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VCE Mathematical Methods ½

Polynomials [1.5]

Workbook

Outline:



Algebra of Polynomial Functions

Pg 2-20

- Terminologies of Polynomials
- Long Division
- Remainder Theorem
- Factor Theorem
- Factorising Polynomials
- Rational Root Theorem
- Sum and Difference of Cubes

Graphs of a Polynomial

Pg 21-30

- Graphing Polynomials in the form of $a(x - h)^n + k$
- Graphing Factorised Polynomials

Learning Objectives:

- MM12 [1.5.1] - Identify the properties of polynomials and solve long division.
- MM12 [1.5.2] - Apply remainder and factor theorem to find remainders and factors.
- MM12 [1.5.3] - Find factored form of polynomials.
- MM12 [1.5.4] - Graph factored and unfactored polynomials.



Section A: Algebra of Polynomial Functions

Sub-Section: Terminologies of Polynomials

Degree of Polynomial Functions



Degree = Highest Power of the Polynomial

Question 1

5s

State the degree of each polynomial.

a. $x^3 - 4x^2 + 5x + 6$

3

b. $3x + 5x^2 - x^7$

7

c. A Quadratic.

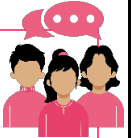
2

Roots of Polynomial Functions



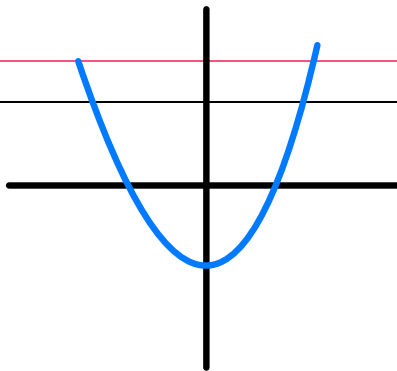
Roots = x-intercept

Discussion: Can a quadratic have more than 2 roots? Hence, can there be more roots than the degree?

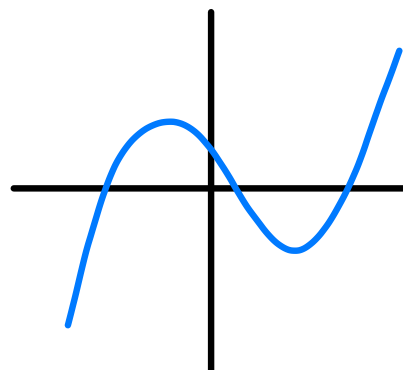


*Degree = Max-its / Roots of
the polynomial*

$$f(x) = ax^2 + bx + c$$



$$g(x) = ax^3 + bx^2 + cx + d$$



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Question 2 **5s**

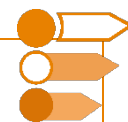
Find the roots of the following polynomial:

$$(x - 1)^2(x + 3)^4$$

$$\therefore x = 1, -3$$

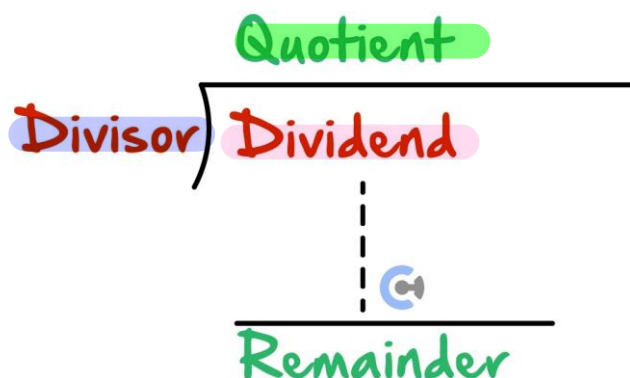
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Sub-Section: Long Division



Polynomial Long Division

➤ Division of polynomials:



$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

Question 3 Walkthrough.

Simplify the following using polynomial long division.

$$\frac{3x^2 + 10x + 20}{2x + 4}$$

$$\begin{array}{r} \frac{3}{2}x + 2 \\ (2x+4) \overline{) 3x^2 + 10x + 20} \\ \underline{-(3x^2 + 6x)} \\ 4x + 20 \\ \underline{-(4x + 8)} \\ 12 \end{array} \Rightarrow \frac{3}{2}x + 2 + \frac{12}{2x+4}$$

$$= \frac{3}{2}x + 2 + \frac{6}{x+2}$$

TIP: Always focus on the highest degree term first.





45s

$$\frac{x^2 - 3x + 5}{x - 1}$$

$$\begin{array}{r}
 x-2 \\
 \hline
 x-1 \sqrt{x^2-3x+5} \\
 \underline{-(x^2-x)} \\
 -2x+5 \\
 \underline{-(-2x+2)} \\
 3
 \end{array}
 = x-2 + \frac{3}{x-1}$$



45s

$$\frac{x^3 + x^2 + 2}{x - 3}$$

$$\begin{array}{r}
 3 \overline{) 1 \ 1 \ 0 \ 2} \\
 \underline{ 3 \ 12 \ 36} \\
 1 \ 4 \ 12 \ 38
 \end{array}$$

$\swarrow \quad \searrow \quad \swarrow \quad \searrow$

$$= x^2 + 4x + 12 + \frac{38}{x-3}$$



TIP: Always remember to fill in any missing powers of x in the numerator or denominator with "placeholders" that have a coefficient of 0.

Question 6 Extension.

45s

Simplify the following using polynomial long division.

$$\frac{x^4 + 4x^3 + 3x^2 - 2x + 3}{x + 3}$$

$$\begin{array}{r} x+3 \overline{) x^4 + 4x^3 + 3x^2 - 2x + 3} \\ \underline{-(x^4 + 3x^3)} \\ x^3 + 3x^2 \\ \underline{-(x^3 + 3x^2)} \\ 0 - 2x + 3 \\ \underline{-(-2x - 6)} \\ 9 \end{array}$$

$$\begin{array}{r} -3 \overline{) 1 \ 4 \ 3 \ -2 \ 3} \\ \underline{-3 \ -3 \ 0 \ 6} \\ 1 \ 1 \ 0 \ -2 \ 9 \\ \Rightarrow x^3 + x^2 - 2 + \frac{9}{x+3} \end{array}$$

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Sub-Section: Remainder Theorem

How can we find the remainder without long division?

Exploration: Derivation of the Remainder Theorem

➤ Consider $\frac{f(x)}{g(x)}$.

$$\frac{f(x)}{g(x)} = q(x) + \frac{R}{g(x)}, \text{ where } R = \text{Remainder}$$

➤ Let's multiply everything by $g(x)$.

$$f(x) = q(x) \cdot g(x) + R$$

➤ Remember, we are trying to find the remainder R before we do long division.

What functions do we already have before long division?

$$f(x) = q(x) \cdot g(x) + R$$

$q(x) = 0$
 $g(x) = 0$

➤ How can we get $f(x)$ to equal to the remainder R ?

🔗 We can substitute a value of x such that, the divisor $g(x)$ is equal to 0.

$$f(a) = q(a) \cdot g(a) + R$$

$$f(a) = R$$

$$g(x) = x - a$$

$$g(a) = 0$$

$$\therefore x = a$$

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Remainder Theorem

➤ **Definition:** Finds the remainder of long division without the need of long division.

when $P(x)$ is divided by $(x - \alpha)$, the remainder is $P(\alpha)$

➤ **Steps:**

1. Find x values which makes the divisor equal to 0.
2. Substitute it into the dividend function.

Discussion: How do we find the remainder of $f(x) \div (x - 2)$?

$$\frac{f(x)}{x-2} \quad \int \begin{matrix} x-2=0 \\ x=2 \end{matrix} \quad \rightarrow \quad R = f(2)$$



Discussion: How do we find the remainder of $f(x) \div (2x + 1)$?

$$\frac{f(x)}{2x+1} \quad \int \begin{matrix} 2x+1=0 \\ x=-\frac{1}{2} \end{matrix} \quad \rightarrow \quad R = f\left(-\frac{1}{2}\right)$$



Question 7 Walkthrough.

Find the remainder of the division, $\frac{f(x)}{g(x)}$, where, $f(x) = x^3 + x^2 - 2x + 5$ and $g(x) = x + 1$.

$$\begin{matrix} g(x) = 0 \\ x = -1 \end{matrix} \quad \rightarrow \quad R = f(-1) = -1 + 1 + 2 + 5 = 7$$

Your turn!

Active Recall: Remainder Theorem

denominator

1. Find x values which makes the divisor equal to 0.

2. Substitute it into the dividend function.

numerator

Question 8

30s

Find the remainder of the division, $\frac{f(x)}{g(x)}$, where, $f(x) = x^3 - 2x^2 + 3x + 1$ and $g(x) = 2x + 4$.

Let $g(x) = 0$:

$\therefore x = -2$

$R = f(-2)$

$= -8 - 8 - 6 + 1$

$= -21$

Question 9 Extension.

45s

For the polynomial $f(x) = 3x^3 - 2x^2 + (7 - 2a)x + 1$, we get a remainder of 14 when $f(x)$ is divided by $g(x) = x - 1$. Find the value of a .

Let $g(x) = 0$:

$\therefore x = 1$

$R = f(1) = 14$

$= 3 - 2 + 7 - 2a + 1$

$= 9 - 2a = 14$

$\therefore -2a = 5$

$\therefore a = -\frac{5}{2}$

Sub-Section: Factor Theorem

Discussion: What division could $f(2)$ be the remainder of?

$$\frac{f(x)}{x-2}, \frac{f(x)}{2x-4} \dots \dots$$

Discussion: Hence, what does it mean when $f(2) = 0$?

$$\hookrightarrow R = 0$$

$\therefore (x-2)$ is a factor
2 is a root (x-int)

This is called the "Factor theorem"

Factor Theorem

► For every x -intercept, there is a factor:

if $P(\alpha) = 0$ then, $(x - \alpha)$ is a factor of $P(x)$

$$\text{Remainder } \frac{P(x)}{x-\alpha} ? \Rightarrow P(-4)$$

Question 10 Walkthrough.

Determine if $x + 4$ is a factor of $P(x) = 3x^3 + 8x^2 - 20x - 16$.

$$P(-4) = 0?$$

$$\begin{aligned} P(-4) &= 3(-64) + 8(16) + 80 - 16 \\ &= -192 + 128 + 80 - 16 \\ &= 0 \end{aligned}$$

$\therefore (x+4)$ is a factor of $P(x)$

Your turn!

Question 11

30s

Determine if $x + 2$ is a factor of $P(x) = 2x^3 - 7x^2 + 7x - 2$.

$$P(-2) = 0?$$

$$\text{Let } x+2=0:$$

$$\therefore x = -2$$

$$R = P(-2) = 2(-8) - 7(4) - 14 - 2$$

$$= -16 - 28 - 14 - 2$$

$$= -60$$

$\therefore (x+2)$ ISNT a factor of $P(x)$

Question 12 Extension.

1 min

Determine if $x - \frac{3}{2}$ is a factor of $P(x) = 6x^3 - x^2 - 20x + 12$.

$$x - \frac{3}{2} = 0$$

$$\therefore x = \frac{3}{2}$$

Remainder when divided by $x - \frac{3}{2}$ is given by:

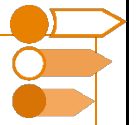
$$R = P\left(\frac{3}{2}\right) = 6\left(\frac{27}{8}\right) - \left(\frac{9}{4}\right) - 30 + 12$$

$$= \frac{81}{4} - \frac{9}{4} - 18$$

$$= \frac{72}{4} - \frac{72}{4} = 0$$

$\therefore (x - \frac{3}{2})$ IS a factor of $P(x)$
as $R = P(\frac{3}{2}) = 0$.

Sub-Section: Factorising Polynomials



Factorising Polynomials

► The steps are:

1. Find a single root by trial and error.

(Factor Theorem: Substitute into the function and see if we get zero.)

2. Use long division to find the quadratic factor.

3. Factorise the quadratic.

Question 13 Walkthrough.

Find all the roots of $f(x) = x^3 + 3x^2 - 6x - 8$.

1. Finding a Single Root (T&E):

Sub $x=1$:

$$f(1) = 1 + 3 - 6 - 8 = -10$$

Sub $x=2$:

$$f(2) = 8 + 12 - 12 - 8 = 0$$

∴ $(x-2)$ is a factor

2. Long Division:

$$\begin{array}{r} x^2 + 5x + 4 \\ (x-2) \overline{) x^3 + 3x^2 - 6x - 8} \\ \underline{-(x^3 - 2x^2)} \\ 5x^2 - 6x \\ \underline{-(5x^2 - 10x)} \\ 4x - 8 \\ \underline{-(4x - 8)} \\ 0 \end{array}$$

NOTE: When the question asks for all roots, you cannot just factorise and end it there!

0



3. Factorise:

$$\begin{aligned} &= (x-2)(x^2 + 5x + 4) \\ &= (x-2)(x+4)(x+1) \end{aligned}$$

$$\therefore x = 2, -4, -1$$

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Your turn!

Question 14 **1min 30s**

Find all the roots of $f(x) = x^3 + 12x^2 + 17x - 90$.

1. Finding a Single Root (T&E):

Sub $x=1$:

$$f(1) = 1 + 12 + 17 - 90 = -60$$

Sub $x=2$:

$$f(2) = 8 + 48 + 34 - 90 = 0$$

$\therefore (x-2)$ is a factor

2. Synthetic Division:

$$\begin{array}{r|rrrr} 2 & 1 & 12 & 17 & -90 \\ & & 2 & 28 & 90 \\ \hline & 1 & 14 & 45 & 0 \end{array}$$

3. Factorise:

$$= (x-2)(x^2 + 14x + 45)$$

$$= (x-2)(x+9)(x+5)$$

$$\therefore x = 2, -9, -5$$

Question 15 **1min 30s**

Find all the roots of $f(x) = -2x^3 - 13x^2 - 5x + 6$.

1. Finding a Single Root (T&E):

Sub $x=1$:

$$f(1) = -2 - 13 - 5 + 6 = -14$$

Sub $x=-1$:

$$f(-1) = 2 - 13 + 5 + 6 = 0$$

$\therefore (x+1)$ is a factor

2. Synthetic Division:

$$\begin{array}{r|rrrr} -1 & -2 & -13 & -5 & 6 \\ & & 2 & 11 & -6 \\ \hline & -2 & -11 & 6 & 0 \end{array}$$

3. Factorise:

$$= (x+1)(-2x^2 - 11x + 6)$$

$$= (x+1)(-2x+1)(x+6)$$

$$\therefore x = -1, \frac{1}{2}, -6$$

Question 16

Find all the roots of $f(x) = 6x^3 - 27x^2 + 21x + 18$.

1. Finding a Single Root (T&E):

Sub $x =$:

$f() =$

Sub $x = 2$:

$f() =$

$\therefore ()$ is a factor

2. Long Division:

$$\begin{array}{r} () \overline{) 6x^3 - 27x^2 + 21x + 18} \\ \underline{-()} \\ - () \\ \underline{-()} \\ - () \end{array}$$

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3. Factorise:

$$\begin{aligned} &= () () () \\ &= () () () () \\ &= () () () () \end{aligned}$$

Sub-Section: Rational Root Theorem

Discussion: Consider $(2x - 1)(3x - 1)(6x - 1)$. What are the roots and could we have gotten that from trial and error?

$$x = \frac{1}{2}, \frac{1}{3}, \frac{1}{6}$$

So, what should we do?

Rational Root Theorem

➤ Rational Root Theorem narrows down the possible roots.

$$\text{Potential root} = \pm \frac{\text{Factors of constant term } a_0}{\text{Factors of leading coefficient } a_n}$$

➤ If the roots are rational numbers, the roots can only be $\pm \frac{\text{factors of constant term } a_0}{\text{factors of leading coefficient } a_n}$.

Question 17 Walkthrough.

Find all the roots of $f(x) = 6x^3 + 13x^2 - 14x + 3$.

1. Finding a Single Root (RRT):

$$\text{RRT} = \pm \frac{\{1, 3\}}{\{1, 2, 3, 6\}} = \pm \left\{ 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{6}, \frac{3}{2} \right\}$$

Sub $x = \frac{1}{2}$:

$$\begin{aligned} f\left(\frac{1}{2}\right) &= 6\left(\frac{1}{2}\right)^3 + 13\left(\frac{1}{2}\right)^2 - 14\left(\frac{1}{2}\right) + 3 \\ &= \frac{3}{2} + \frac{13}{4} - 7 + 3 \\ &= \frac{6}{4} + \frac{13}{4} - 7 + 3 \\ &= \frac{19}{4} - 7 + 3 \\ &= \frac{19}{4} - \frac{28}{4} + \frac{12}{4} \\ &= \frac{3}{4} \neq 0 \end{aligned}$$

2. Long Division:

$$\begin{array}{r} 3x^2 + 8x - 3 \\ (2x-1) \overline{) 6x^3 + 13x^2 - 14x + 3} \\ \underline{-(6x^3 - 3x^2)} \\ 16x^2 - 14x \\ \underline{-(16x^2 - 8x)} \\ -6x + 3 \\ \underline{-(-6x + 3)} \\ 0 \end{array}$$

3. Factorise:

$$\begin{aligned} &= (2x-1)(3x^2 + 8x - 3) \\ &= (2x-1)(3x-1)(x+3) \end{aligned}$$

4. Find Root:

$$\therefore x = \frac{1}{2}, \frac{1}{3}, -3$$

$\therefore (2x-1)$ is a factor

NOTE: All the roots are part of the suggestion given by the rational root theorem.



Question 18

Find all the roots of $f(x) = 2x^3 - x^2 - 22x - 24$.

1. Finding a Single Root (RRT):

$$RRT = \pm \frac{\{1, 2, 3, 4, 6, 8, 12, 24\}}{\{1, 2\}} = \pm \{1, 2, 3, 4, 6, 8, 12, 24, \frac{1}{2}, \frac{3}{2}\}$$

Sub $x = 4$:

$$\begin{aligned} f(4) &= 2(4)^3 - (4)^2 - 22(4) - 24 \\ &= 128 - 16 - 88 - 24 \\ &= 0 \end{aligned}$$

$\therefore (x-4)$ is a factor

2. Synthetic Division:

$$\begin{array}{r|rrrr} 4 & 2 & -1 & -22 & -24 \\ & & 8 & 28 & 24 \\ \hline & 2 & 7 & 6 & 0 \end{array}$$

3. Factorise:

$$\begin{aligned} &= (x-4)(2x^2 + 7x + 6) \\ &= (x-4)(2x+3)(x+2) \end{aligned}$$

4. Find Root:

$$\therefore x = 4, -\frac{3}{2}, -2$$

Discussion: Why is rational root theorem called a rational root theorem?



→ rational no. is a no. that can be written as a

Factor: $(x - \sqrt{3})$ fraction (of integers)

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Roots: $\sqrt{3} \Rightarrow$ Irrational
 $\pi \Rightarrow$

ONLY finds RATIONAL ROOTS!

Question 19 Extension.

Find all the roots of $f(x) = 6x^3 + 19x^2 - 24x - 16$.

1. Finding a Single Root (RRT):

$$RRT = \pm \frac{\{1, 2, 4, 8, 16\}}{\{1, 2, 3, 6\}}$$

$$= \pm \left\{ 1, 2, 4, 8, 16, \frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{8}{3}, \frac{16}{3}, \frac{1}{6} \right\}$$

2. Synthetic Division:

$$\begin{array}{r|rrrr} -4 & 6 & 19 & -24 & -16 \\ & & -24 & 20 & 16 \\ \hline & 6 & -5 & -4 & 0 \end{array}$$

Sub $x = -4$:

$$f(-4) = 6(-4)^3 + 19(-4)^2 - 24(-4) - 16$$

$$= 6(-64) + 19(16) + 96 - 16$$

$$= -384 + 304 + 80 = 0$$

$\therefore (x+4)$ is a factor

3. Factorise:

$$= (x+4)(6x^2 - 5x - 4)$$

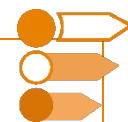
$$= (x+4)(3x-4)(2x+1)$$

4. Find Root:

$$\therefore x = -4, \frac{4}{3}, -\frac{1}{2}$$

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Sub-Section: Sum and Difference of Cubes



Sum and Difference of Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Question 20 Walkthrough.

Factorise the following polynomial as much as possible.

$$\begin{array}{cc} x^3 + 125 \\ \sqrt[3]{x^3} & \sqrt[3]{125} \\ x & 5 \end{array}$$

$$= (x + 5)(x^2 - 5x + 25)$$

Question 21

Factorise the following polynomial as much as possible.

$$8x^3 - 216$$

$$= 8(x^3 - 27)$$

$$= 8(x - 3)(x^2 + 3x + 9)$$

Question 22 Extension.

Factorise the following polynomial as much as possible.

$$32x^3 - 256$$

$$= 32(x^3 - 8)$$

$$= 32(x-2)(x^2+2x+4)$$

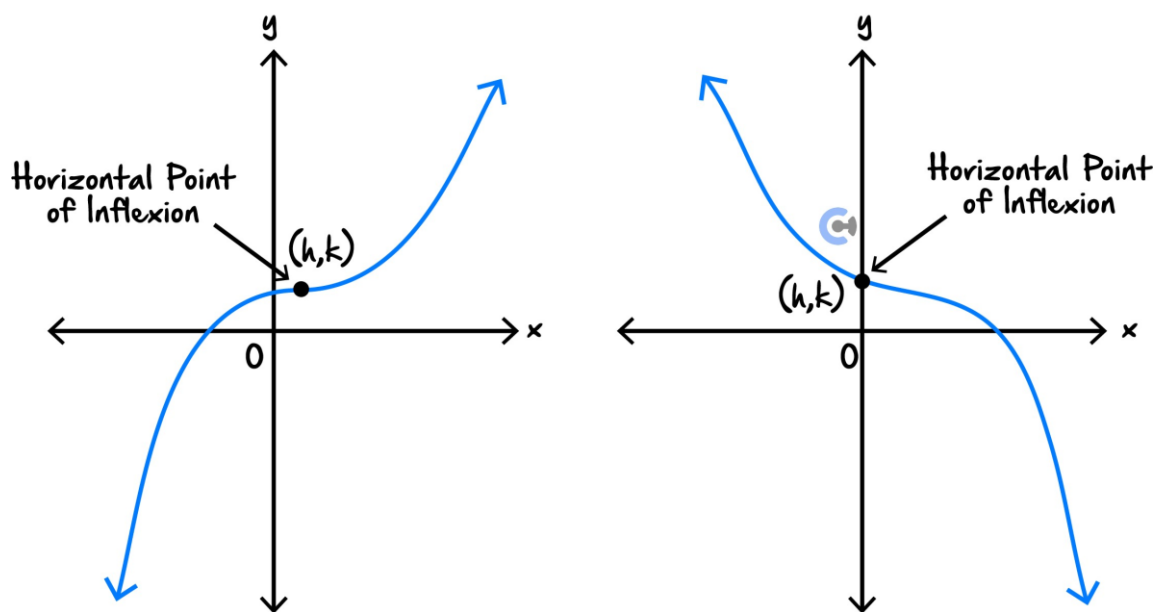
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Section B: Graphs of a Polynomial

Sub-Section: Graphing Polynomials in the Form of $a(x - h)^n + k$

Graphs of $a(x - h)^n + k$, where n is an Odd Positive Integer

- All graphs look like a "cubic".

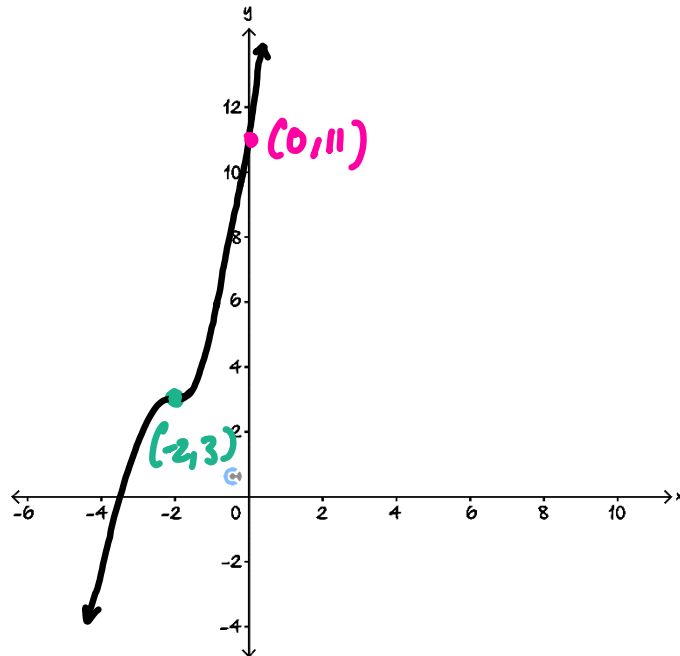


- The point (h, k) gives us the stationary point of inflection.
- n cannot be 1 for this shape to occur!

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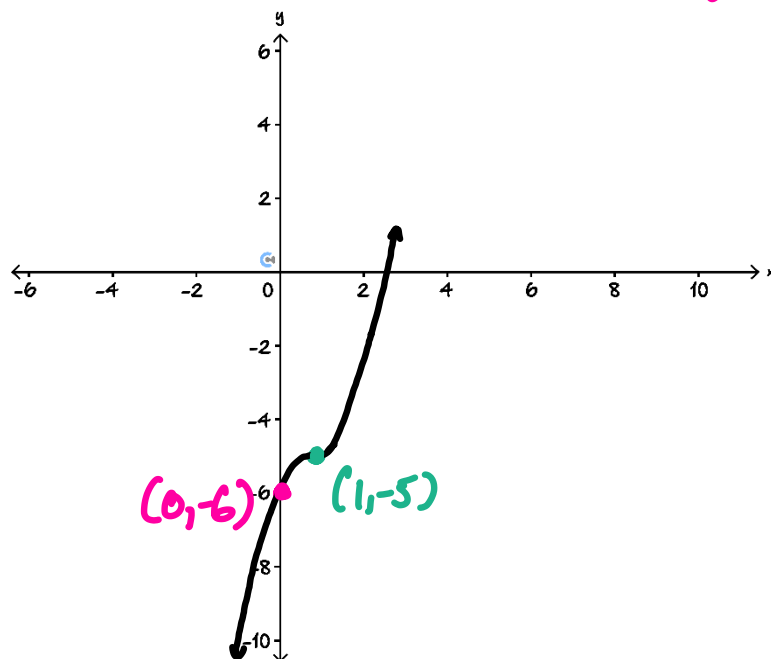
Question 23 Walkthrough.

Sketch the graph of $y = (x + 2)^3 + 3$ on the axes below.



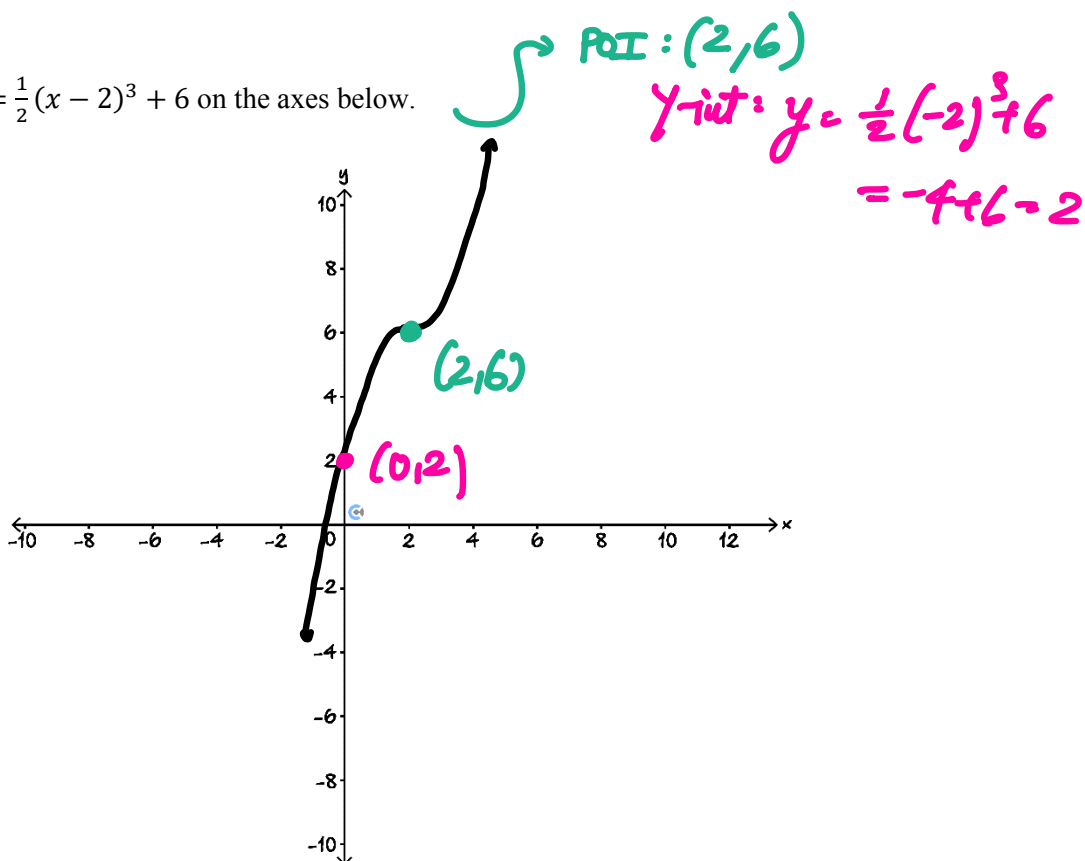
Question 24

Sketch the graph of $y = (x - 1)^3 - 5$ on the axes below.



Question 25

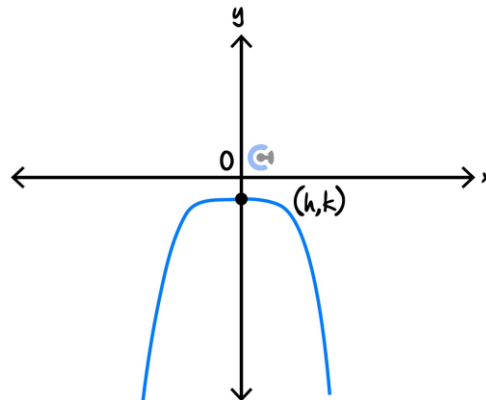
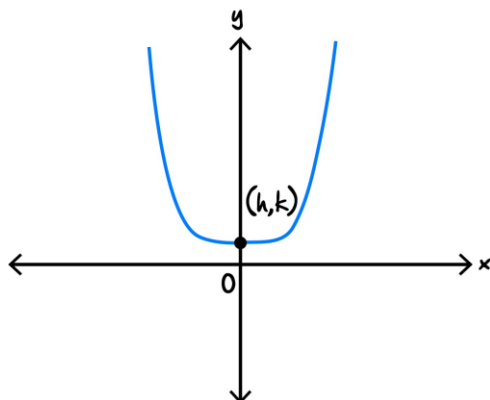
Sketch the graph of $y = \frac{1}{2}(x - 2)^3 + 6$ on the axes below.



What about even powers?

Graphs of $a(x - h)^n + k$, where n is an Even Positive Integer

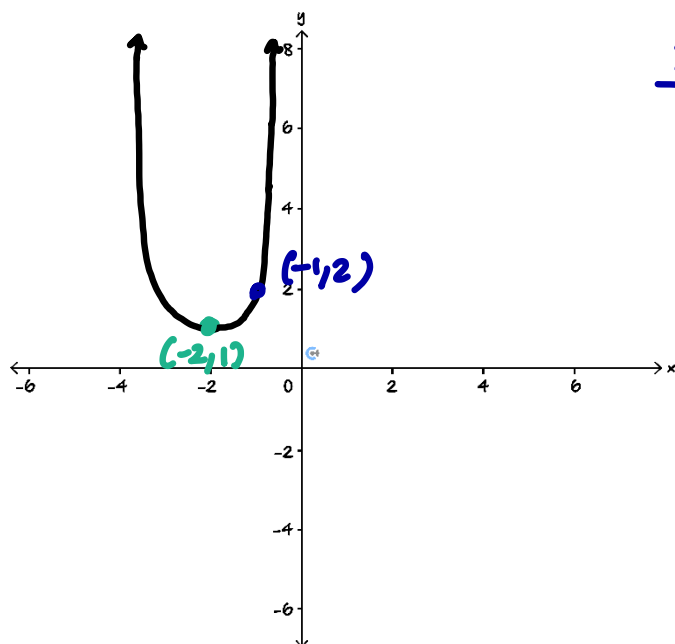
➤ All graphs look like a "quadratic".



➤ The point (h, k) gives us the turning point.

Question 26 Walkthrough.

Sketch the graph of $y = (x + 2)^4 + 1$ on the axes below.

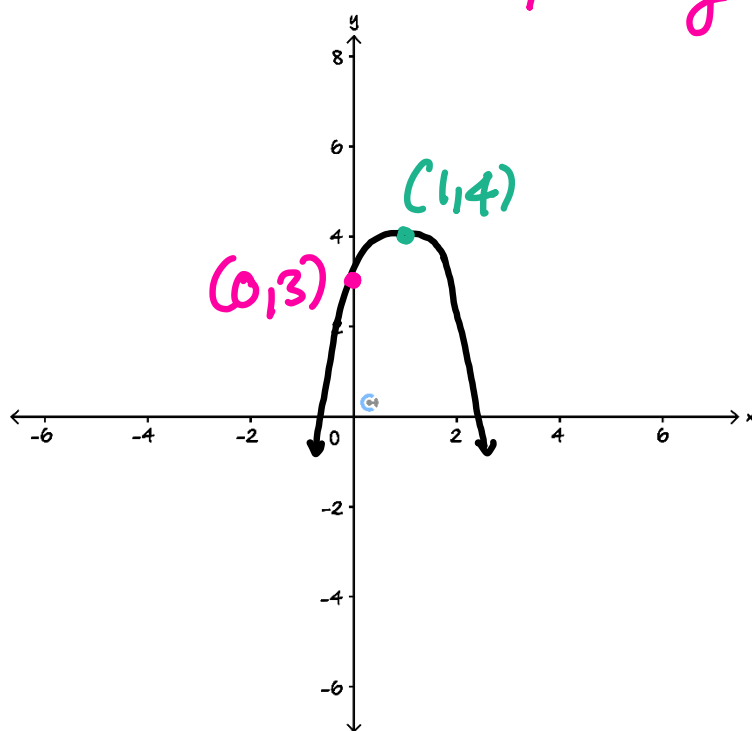


TP: $(-2, 1)$ y-int: $y = (-2)^4 + 1 = 17$

$x = -1:$
 $y = (-1)^4 + 1 = 2$
 $(-1, 2)$

Question 27

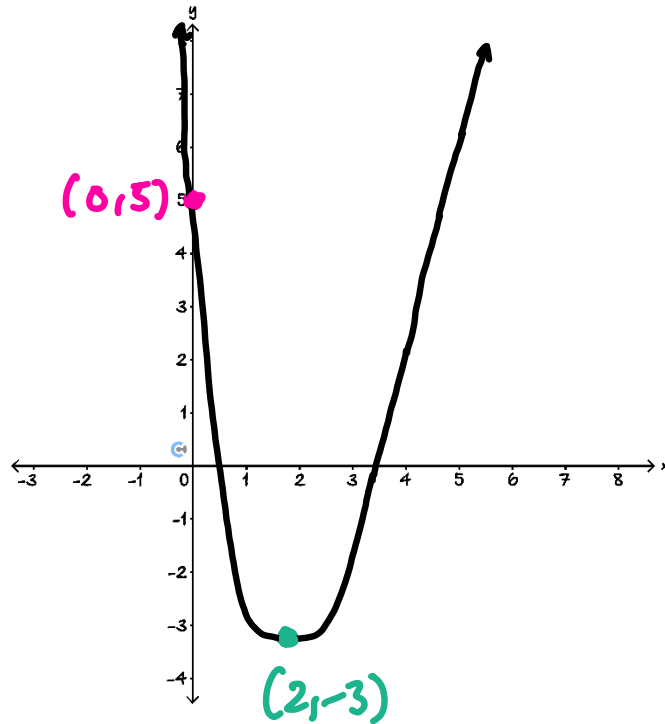
Sketch the graph of $y = -(x - 1)^4 + 4$ on the axes below.



TP: $(1, 4)$ y-int: $y = -(-1)^4 + 4 = 3$

Question 28 Extension.

Sketch the graph of $y = \frac{1}{2}(x - 2)^4 - 3$ on the axes below.



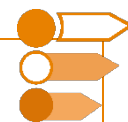
$$y_{int}: y = \frac{1}{2}(-2)^4 - 3$$

$$= 8 - 3$$

$$= 5$$

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Sub-Section: Graphing Factorised Polynomials



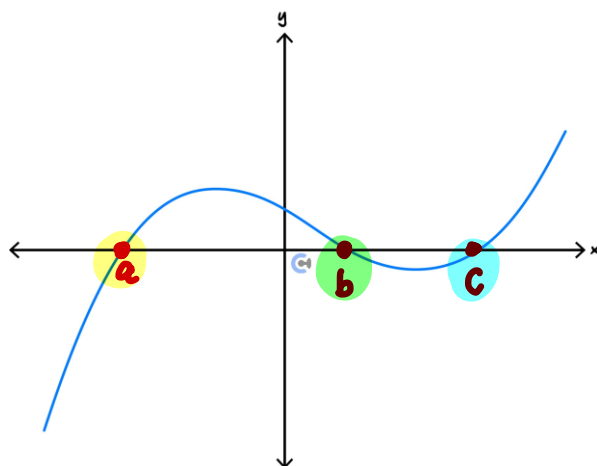
What about the graph of a factorised polynomial?



Exploration: Graphs of Factorised Polynomials

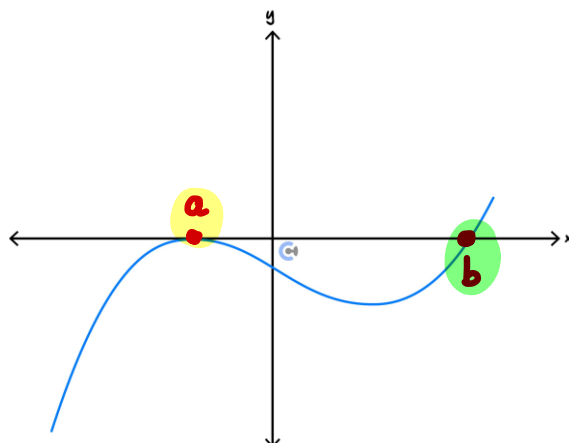


All nonrepeated linear factors correspond to x-intercepts of the graph.



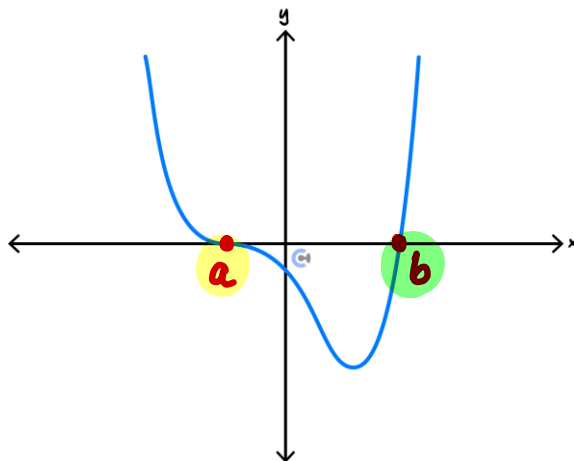
➤ E.g., $f(x) = (x - a)(x - b)(x - c)$ results in x -intercepts at $(a, 0)$, $(b, 0)$ and $(c, 0)$.

All Squared linear factors correspond to TP & x-intercept of the graph.



➤ E.g., $f(x) = (x - a)^2(x - b)$ will have an x -intercept $(a, 0)$ which is also a local minimum/maximum.

All Cubed linear factors correspond to SPOI & x -intercept of the graph.



➤ E.g., $f(x) = (x - a)^3(x - b)$ has an x -intercept $(a, 0)$ which is also a stationary point of inflection.

Graphs of Factorised Polynomials

➤ Steps:

1. Plot x -intercepts.
2. Determine whether the polynomial is positive or negative.
3. Use the repeated factors to deduce the shape.

Non-Repeated: Only x -intercept.

Even Repeated: x -intercept and a turning point.

Odd Repeated: x -intercept and a stationary point of inflection.



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Question 29 Walkthrough.

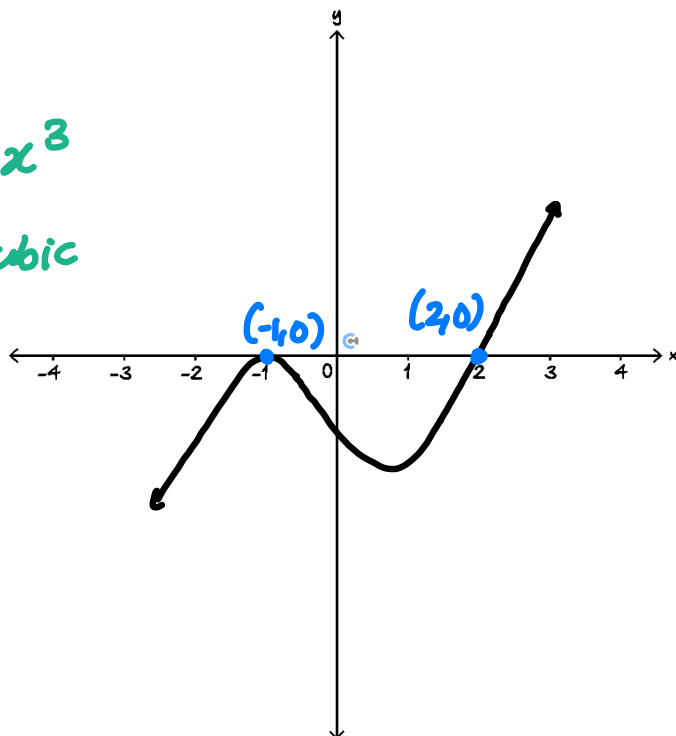
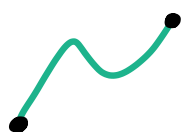
Sketch the graphs of the following functions on the axes provided. Ignore the y-axis scale.

a. $y = (x + 1)^2(x - 2)$

$x = -1, 2$

$x^2 \cdot x = +x^3$

Shape = +ve cubic

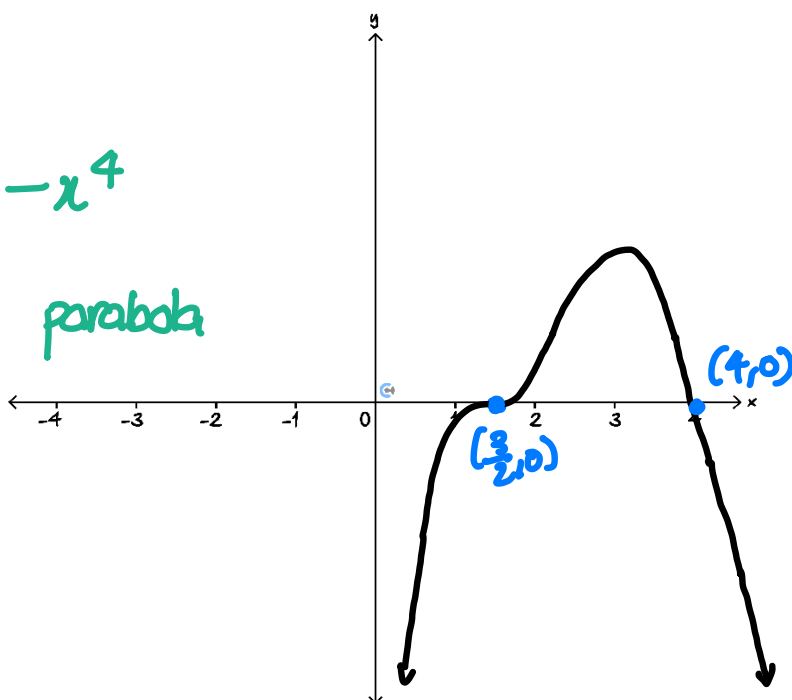


b. $y = \left(x - \frac{3}{2}\right)^2(4 - x)$

$x = \frac{3}{2}, 4$

$x^3 \cdot -x = -x^4$

Shape = -ve parabola



Your turn!



Question 30

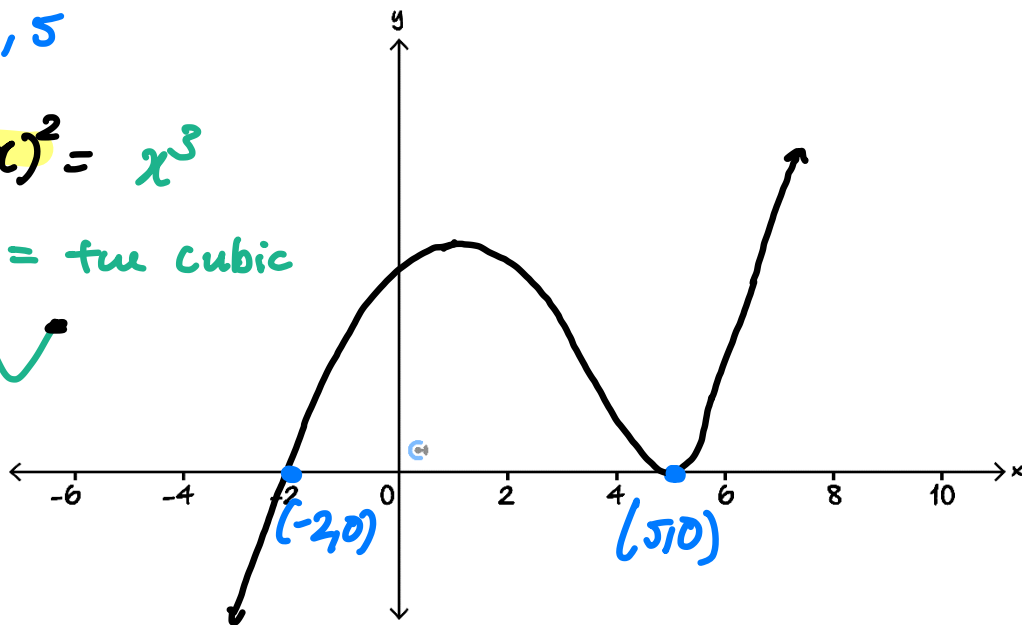
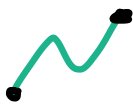
Sketch the graphs of the following functions on the axes provided. Ignore the y-axis scale.

a. $y = (2 + x)(5 - x)^2$

$x = -2, 5$

$x \cdot (-x)^2 = x^3$

Shape = true cubic

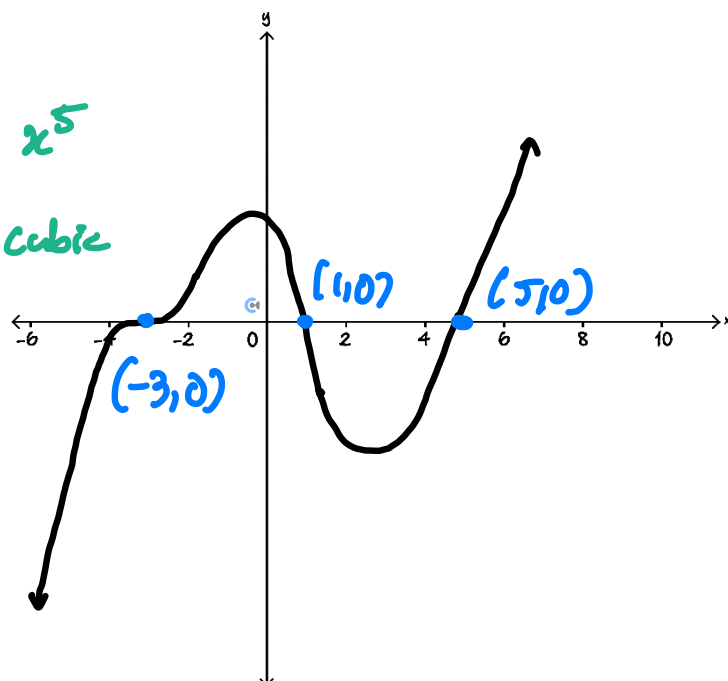


b. $y = (x + 3)^3(x - 1)(x - 5)$

$x = -3, 1, 5$

$x^3 \cdot x \cdot x = x^5$

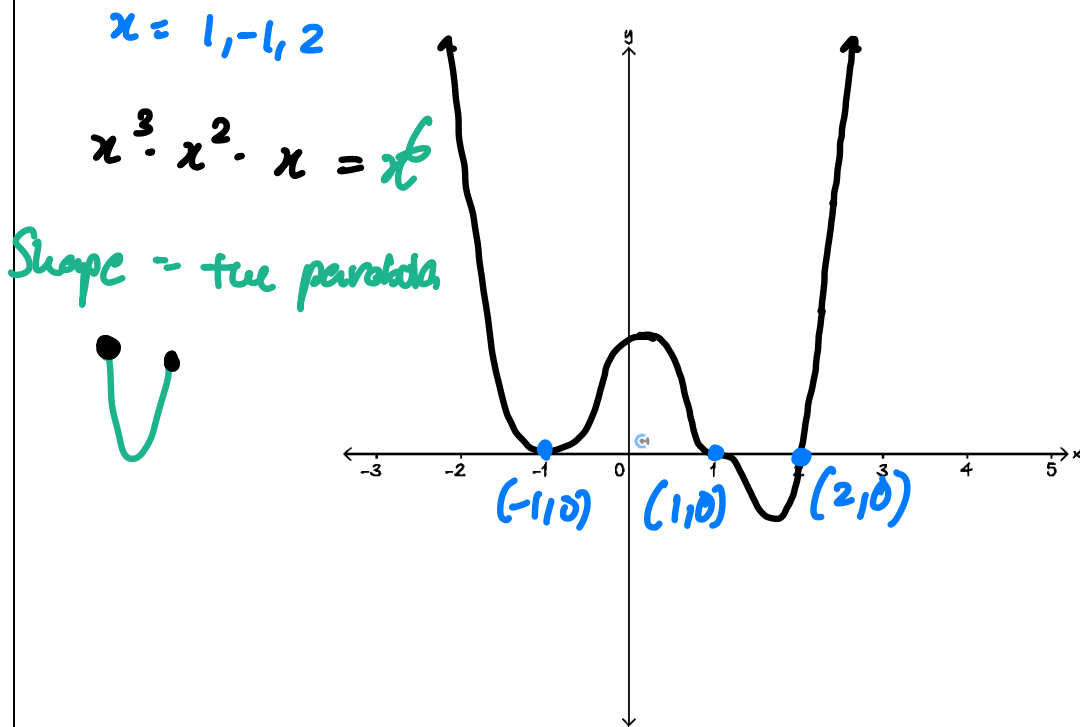
Shape \Rightarrow true cubic



Question 31

Sketch the graph of the following function on the axes provided. Ignore the y-axis scale.

$$y = (x - 1)^3(x + 1)^2(x - 2)$$



Space for Personal Notes



Contour Check

Learning Objective: [1.5.1] - Identify the properties of polynomials and solve long division.

Key Takeaways

- The degree of a polynomial is the polynomial's highest power.
- The roots of a polynomial are its x-intercepts.
- For polynomial long division:

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

Learning Objective: [1.5.2] - Apply remainder and factor theorem to find remainders and factors.

Key Takeaways

- When $P(x)$ is divided by $(x - \alpha)$, the remainder is $P(\alpha)$.
- If $P(\alpha) = 0$ then $(x - \alpha)$ is a factor of $P(x)$.

Learning Objective: [1.5.3] - Find factored form of polynomials.

Key Takeaways

- ▣ Steps to factor a cubic polynomial are:

- Find a single root by trial and error.

(Factor Theorem: Substitute into the function and see if we get zero.)

- Use long division to find the quadratic factor.

- Factorise the quadratic.

- ▣ Rational Root Theorem **narrows down** the possible roots. If the roots are rational numbers, it must be that any.

$$\text{Potential root} = \pm \frac{\text{Factors of } \text{Constant term } a_0}{\text{Factors of } \text{Leading coefficient } a_n}$$

- ▣ Sum and difference of cubes:

$$a^3 + b^3 = (\text{ } a+b \text{ })(a^2 - ab + b^2)$$

$$a^3 - b^3 = (\text{ } a-b \text{ })(a^2 + ab + b^2)$$

Learning Objective: [1.5.4] - Graph factored and unfactored polynomials.

Key Takeaways

□ Graphs of $a(x - h)^n + k$, where n is an Odd Positive Integer that is not equal to 1:

- The point (h, k) gives us the stationary point of inflection.

□ Graphs of $a(x - h)^n + k$, where n is an Even Positive Integer:

- The point (h, k) gives us the TP.
- These graphs look like a parabola.

□ Steps to graphing factorised polynomials:

1. Plot x -intercepts.
2. Determine whether the polynomial is positive or negative.
3. Use the repeated factors to deduce the shape:

Non-Repeated: Only x -intercept.

Even Repeated: x -intercept and a TP.

Odd Repeated: x -intercept and a SPOI.

Stationary
Point of
Inflection //



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