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VCE Mathematical Methods ½ Quadratics Exam Skills [1.4] Workbook

Outline:



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- Perfect Squares
- Difference of Squares
- Completing the Square
- Solving by Factorisation
- Quadratic Formula
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- Graphing Quadratics
- Finding a Rule of a Quadratic From a Graph
- Quadratic Inequalities
- Hidden Quadratics

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- Finding Turning Point efficiently
- Modelling using Quadratics
- Family of Functions

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Exam 2

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Section A: Recap

Sub-Section: Factorising Quadratics

Let's quickly revise how we factorised quadratics!

Factorising Quadratics



$$y = (x - a)(x - b)$$

► Steps:

1. Divide by the coefficient of the leading term. (If applicable)
2. Consider the factors of the constant term.
3. (If Positive Constant Term): See which pair of factors can add up to the coefficient of the x term.
(If Negative Constant Term): See which pair of factors can subtract from the coefficient of the x term.
4. Construct the linear factors.

Question 1

Factorise the following expressions:

a. $x^2 + 6x + 8$

b. $x^2 - 5x + 6$

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Sub-Section: Perfect Squares



Let's quickly revise perfect squares!



Perfect Squares



$$(a + b)^2 = \underline{\hspace{10cm}}$$

$$(a - b)^2 = \underline{\hspace{10cm}}$$

- Perfect squares are special quadratic expressions that are made up of two **identical** linear factors.
- In other words, when a linear factor is squared, it becomes a perfect square.

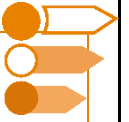
Question 2

Factorise the following expressions using the perfect square formula.

a. $x^2 + 6x + 9$

b. $4x^2 - 12x + 9$

Sub-Section: Difference of Squares



Let's quickly revise the difference between squares!



Difference of Squares



$$a^2 - b^2 = \underline{\hspace{4cm}}$$

Question 3

Factorise the following expressions:

a. $x^2 - 16$

b. $9x^2 - 4$

Sub-Section: Completing the Square



Let's quickly revise completing the square!



Completing the Square



➤ When we complete the square of a quadratic $x^2 + bx + c$, we write it in the form:

$$x^2 + bx + c = (\text{_____})^2 - \left(\frac{b}{2}\right)^2 + c$$

➤ Steps:

1. We halve the coefficient of x .
2. Subtract the half of the coefficient of x squared outside the square bracket.

Question 4

Complete the square for each quadratic.

a. $x^2 - 6x + 4$

b. $2x^2 + 16x + 17$

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Sub-Section: Solving by Factorisation



Solving by Factorisation

$$(x - a)(x - b) = 0$$

$$x = a \text{ or } b$$

➤ Steps:

1. Factorise the quadratic.
2. Equate each factor to 0 and solve for x .

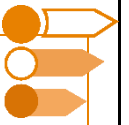
Question 5

Solve each of the following quadratic equations for x :

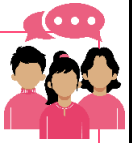
a. $x^2 - 6x + 8 = 0$

b. $2x^2 - 3x - 5 = 0$

Sub-Section: Quadratic Formula



Discussion: What do we do if the quadratic is not easy to factorise?



The Quadratic Formula



for $ax^2 + bx + c = 0$

$x =$ _____

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Question 6

Solve each of the following quadratic equations for x .

a. $x^2 - 4x - 8 = 0$

b. $2x^2 + 4x + 1$

Sub-Section: Discriminant



The Discriminant

➤ Definition:

- The discriminant, often denoted by Δ (Delta), is the part **inside** the square root of the quadratic formula.

$$\text{Discriminant} = \Delta = b^2 - 4ac$$

if $\Delta > 0$, there are _____.

if $\Delta = 0$, there is _____.

if $\Delta < 0$, there are _____.

Question 7

Determine how many unique roots exist in each of the following quadratic equations:

$$x^2 - 7x + 5 = 0$$

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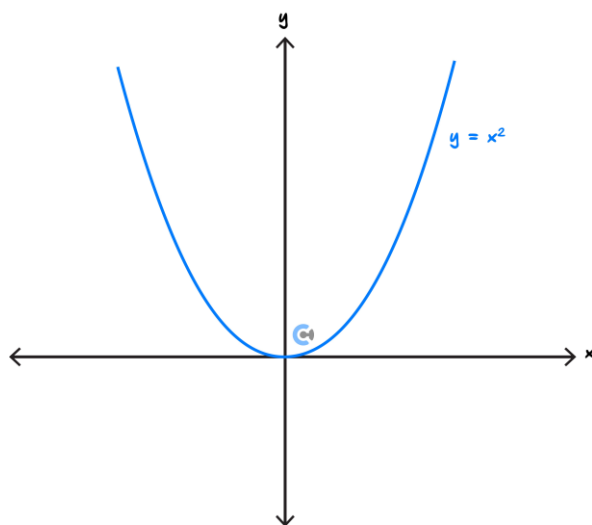
Sub-Section: Parabola and Symmetry



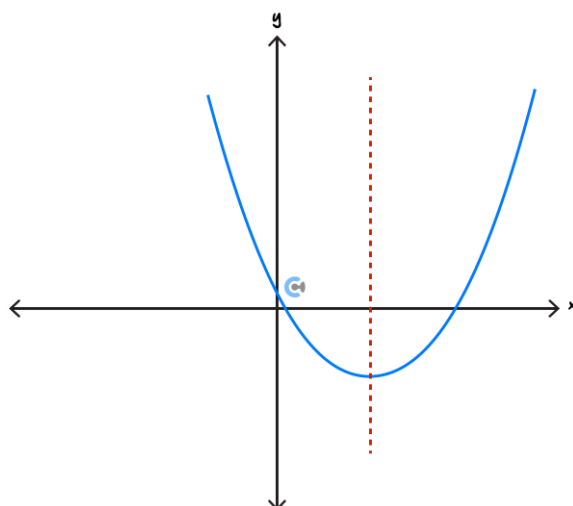
Parabola

► Definition:

The shape of the graph of a quadratic is known as a _____.



Axis of Symmetry



Axis of symmetry: $x = -\frac{b}{2a}$

Question 8

Find the axis of symmetry of each of the following quadratic hence, the coordinate of turning point.

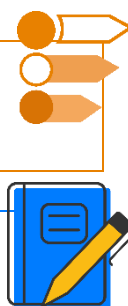
$$y = 2x^2 - 6x + 5$$

NOTE: When a question asks for coordinates, you must mention both the x and y -value of the point.



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Sub-Section: Graphing Quadratics



Turning Point Form

- The turning point form of a quadratic is given by:

$$y = a(x - h)^2 + k$$

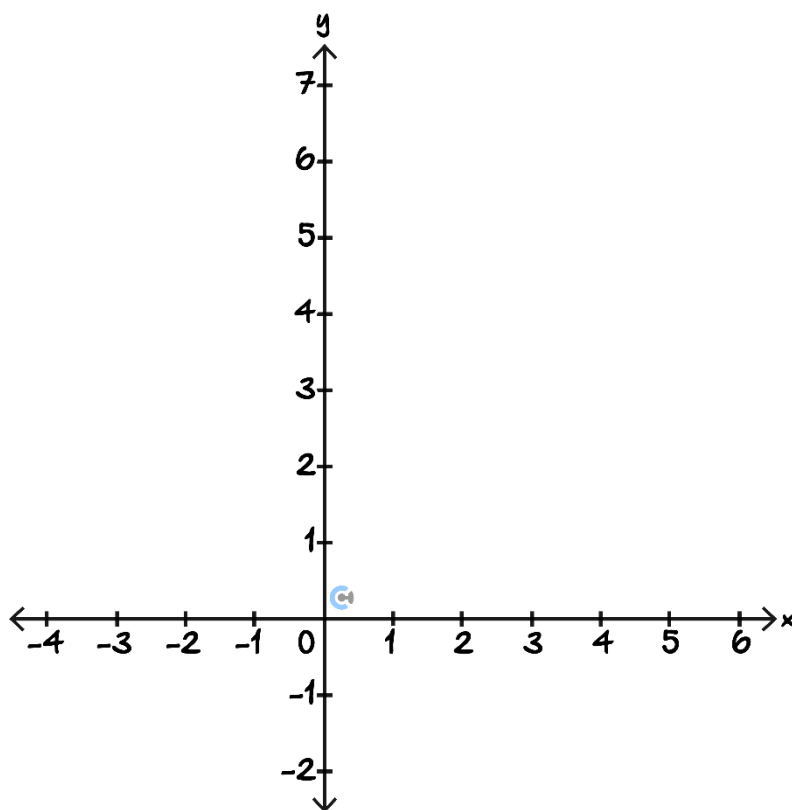
Turning point = _____

- The turning point form is obtained by **completing the square**.

Question 9

Sketch the graph of the following quadratic equation, labelling the coordinates of all axes intercepts and the turning point.

$$y = (x - 2)^2 + 1$$





Intercept Form

- The x -intercept form of a quadratic is given by:

$$y = a(x - b)(x - c)$$

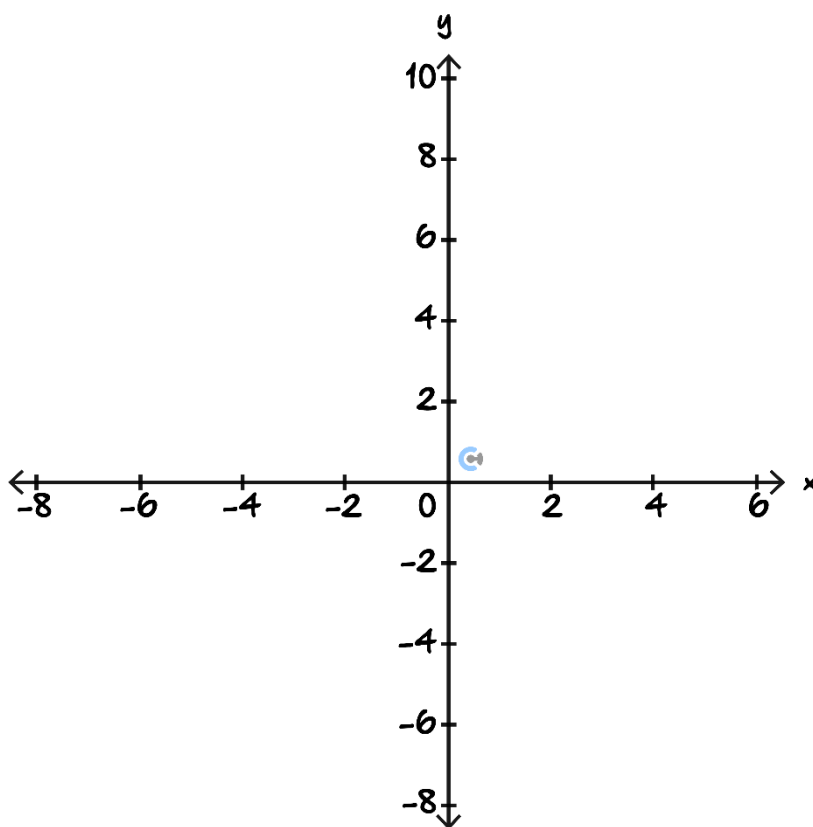
x -intercepts: $(b, 0)$ and $(c, 0)$

- The axis of symmetry is located exactly in the middle of the two x -intercepts.

Question 10

Sketch the graph of the following quadratic equation, labelling the coordinates of all axes intercepts, as well as the turning point.

$$y = -x^2 + 2x + 8$$



NOTE: When a is negative, the x -intercepts stay the same, but the **shape** of the parabola becomes a **negative** parabola instead.



Sub-Section: Finding a Rule of a Quadratic from a Graph



Let's try to do it the other way around!



Finding the Equation of a Quadratic




➤ Form 1: Turning Point Form

$$y = a(x - h)^2 + k$$

 Recommended when a turning point is easy to identify.

➤ Form 2: x -intercept Form

$$y = a(x - b)(x - c)$$

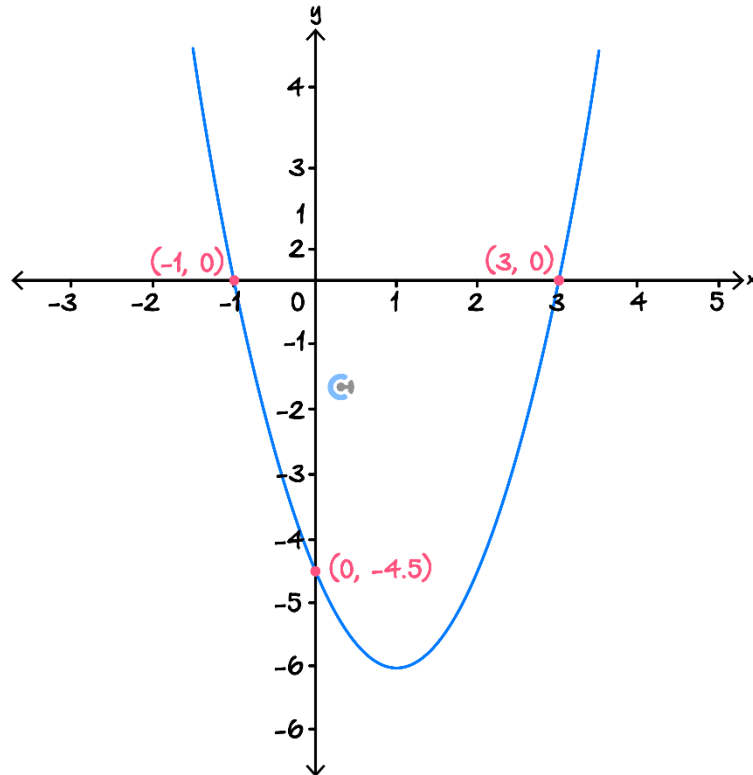
 Recommended when both x -intercepts are easy to identify.

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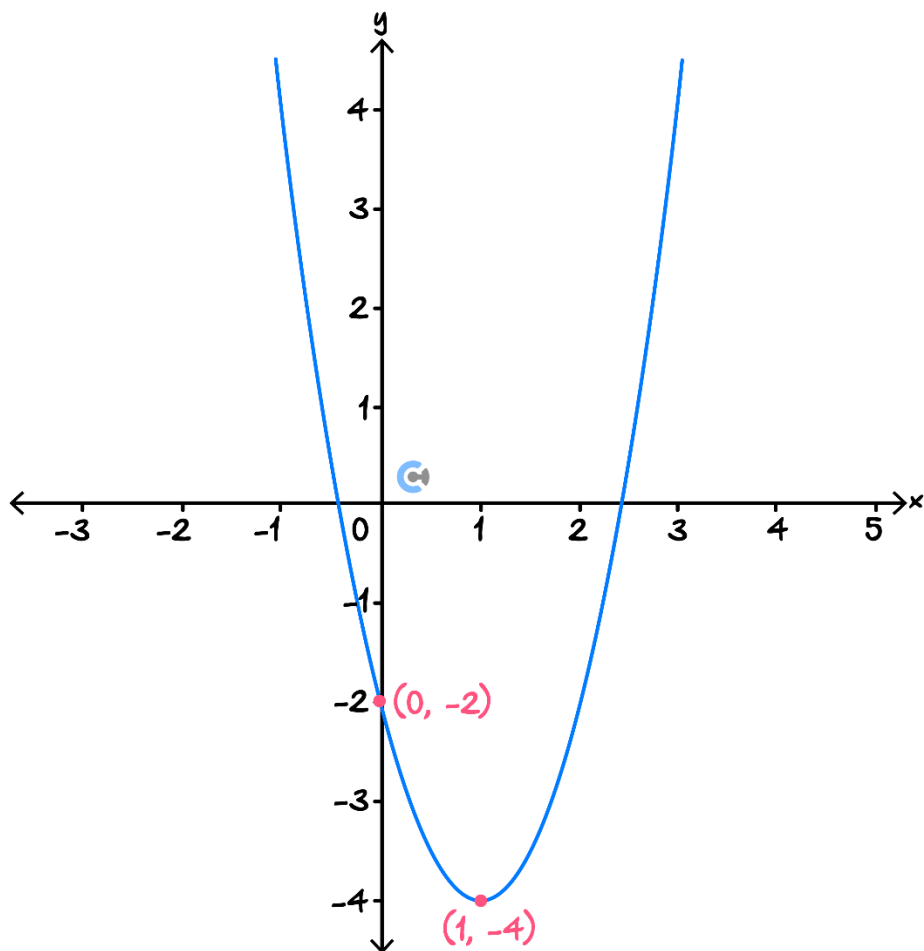
Question 11

Find the equations of the quadratics graphed below. Show your working.

a.



b.



NOTE: Never forget the a coefficient!



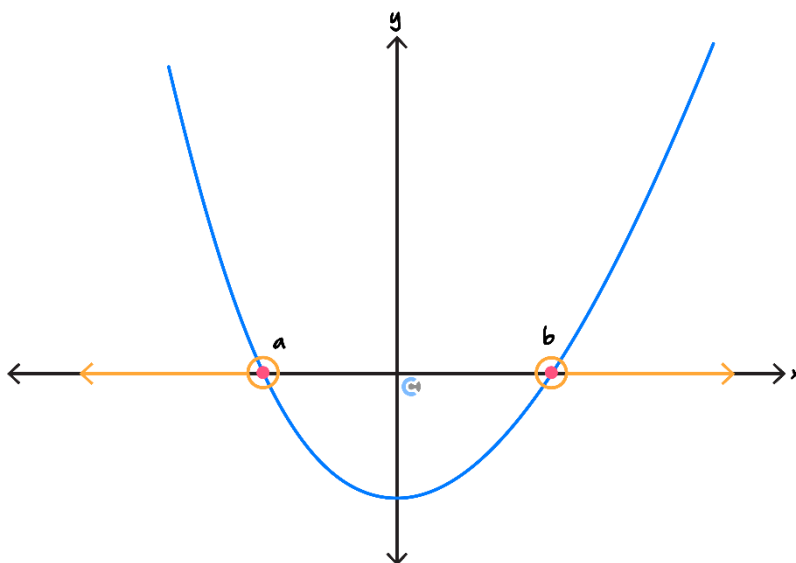
Sub-Section: Quadratic Inequalities



How can we tackle quadratic inequalities?



Quadratic Inequalities



➤ For quadratic inequalities, we always _____ the function.

➤ Steps:

1. Sketch the function.
2. See where the y -value is within the inequality.
3. Find the corresponding x -values.

Discussion: Why do we look at $y\text{-value} < 1$ if the function < 1 ?



Question 12

Solve each of the following for x :

a. $(x - 2)(x - 5) > 0$

b. $2x^2 - 7x + 6 \leq 0$

Sub-Section: Hidden Quadratics



Let's take a look at hidden quadratics!



Hidden Quadratics



➤ Instead of:

$$af(x)^2 + bf(x) + c = 0$$

➤ We can let $f(x) = X$ to have:

$$aX^2 + bX + c = 0$$

Question 13

a. Solve $(x - 2)^2 + 5(x - 2) + 6 = 0$ for x .

b. Solve $x^4 - 13x^2 + 36 = 0$ for x .

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Section B: Quadratics Exam Skills

Sub-Section: Finding Turning Point efficiently

How do we complete the square very quickly?

Completing the square quickly.

$$y = a(x - h)^2 + k$$

► Steps

1. Find the turning point using $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.
2. Use the leading coefficient as a .

Question 14 Walkthrough

Complete the square for $y = 2x^2 - 4x + 5$.



Active Recall: Completing the square quickly

1. Find the turning point using _____.
2. Use the leading coefficient as _____.

Question 15

Complete the square for $y = -3x^2 + 12x - 1$.

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Sub-Section: Modelling using Quadratics



Modelling with Quadratics

- Focus on key points such as turning points, x -intercepts and y -intercepts.

Question 16 Walkthrough.

A quadratic bridge has the highest point at $(2,10)$ and hits the ground at $x = 10$.

Find the equation of the quadratic bridge.

Question 17

A ramp is modelled by a quadratic equation and passes through $(1,3)$, $(2,6)$ and $(-1,9)$.

Find the equation of the ramp.

Sub-Section: Family of Functions



Family of Functions

- **Definition:** Functions with unknowns.
- **Question Type:** Find the unknown value to satisfy a certain condition.

Question 18 Walkthrough.

Consider the function $f(x) = -x^2 + 3x - k$.

Find the value(s) of k if the maximum is less than 6.

NOTE: When VCAA writes value(s), the number of solutions should be more than one.

ALSO NOTE: For maximum and minimum, focus on the turning points.



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Question 19

Consider the function $f(x) = 2x^2 - kx - 4$.

- a. Find the x -intercepts of f in terms of k .
- b. Find the values of k such that the distance between two x -intercepts is less than 5.

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Section C: Exam 1 (20 Marks)**Question 1 (3 marks)**

The sum of two children's age is 7 and the product of their ages is 12.

- a. Write down a quadratic equation in the form $ax^2 + bx + c = 0$ that can be solved to find the ages of the children. (1 mark)

- b. Find the ages of the children. (2 marks)

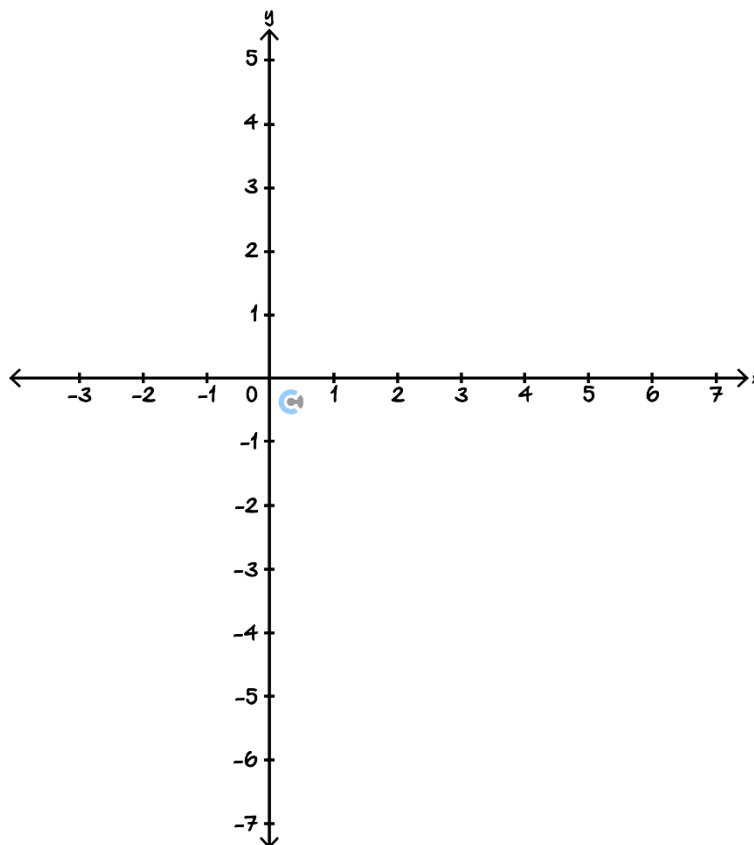
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Question 2 (4 marks)

Consider the function $f(a) = a^2 - 3a - 4$.

- a. Solve the equation $f(x) = 0$. (1 mark)

- b. Sketch the graph of $y = f(x)$ on the axes below. Label the turning point and all axes intercept with coordinates. (2 marks)



c. Hence, find the value(s) of x such that $f(x) + 4 < 0$. (1 mark)

Question 3 (2 marks)

Solve the inequality $x^2 - 8x + 12 \leq 0$.

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Question 4 (3 marks)

Solve the equation $x^4 + 5x^2 - 36 = 0$, for real values of x .

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Question 5 (3 marks)

Consider the function $f(x) = x^2 - 2kx + 5$, where k is a real number. Find all possible values of k if $f(x)$ is always greater than 1.

Question 6 (5 marks)

Consider the function $f(x) = x^2 - kx - 1$, where k is a real number.

- a. Show that the graph $y = f(x)$ always has two x -intercepts.(1 mark)

- b. Find the values of k such that the distance between the two x -intercepts is less than 4. (3 marks)

- c. Find the minimum possible distance between the two x -intercepts. (1 mark)

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Section D: Tech Active Exam Skills

Calculator Commands: Solving equations



➤ Mathematica

 Solve[

In[122]:= **Solve**[$x^2 - 4x - 9 == 0$, x]
 Out[122]= $\left\{\left\{x \rightarrow 2 - \sqrt{13}\right\}, \left\{x \rightarrow 2 + \sqrt{13}\right\}\right\}$

➤ TI-Nspire

 Menu→3→1.

solve($x^2 - 4x - 9 = 0, x$)
 $x = -(\sqrt{13} - 2)$ or $x = \sqrt{13} + 2$

➤ Casio Classpad


 Action→Advanced→Solve.

solve($x^2 - 4x - 9 = 0, x$)
 $\{x = -\sqrt{13} + 2, x = \sqrt{13} + 2\}$

Calculator Commands: Completing the Square




➤ TI-Nspire

 Menu→3→5
completeSquare
 (func, var).

completeSquare($x^2 - 6x + 8, x$) $(x - 3)^2 - 1$

➤ Mathematica


 no inbuilt function need
 udf.

Compisq
Compisq[$a x^2 + b x + c, x$]
 Converts a standard form quadratic to turning point form.
Compisq[a, b, c]
 Uses the coefficients of a quadratic to return the turning point form.

In[121] **Compisq**[2, 2, 3]
 Out[121] $2(x - 0.5)^2 - 0.5$

In[122] **Compisq**[7 + 7 + 3 + 5, x]
 Out[122] **Compisq**[15 + 3 + 5 + 5, x]

➤ Casio Classpad

 No function.

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Section E: Exam 2 (19 Marks)**Question 7 (1 mark)**

Find the value(s) of k for which the quadratic equation below has exactly one unique real solution.

$$3x^2 - \frac{2}{5}kx + k = 0$$

- A. $k = -75$
- B. $k = 0,75$
- C. $k = \frac{2}{5}$
- D. $k = 0,50$

Question 8 (1 mark)

A quadratic function has a turning point at $(3, -4)$ and goes through the point $(5, -8)$. What is the equation of the function?

- A. $-(x - 3)^2 + 4$
- B. $(x - 3)^2 - 4$
- C. $-(x + 3)^2 - 4$
- D. $-(x - 3)^2 - 4$

Question 9 (1 mark)

The function $f(x) = 3x^2 + mx + 5$ is always greater than 2. The possible values of m are:

- A. $m = \pm 10$
- B. $0 < m < 12$
- C. $-6 < m < 6$
- D. $-9 < m < 9$

Question 10 (1 mark)

If one root of the quadratic equation $3x^2 + px - 12 = 0$ is -4 , the value of p is:

- A. -9
- B. 9
- C. -4
- D. 4

Question 11 (1 mark)

The equation $ax^2 + 2x + c = 0$ has only one real solution if:

- A. $c^2 = 4a$
- B. $c^2 < 4a$
- C. $c^2 > 4a$
- D. $ac = 1$

Question 12 (7 marks)

Emily is standing at the top of a cliff overlooking the ocean. She is 80 metres above the ocean. She tosses a ball into the air. The height, h , of the ball after t seconds is given by:

$$h = -12t^2 + 48t + 80.$$

- a. What is the height of the ball above the ocean after $t = 1$ second? (1 mark)

b. What is the maximum height above the ocean that the ball reaches? (1 mark)

c. What is the time taken for the ball to reach its maximum height? (1 mark)

d. How many seconds does it take for the ball to land in the ocean? Give an exact value. (2 marks)

e. After Emily throws the ball upwards, when exactly does the ball pass her on its way down? (2 marks)

Question 13 (7 marks)

A cricket player hits a ball, and the ball's trajectory is modelled by the quadratic equation:

$$h(x) = ax^2 + bx + c,$$

Where $h(x)$ is the height of the ball (in metres) above the ground, and x is the horizontal distance (in metres) from where the ball was hit.

The following conditions are given:

- The ball is hit from a height of 1.2 metres, i.e., $h(0) = 1.2$.
- The ball reaches a height of 25 metres when it has traveled 15 metres horizontally.
- The ball hits the ground after travelling 30 metres horizontally, i.e., $h(30) = 0$.

- a.** Using the given conditions, set up and solve a system of equations to determine the values of a , b , and c . (3 marks)

- b.** Determine the horizontal distance the ball has travelled when its height is 10 metres. Provide both possible values of x correct to two decimal places. (2 marks)

Another player hits a ball, and the ball's trajectory is modelled by a quadratic equation $h_2(x)$.

This player's ball:

- ▶ Is hit from a height of 1.5 metres, i.e., $h_2(0) = 1.5$,
- ▶ Reaches its maximum height of 20 metres at a horizontal distance of 10 metres.

- c.** Find the equation of $h_2(x)$ in turning point form. (2 marks)

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Contour Check

Learning Objective: [1.1.1] - Find factorised form of quadratics

Key Takeaways

- ☐ Perfect square is in the form of _____.
- ☐ Differences of squares are in the form of $a^2 - b^2$ _____.
- ☐ Complete the square form of $x^2 + bx + c =$ _____.

Learning Objective: [1.1.2] - Find solutions and number of solutions to quadratic equations

Key Takeaways

- ☐ We can solve for quadratic equations by first _____.
- ☐ Alternatively, we can use the quadratic formula given by $x =$ _____.
- ☐ The discriminant is given by _____ which dictates the number of solutions.

Learning Objective: [1.1.3] - Graph and find rules from the graph of quadratic equations

Key Takeaways

- ☐ Every quadratic can be put into the turning point given by $y =$ _____.
- ☐ _____ all quadratic can be put into the x -intercept form given by $y =$ _____.
- ☐ We can use x -intercept form or turning point form to find the rule.

Learning Objective: [1.1.4] - Solving Quadratic Inequalities and hidden quadratics

Key Takeaways

- ☐ For quadratic inequalities, we always _____.
- ☐ For hidden quadratics, look for the pattern of something and something _____.

VCE Mathematical Methods ½

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