k$ has:

a. Two solutions.

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b. Three solutions.

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Question 39 Tech-Active.

Consider the function $f(x) = x^4 - 8x^3 + 6x^2 + 40x - 14$.

Determine the real values of k for which $f(x) = k$ has:

a. Three solutions.

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b. Two solutions.

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Question 40

Consider the function $f(x) = 3x^3 + k$.

Determine the real value of k for which $f(x) = f^{-1}(x)$ has three solutions.

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Section D: [2.4] – Transformations (Checkpoints)

Sub-Section [2.4.1]: Applying x' and y' Notation to Find Transformed Points, Find Interpretation of Transformations and Altered Order of Transformation

Question 41

Find the coordinates of the image point for the following:

a. The point $(2, 3)$ undergoes a dilation by a factor of 6 from the y-axis, a reflection in the x-axis, followed by a translation 1 unit up.

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b. The point $(1,5)$ undergoes a translation 2 units left, a dilation by a factor of $\frac{1}{4}$ from the y-axis, a translation 3 units up, followed by a reflection in the x-axis.

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c. The point $(-4,2)$ is dilated by a factor of 3 from the x-axis, translated 1 unit right, reflected in the x-axis, reflected in the y-axis, dilated by a factor of 2 from the y-axis and then translated 5 units down.

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Question 42

Consider the sequence of transformations:

➤ A dilation by a factor of $\frac{1}{2}$ from the y-axis.

➤ A reflection in the x-axis.

➤ A dilation by a factor of 6 from the x-axis.

➤ A translation 4 units down.

➤ A translation 1 unit right.

➤ A translation 9 units up.

a. Rewrite the transformations in the order of a dilation, a translation, a dilation, a reflection and then a translation.

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b. Express the transformations as a sequence of two translations, followed by two dilations and a reflection.

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c. Express the transformations in the order of a dilation, a translation, a dilation, a translation and then a reflection.

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Question 43

The transformation T is defined as $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (5 - 2x, 6y + 1)$.

a. Evaluate $T(-3,8)$.

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b. Find the pre-image of $(7, -35)$ under the transformation T.

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c. Express T as a sequence of two translations, two dilations and a reflection.

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d. Identify a sequence of transformations that maps the point $(-3, 8)$ to the image of $(-3, 8)$ under T and also maps the point $(1, -2)$ to the point $(23, -1)$.

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Question 44

a. Consider the transformation T described by:

➤ A translation 2 units left.

➤ A dilation by a factor of 3 from the x-axis.

➤ A dilation by a factor of $\frac{1}{4}$ from the y-axis.

➤ A reflection in the x-axis.

➤ A translation 1 unit up.

➤ A reflection in the line $y = x$.

➤ A translation 4 units right.

i. Apply T to the point $(5,2)$.

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ii. Express T as a sequence of 2 dilations followed by 2 reflections and then 2 translations.

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b. Consider the transformation S described by:

➤ A dilation by a factor of 2 from the x-axis.

➤ A reflection in the y-axis.

➤ A dilation by a factor of $\frac{1}{3}$ from the y-axis.

➤ A reflection in the line $y = 4$.

➤ A translation 5 units down.

➤ A translation 1 unit right.

    i. S can also be defined $S : \mathbb{R}^2 \to \mathbb{R}^2, S(x, y) = (ax + b, cy + d)$. Find the values of a, b, c and d.

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    ii. Hence, evaluate $S(-2,4)$.

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c. A point (x, y) undergoes the transformations T followed by S. Find the image point.

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d. Given that the image point from **part c.** is $(-4, 6)$, find the pre-image.

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Sub-Section [2.4.2]: Find Transformed Functions

Question 45

Find the resultant function when:

a. $y = x^2$ is dilated by a factor of 2 from the y-axis, reflected in the x-axis, translated 3 units up and translated 1 unit left.

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b. $y = \frac{1}{x}$ is reflected in the y-axis, translated 3 units up, dilated by a factor of 2 from the x-axis, dilated by a factor of $\frac{1}{4}$ from the y-axis and translated 2 units right.

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c. $y = \sqrt{x}$ is translated 3 units down, translated 5 units right, reflected in the y-axis, dilated by a factor of 3 from the x-axis, dilated by a factor of 2 from the y-axis, and reflected in the x-axis.

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Question 46

Find the resultant function when:

a. $y = -2(x + 5)^2 + 1$ is dilated by a factor of $\frac{1}{3}$ from the x-axis, translated 4 units right, translated 1 unit down, reflected in the y-axis and dilated by a factor of 2 from the y-axis.

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b. $y = \frac{2}{(5-x)^2} + 7$ is reflected in the x-axis, translated 2 units up, dilated by a factor of 3 from the y-axis, reflected in the y-axis, translated 4 units right, and dilated by a factor of $\frac{1}{4}$ from the x-axis.

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c. $y = 4 - 2(x + 1)^3$ is translated 4 units right, dilated by a factor of 3 from the x-axis, reflected in the y-axis, translated 5 units up, reflected in the x-axis and dilated by a factor of 2 from the y-axis.

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Question 47

Find the resultant function when:

a. $(x - 2)^2 + (y + 5)^2 = 9$ is dilated by a factor of 3 from the y-axis, reflected in the x-axis, translated 4 units up, translated 1 unit left and dilated by a factor of 3 from the x-axis.

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b. $y = 2x^2 + 3x - 6$ is reflected in the y-axis, dilated by a factor of 4 from the x-axis, translated 5 units down, translated 1 unit right, dilated by a factor of $\frac{1}{2}$ from the y-axis.

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c. $x = -\sqrt{-y^2 + 6y + 15} + 4$ is translated 2 units down, dilated by a factor of 3 from the x-axis, reflected in the y-axis, translated 5 units left, dilated by a factor of $\frac{1}{4}$ from the y-axis, translated 5 units up, reflected in the y-axis and dilated by a factor of 2 from the x-axis.

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Question 48

a. When the graph $y = 6 - 2(x + 1)^2$ undergoes the transformation T, described as:

➤ A translation 4 units right.

➤ A dilation by a factor of 2 from the y-axis.

➤ A translation 4 units down.

➤ A reflection in the y-axis.

➤ A dilation by a factor of 3 from the x-axis.

It is mapped onto an equation $y = a(x - h)^2 + k$, where $a, h, k \in \mathbb{R}$.

Find the values of a, h and k.

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b. The graph $y = 6 - 2(x + 1)^2$ can also be mapped to the same equation from **part a**., by a sequence of 2 dilations, a reflection and 2 translations. Describe this sequence of transformations.

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c. Find the pre-image, that when undergoes the transformation T, results in the equation $y = 6 - 2(x + 1)^2$.

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d. The graph of $y = 6 - 2(x + 1)^2$ undergoes the transformation T, followed by a dilation by a factor of 2 from the x-axis, a reflection in the line $x = 6$, a reflection in the line $y = x$, and a translation 1 unit up. Find the image equation.

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Sub-Section [2.4.3]: Find Transformations from Transformed Function (Reverse Engineering)

Question 49

Find the sequence of transformations that map:

a. $y = x^2$ to $y = -3(x + 1)^2 + 7$.

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b. $y = \dfrac{1}{x}$ to $y = \dfrac{3}{5 - 2x} + 6$.

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c. $y = \sqrt{x}$ to $y = 1 - \frac{\sqrt{4-3x}}{2}$.

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Question 50

Find the sequence of transformations that map:

a. $y = 4(x + 8)^3 - 5$ to $y = 5 - 2(6x - 1)^3$.

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b. $y = 3\sqrt{16 - (x + 1)^2} + 5$ to $y = 1 - 2\sqrt{16 - (3x + 5)^2}$.

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c. $y = \frac{3}{(4-2x)^2} + 7$ to $y = -\frac{6}{(x+1)^2} + 5$.

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Question 51

a. The function $y = -2(3(x-1))^4 + 5$ undergoes a sequence of 2 transformations, a reflection, and 2 dilations to become the graph $y = 6(2-x)^4 - 1$.

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b. The transformation $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x,y) = (ax + b, y + c)$ maps the equation $y = 11 + 5(x+3)^2$ onto the equation $y = 20(x-6)^2 + 9$. Find the values of a, b and c.

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c. The graph $y = \frac{\sqrt{6x-4}}{3} + 2$ is mapped onto $y = 5 - 2\sqrt{-1-x}$ by a sequence of 2 dilations and 2 reflections, followed by a translation.

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Question 52

Find the sequence of transformations that map:

a. $y = x^2 - 4x + 6$ onto $y = -2x^2 + 10x - 7$.

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b. $y = 2\sqrt{(x+4)^2 + 1} - 5$ onto $y = 3 - \sqrt{(2x-6)^2 + 9}$.

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c. $f : [-4, \infty) \to \mathbb{R}, f(x) = -x^2 - 8x + 9$ onto $g : (-\infty, 5] \to \mathbb{R}, g(x) = 2x^2 - 20x + 13.$

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Section E: [2.5] – Transformations Exam Skills (Checkpoints)

Sub-Section [2.5.1]: Apply Quick Method to Find Transformations

Question 53

Find the image of the graph of $y = x^2$ under the transformation, $T : \mathbb{R}^2 \to \mathbb{R}^2$, $T(x, y) = (1 - 2x, y + 5)$.

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Question 54

Describe a sequence of transformations that maps the graph of $y = x^3$ onto the graph of $y = 2(3x + 2)^3 - 3$.

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Question 55

Find the image of the graph of $y = \log_2(x)$ under the following sequence of transformations:

➤ A dilation by a factor of 3 from the x-axis, followed by,

➤ A translation of 2 units left and 3 units up, followed by,

➤ A reflection in the y-axis, followed by,

➤ A dilation by a factor of 5 from the y-axis.

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Sub-Section [2.5.2]: Find Opposite Transformations

Question 56

Describe a sequence of transformations that maps the graph of $y = 4(x - 2)^2 - 3$ onto the graph of $y = x^2$.

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Question 57

The transformation, $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = \left(2x + 3, \frac{1}{3}y - 4\right)$ maps the graph of $y = f(x)$ onto the graph of $y = x^3$.

Find the rule of f.

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Question 58

The following sequence of transformations maps the graph of f onto the graph of $y = \sqrt{x}$, for $x \in (2, \infty)$:

A dilation by a factor of 3 from the x-axis, followed by,

A translation of 2 units left and 4 units up, followed by,

A reflection in both the x-axis and the y-axis.

State the rule and domain of f.

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Sub-Section [2.5.3]: Apply Transformations of Functions to Find its Domain, Range, Transformed Points

Question 59

The function $f : \mathbb{R} \to \mathbb{R}$ has a range of $[2, \infty]$.

The transformation, $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (5 - 2x, 3 + y)$ maps the graph of f onto the graph of g. State the domain and range of g.

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Question 60

The function $f : (-\infty, -1) \to \mathbb{R}$ has a range of $(-2, \infty)$.

Describe a sequence of transformations that maps the graph of f onto a graph of a function with a domain of $[0, \infty]$ and a range of $(-\infty, 2)$.

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Question 61

Consider the function, $f : [-2, \infty] \to R, f(x) = 3\sqrt{x + 2} - 5$.

The following sequence of transformations maps the graph of f onto the graph of g:

➤ A reflection in the x-axis, followed by,

➤ A dilation by a factor of 3 from the x-axis, followed by,

➤ A dilation by a factor of $\frac{1}{2}$ from the y-axis, followed by,

➤ A translation of 3 units up and 2 units left.

State the domain and range of g.

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Sub-Section [2.5.4]: Find Transformations of Inverse Functions

Question 62

Consider the function, $f: \mathbb{R}\backslash\{1\} \to \mathbb{R}, f(x) = \frac{2}{x-1} + 4$. The transformation, $T: \mathbb{R}^2 \to \mathbb{R}^2$, $T(x, y) = (x + a, y + b)$ maps the graph of f onto the graph of its inverse function. Find the values of a and b.

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Question 63

Consider the one-to-one functions, $f(x)$ and $g(x)$. The transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$, $T(x, y) = (3 - x, 2y + 7)$ maps the graph of f onto the graph of g.

Describe a sequence of transformations that maps the graph of f^{-1} onto the graph of g^{-1}.

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Question 64

Let $f : [1, \infty] \to \mathbb{R}, f(x) = 3x^2 - 6x + 8$ and $g : [-3, \infty] \to \mathbb{R}, \ g(x) = \sqrt{x + 3} + 4$.

Describe a sequence of transformations that maps the graph of f onto the graph of g^{-1}.

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Sub-Section [2.5.5]: Find Multiple Transformations for the Same Functions

Question 65

Describe a sequence of transformations that map the graph of $f(x) = 4(x-3)^2 + 5$ to $g(x) = x^2$ without using a dilation from the x-axis.

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Question 66

Consider the functions $f(x) = x^2 - 8x + 10$ and $g(x) = 4(x+2)^2 - 5$. Find 2 different sets of transformations, one using a dilation from the x-axis and one using a dilation from the y-axis to map the graph of $f(x)$ to the graph of $g(x)$.

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Question 67

Consider the functions $f(x) = x^2 + 6x + 7$ and $g(x) = 16x^2 - 32x + 6$. Find 2 different sequences of 3 transformations, one using a dilation from the x-axis and one using a dilation from the y-axis to map the graph of $f(x)$ to the graph of $g(x)$.
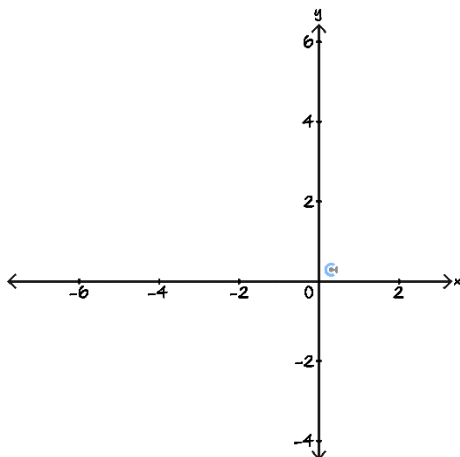
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Section F: [2.1-2.5] – Exam 1 Questions (Checkpoints) (14 Marks)

Question 68

Let $f(x) = \frac{2x+6}{x+4}$ be defined on its maximal domain.

a. Sketch the graph of $f(x)$ on the axes below. Label all asymptotes with their equations and axial intercepts with their coordinates.
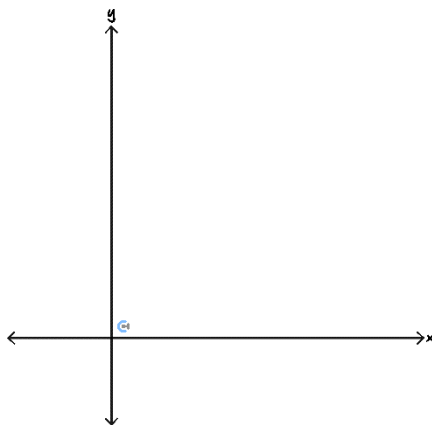
b. State the domain and range of f^{-1}.

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c. Find the values of x for which $f(x) > 1$.

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Question 69

Consider the function $f : \mathbb{R}\backslash\{h\} \rightarrow \mathbb{R}, \ f(x) = \frac{a}{(x-h)^2} + k$.

The graph of f is drawn below.

a. Show that $a = -2, h = 2$, and $k = 5$.

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b. Find the maximal domain of $g(x) = \sqrt{4 - (f(x) - 1)^2}$.
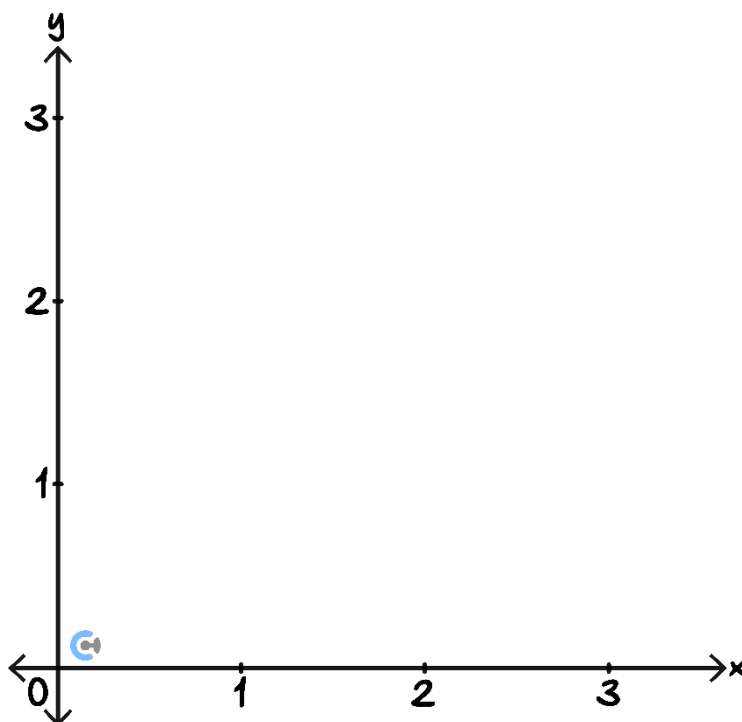
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Question 70

Consider the function $f : [a, \infty) \to \mathbb{R}, f(x) = x^2 - 2x + 2$.

a. Find the smallest value of a for which the inverse function of f, f^{-1} exists.

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b. State the domain and range of f^{-1}.

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c. The graph of $y = f(x)$ is drawn on the axis below, sketch the graph of $y = f^{-1}(x)$ on the same axis, labelling points of intersection with their coordinates.

d. Let $g: [1, \infty) \to \mathbb{R}, g(x) = (x-1)^2 + k$.

i. Find the values of k for which $g(x) = g^{-1}(x)$ has no solutions.

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ii. Find the values of k for which $g(x) = g^{-1}(x)$ has two solutions.

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iii. Find the values of k for which $g(x) = g^{-1}(x)$ has one solution.

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Space for Personal Notes

Question 71

Consider the transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2, T(x, y) = \left(\frac{1}{2}x - 3, 4y + 2\right)$.

a. Find the image of the point $(4, 1)$ under T.

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b. Write out what the transformation T does in the order DRT.

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c. Find the image of the curve $y = x^3$ under the transformation T. Give your answer in the form $y = a(x + b)^3 + c$.

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Question 72

Consider the function $f: \mathbb{R} \rightarrow \mathbb{R}, \; f(x) = 4x^2 - 16$.

a. Find the coordinates of all axes intercepts of f.

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b. Let the graph of g be a transformation of the graph of f where the transformations have been applied in the following order:

    1. Dilation by a factor of 2 from the y-axis.

    2. Dilation by a factor of 3 from the x-axis.

    3. Translation 6 units to the right.

Find the rule for $g(x)$.

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c. State the coordinates of the axes intercepts of g.

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Question 73

Consider the function $f(x) = 4\sqrt{3x + 7} + 2$.

Apply the following transformations to $f(x)$:

1. Dilation by a factor of $\frac{1}{2}$ from the x-axis.

2. Translated 3 units in the positive direction of the y-axis.

3. Reflection in the x-axis.

4. Translated 2 units in the negative direction of the x-axis.

5. Dilated by a factor of 2 from the y-axis.

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Question 74 (4 marks)

a. Sketch the graph of $y = 1 - \frac{2}{x-2}$ on the axes below. Label asymptotes with their equations and axis intercepts with their coordinates. (3 marks)
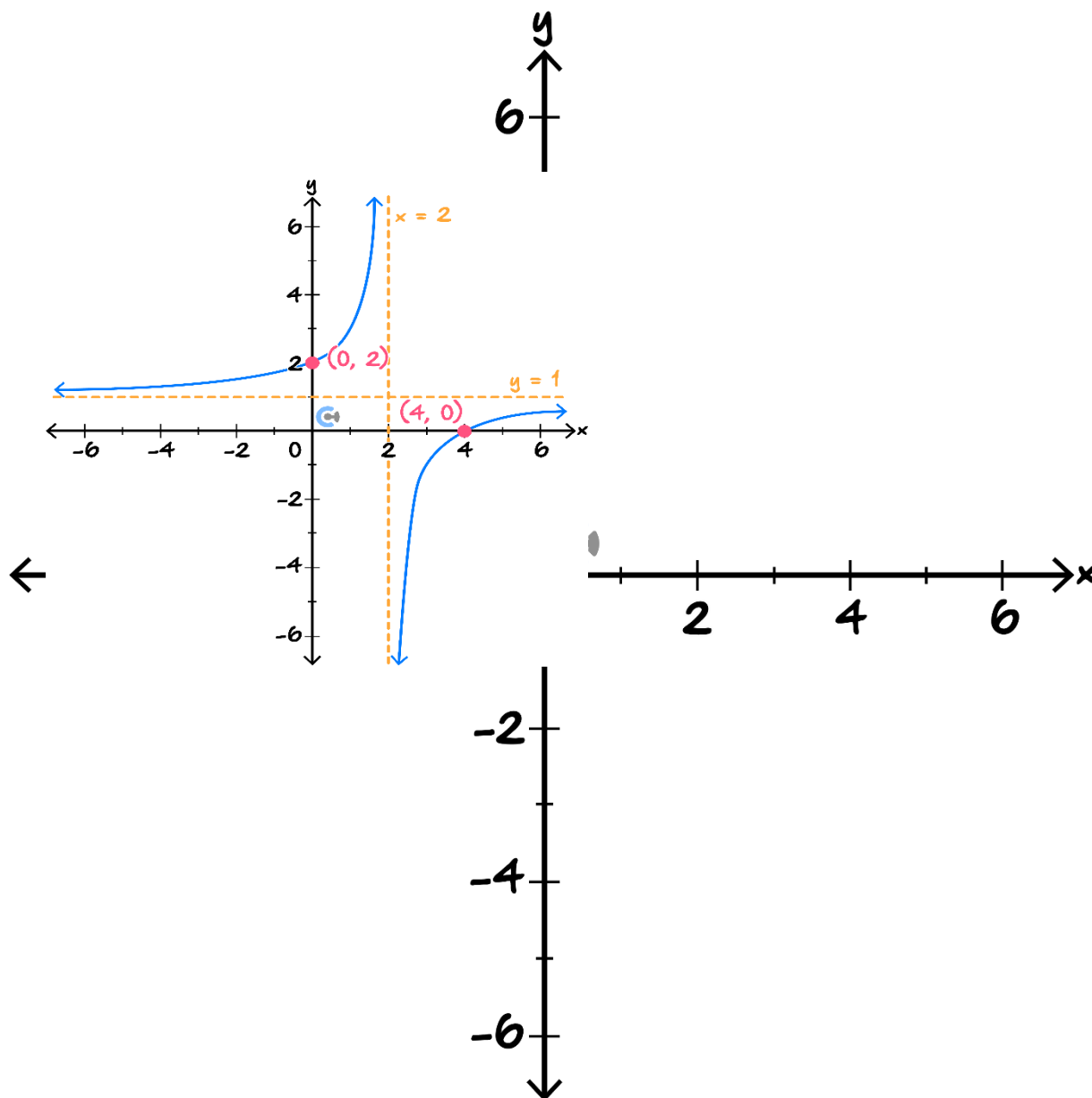
b. Find the values of x for which $1 - \frac{2}{x-2} \geq 3$. (1 mark)

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Question 75 (3 marks)

The transformation $T: R^2 \rightarrow R^2$ with rule $T(x, y) = (x + b, ay + c)$, where a, b, and c are integers, maps the graph of $y = 2 - 4x^3$ onto the graph of $y = 1 - 16(x - 2)^3$.

Find the values of a, b and c.

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Space for Personal Notes

Question 76 (3 marks)

Let $f: R \to R, f(x) = 2e^x + 1$ and let $g: (-2, \infty) \to R, g(x) = \log_e(x + 2)$.

Let $T: R^2 \to R^2, T(x, y) = (x + c, y + d)$ and let the graph of the function h be the transformation of the graph of the function g under T.

If $h = f^{-1}$, then find the values of c and d.

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Question 77 (4 marks)

a. Let $f: R \setminus \left\{\frac{1}{3}\right\} \to R, f(x) = \frac{1}{3x - 1}$.

Find the rule of f^{-1}. (2 marks)

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b. State the domain of f^{-1}. (1 mark)

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c. Let g be the function obtained by applying the transformation T to the function f, where:

$$T(x, y) = (x + c, y + d)$$

and $c, d \in R$.

Find the values of c and d given that $g = f^{-1}$. (1 mark)

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