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VCE Mathematical Methods ½
Polynomials Exam Skills [1.6]
Homework

Homework Outline:

Compulsory Questions	Pg 2 – Pg 25
Supplementary Questions	Pg 26 – Pg 47

Section A: Compulsory Questions

Sub-Section [1.6.1]: Solve Polynomial Inequalities

Question 1

Solve the following inequalities for x:

a. $(x-5)(x+2)(x-1) > 0$

b. $(x-1)(2-x)(x+3) < 0$

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Question 2

Solve the following inequalities for x:

a. $x(x^2 - 4x + 6) > 0$

b. $(3 - x)(x^2 - 5x + 4) < 0$

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Question 3

Solve the following inequalities for x:

a. $x^3 - x^2 - 14x + 24 \leq 0$

b. $2x^3 - 7x^2 - 33x + 18 > 0$

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Sub-Section [1.6.2]: Solve Number of Solution Problems

Question 4

Find the values of k, for which the equation $x^3 + 3kx^2 + 9x = 0$ has:

a. 1 solution.

b. 2 solutions.

c. 3 solutions.

Question 5

Find the values of k, for which the equation $x^3 + 3x^2 - 4kx = 0$ has:

a. 1 solution.

b. 2 solutions.

c. 3 solutions.

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Question 6

Find the values of k, for which the equation $(x^2 - 4kx + 8)(x^2 - 4x + 4k) = 0$ has:

a. 4 solutions.

b. 3 solutions.

c. 2 solutions.

d. 1 solution.

e. No solutions.

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Sub-Section [1.6.3]: Apply Bisection Method to Approximate x-Intercepts

Question 7 CAS-Active.

Use the bisection method to find the approximate real solution to the equation $x^3 - 3x^2 + 3x + 2 = 0$. Use the interval $[-1, 1]$ for the first iteration and a maximum error of 0.1. Give your approximation correct to two decimal places.

Question 8 CAS-Active.

Use the bisection method to find the approximate real solution to the equation $x^2 \log_2(x) - 3x - 2 = 0$. Use the interval $[1, 4]$ for the first iteration and a maximum error of 0.1. Give your approximation correct to two decimal places.

Question 9 CAS-Active.

Use the bisection method to approximate $\sqrt[3]{5}$ correct to two decimal places.

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Sub-Section: Exam 1 Questions

Question 10

Consider the polynomial $f(x) = x^3 + ax^2 + bx + 4$. It is known that $x - 1$ is a factor of f and when f is divided by $x - 2$ the remainder is 6. Find the values of a and b.

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Question 11

Solve the equation $2x^3 - 4x^2 - 22x + 24 = 0$.

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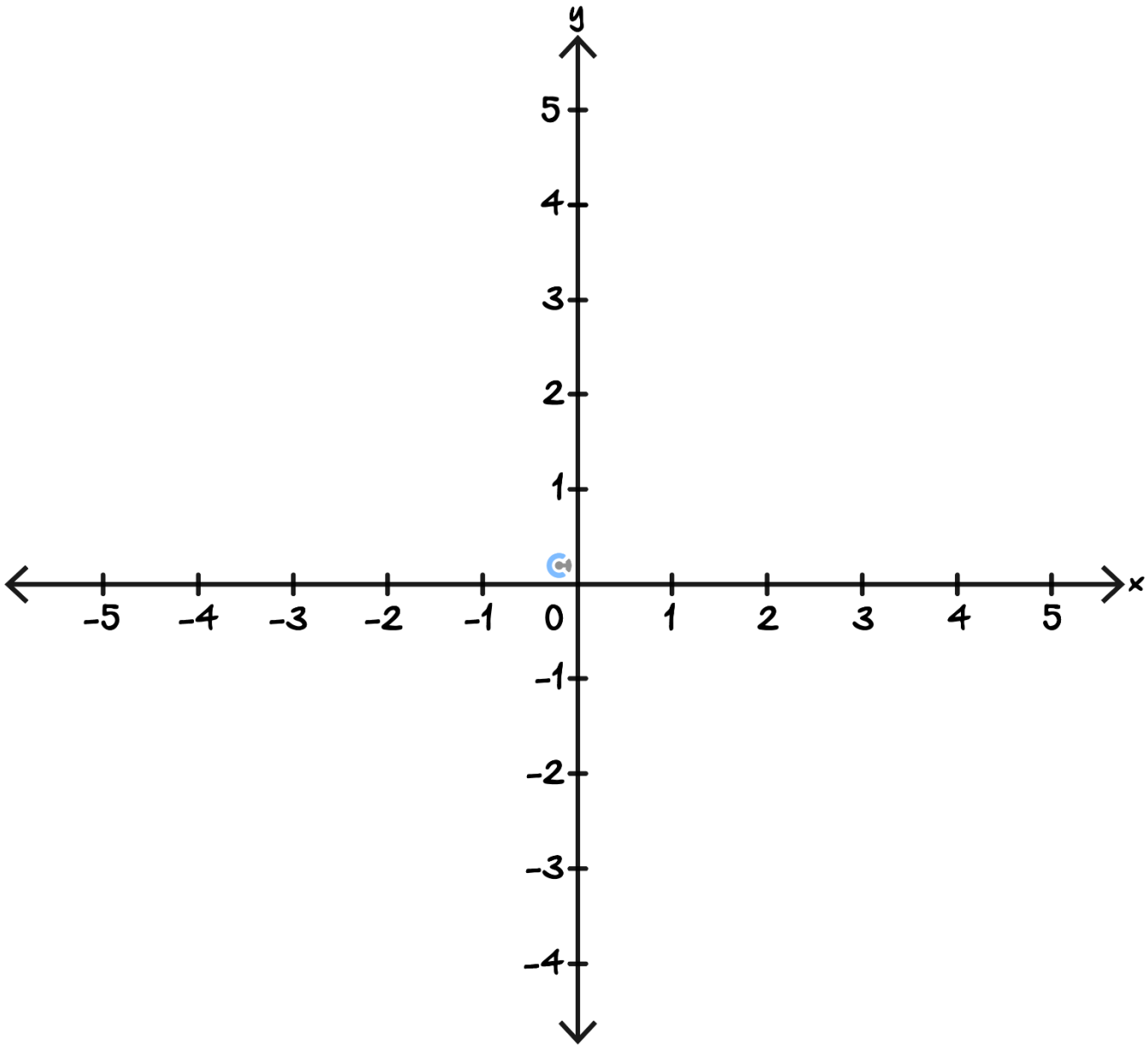
Question 12

Sketch the graph of $y = -(x - 1)^3 + 1$ on the axes below. Label all axis intercepts and the inflection point with coordinates.



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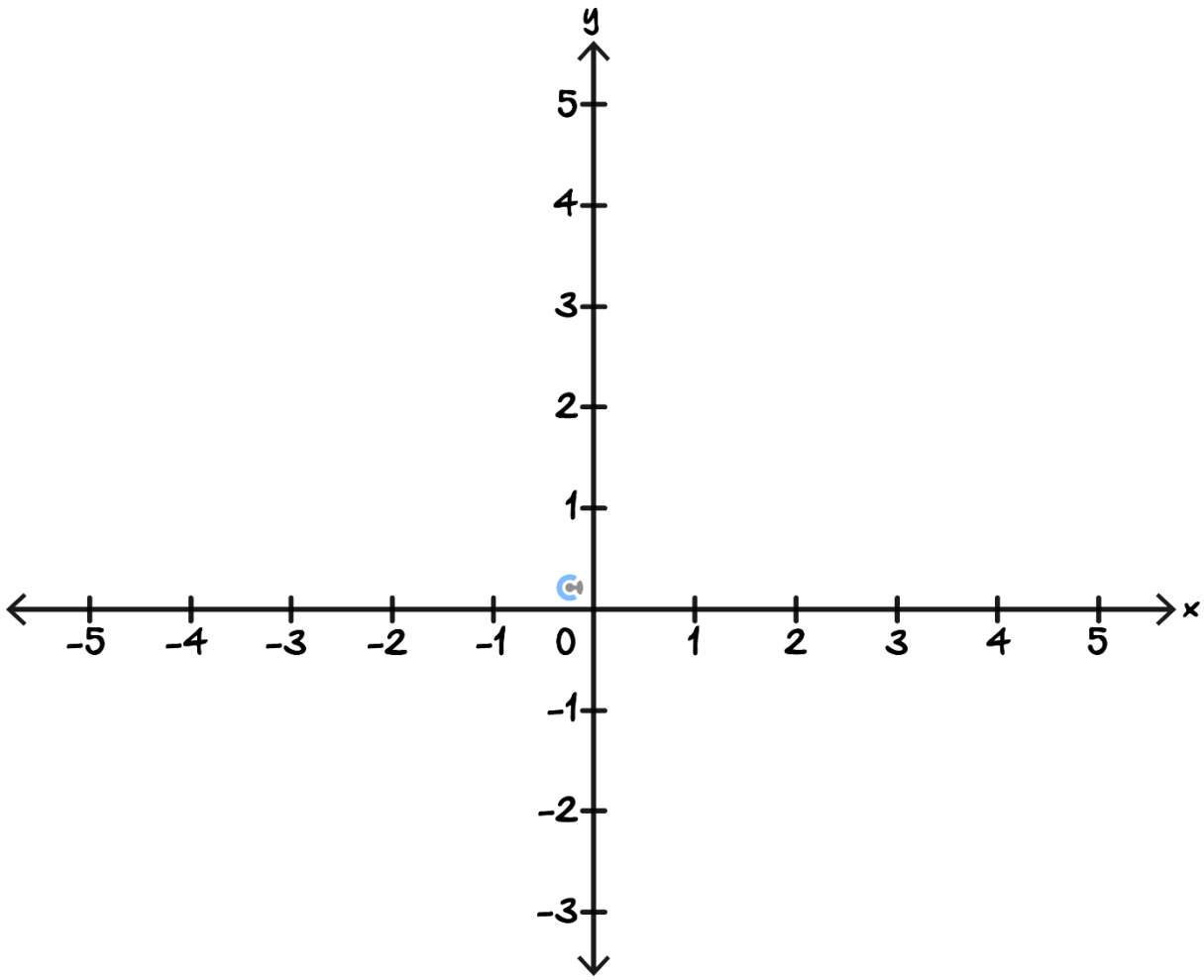
Question 13

Consider the function $f(x) = x^4 + x^3 - 3x^2 - x + 2$.

a. Show that $x + 2$ is a factor of $f(x)$.

b. Fully factorise $f(x)$.

c. Hence, sketch the graph of $y = f(x)$. Label all axis intercepts with coordinates. Note that some turning points occur at approximately $(-1.59, -1.63)$ and $(-0.16, 2.08)$.



d. Solve the inequality $f(x) \leq 0$.

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Question 14

Consider $f(x) = 2x^3 + 2kx^2 + 5x$, where k is a real constant.

Find the values of k, such that $f(x) = 0$ has:

a. One solution.

b. Two solutions.

c. Three solutions.

Sub-Section: Exam 2 Questions

Question 15

The equation $3x^2 + 2x - 8 = 0$ has one real solution, which lies in the interval $[0,2]$. Approximate the solution using the bisection method with a maximum error of 0.1. The approximate solution correct to two decimal places is:

A. $x \approx 1.25$

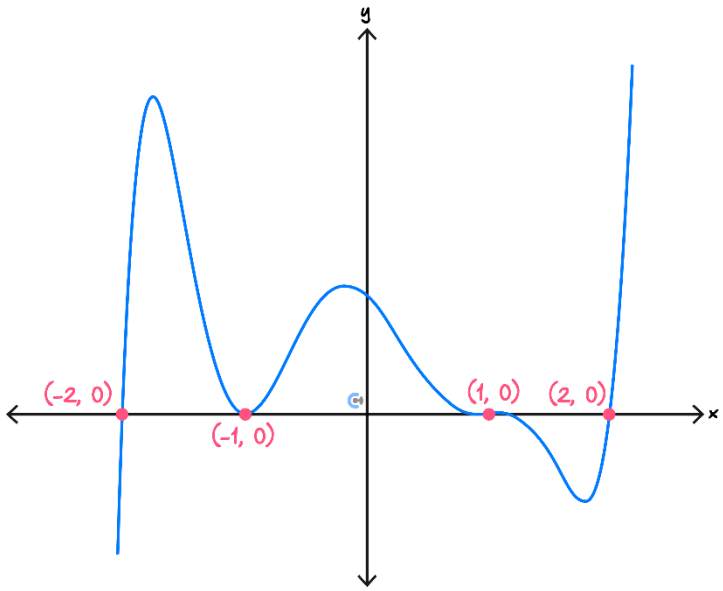
B. $x \approx 1.13$

C. $x \approx 1.19$

D. $x \approx 1.15$

Question 16

The minimum degree of the polynomial sketched below is:



A. 5

B. 6

C. 7

D. 8

Question 17

The polynomial $ax^3 + 3x^2 + bx + 5$ is perfectly divisible by $x-1$ and has a remainder of 6 when divided by $x + 2$. The values (a, b) are:

A. $(8, -12)$

B. $\left(-\frac{1}{2}, -\frac{9}{2}\right)$

C. $\left(-\frac{3}{2}, -\frac{5}{2}\right)$

D. $\left(\frac{9}{2}, -\frac{25}{2}\right)$

Question 18

The equation $x^3 - 5kx^2 + 9x = 0$ has exactly one solution when:

A. $k = \pm\frac{6}{5}$

B. $-\frac{6}{5} < k < \frac{6}{5}$

C. $k > \frac{6}{5}$

D. $k < -\frac{6}{5}$

Question 19

A graph with rule $f(x) = x^3 - 3x^2 - 4c$, where c is a real number, has three distinct x-intercepts. All possible values of c are:

A. $c > 1$

B. $-1 < c < 0$

C. $0 < c < 1$

D. $c < 1$

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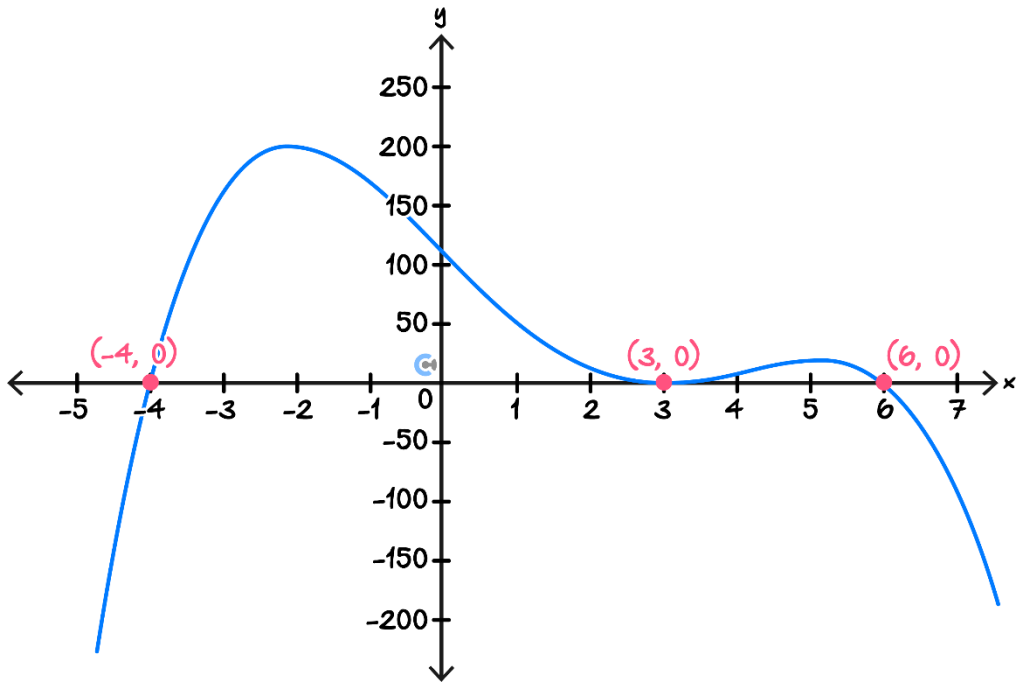
Question 20

Consider the function f that is sketched on the axes below. It is given that the point $(2,12)$ lies on the graph.



a.

i. State the degree of f.

ii. Find a rule for $f(x)$.

b. Consider the function $g(x) = f(x) + 10(k^2 - 4k + 3)$, where k is a real constant.

i. Find the values of k such that $g(x) = f(x)$.

ii. Find the values of k, such that $g(x) > f(x)$.

It is known that the function $f(x)$ has a turning point when $x = \frac{3 \pm \sqrt{51}}{2}$.

Let $h(x) = -\frac{4}{3}(x-3)^2(x+4)(x-6)$.

c. Find all values of k, such that $h(x) = k$ has two solutions.

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Question 21

Consider the cubic polynomial $f(x) = x^3 + 2x^2 - 7x - 2$.

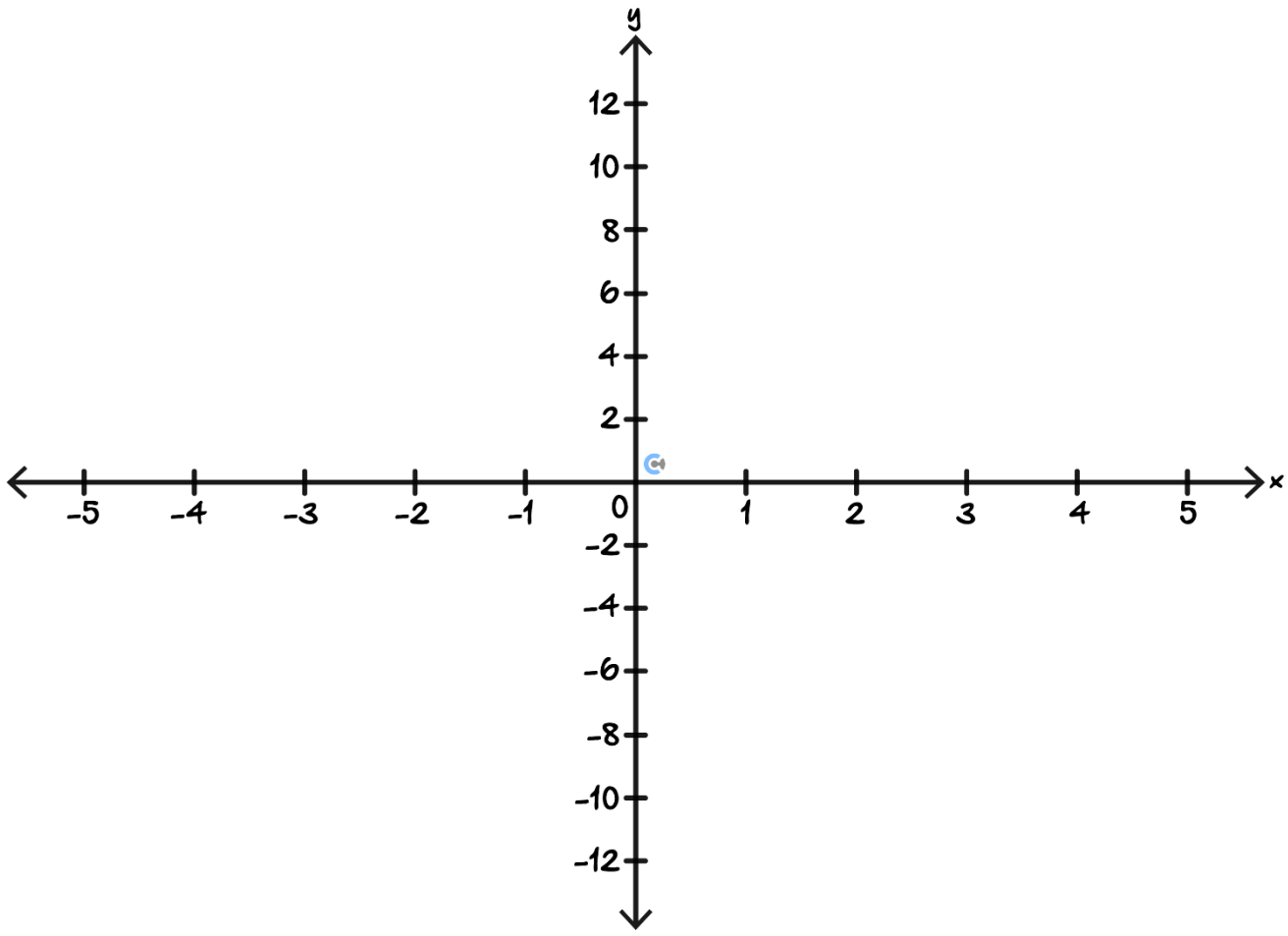
a.

i. Explain why $f(x)$ has a root between $x = -1$ and $x = 0$.

ii. Approximate the root in the interval $[-1,0]$ using the bisection method with a maximum error of 0.05. Give your answer correct to two decimal places.

iii. Find the distance between our approximate root and the actual root that lies in the interval $[-1,0]$. Give your answer correct to two decimal places.

b. Sketch the graph of $y = f(x)$ on the axes below. Label all turning points and axis intercepts with coordinates.



c. Find the values of k, such that $f(x) + k = 0$, where k is a positive constant, has one solution.

d. Let a be a real constant.

Find the values of a such that the equation $x^3 - (4a + 2)x^2 + (8a + 3)x - 6 = 0$ has three real solutions.

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Section B: Supplementary Questions

Sub-Section [1.6.1]: Solve Polynomial Inequalities

Question 22

Solve the following inequalities for x:

a. $x(x-1)(x+2) \leq 0$

b. $(x-2)(x+1)(x+3) > 0$

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Question 23

Solve the following inequalities for x:

a. $(x-5)(x^2+x-2) \leq 0$

b. $(1-x)(x^2-4x+4) \geq 0$

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Question 24

Solve the following inequalities for x:

a. $x^3 - 5x^2 - 8x + 12 > 0$

b. $-x^3 + 4x^2 + x - 4 \leq 0$

Question 25

Solve the inequality $4x^5 - 16x^4 + 13x^3 - 3x^2 > 4x^3 - 16x^2 + 13x - 3$.

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Sub-Section [1.6.2]: Solve Number of Solution Problems

Question 26

Find the values of k, for which the equation $x(x^2 + 4) = 4kx^2$ has:

a. 1 solution.

b. 2 solutions.

c. 3 solutions.

Question 27

Find the values of k, for which the equation $kx^9 + 2x^6 + x^3 = 0$ has:

a. 1 solution.

b. 2 solutions.

c. 3 solutions.

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Question 28

Find the values of k, for which the equation $x(x-2k-2)(x^2+kx+4) = -x^2-kx-4$ has:

a. 4 solutions.

b. 3 solutions.

c. 2 solutions.

d. 1 solution.

e. No solutions.

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Question 29

Consider the polynomial $P(x) = x^3 + ax + b$.

Show that if $\Delta = -4a^3 - 27b^2 = 0$, that $P(x) = 0$ has less than 3 solutions.

Hint: If r_1, r_2, r_3 are the roots of $P(x)$, show that $\Delta = (r_1 - r_2)^2(r_2 - r_3)^2(r_3 - r_1)^2$.

Please use a calculator.

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Sub-Section [1.6.3]: Apply Bisection Method to Approximate x-Intercepts

Question 30 CAS-Active.

Use the bisection method to find the approximate real solution to the equation $x^3 + 2x^2 - 5x + 3 = 0$. Use the interval $[-4, -3]$ for the first iteration and a maximum error of 0.1. Give your approximation correct to two decimal places.

Question 31 CAS-Active.

Use the bisection method to find the approximate real solution to the equation $x\log_2(x) + 3x = 4$. Use the interval $[0.1,2]$ for the first iteration and a maximum error of 0.01. Give your approximation correct to two decimal places.

Question 32 CAS-Active.

Use the bisection method to approximate π correct to three decimal places.

Question 33

Explain why you cannot use the bisection method to approximate the solution to the equation $x^4 - 2x^2 + 1 = 0$.

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Sub-Section: Exam 1 Questions

Question 34

Consider the polynomial $f(x) = x^3 - 7x + 6$.

a. Show that $f(1) = 0$.

b. Solve $f(x) = 0$ for x.

c. Hence, solve $f(x) \geq 0$ for x.

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Question 35

For what values of k does the equation $k(x^3 + x^2) = x$ have exactly one solution.

Question 36

Consider the polynomial $f(x) = x^3 - 3x^2 + x + 1$.

a. Fully factorise $f(x)$ into linear factors.

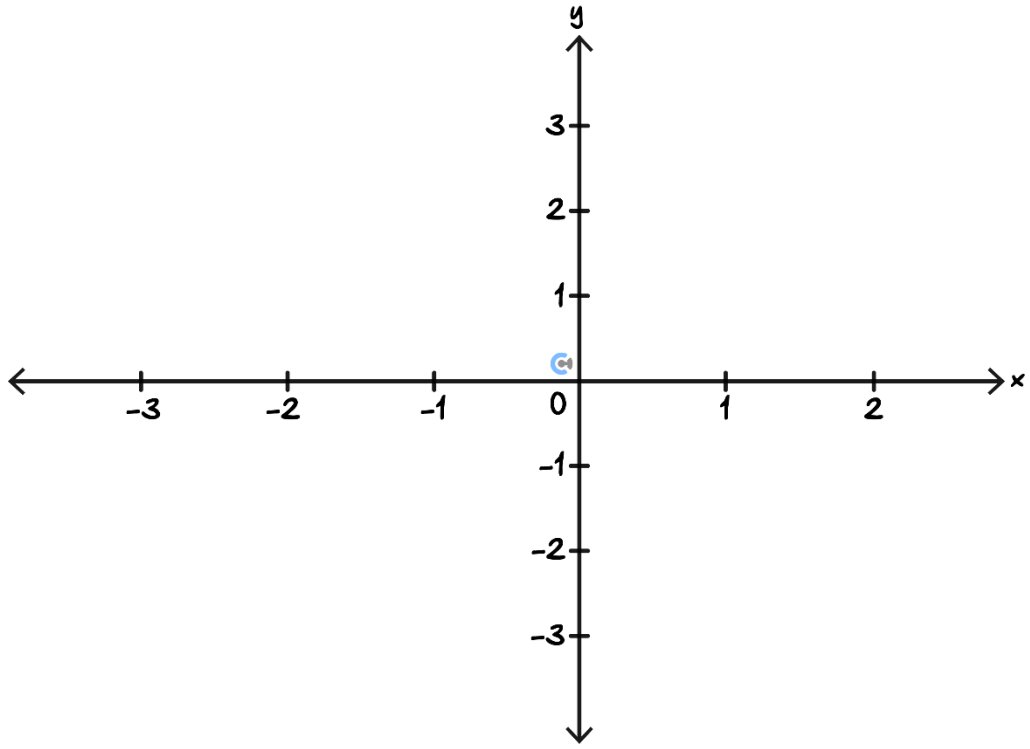
b. A bisection method is used to solve $f(x) = 0$ with the first interval being [2,3].
Use the fact that $\sqrt{2} \approx 1.4$ to write down the next 3 intervals.

Question 37

Let $f(x) = x^4 + 3x^3 + x^2 - 3x - 2$.

a. Show that $x^2 - 1$ is a factor of f.

b. Sketch the graph of $y = f(x)$ on the axis below. Label all axis intercepts with their coordinates. Note that some turning points occur at $(-1.69, -0.40)$ and $(0.44, -2.83)$.



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Sub-Section: Exam 2 Questions

Question 38

The equation $x^2(x - 2k) = -2x$ has exactly two solutions when,

A. $k < -\sqrt{2}$ or $k > \sqrt{2}$.

B. $k = \pm\sqrt{2}$

C. $-\sqrt{2} < k < 0$ or $0 < k < \sqrt{2}$.

D. $-\sqrt{2} < k < \sqrt{2}$

Question 39

The polynomial $x^3 + ax^2 - 2x + b$ has a factor of $x + 1$, and has a remainder of 12 when divided by $x - 2$. The values of a and b are:

A. $a = 3$ and $b = -4$.

B. $a = \frac{7}{3}$ and $b = -\frac{4}{3}$.

C. $a = \frac{17}{3}$ and $b = -\frac{20}{3}$.

D. $a = 5$ and $b = -4$.

Question 40

A bisection method is used to solve the equation $x^3 = 7$. The initial interval is $[1,2]$. The bisection reduces this interval down four times, and then takes the midpoint of the final interval. The result of this method is closest to:

A. 1.94

B. 1.92

C. 1.91

D. 1.88

Question 41

The equation $kx^3 - 3kx = 1$ has exactly one solution.

The possible values of k are:

A. $k < -2$ or $k > 2$.

B. $-2 < k < 2$

C. $k < -\frac{1}{2}$ or $k > \frac{1}{2}$.

D. $-\frac{1}{2} < k < \frac{1}{2}$

Question 42

The maximum number of x-intercepts a quartic can have is:

A. 2

B. 3

C. 4

D. 5

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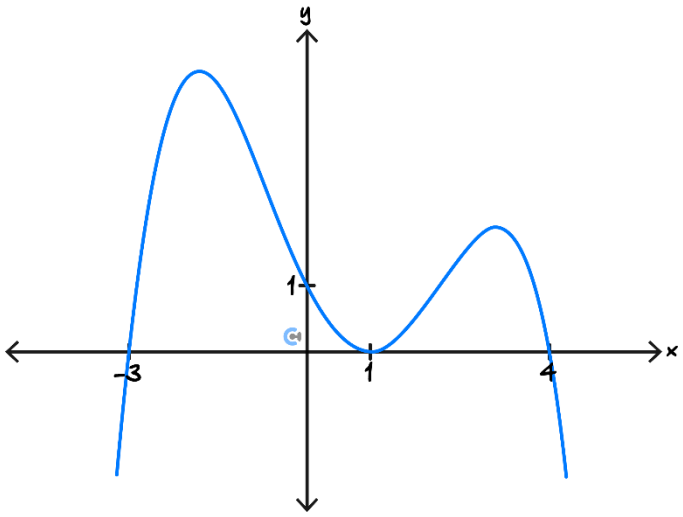
Question 43

The graph of $f(x) = ax^4 + bx^3 + cx^2 + dx + 1$ is drawn below.



a. Find the values of a, b, c and d.

b. Hence or otherwise, solve $f(x) > 1$. Give your answers correct to 2 decimal places.

c. Find all values of a correct to 3 decimal places such that $f(x) = a$ has exactly three solutions.

d. Consider the polynomial $g(x) = (x - a)^2(x + 3)(x - 4)$.

i. For what values of a is the solution to $g(x) \leq 0$ an interval.

ii. For what values of a is the solution to $g(x) \geq 0$ an interval.

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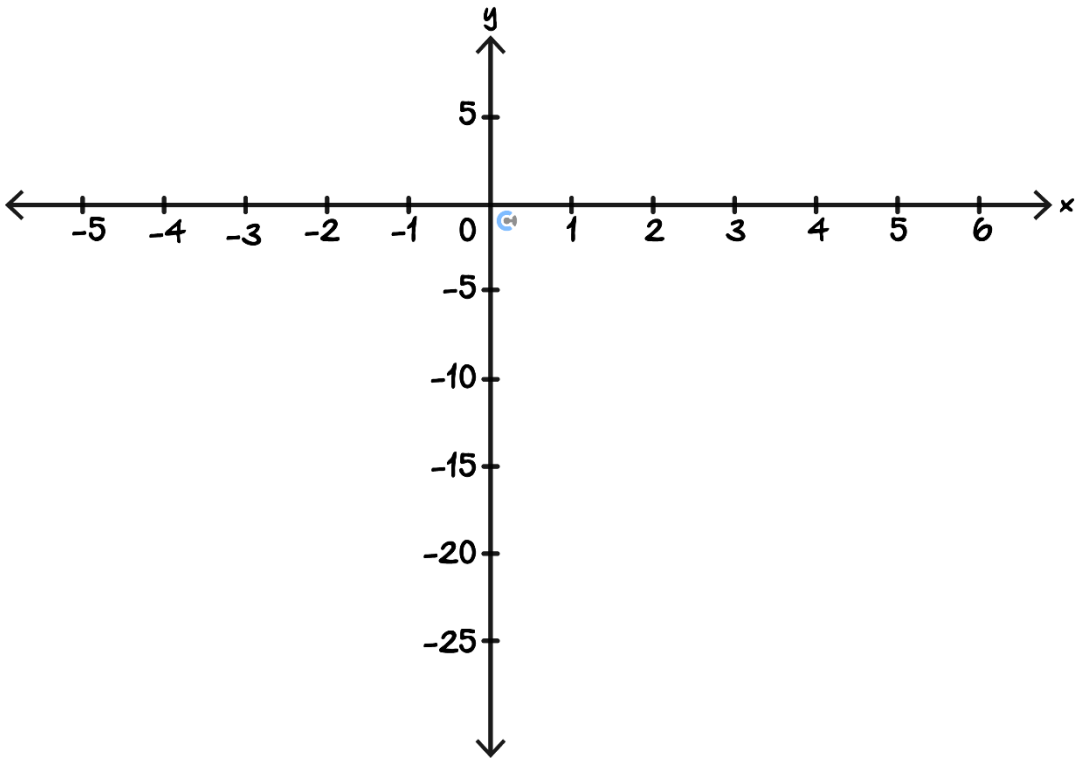
Question 44

Consider the polynomial $f(x) = x^3 - 2x^2 - 9x - 2$.

a. State the co-ordinates of the axis intercepts of f.

b. Hence, sketch the graph of f, labelling all axis intercepts with their co-ordinates.



c. A bisection method with an initial interval of $[3,5]$ is used to approximate the solution to $f(x) = 0$.

First, the interval is refined n times, before the midpoint of the last interval is taken as an answer.

i. If $n = 3$, what answer will this approach yield?

ii. What is the smallest value of $n > 2$ which gives a better approximation to the actual solution than $n = 2$ does?

d. If the bisection method is instead applied with an initial interval of $[-11,5]$, what root will be approximated?

Justify your answer.

e. Use the rational root theorem to show that $\sqrt{7}$ cannot be rational.

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