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VCE Mathematical Methods ½
Polynomials [1.5]
Homework Solutions

Homework Outline:

Compulsory Questions	Pg 2 – Pg 18
Supplementary Questions	Pg 19 – Pg 35

Section A: Compulsory Questions

Sub-Section [1.5.1]: Identify the Properties of Polynomials and Solve Long Division

Question 1

Consider the polynomial $f(x) = 4x^3 - 2x^2 - x + 5$.

a. State the degree of $f(x)$.

3

b. State the leading coefficient of $f(x)$.

4

c. State the constant term of $f(x)$.

5

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Question 2

Simplify the following using polynomial long division:

$$\frac{x^3 - 10x^2 + 8x + 3}{x - 2}$$

$$x^2 - 8x - \frac{13}{x-2} - 8$$

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Question 3

Simplify the following using polynomial long division:

$$\frac{2x^4 - 8x^2 + 6x}{x + 3}$$

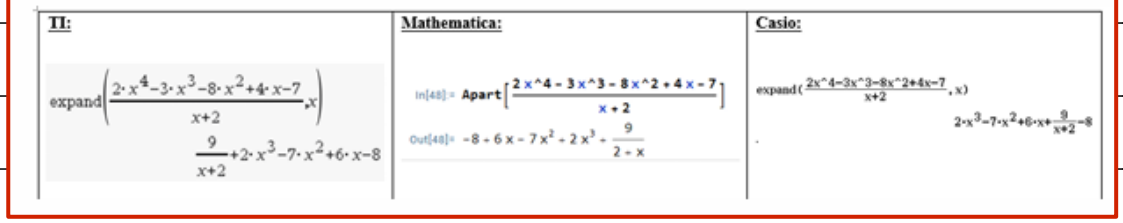
$$2x^3 - 6x^2 + 10x + \frac{72}{x+3} - 24$$

Question 4 Tech-Active.

Simplify the following using polynomial long division:

$$\frac{2x^4 - 3x^3 - 8x^2 + 4x - 7}{x + 2}$$

$$2x^3 - 7x^2 + 6x + \frac{9}{x+2} - 8$$



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Sub-Section [1.5.2]: Apply Reminder and Factor Theorem to Find Reminders and Factors

Question 5

Find the remainder of the division, $\frac{f(x)}{g(x)}$, where:

a. $f(x) = x^3 - 3x^2 + 2x + 1$ and $g(x) = x - 2$.

$f(2) = 1$

b. $f(x) = 2x^3 - x^2 + 3x + 4$ and $g(x) = x + 1$.

$f(-1) = -2$

c. $f(x) = -x^3 + 6x^2 + 3x + 2$ and $g(x) = x - 3$.

$f(3) = 38$

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Question 6

For the polynomial $f(x) = 3x^3 - 2x^2 + (9 - 2a)x + 2$, we get a remainder of 8 when $f(x)$ is divided by $g(x) = x - 1$. Find the value of a.

We have that $f(1) = 8 \implies a = 2.$

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Question 7

Consider the expression:

$$f(x) = x^3 + ax^2 - 2x + b$$

Where a and b are non-zero constants.

It is known that $x - 2$ is a factor of $f(x)$ and that the remainder when $f(x)$ is divided by $x - 3$ is 2. Find the values of a and b.

We must have that $f(2) = 0$ and $f(3) = 2$ so we get the simultaneous equations

$$4 + 4a + b = 0$$
$$21 + 9a + b = 2$$

which yield $a = -3$ and $b = 8$.

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Sub-Section [1.5.3]: Find Factored Form of Polynomials

Question 8

Factorise $f(x) = x^3 - 4x^2 + x + 6$ as the product of three linear factors.

$$f(x) = (x+1)(x-2)(x-3)$$

Question 9

Factorise $f(x) = x^3 - 4x^2 - 17x + 60$ as the product of three linear factors.

$$f(x) = (x-3)(x-5)(x+4)$$

Question 10

Factorise $f(x) = 8x^3 - 56x^2 - 70x + 48$ as the product of three linear factors.

$$f(x) = (2x - 1)(4x + 6)(x - 8)$$

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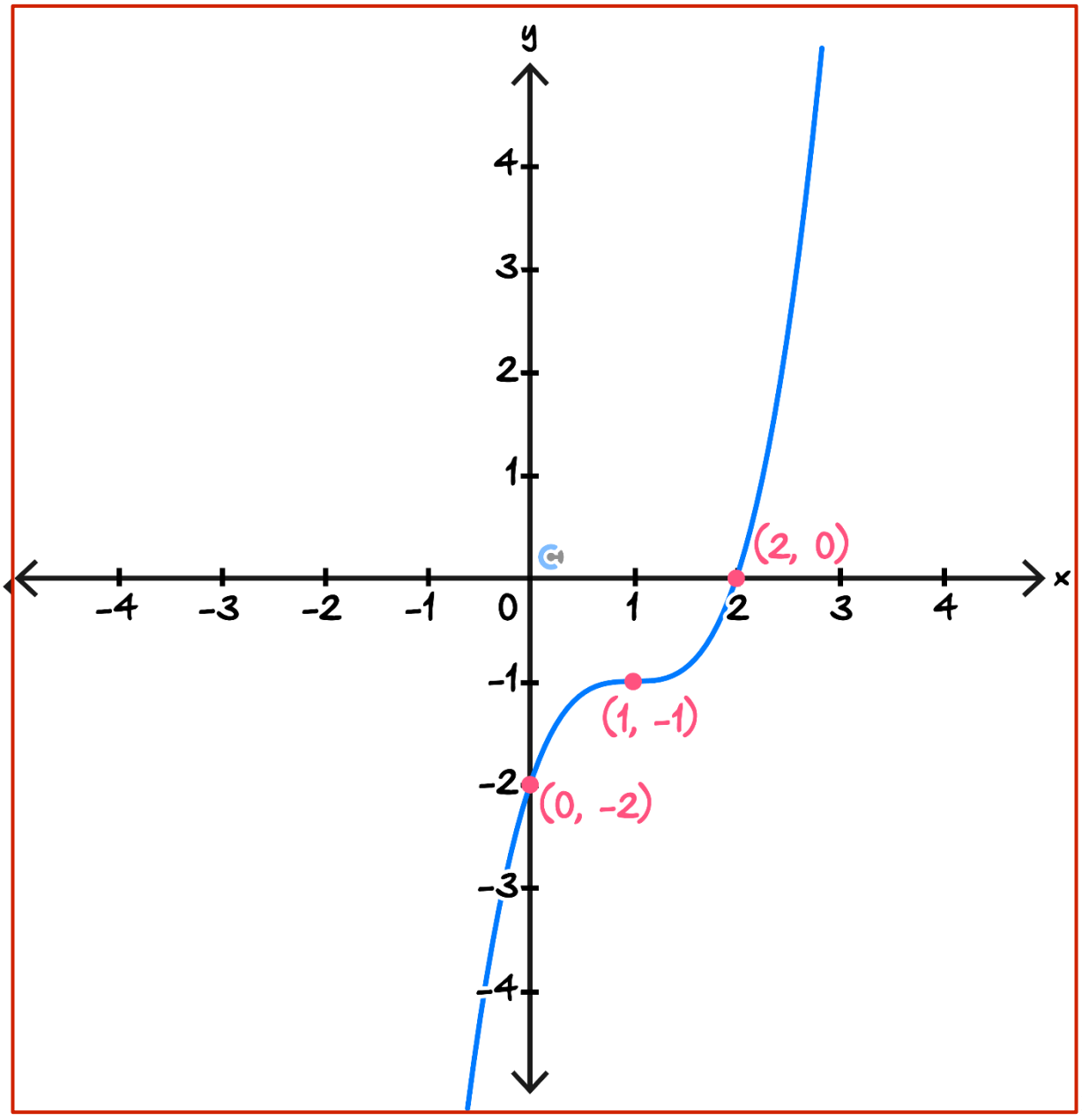
Sub-Section [1.5.4]: Graph Factored and Unfactored Polynomials

Question 11

Sketch the graphs of each of the functions on the axes provided.

a. $y = (x-1)^3 - 1$



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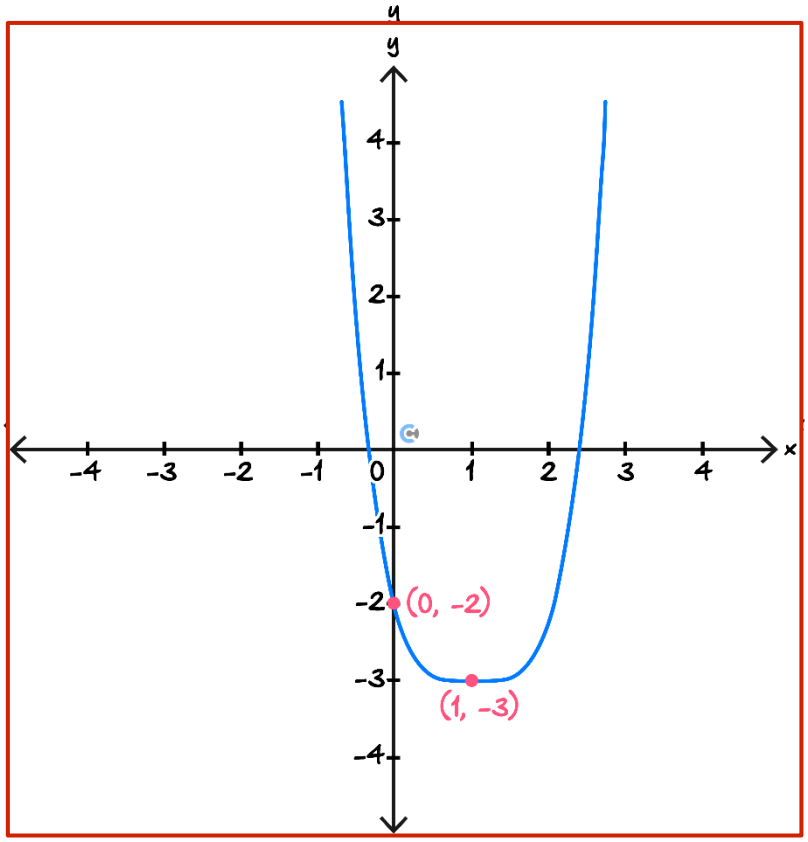
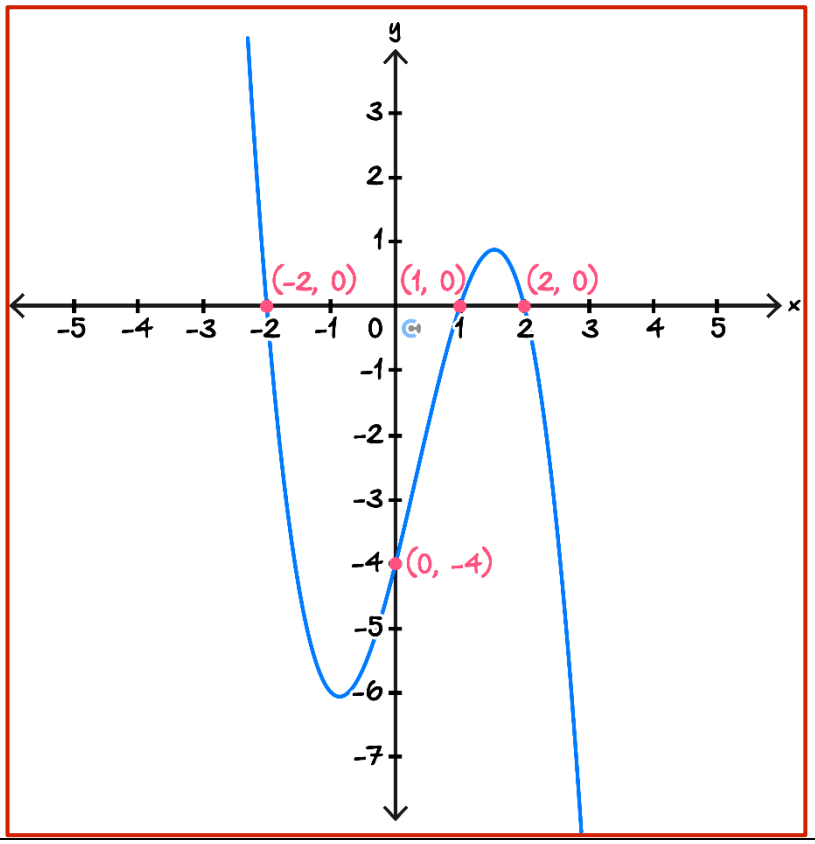
b. $y = (x-1)^4 - 3$



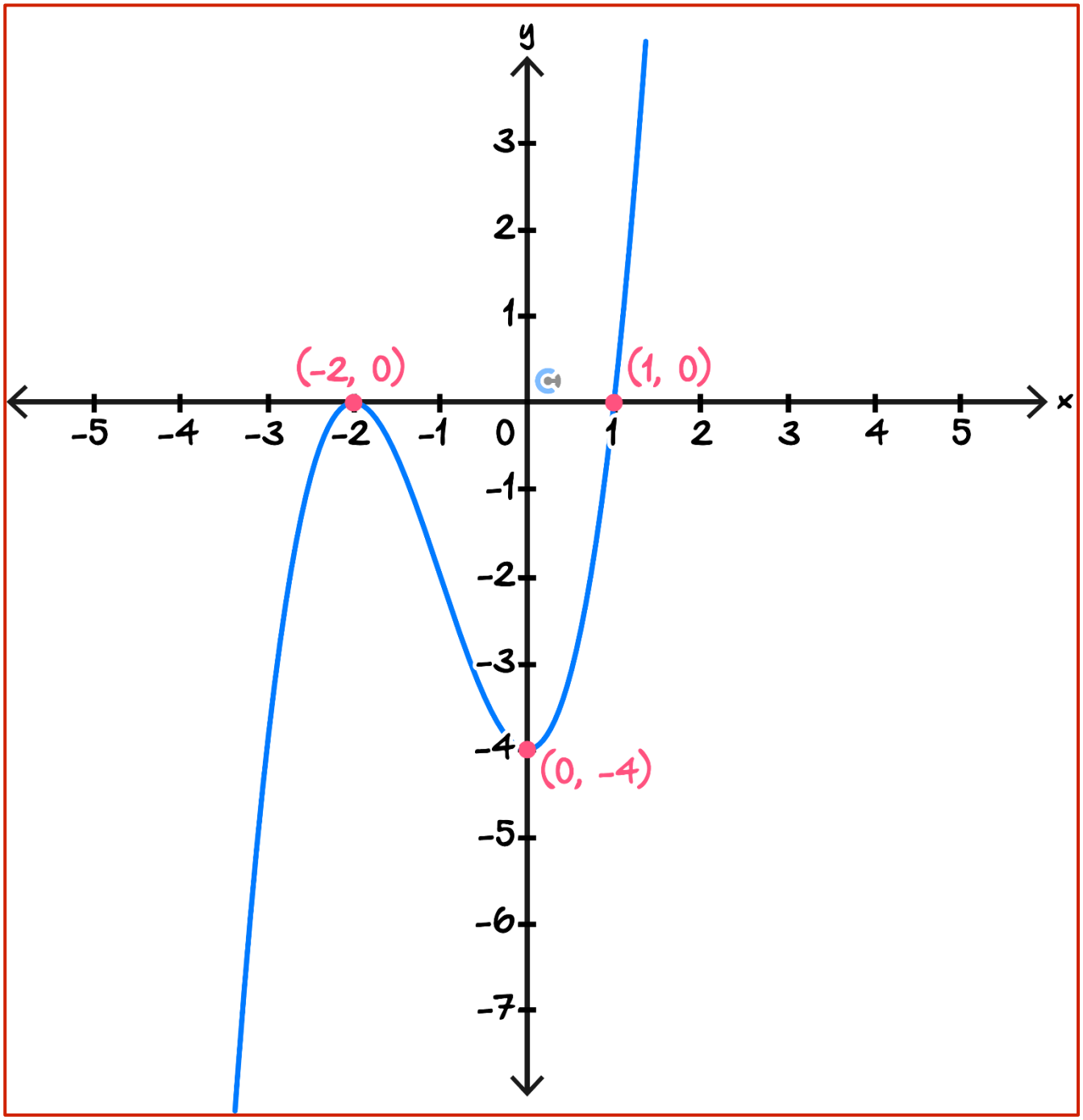
c. $y = -(x+2)(x-1)(x-2)$



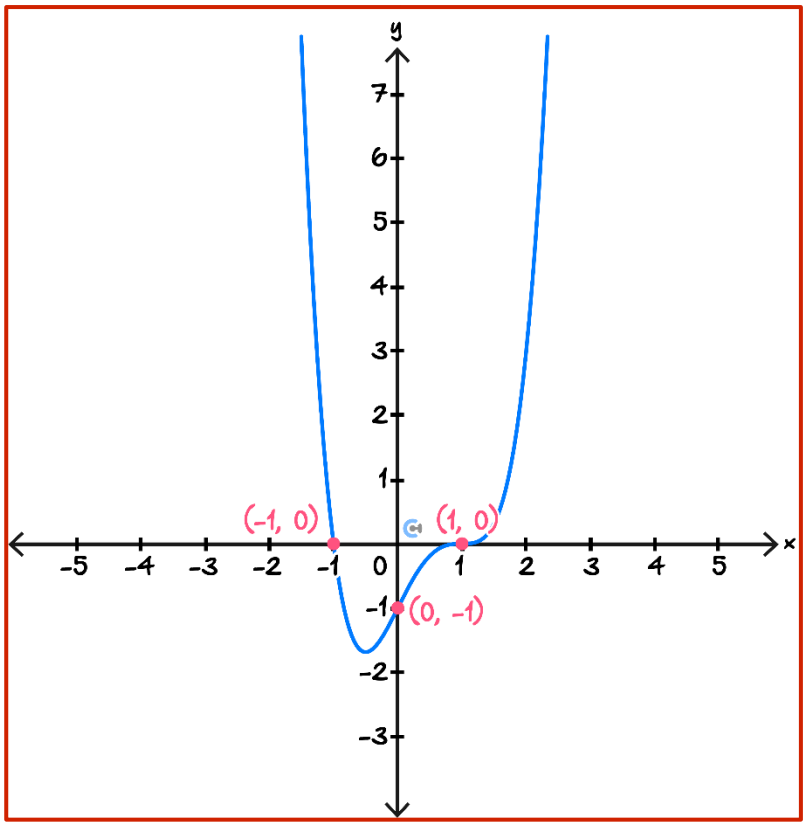
Question 12

Sketch the graphs of each of the functions on the axes provided.

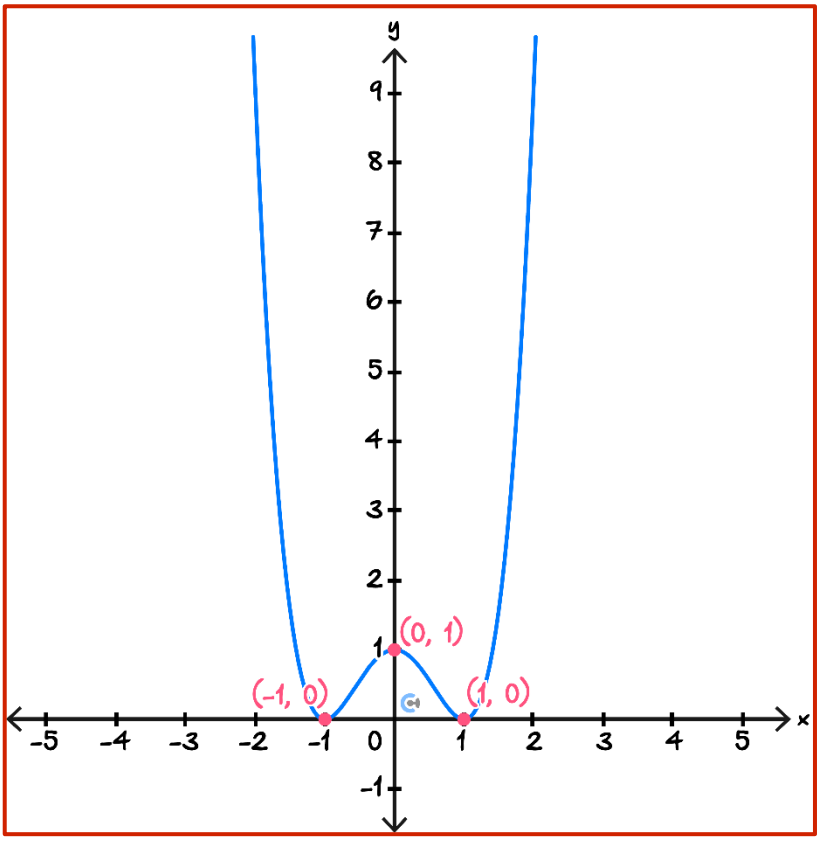
a. $y = (x + 2)^2(x - 1)$



b. $y = (x+1)(x-1)^3$



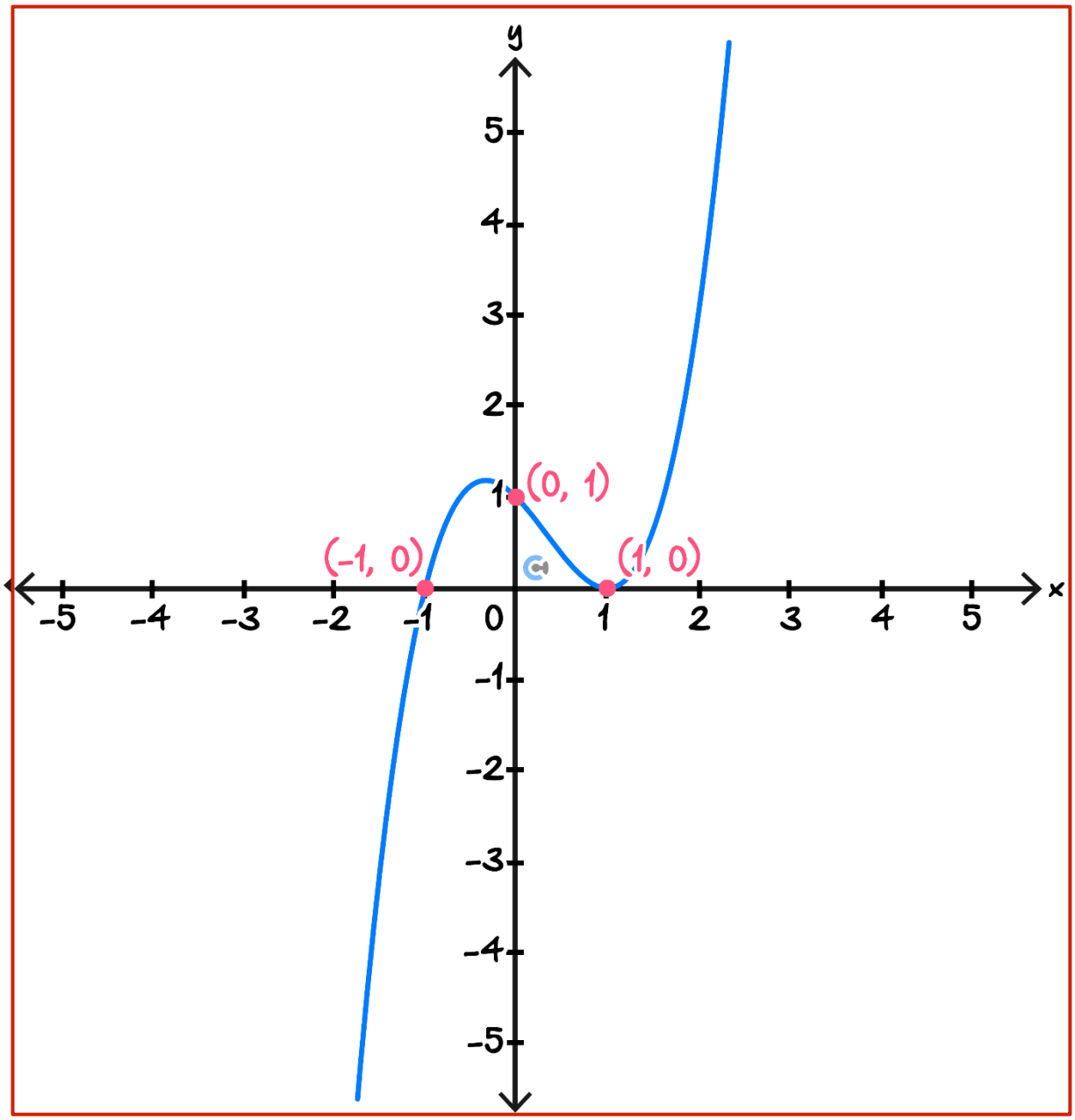
c. $y = (x+1)^2(x-1)^2$



Question 13

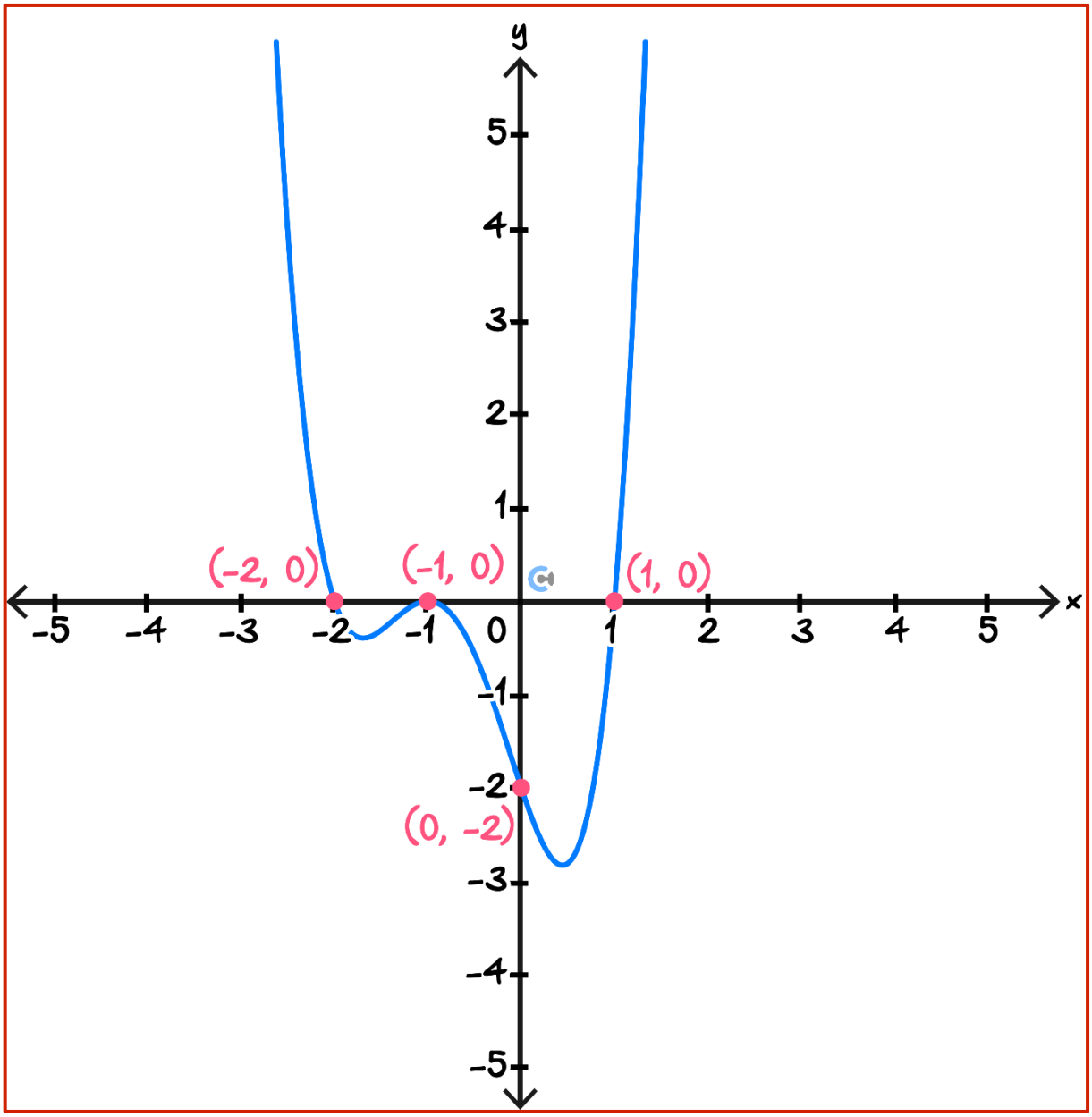
Factorise and hence, sketch the graphs of each of the functions on the axes provided.

a. $y = x^3 - x^2 - x + 1$



Solution Pending

b. $y = x^4 + 3x^3 + x^2 - 3x - 2$



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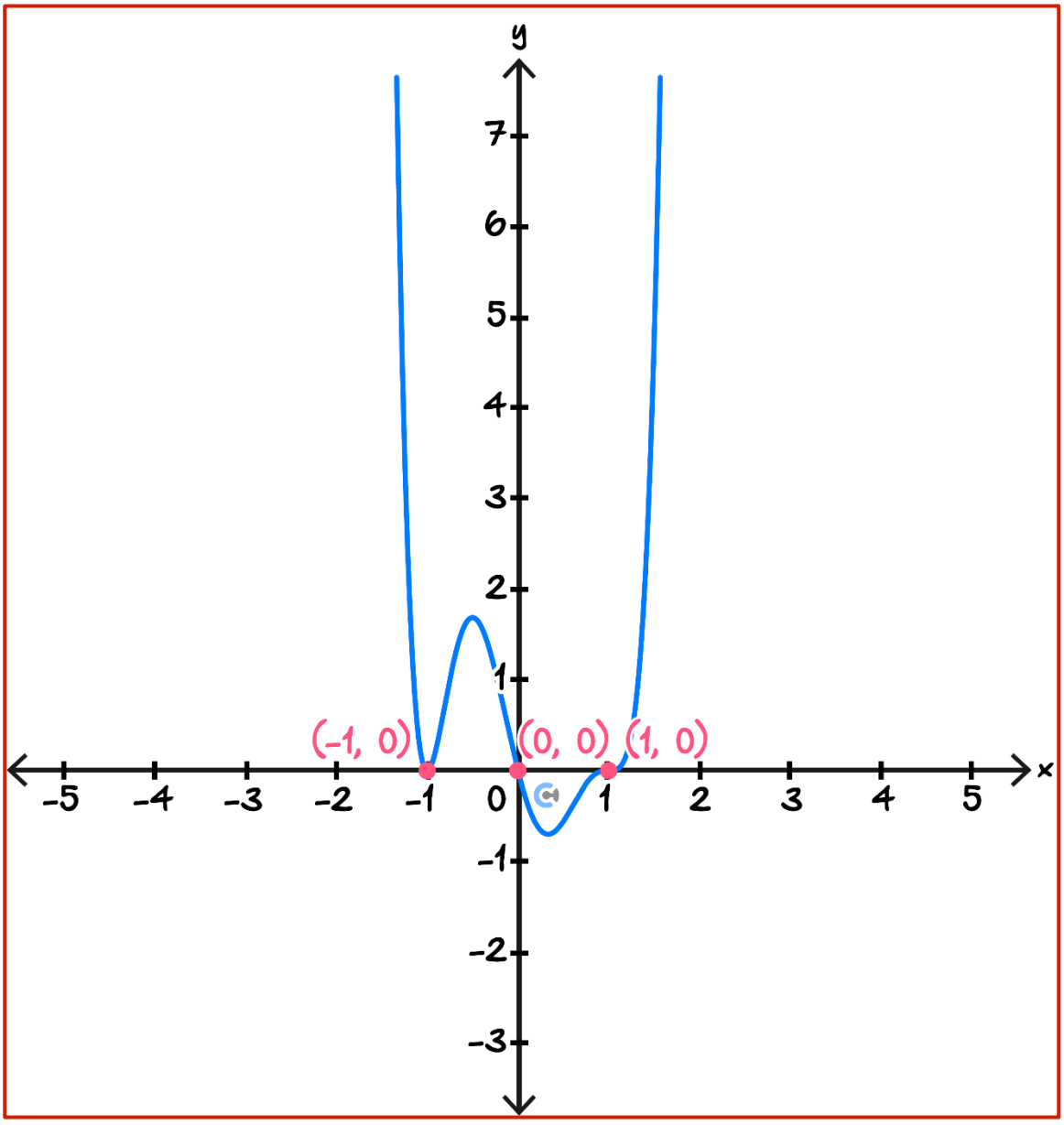
c. $y = 4x^6 - 4x^5 - 8x^4 + 8x^3 + 4x^2 - 4x$

HINT: $(x-1)^3$ is a factor.



Solution Pending

Sub-Section: Final Boss

Question 14

Consider the polynomial $f(x) = ax^3 - bx^2 + x + 2$, where a and b are integers. It is known that $(x - 1)$ is a factor of $f(x)$ and that there is a remainder of -6 when $f(x)$ is divided by $(x + 1)$.

a. Show that $a = 2$ and $b = 5$.

By the remainder theorem we must have that $f(1) = 0$ and $f(-1) = -6$, therefore,

$$\begin{aligned} a - b + 3 &= 0 \\ -a - b + 1 &= -6 \\ \implies -2b + 4 &= -6 \\ \implies b &= 5 \\ \implies a &= 2 \end{aligned}$$

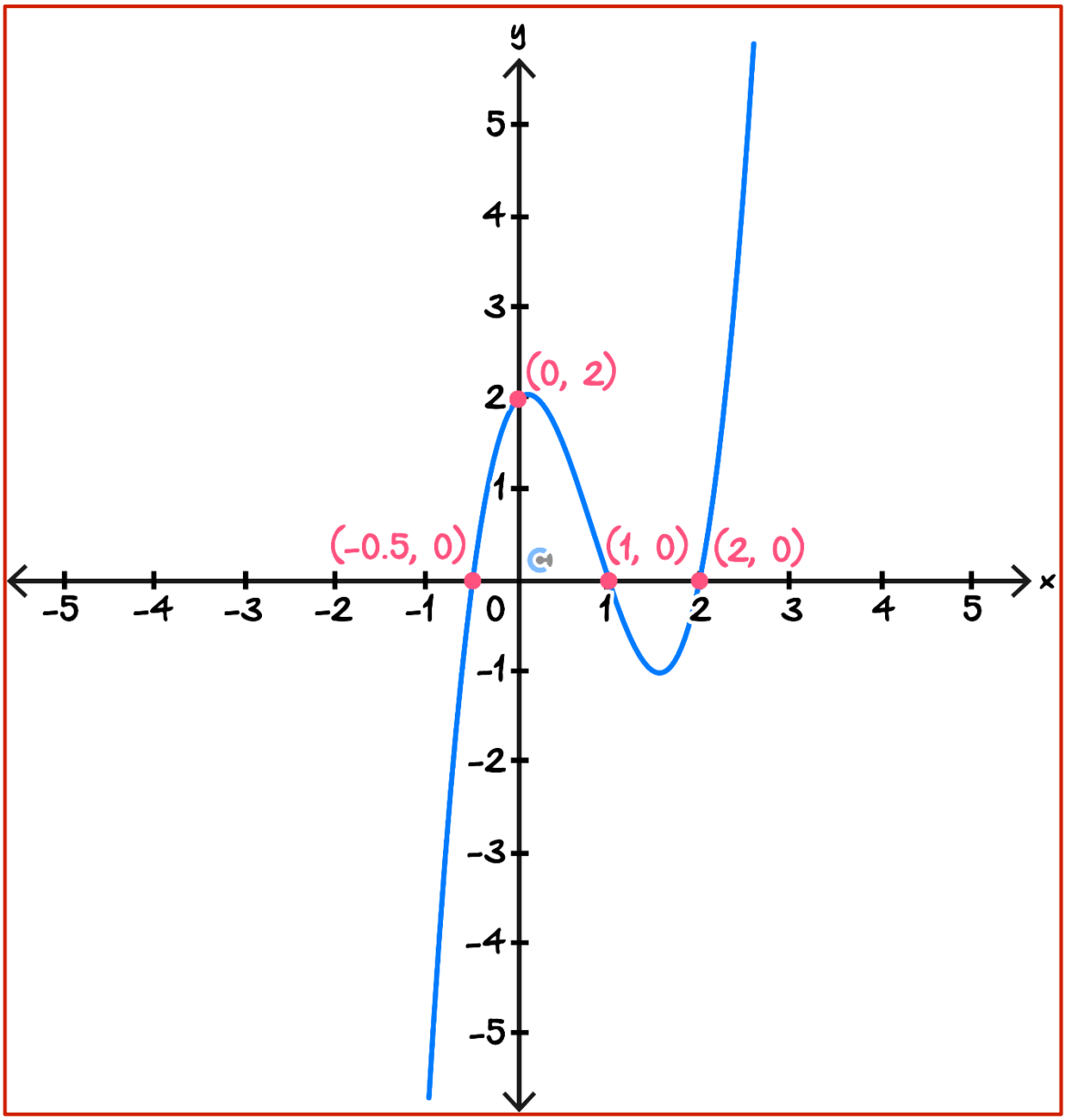
b. Solve the equation $f(x) = 0$.

We must solve $f(x) = 2x^3 - 5x^2 + x + 2 = 0$. Factorise $f(x)$.

$$\begin{aligned} f(x) &= (x - 1)(2x^2 - 3x - 2) \\ &= (x - 1)(2x + 1)(x - 2) \end{aligned}$$

Therefore, $f(x) = 0 \implies x = -\frac{1}{2}, 1, 2$

c. Sketch the graph of $y = f(x)$ on the axes below. Label all axes intercepts with coordinates.



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Section B: Supplementary Questions

Sub-Section [1.5.1]: Identify the Properties of Polynomials and Solve Long Division

Question 15

Consider the polynomial $f(x) = 3x^2 - 4x^4 + 1 - 2x$.

a. State the degree of $f(x)$.

4

b. State the leading coefficient of $f(x)$.

-4

c. State the constant term of $f(x)$.

1

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Question 16

Simplify the following using polynomial long division:

$$\frac{x^3 + 2x^2 - 5x - 6}{x - 2}$$

$x^2 + 4x + 3$

Question 17

The polynomial $P(x) = x^4 - 2x^2 - 5x + 3$ can be written in the form $P(x) = Q(x)(x - 2) + r$, where $r \in R$ and $Q(x)$ is a real valued polynomial. Find $Q(x)$ and r.

We divide $P(x)$ by $(x - 2)$ to get $Q(x)$. The remainder is r.
Thus $Q(x) = x^3 + 2x^2 + 2x - 1$ and $r = 1$.

Question 18

The polynomial $P(x) = 2x^4 + 3x^3 - 5x + 1$ can be written in the form $P(x) = Q(x)(x^2 - 2x + 3) + R(x)$, where $R(x)$ is a polynomial of degree 1 and $Q(x)$ is a polynomial.

a. State the degree of $Q(x)$.

Since the degree of $R(x)$ is 1, $Q(x)$ needs an x^2 term so that $P(x)$ has an x^4 term.
Similarly $Q(x)$ cannot have an x^n term for $n > 2$.
Hence the degree of $Q(x)$ is 2.

b. Find $Q(x)$ and $R(x)$.

$Q(x) = 2x^2 + 7x + 8$ and $R(x) = -10x - 23$.

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Sub-Section [1.5.2]: Apply Reminder and Factor Theorem to Find Reminders and Factors

Question 19

Find the remainder of the division $\frac{f(x)}{g(x)}$, where:

a. $f(x) = x^3 - 7x + 8$ and $g(x) = x + 3$.

Our remainder is $f(-3) = -27 + 21 + 8 = 2$.

b. $f(x) = 2x^3 - 6x^2 - 2x + 4$ and $g(x) = x - 1$.

Our remainder is $f(1) = 2 - 6 - 2 + 4 = -2$

c. $f(x) = -3x^3 + 8x^2 - 3x + 2$ and $g(x) = 3x + 1$.

Our remainder is $f\left(-\frac{1}{3}\right) = \frac{1}{9} + \frac{8}{9} + 1 + 2 = 4$.

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Question 20

For the polynomial $f(x) = ax^3 + 2x^2 - 3ax + 1$, we get a remainder of 5 when $f(x)$ is divided by $x + 2$. Find the value of a.

We have that $f(-2) = -8a + 8 + 6a + 1 = 9 - 2a = 5 \implies a = 2.$

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Question 21

Consider the expression:

$$f(x) = 2x^3 - ax^2 + b$$

Where a and b are non-zero constants.
It is known that $x + 1$ is a factor of $f(x)$ and that the remainder when $f(x)$ is divided by $x - 2$ is 3. Find the values of a and b.

Since $x + 1$ is a factor of $f(x)$ we know that $f(-1) = -2 - a + b = 0$.
Since remainder of $\frac{f(x)}{x-2}$ is 3, we know that $f(2) = 16 - 4a + b = 3$. We get the following pair of simultaneous equations.

$$-2 - a + b = 0$$
$$16 - 4a + b = 3$$

Solving these equations yields $a = 5$ and $b = 7$.

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Question 22

Find a cubic polynomial $f(x)$ which has the following properties:

- $f(x)$ has a leading coefficient of -2.
- $f(x)$ divided by $x^2 - 1$ leaves a remainder of 1.
- $x - 3$ is a factor of $f(x)$.

$f(x)$ will be of the form $f(x) = -2x^3 + ax^2 + bx + c$.
Since when $f(x)$ is divided by $x^2 - 1$ it leaves a remainder of 1 we know that $f(1) = f(-1) = 1$. We also know that $f(3) = 0$.
Thus we have the following equations.

$$\begin{aligned} -2 + a + b + c &= 1 \\ 2 + a - b + c &= 1 \\ -54 + 9a + 3b + c &= 0 \end{aligned}$$

By subtracting the first equation from the second we get $4 - 2b = 0 \implies b = 2$.
We thus know that $a + c = 1 \implies c = 1 - a$ and $-48 + 9a + c = 0$. From here we see that $-47 + 8a = 0 \implies a = \frac{47}{8}$.
Thus $c = 1 - \frac{47}{8} = -\frac{39}{8}$.
Hence our polynomial is $-2x^3 + \frac{47}{8}x^2 + 2x - \frac{39}{8}$

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Sub-Section [1.5.3]: Find Factored Form of Polynomials

Question 23

Factorise $x^3 - 2x^2 - x + 2$ as a product of three linear factors.

$(x-1)(x+1)(x-2)$

Question 24

Factorise $x^3 - 6x^2 + 3x + 10$ as a product of three linear factors.

$(x-5)(x+1)(x-2)$

Question 25

Factorise $2x^3 + \frac{25x^2}{3} + x - \frac{4}{3}$ as a product of three linear factors.

$$\frac{1}{3}(2x+1)(3x-1)(x+4)$$

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Question 26

Use the fact that $x^n - 1 = (1 + x + x^2 + \cdots + x^{n-1})(x - 1)$ to factorise $1 + x^2 + x^4 + x^6 + x^8$ as a product of two-degree four polynomials.

By our hint we know that $(1 + x^2 + x^4 + x^6 + x^8)(x^2 - 1) = x^{10} - 1$.
We also know that $x^{10} - 1 = (x^5 - 1)(x^5 + 1)$.
Again applying our fact we see that $x^5 - 1 = (x - 1)(1 + x + x^2 + x^3 + x^4)$ and that $x^5 + 1 = -((-x)^5 - 1) = -(-x - 1)(1 - x + x^2 - x^3 + x^4) = (x + 1)(1 - x + x^2 - x^3 + x^4)$.
Hence,

$$\begin{aligned} x^{10} - 1 &= (x - 1)(1 + x + x^2 + x^3 + x^4)(x + 1)(1 - x + x^2 - x^3 + x^4) \\ &= (x^2 - 1)(1 + x + x^2 + x^3 + x^4)(1 - x + x^2 - x^3 + x^4) \end{aligned}$$

Dividing both sides by $x^2 - 1$ we get that,

$$1 + x^2 + x^4 + x^6 + x^8 = (1 + x + x^2 + x^3 + x^4)(1 - x + x^2 - x^3 + x^4).$$

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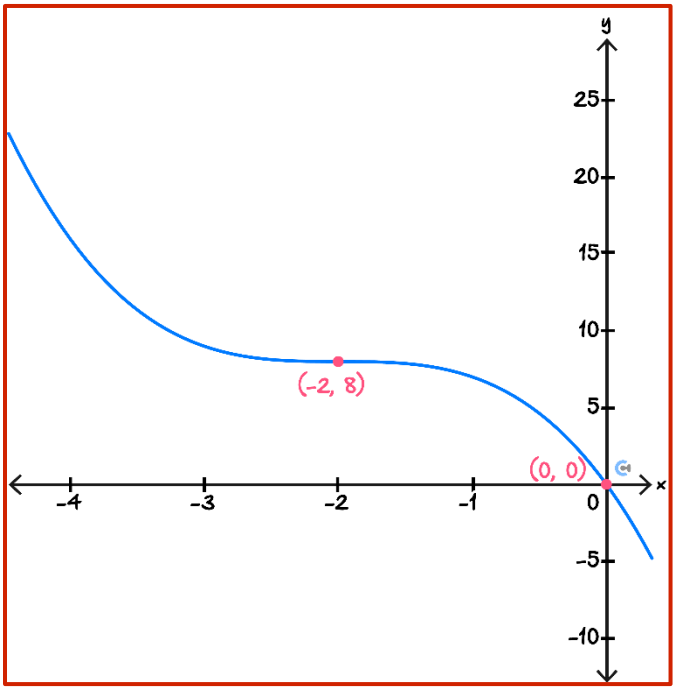
Sub-Section [1.5.4]: Graph Factored and Unfactored Polynomials

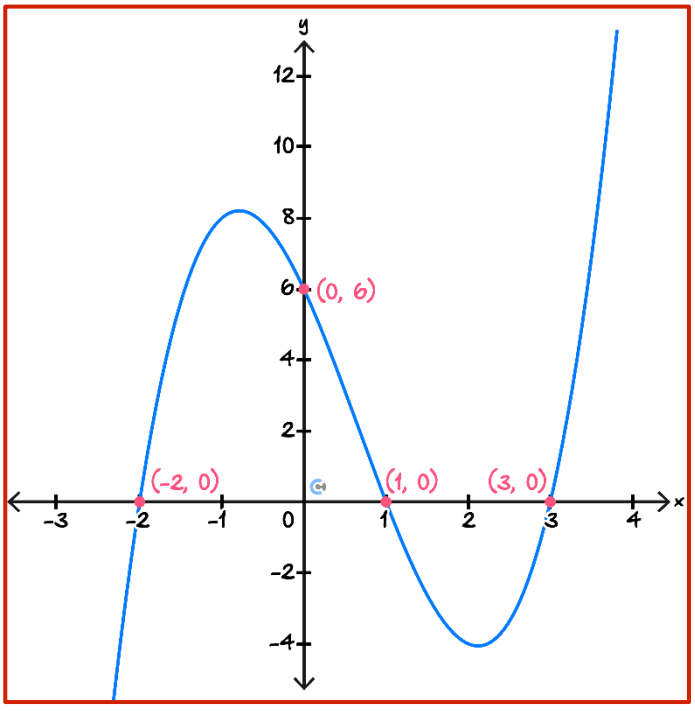
Question 27

Sketch the graphs of each of the functions on the axes provided.

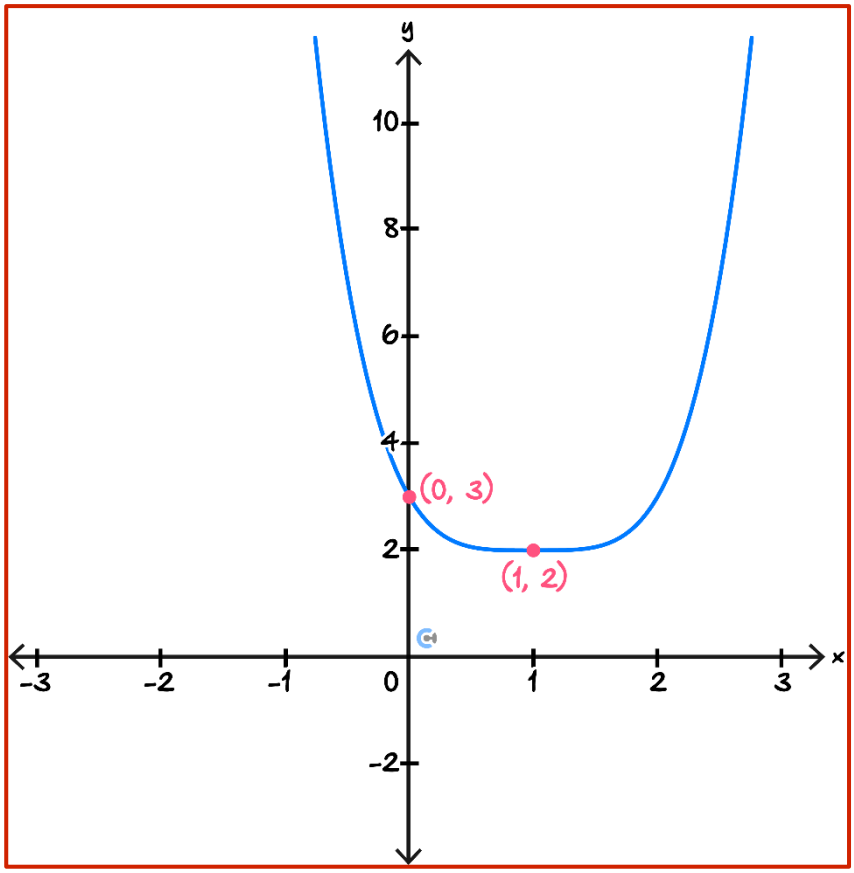
a. $y = 8 - (x + 2)^3$



b. $y = (x - 1)(x + 2)(x - 3)$



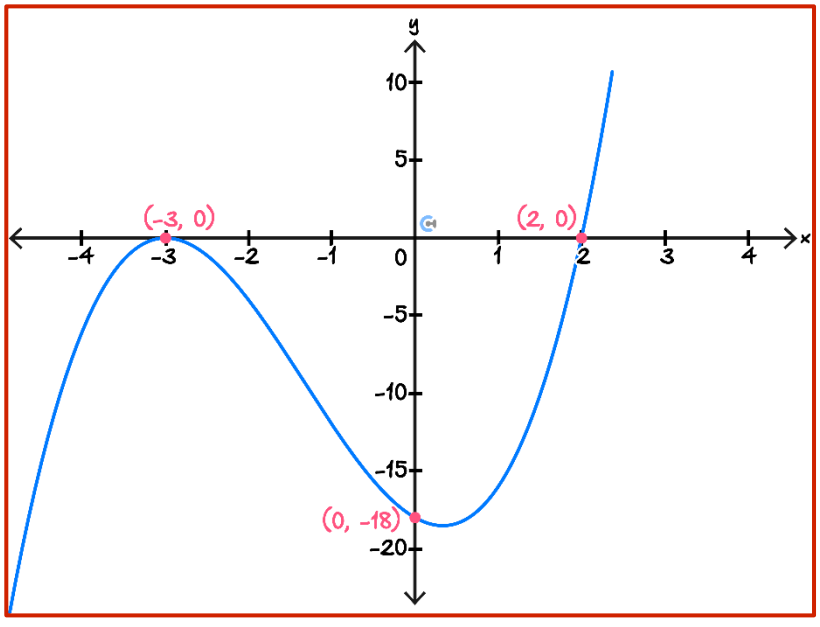
c. $y = 2 + (x - 1)^4$



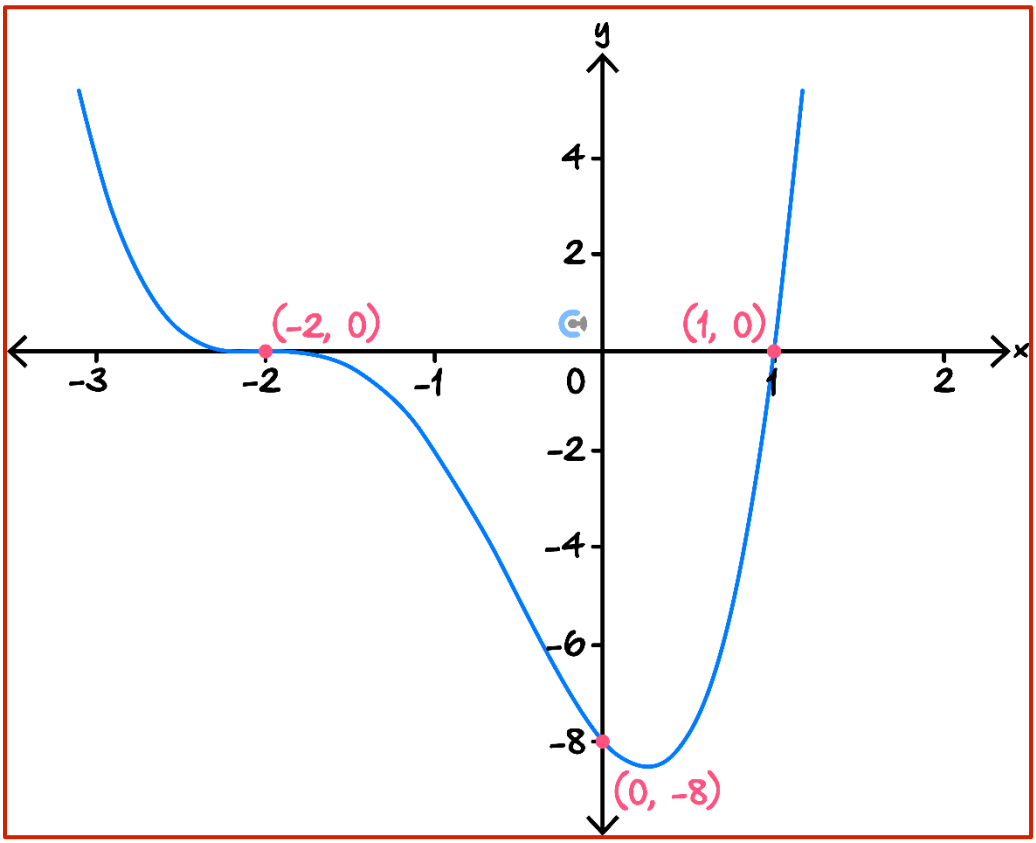
Question 28

Sketch the graphs of each of the functions on the axes provided.

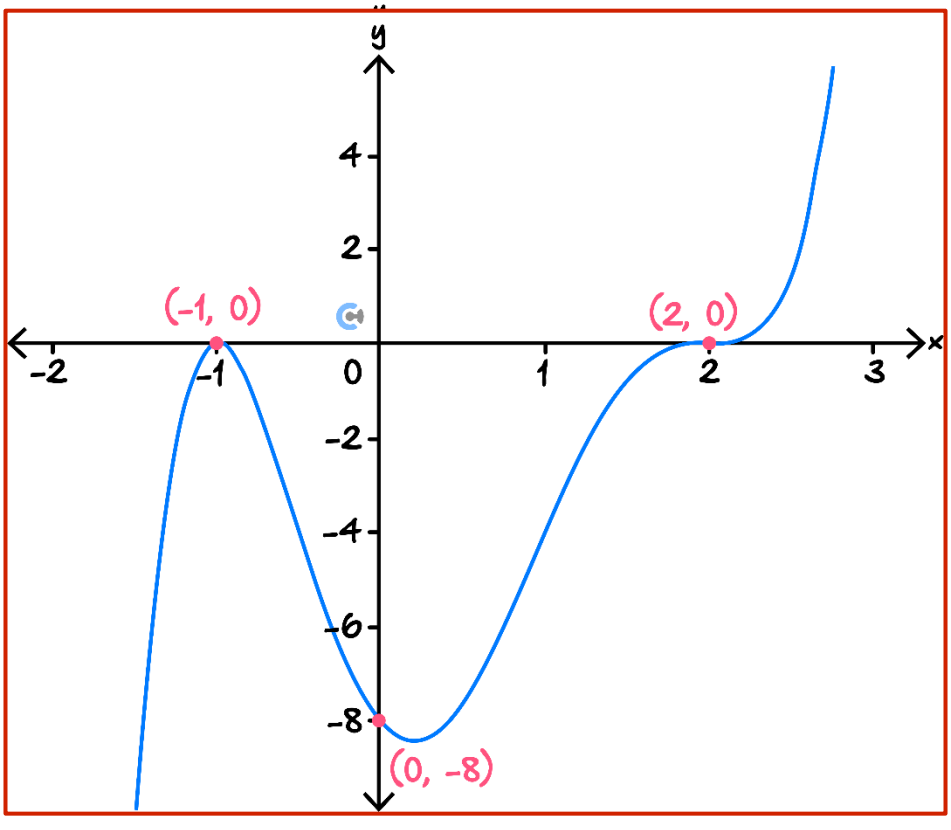
a. $y = (x + 3)^2(x - 2)$



b. $y = (x-1)(x+2)^3$



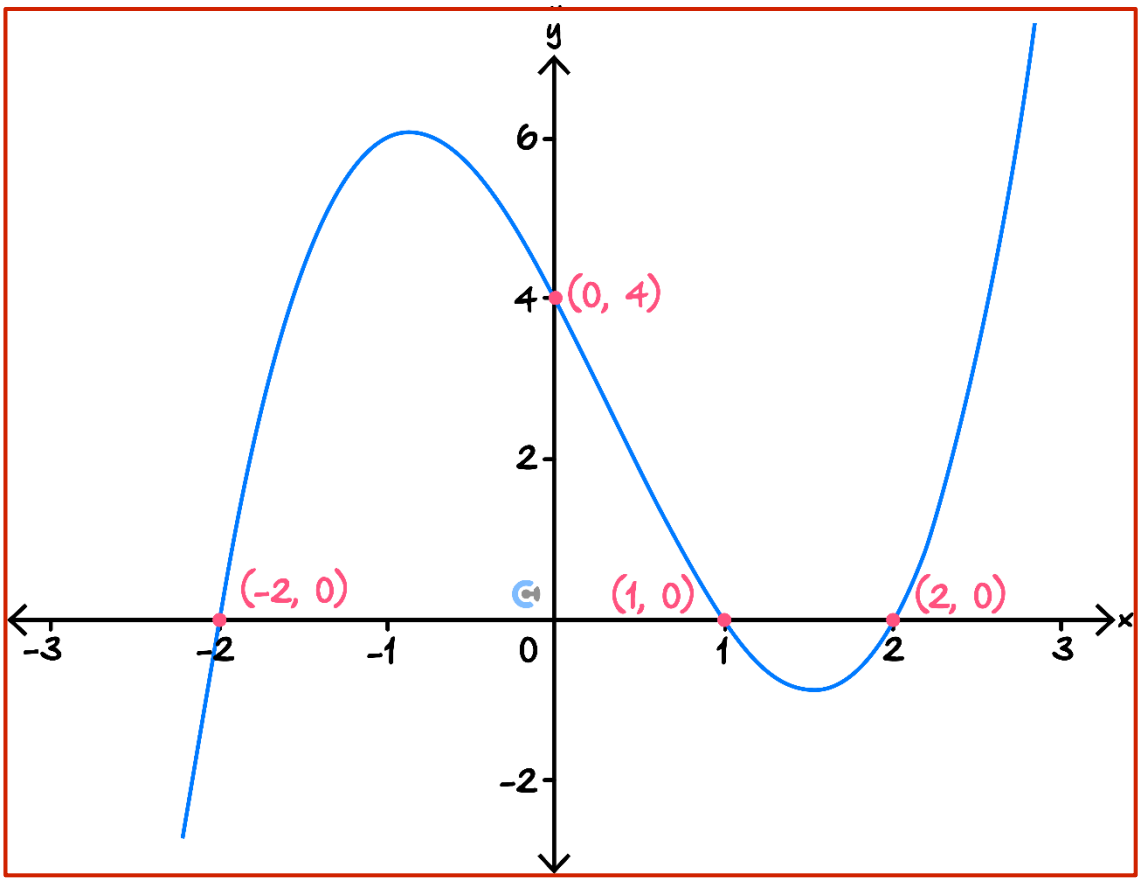
c. $y = (x+1)^2(x-3)^3$



Question 29

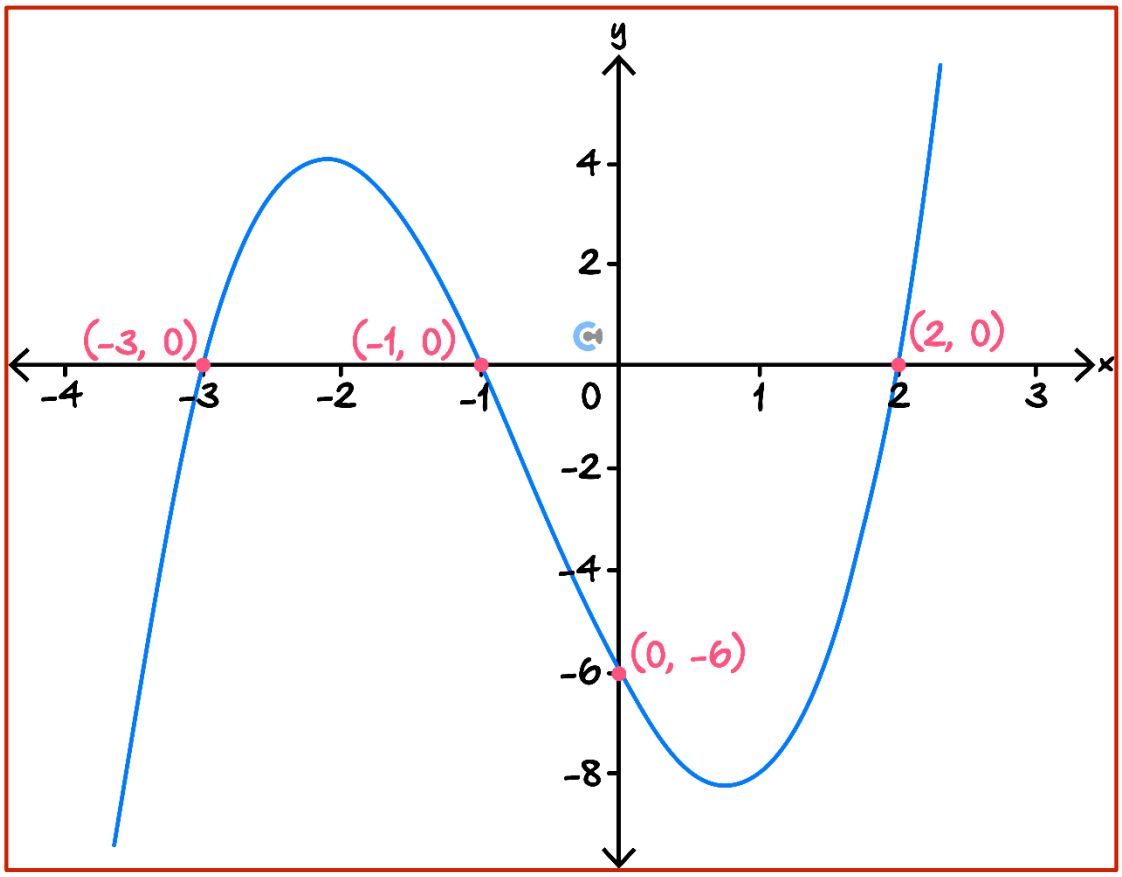
Factorise and hence, sketch the graphs of each of the functions on the axes provided.

a. $y = x^3 - x^2 - 4x + 4$



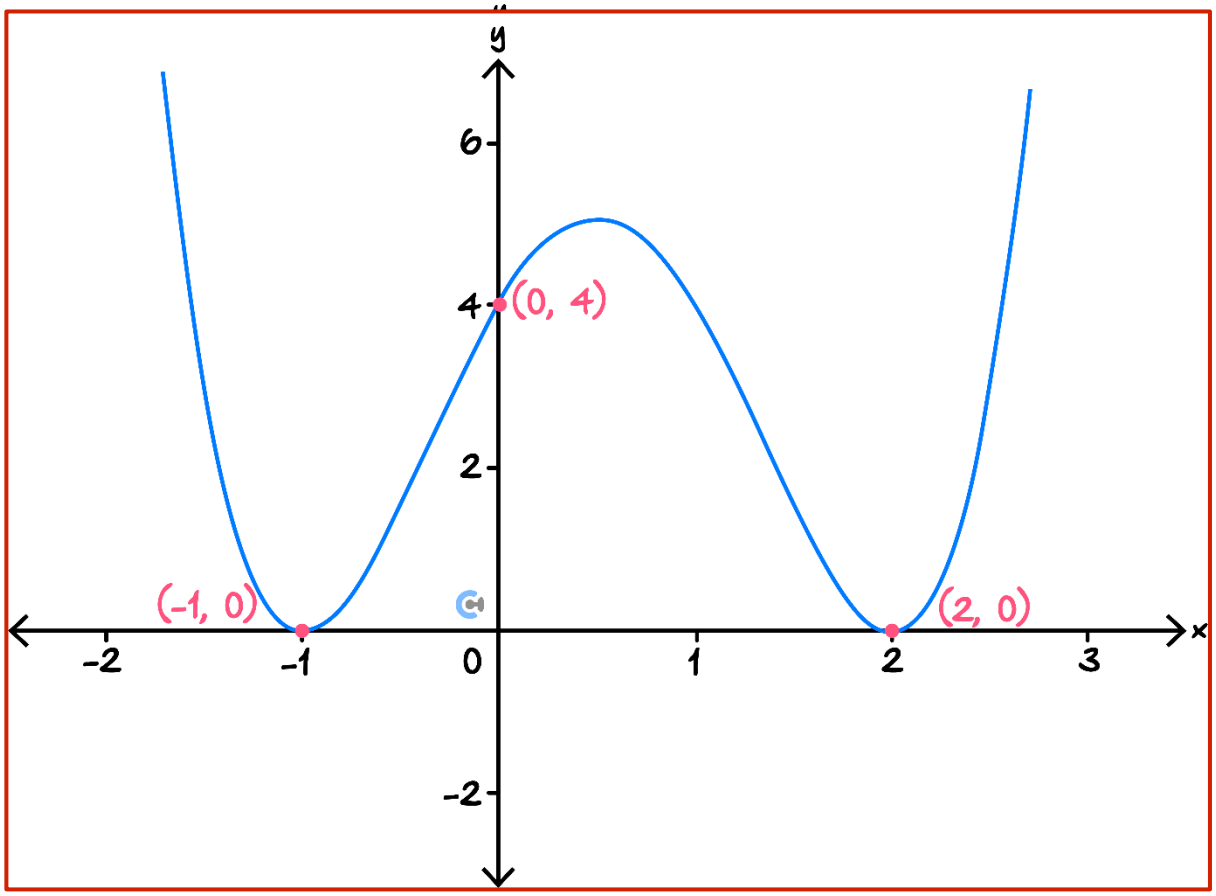
Solution Pending

b. $y = x^3 + 2x^2 - 5x - 6$



Solution Pending

c. $y = x^4 - 2x^3 - 3x^2 + 4x + 4$



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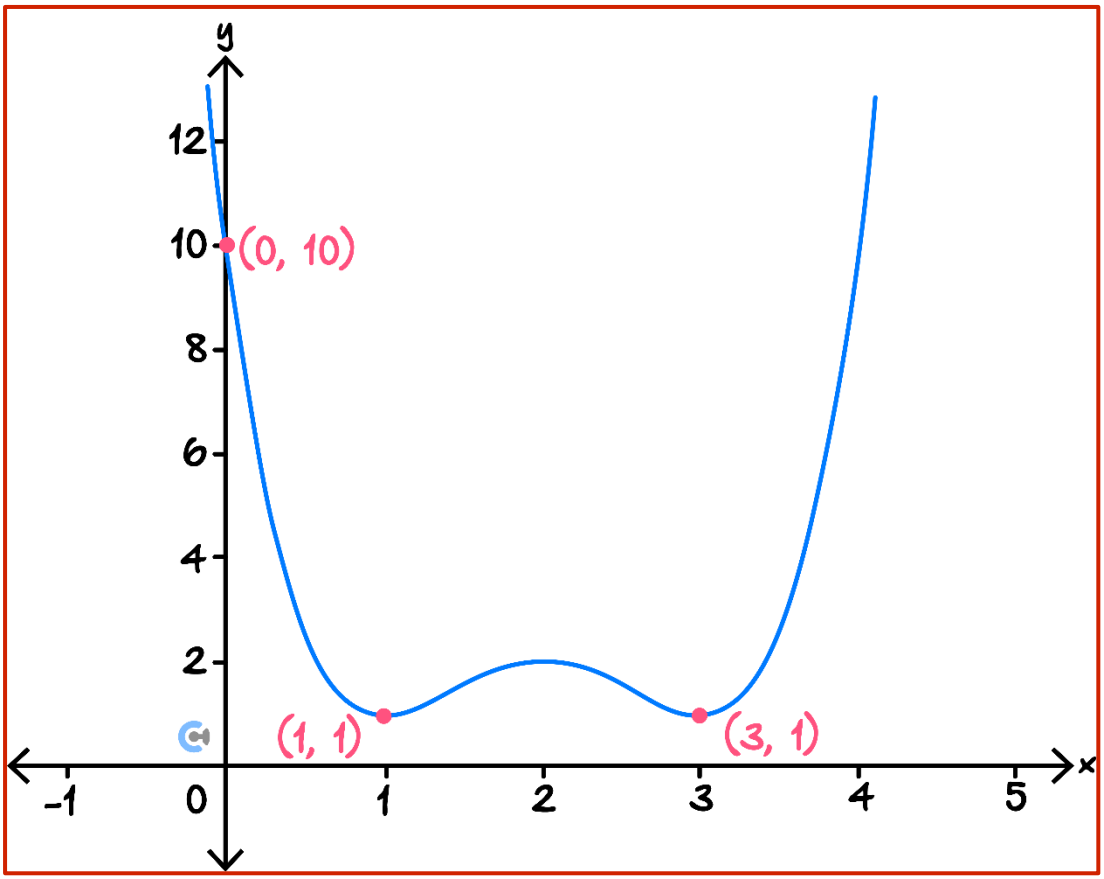
Question 30

Sketch the graph of $y = x^4 - 8x^3 + 22x^2 - 24x + 10$ on the axis below.

Hint: Factorise $x^4 - 8x^3 + 22x^2 - 24x + 9$ instead.



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