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VCE Mathematical Methods ½
Polynomials [1.5]
Homework

Homework Outline:

Compulsory Questions	Pg 2 – Pg 18
Supplementary Questions	Pg 19 – Pg 35

Section A: Compulsory Questions

Sub-Section [1.5.1]: Identify the Properties of Polynomials and Solve Long Division

Question 1

Consider the polynomial $f(x) = 4x^3 - 2x^2 - x + 5$.

a. State the degree of $f(x)$.

_____

b. State the leading coefficient of $f(x)$.

_____

c. State the constant term of $f(x)$.

_____

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Question 2

Simplify the following using polynomial long division:

$$\frac{x^3 - 10x^2 + 8x + 3}{x - 2}$$

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Question 3

Simplify the following using polynomial long division:

$$\frac{2x^4 - 8x^2 + 6x}{x + 3}$$

Question 4 Tech-Active.

Simplify the following using polynomial long division:

$$\frac{2x^4 - 3x^3 - 8x^2 + 4x - 7}{x + 2}$$

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Sub-Section [1.5.2]: Apply Reminder and Factor Theorem to Find Reminders and Factors

Question 5

Find the remainder of the division, $\frac{f(x)}{g(x)}$, where:

a. $f(x) = x^3 - 3x^2 + 2x + 1$ and $g(x) = x - 2$.

b. $f(x) = 2x^3 - x^2 + 3x + 4$ and $g(x) = x + 1$.

c. $f(x) = -x^3 + 6x^2 + 3x + 2$ and $g(x) = x - 3$.

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Question 6

For the polynomial $f(x) = 3x^3 - 2x^2 + (9 - 2a)x + 2$, we get a remainder of 8 when $f(x)$ is divided by $g(x) = x - 1$. Find the value of a.

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Question 7

Consider the expression:

$$f(x) = x^3 + ax^2 - 2x + b$$

Where a and b are non-zero constants.

It is known that $x - 2$ is a factor of $f(x)$ and that the remainder when $f(x)$ is divided by $x - 3$ is 2. Find the values of a and b.

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Sub-Section [1.5.3]: Find Factored Form of Polynomials

Question 8

Factorise $f(x) = x^3 - 4x^2 + x + 6$ as the product of three linear factors.

Question 9

Factorise $f(x) = x^3 - 4x^2 - 17x + 60$ as the product of three linear factors.

Question 10

Factorise $f(x) = 8x^3 - 56x^2 - 70x + 48$ as the product of three linear factors.

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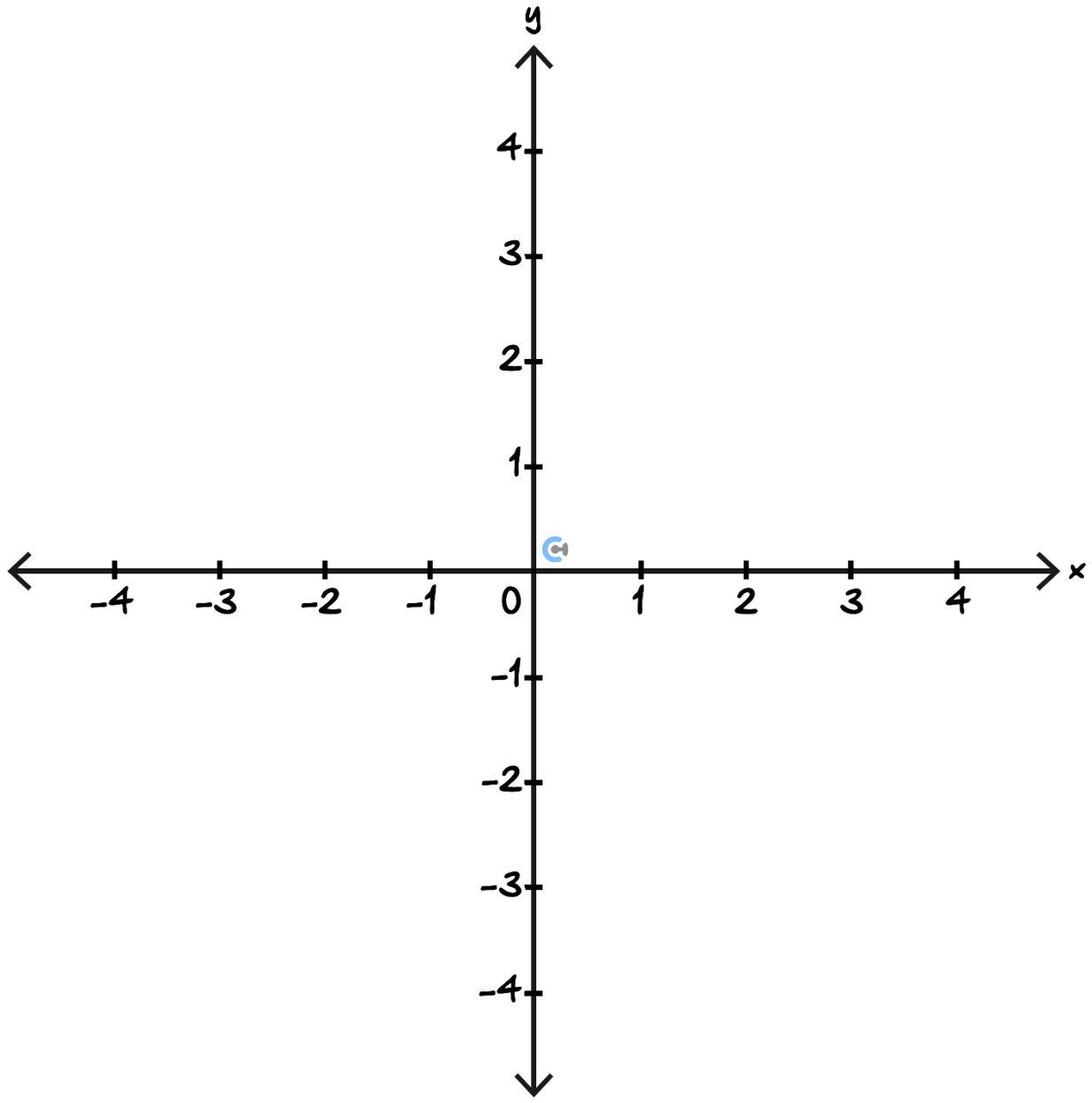
Sub-Section [1.5.4]: Graph Factored and Unfactored Polynomials

Question 11

Sketch the graphs of each of the functions on the axes provided.

a. $y = (x-1)^3 - 1$



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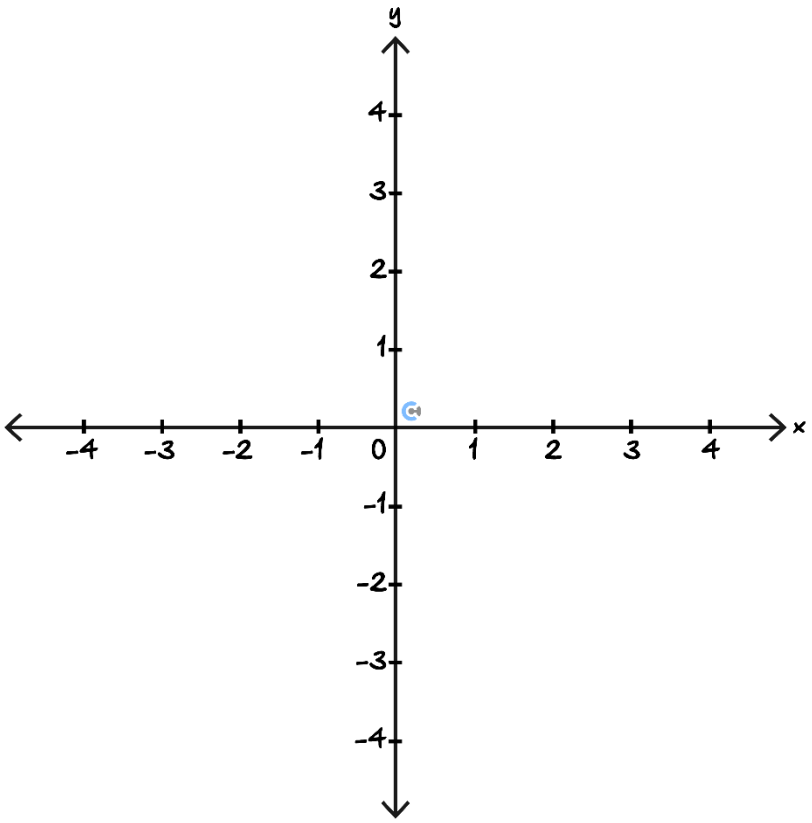
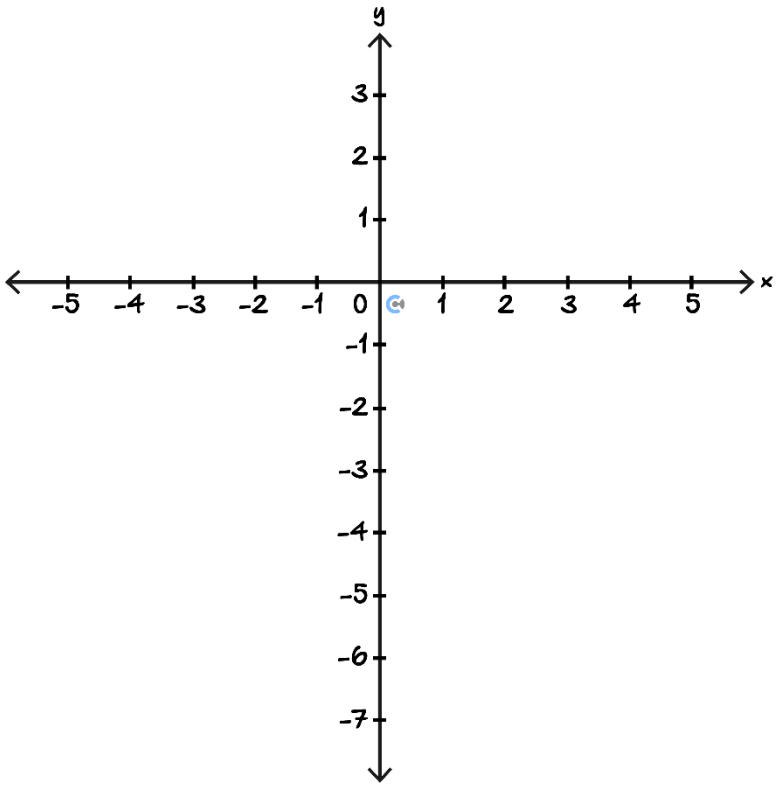
b. $y = (x-1)^4 - 3$



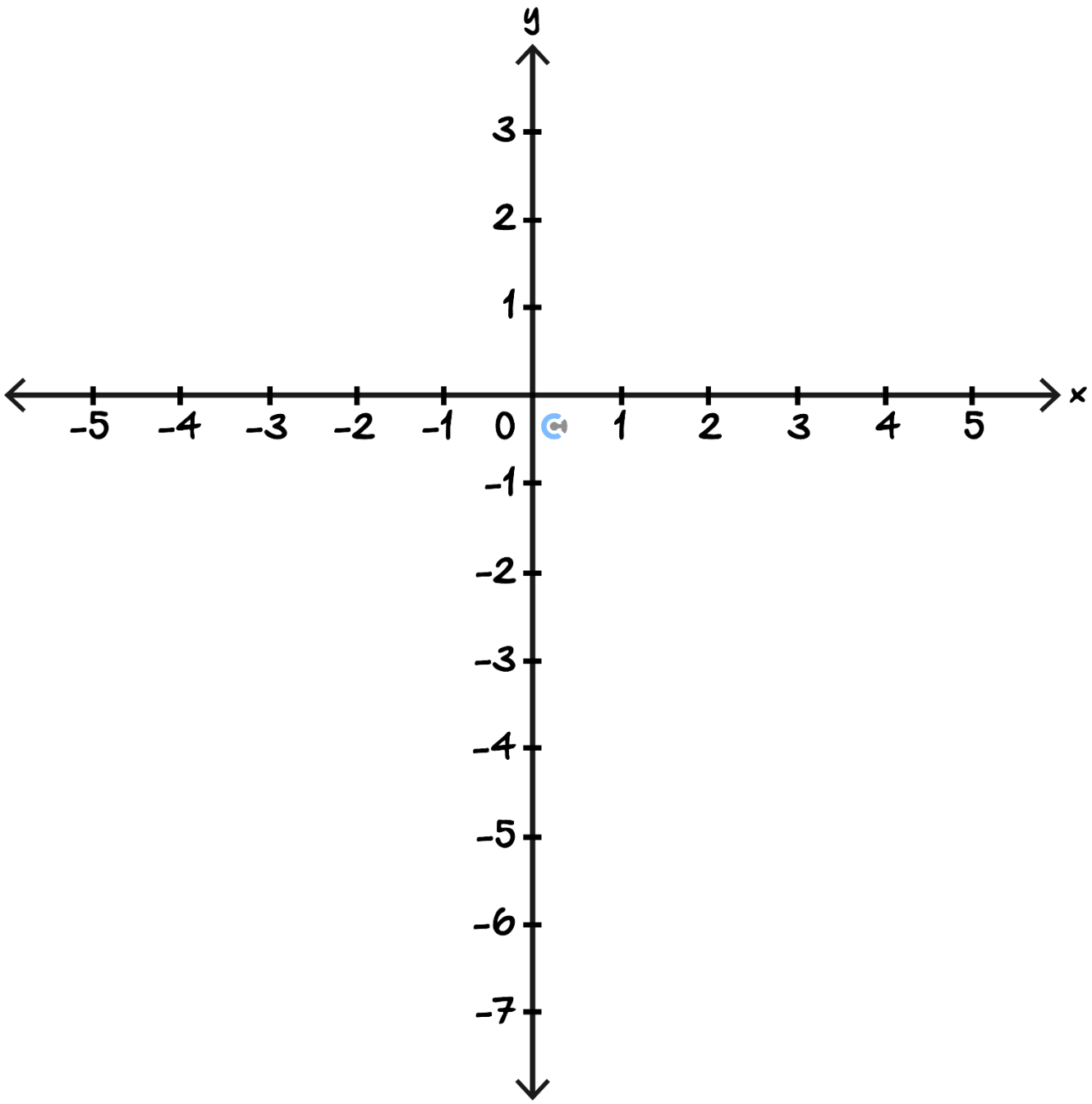
c. $y = -(x+2)(x-1)(x-2)$



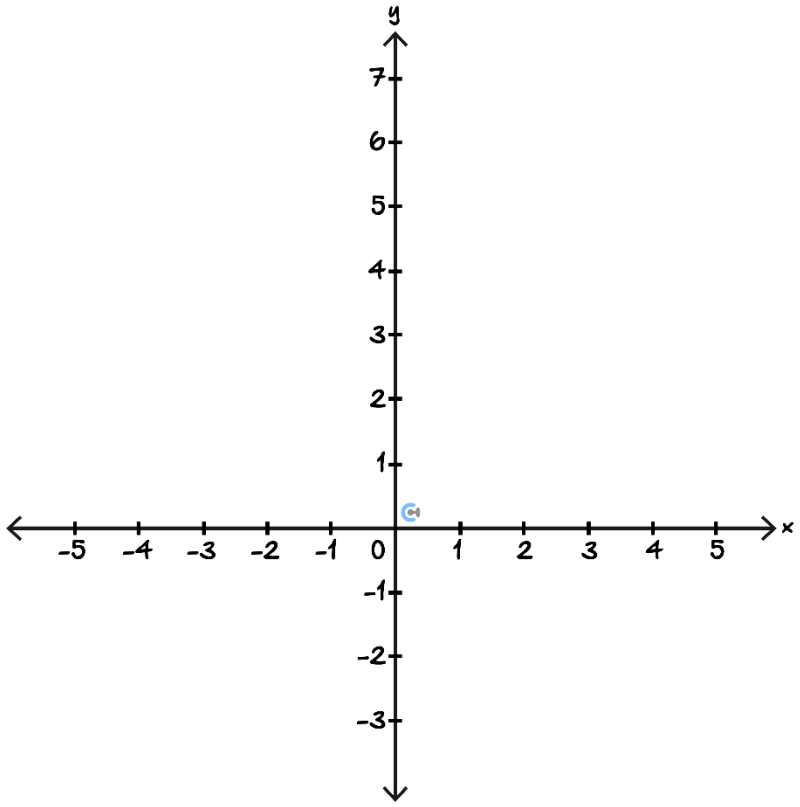
Question 12

Sketch the graphs of each of the functions on the axes provided.

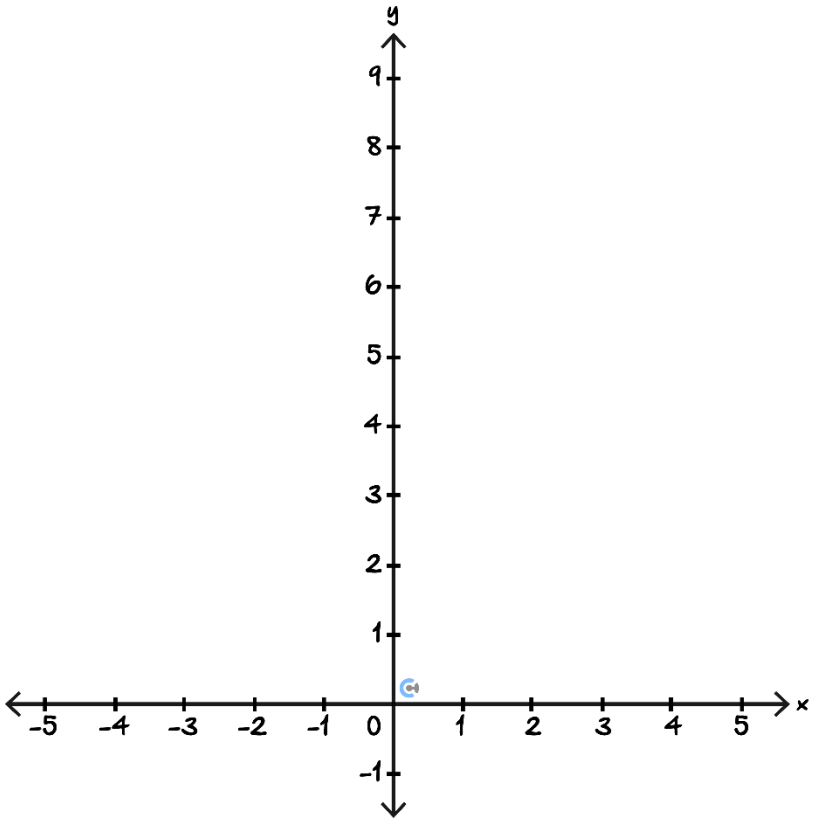
a. $y = (x + 2)^2(x - 1)$



b. $y = (x+1)(x-1)^3$



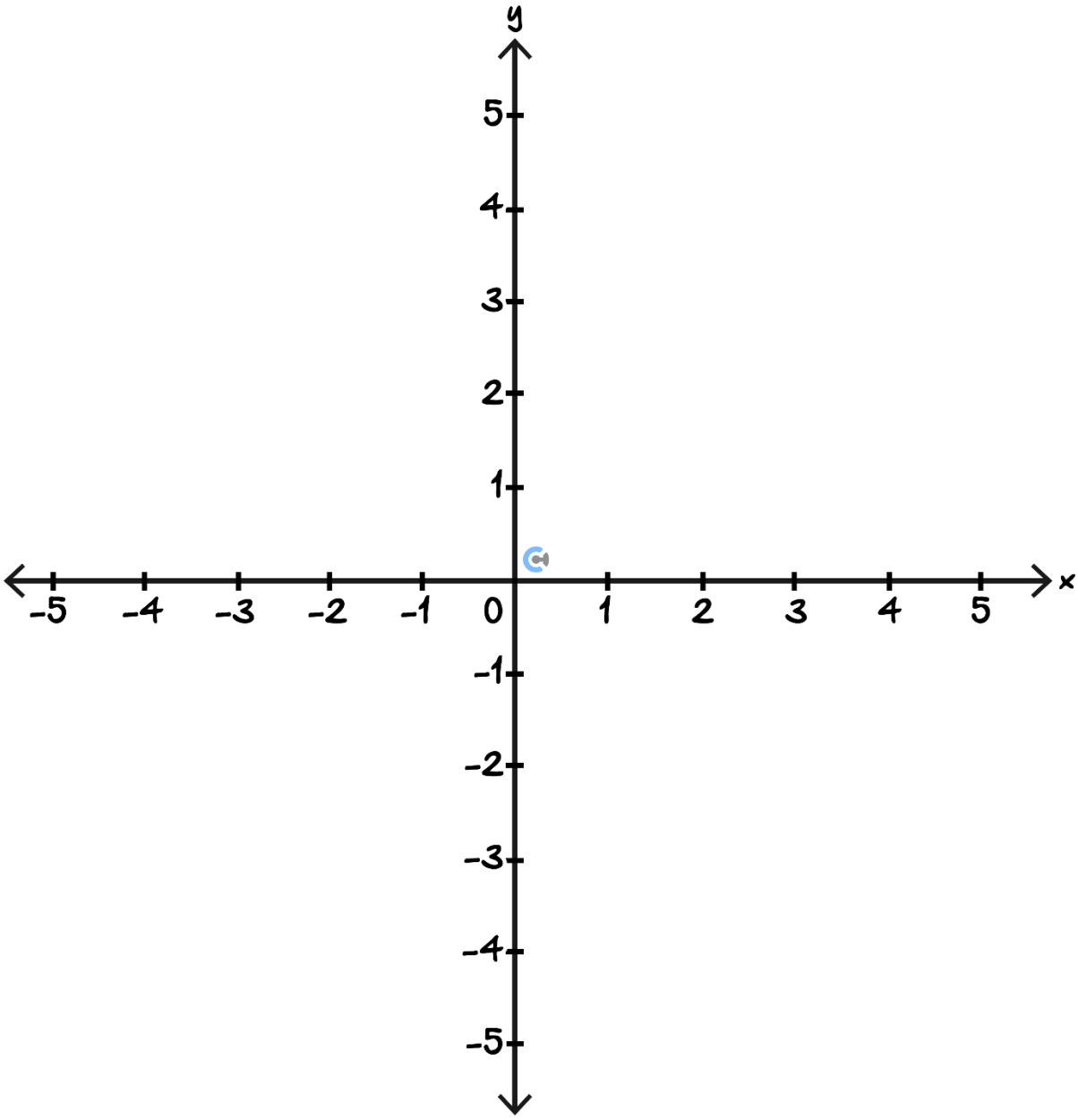
c. $y = (x+1)^2(x-1)^2$



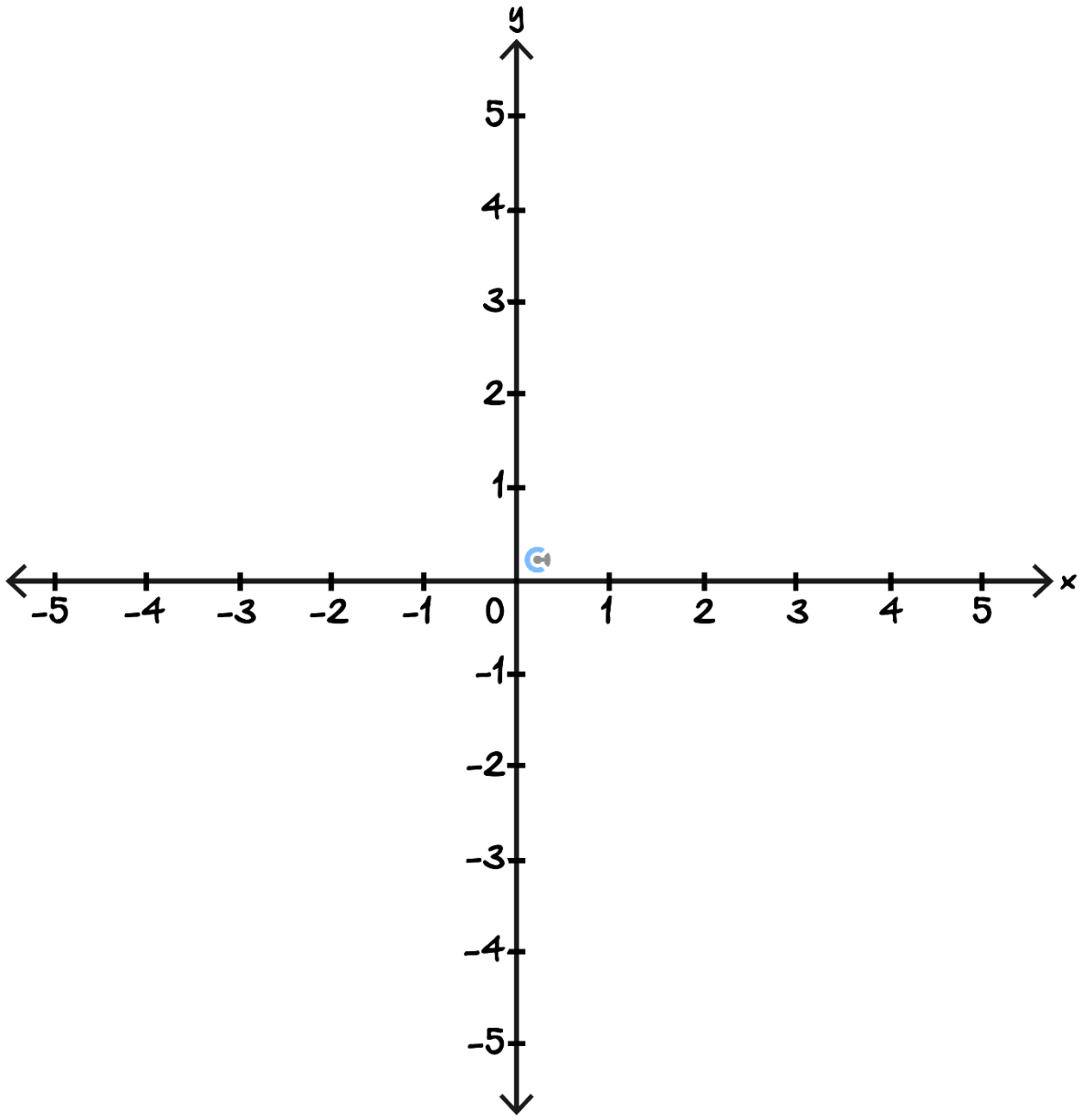
Question 13

Factorise and hence, sketch the graphs of each of the functions on the axes provided.

a. $y = x^3 - x^2 - x + 1$

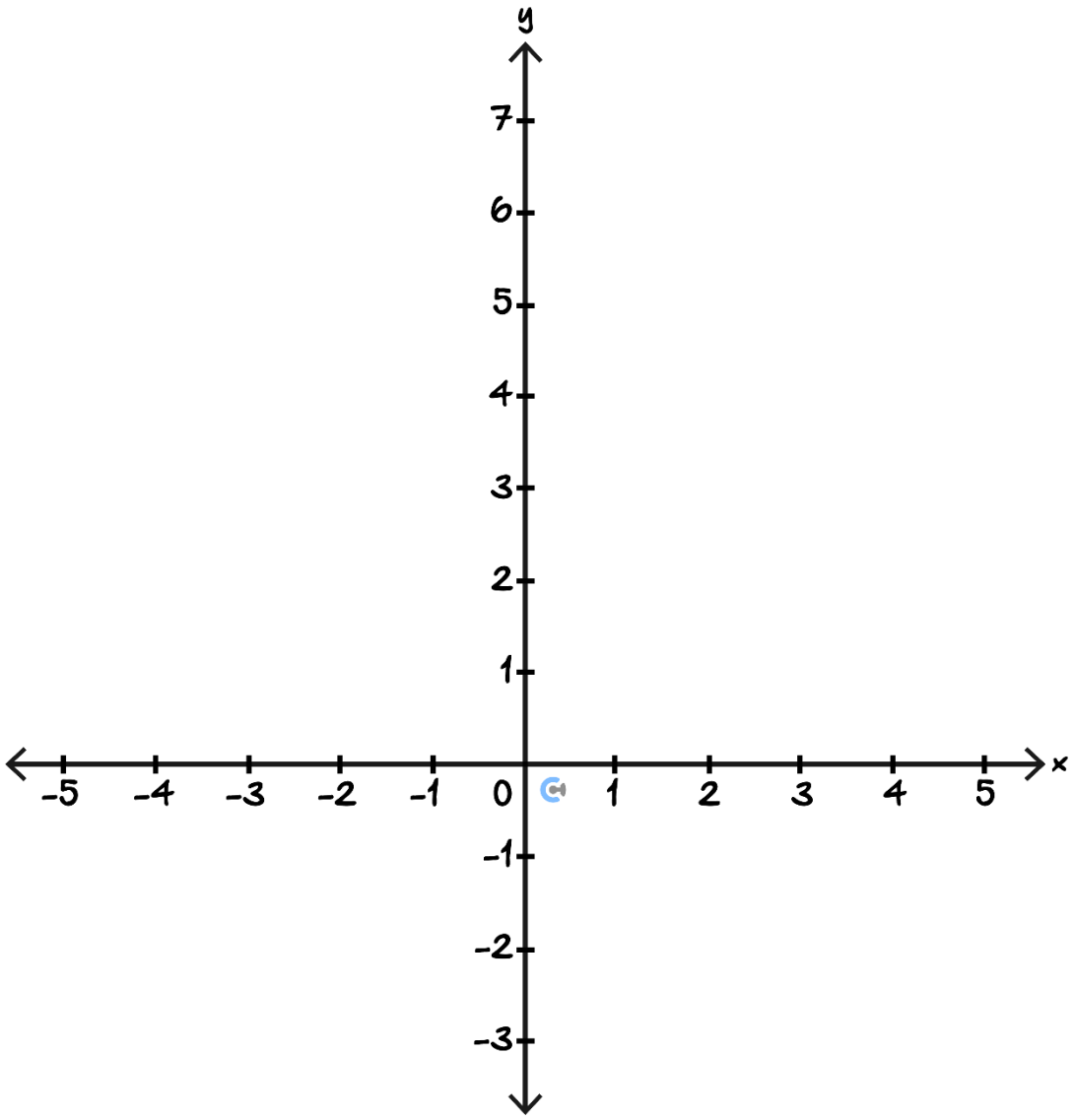


b. $y = x^4 + 3x^3 + x^2 - 3x - 2$



c. $y = 4x^6 - 4x^5 - 8x^4 + 8x^3 + 4x^2 - 4x$

HINT: $(x-1)^3$ is a factor.



Sub-Section: Final Boss

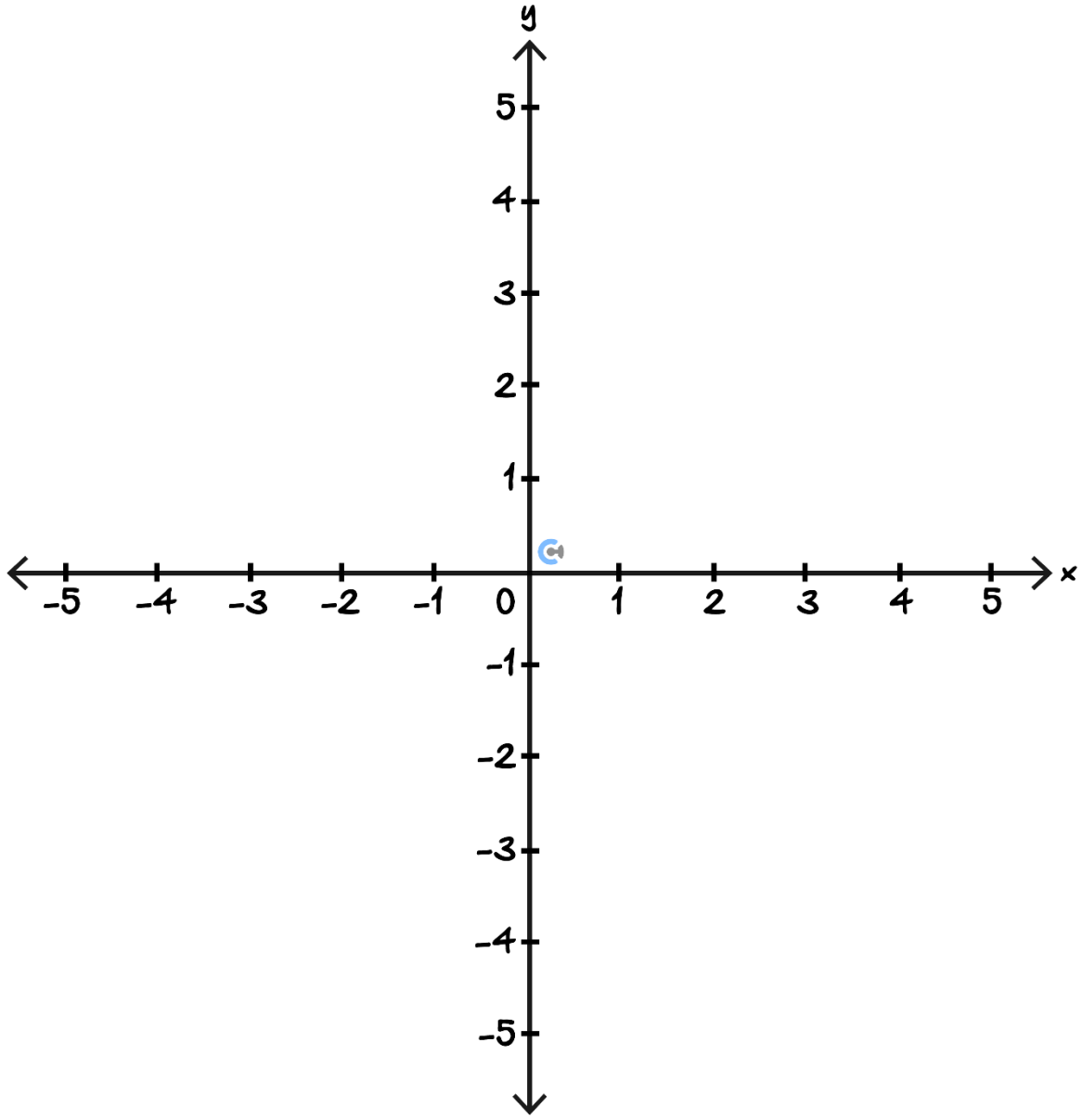
Question 14

Consider the polynomial $f(x) = ax^3 - bx^2 + x + 2$, where a and b are integers. It is known that $(x - 1)$ is a factor of $f(x)$ and that there is a remainder of -6 when $f(x)$ is divided by $(x + 1)$.

a. Show that $a = 2$ and $b = 5$.

b. Solve the equation $f(x) = 0$.

c. Sketch the graph of $y = f(x)$ on the axes below. Label all axes intercepts with coordinates.



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Section B: Supplementary Questions

Sub-Section [1.5.1]: Identify the Properties of Polynomials and Solve Long Division

Question 15

Consider the polynomial $f(x) = 3x^2 - 4x^4 + 1 - 2x$.

a. State the degree of $f(x)$.

_____

b. State the leading coefficient of $f(x)$.

_____

c. State the constant term of $f(x)$.

_____

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Question 16

Simplify the following using polynomial long division:

$$\frac{x^3 + 2x^2 - 5x - 6}{x - 2}$$

Question 17

The polynomial $P(x) = x^4 - 2x^2 - 5x + 3$ can be written in the form $P(x) = Q(x)(x - 2) + r$, where $r \in R$ and $Q(x)$ is a real valued polynomial. Find $Q(x)$ and r.

Question 18

The polynomial $P(x) = 2x^4 + 3x^3 - 5x + 1$ can be written in the form $P(x) = Q(x)(x^2 - 2x + 3) + R(x)$, where $R(x)$ is a polynomial of degree 1 and $Q(x)$ is a polynomial.

a. State the degree of $Q(x)$.

b. Find $Q(x)$ and $R(x)$.

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Sub-Section [1.5.2]: Apply Reminder and Factor Theorem to Find Reminders and Factors

Question 19

Find the remainder of the division $\frac{f(x)}{g(x)}$, where:

a. $f(x) = x^3 - 7x + 8$ and $g(x) = x + 3$.

b. $f(x) = 2x^3 - 6x^2 - 2x + 4$ and $g(x) = x - 1$.

c. $f(x) = -3x^3 + 8x^2 - 3x + 2$ and $g(x) = 3x + 1$.

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Question 20

For the polynomial $f(x) = ax^3 + 2x^2 - 3ax + 1$, we get a remainder of 5 when $f(x)$ is divided by $x + 2$. Find the value of a.

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Question 21

Consider the expression:

$$f(x) = 2x^3 - ax^2 + b$$

Where a and b are non-zero constants.
It is known that $x + 1$ is a factor of $f(x)$ and that the remainder when $f(x)$ is divided by $x - 2$ is 3. Find the values of a and b.

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Question 22

Find a cubic polynomial $f(x)$ which has the following properties:

- $f(x)$ has a leading coefficient of -2.
- $f(x)$ divided by $x^2 - 1$ leaves a remainder of 1.
- $x - 3$ is a factor of $f(x)$.

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Sub-Section [1.5.3]: Find Factored Form of Polynomials

Question 23

Factorise $x^3 - 2x^2 - x + 2$ as a product of three linear factors.

Question 24

Factorise $x^3 - 6x^2 + 3x + 10$ as a product of three linear factors.

Question 25

Factorise $2x^3 + \frac{25x^2}{3} + x - \frac{4}{3}$ as a product of three linear factors.

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Question 26

Use the fact that $x^n - 1 = (1 + x + x^2 + \cdots + x^{n-1})(x - 1)$ to factorise $1 + x^2 + x^4 + x^6 + x^8$ as a product of two-degree four polynomials.

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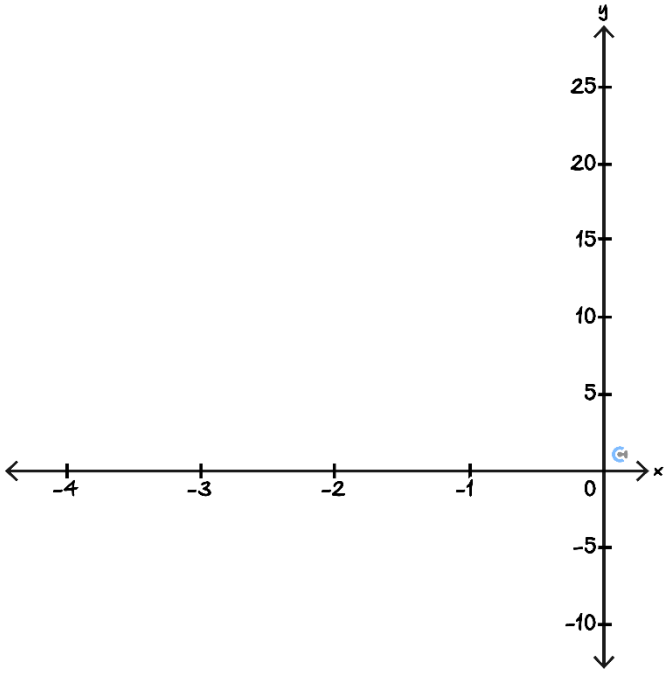
Sub-Section [1.5.4]: Graph Factored and Unfactored Polynomials

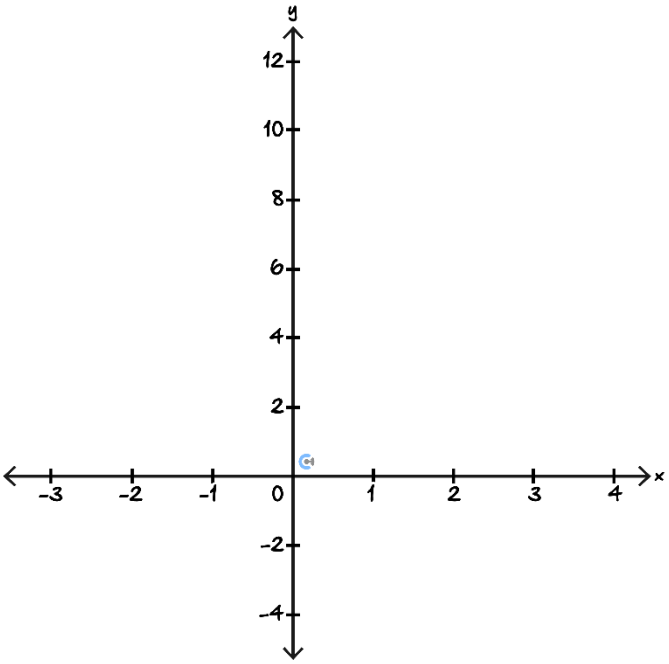
Question 27

Sketch the graphs of each of the functions on the axes provided.

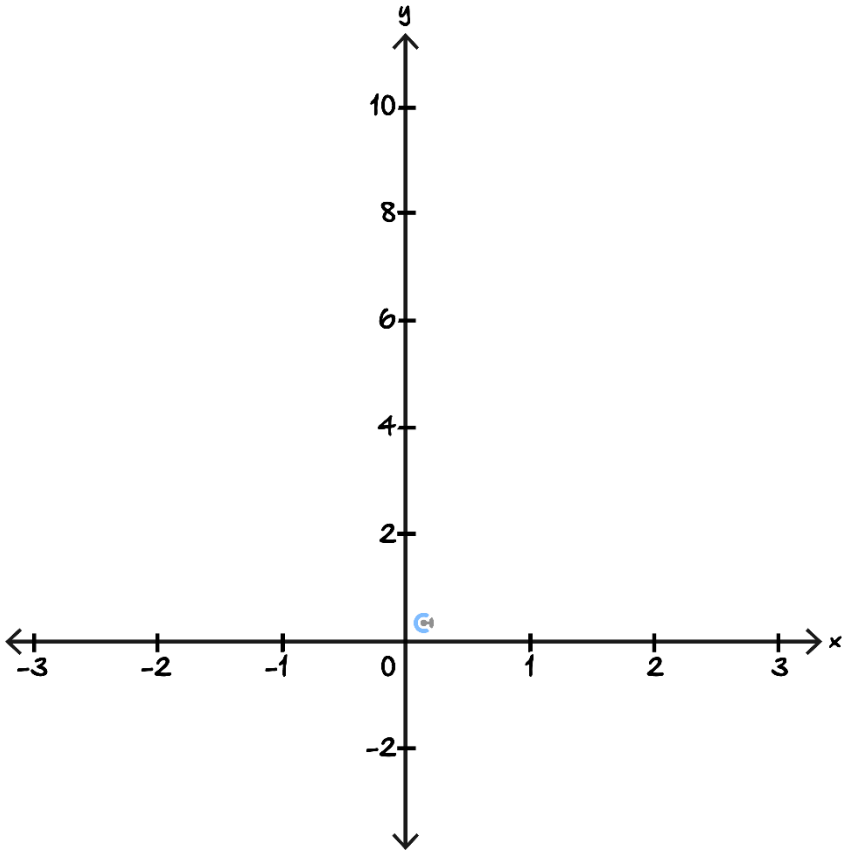
a. $y = 8 - (x + 2)^3$



b. $y = (x - 1)(x + 2)(x - 3)$



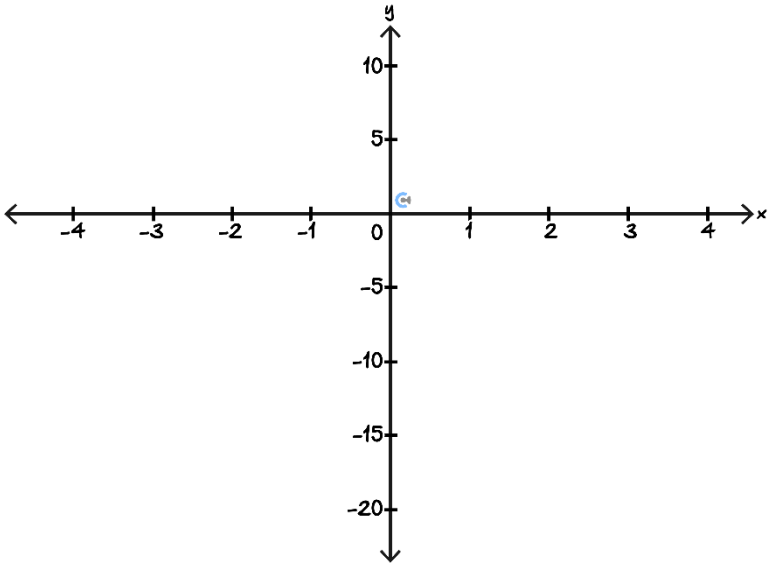
c. $y = 2 + (x - 1)^4$



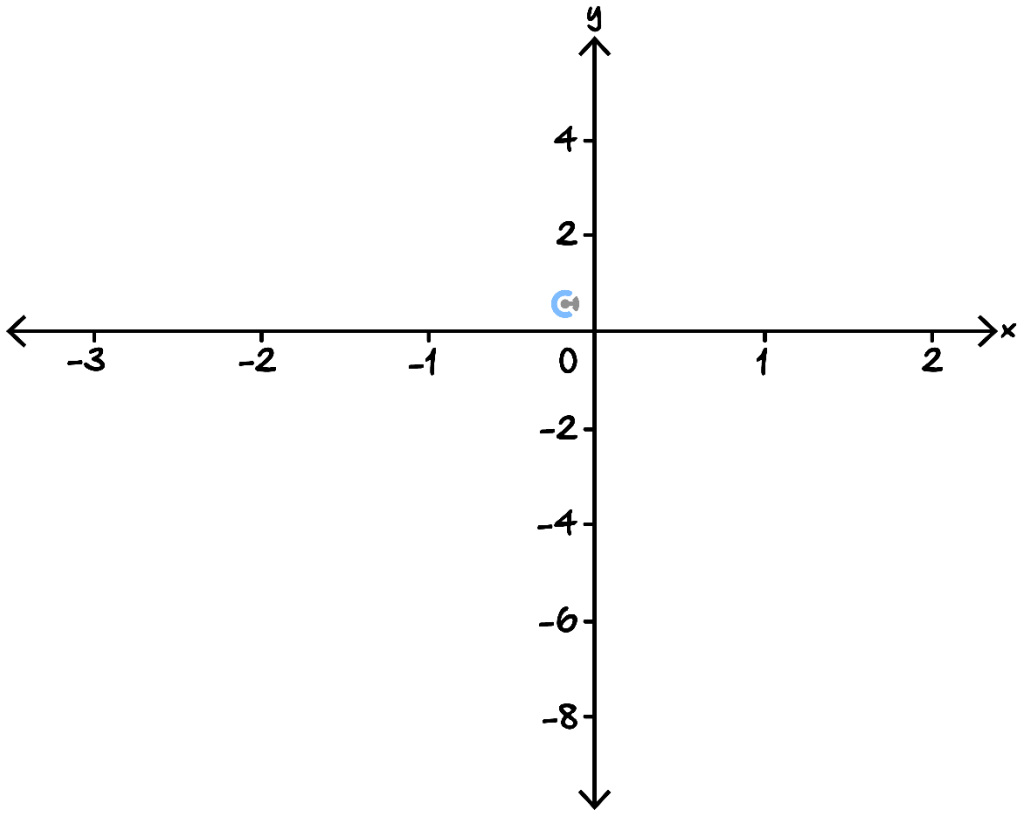
Question 28

Sketch the graphs of each of the functions on the axes provided.

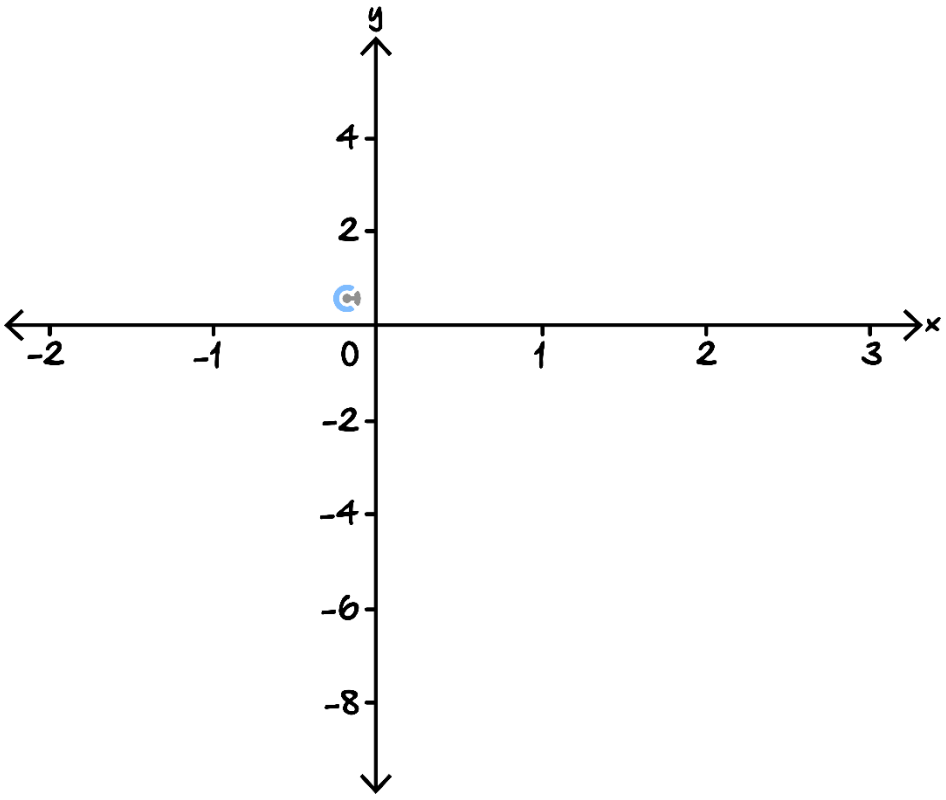
a. $y = (x + 3)^2(x - 2)$



b. $y = (x-1)(x+2)^3$



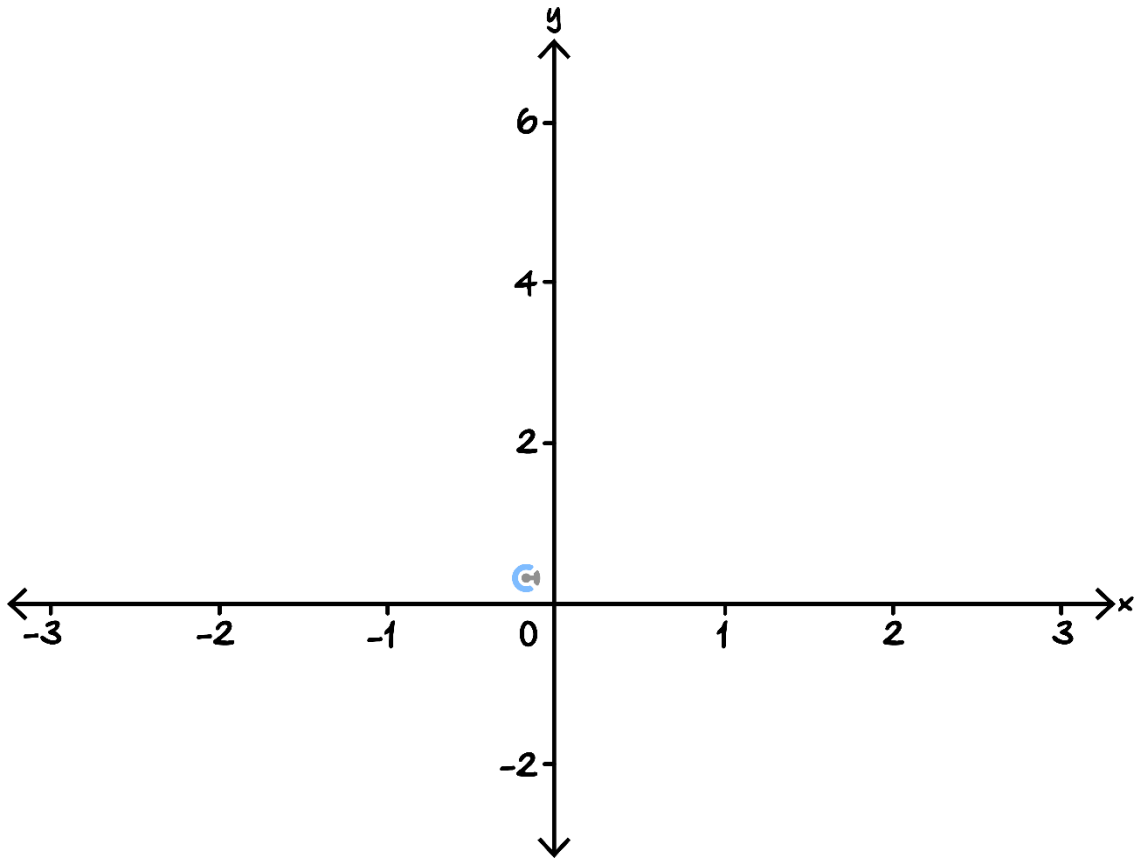
c. $y = (x+1)^2(x-3)^3$



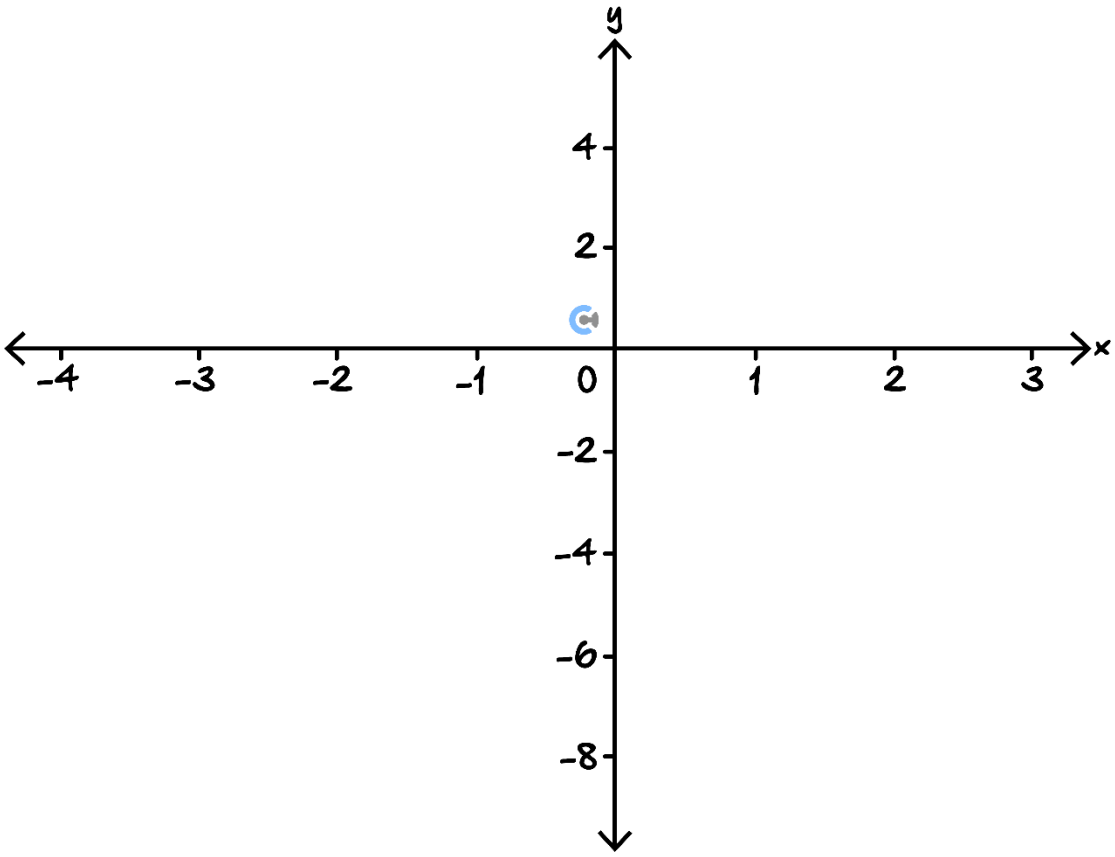
Question 29

Factorise and hence, sketch the graphs of each of the functions on the axes provided.

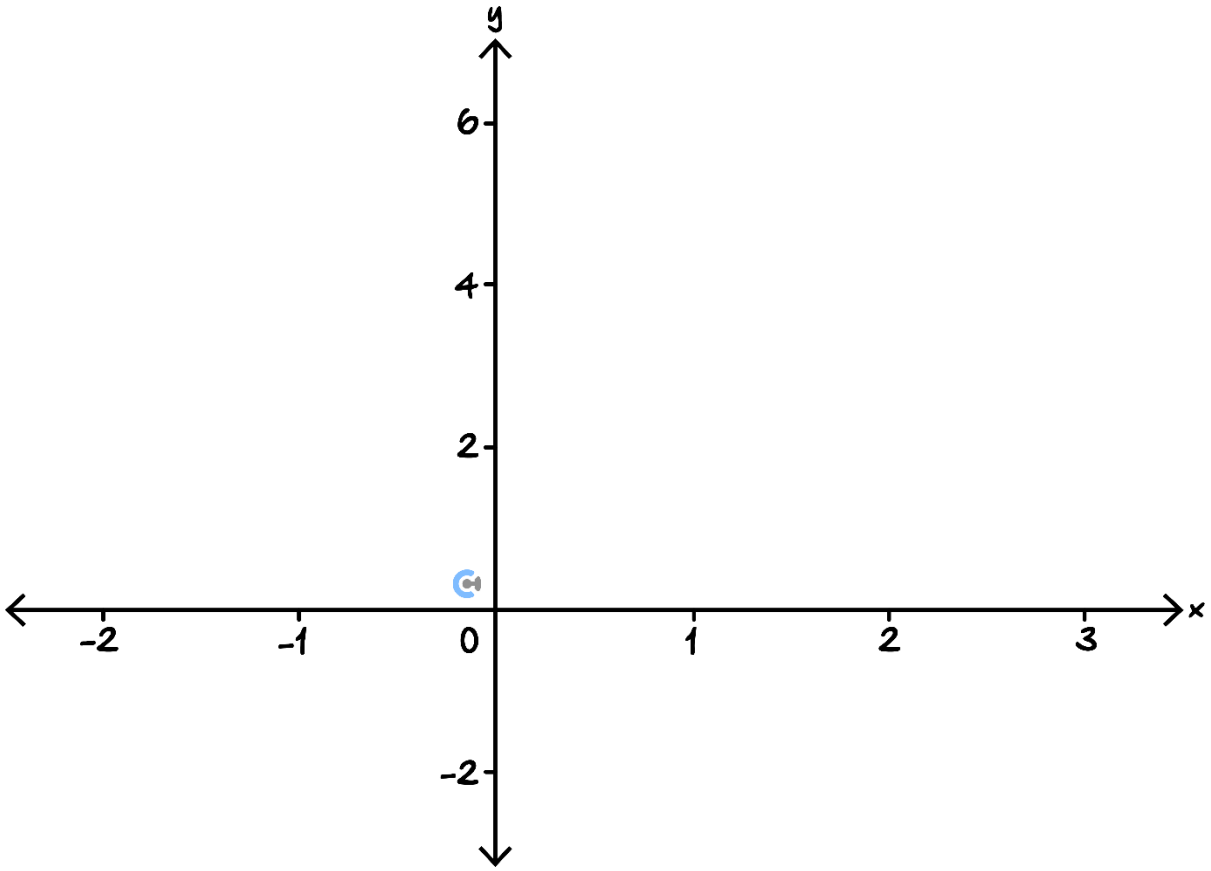
a. $y = x^3 - x^2 - 4x + 4$



b. $y = x^3 + 2x^2 - 5x - 6$



c. $y = x^4 - 2x^3 - 3x^2 + 4x + 4$



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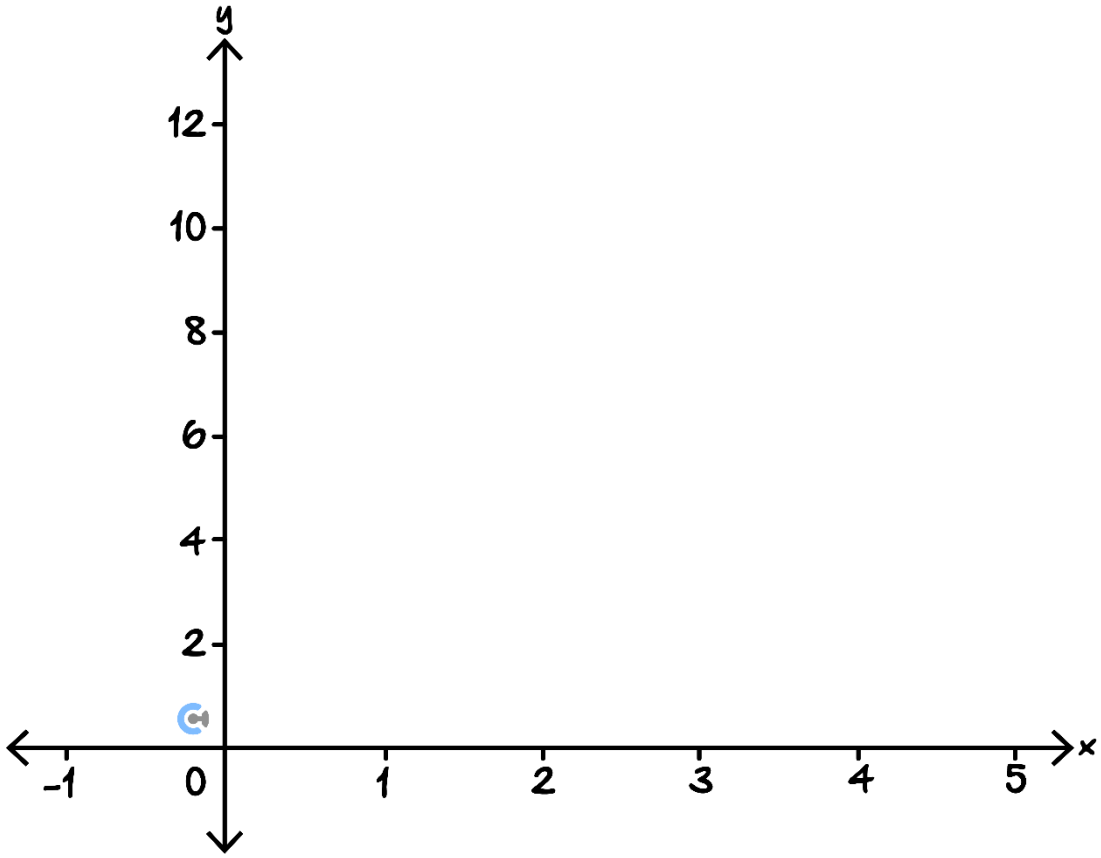
Question 30

Sketch the graph of $y = x^4 - 8x^3 + 22x^2 - 24x + 10$ on the axis below.

Hint: Factorise $x^4 - 8x^3 + 22x^2 - 24x + 9$ instead.



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