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VCE Mathematical Methods ½
Quadratics Exam Skills [1.4]
Homework Solutions

Homework Outline:

Compulsory Questions	Pg 2 – Pg 22
Supplementary Questions	Pg 23 – Pg 43

Section A: Compulsory Questions

Sub-Section [1.4.1]: Find Turning Point Form Using Turning Points

Question 1

Find the turning point of the parabola $y = 3(x-2)^2 + 5$.

Recall that the turning point of any parabola in turning point form, $y = a(x-h)^2 + k$ is (h, k).
Thus our turning point is $(2, 5)$.

Question 2

Find the equation of a parabola that has a turning point at (3,4) and has a y-axis intercept of -5.

Since the parabola has a turning point of $(3, 4)$ its equation will be of the form,

$$y = a(x-3)^2 + 4$$

We now solve for a using the information about the y-axis intercept, specifically that when x is equal to 0, y is equal to -5. Thus,

$$-5 = a(-3)^2 + 4 \implies -9 = 9a \implies a = -1$$

Hence our parabola has an equation of $y = -(x-3)^2 + 4$

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Question 3

Find the turning point of the parabola, $y = -2x^2 - 4x + 4$.

We first get our parabola into turning point form by completing the square.

$$\begin{aligned} y &= -2x^2 - 4x + 4 \\ &= -2(x^2 + 2x) + 4 \\ &= -2((x+1)^2 - 1) + 4 \\ &= -2(x+1)^2 + 6 \end{aligned}$$

From here we can read off our turning point as $(-1, 6)$.

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Sub-Section [1.4.2]: Apply Quadratics to Model a Scenario

Question 4

A ground-based contraption flings a ball straight up into the air. 4 seconds later, the ball lands on the ground. The height in metres of the ball h, t seconds after the ball is launched is:

$$h(t) = -10t^2 + bt + c$$

Find the values of b and c.

Our two pieces of information are that $h(0) = h(4) = 0$.
From here we see that $h(t) = at(t-4) = at^2 - 4at$.
Now by comparing our x^2 coefficient with the formula $h(t) = -10t^2 + bt + c$ we know that $a = -10$.
Thus $h(t) = -10t^2 + 40t \implies b = 40$ and $c = 0$.

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Question 5

A parabola-shaped bridge is used to cross a long river. The height of the bridge above the water level in metres, h, is a quadratic function of the horizontal distance of a point of a bridge from the starting river bank, x.

At the starting river bank, the height of the bridge is 1 metre above water level, and 3 metres away from the starting point ($x = 3$), the bridge is at its highest point, 4 metres above water level ($h = 4$).

Relate x and h.

Since we are given the turning point of the bridge of $(3, 4)$ we can say that, $h(x) = a(x-3)^2 + 4$.
We also know that $h(0) = 1$, thus we can solve for a.

$$1 = a(-3)^2 + 4 \implies -3 = 9a \implies a = -\frac{1}{3}$$

Hence, $h(x) = -\frac{1}{3}(x-3)^2 + 4$

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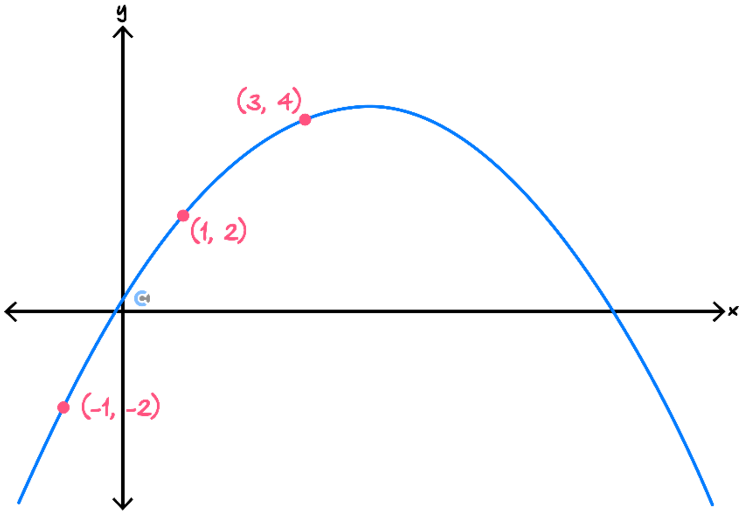
Question 6

A river passes through 3 points in a park as shown below:



Where the x-axis represents the position due east from the centre of the park, and the y-axis represents the position due north from the centre of the park.

We can relate the north position (y) of the river to the east position (x) of the river through the equation:

$$y = ax^2 + bx + c$$

Find the values of a, b and c.

We can construct 3 equations from the information provided in by the graph.

$$\begin{aligned} -2 &= a - b + c && (-1,-2) && (1) \\ 2 &= a + b + c && (1,2) && (2) \\ 4 &= 9a + 3b + c && (3,4) && (3) \end{aligned}$$

By adding (1) and (2) we get $0 = 2a + 2c \implies a = -c$.
By subtracting (3) from $3 \times (2)$ we get $2 = -6a + 2c$. We can substitute $a = -c$ into this equation to get $2 = 8c \implies c = \frac{1}{4} \implies a = -\frac{1}{4}$.
We can substitute these two values of a and c into (2) to get $b = 2$.

Sub-Section [1.4.3]: Apply Family of Functions to Find an Unknown of Function

Question 7

Consider the parabola $y = x^2 + 2x - 4k^2 + 1$.
Find the value of k such that the horizontal distance between x-axis intercepts of the parabola is less than 8.

To find the x-axis intercepts of the parabola we set $y = 0$ and solve for x. Thus

$$4k^2 = x^2 + 2x + 1 = (x+1)^2 \implies x = -1 \pm 2k$$

The difference between the two x-values of the x-axis intercepts is $2k - 1 - (-2k - 1) = 4k$. The magnitude of this quantity is less than 8 if $-2 < k < 2$.

Question 8

Let $y = x^2 + 2kx - 2$.
Find the values of k such that $y \geq -6$ for all x.

We can complete the square to get the parabola in turning point form.

$$y = x^2 + 2kx + k^2 - k^2 - 2 = (x+k)^2 - 2 - k^2$$

We know that the range of $(x+k)^2$ is $[0, \infty)$, thus for $y \geq -6$ we simply require $-2-k^2 \geq -6$. Thus $k^2 \leq 4 \implies -2 \leq k \leq 2$.

Question 9

Find all values of k such that the equation $\left(x - k - \frac{1}{2}\right)^2 = -k$ has two real solutions for x, one positive and one negative.

We can solve this equation for x to get,

$$x = k + \frac{1}{2} \pm \sqrt{-k}$$

From here we see that if our equation has 2 real solutions, $k < 0$.
Substituting $a = \sqrt{-k}$ we see that $k + \frac{1}{2} + \sqrt{-k} = -a^2 + a + \frac{1}{2}$.
We solve it equal to 0 to get,

$$a = \frac{-1 \pm \sqrt{1+2}}{-2} = \frac{1 \pm \sqrt{3}}{2}$$

Since $-a^2 + a + \frac{1}{2}$ is a negative parabola we see that it is greater than 0 if $1 - \sqrt{3} < 2a < 1 + \sqrt{3}$. Due to domain constraints we get $0 \leq a < \frac{1+\sqrt{3}}{2}$
We repeat a similar process using $k + \frac{1}{2} - \sqrt{-k} = -a^2 - a + \frac{1}{2}$. This is equal to 0 if,

$$a = \frac{1 \pm \sqrt{1+2}}{-2} = \frac{-1 \pm \sqrt{3}}{2}$$

Thus after taking in domain constraints, $-a^2 - a + \frac{1}{2} > 0$ if $0 \leq a < \frac{-1+\sqrt{3}}{2}$.
Thus our two solutions of x have opposite signs if $\frac{-1+\sqrt{3}}{2} < a < \frac{1+\sqrt{3}}{2}$.
Since $a = \sqrt{-k}$ our solutions will be of opposite signs if $-\frac{(1+\sqrt{3})^2}{4} < k < -\frac{(\sqrt{3}-1)^2}{4}$.
We can simplify this interval to,

$$-1 - \frac{\sqrt{3}}{2} < k < -1 + \frac{\sqrt{3}}{2}$$

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Sub-Section [1.4.4]: Harder Quadratic Inequalities

Question 10

Solve $x(x+1) > 12$ for x.

We rearrange our expression to get $x^2 + x - 12 = (x-3)(x+4) > 0$.
Since $x^2 + x - 12$ is a positive parabola with roots of $-4, 3$ we see that it is greater than 0 if $x < -4$ or $x > 3$.
Thus $x(x+1) > 12 \implies x < -4$ or $x > 3$.

Question 11

Solve $1 + \frac{3}{x-1} \leq \frac{10}{(x-1)^2}$ for x.

We multiply our inequality by $(x-1)^2$ to get $(x-1)^2 + 3(x-1) - 10 \leq 0$.
For simplicity set $x - 1 = a$, thus our inequality becomes

$$a^2 + 3a - 10 = (a+5)(a-2) \leq 0$$

Since $a^2 + 3a - 10$ is a positive parabola with roots of $2, -5$ we see that it is ≤ 0 when $-5 \leq a \leq 2$.
Since $a = x - 1$ we can rearrange our expression to be in terms of x, specifically, $-4 \leq x \leq 3$. However since in our original expression we divide by $x - 1$ we cannot have $x = 1$. Thus our solution is,

$$-4 \leq x < 1 \quad \text{or} \quad 1 < x \leq 3.$$

Question 12

Solve $(x^2+1)^2+3 \geq 7x^2$ for x.

Hint: Break this down into 2 quadratic inequalities.

For our first inequality we wish to substitute $a = x^2+1$, thus we should rearrange everything to be in terms of x^2+1.

$$(x^2+1)^2 - 7x^2 - 7 + 7 + 3 = (x^2+1)^2 - 7(x^2+1) + 10 \geq 0$$

Thus we will first solve the inequality $a^2 - 7a + 10 = (a-5)(a-2) \geq 0$.
As $a^2 - 7a + 10$ is a positive parabola with roots of $2, 5$ we see that it is ≥ 0 if $a \leq 2$ or $a \geq 5$.
Now we substitute back $x^2 + 1 = a$.
If $x^2 + 1 \leq 2$ then $x^2 \leq 1 \implies -1 \leq x \leq 1$.
If $x^2 + 1 \geq 5$, then $x^2 \geq 4 \implies x \leq -2$ or $x \geq 2$.
Thus our solution is $x \leq -2$ or $-1 \leq x \leq 1$ or $x \geq 2$.

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Sub-Section: Exam 1 Questions

Question 13 (4 marks)

a. Solve $6 - \frac{5}{x} = x$ for $x \in \mathbb{R}$. (2 marks)

We multiply our expression by x to get, $6x - 5 = x^2$.
We rearrange this to be in the form $x^2 - 6x + 5 = (x-1)(x-5) = 0$.
From here we see that $x = 1, 5$.

b. Solve $6 - \frac{5}{x} < x$ for $x > 0$. (2 marks)

Since $x > 0$ we see that the inequality $6 - \frac{5}{x} < x$ is equivalent to $6x - 5 < x^2$.
We rearrange our second inequality to get $x^2 - 6x + 5 > 0$.
Now as $x^2 - 6x + 5$ is a positive parabola with roots of $1, 5$, we see that it is greater than 0 if $x < 1$ or $x > 5$. As $x > 0$ or solution is,

$$0 < x < 1 \quad \text{or} \quad x > 5.$$

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Question 14 (3 marks)

Solve the equation $x^4 - 8x^2 - 9 = 0$, for real values of x.

Let $a = x^2$. Thus our equation in terms of a becomes,

$$a^2 - 8a - 9 = (a-9)(a+1) = 0$$

Since $a = x^2 \geq 0$ we see that $a = 9$.
Hence $x^2 = 9 \implies x = \pm 3$.

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Question 15 (3 marks)

To abate Alex's loneliness, Contour is building a parabolic bridge from their CBD office to their Box-Hill office.

The distance between the two offices is 14 kilometres.

The height, h on the bridge in metres, x kilometres away from the CBD office satisfies the following equation:

$$h = ax^2 + bx + c$$

Whilst walking on the bridge, when Alex is 2 kilometres away from the CBD office, he is 4 metres above ground level.

a. Find the values of a, b and c. (2 marks)

The bridge will be at ground level at both offices, hence when $x = 0$ or $x = 14$ we have that $h = 0$.
From here we see that $h = ax(x - 14)$.
We can solve for a by using the fact that when $x = 2$, $h = 4$. Thus,

$$4 = a \times 2 \times (-12) = -24a \implies a = -\frac{1}{6}$$

.
Hence $h = -\frac{1}{6}x^2 + \frac{14}{6}x$, i.e. $a = -\frac{1}{6}$, $b = \frac{14}{6}$ and $c = 0$.

b. What is the maximum height of the bridge above the ground? (2 marks)

The maximum height will occur at the middle of the offices, specifically when $x = 7$.
Thus $h_{\max} = -\frac{1}{6} \times 7 \times (-7) = \frac{49}{6}$ meters.

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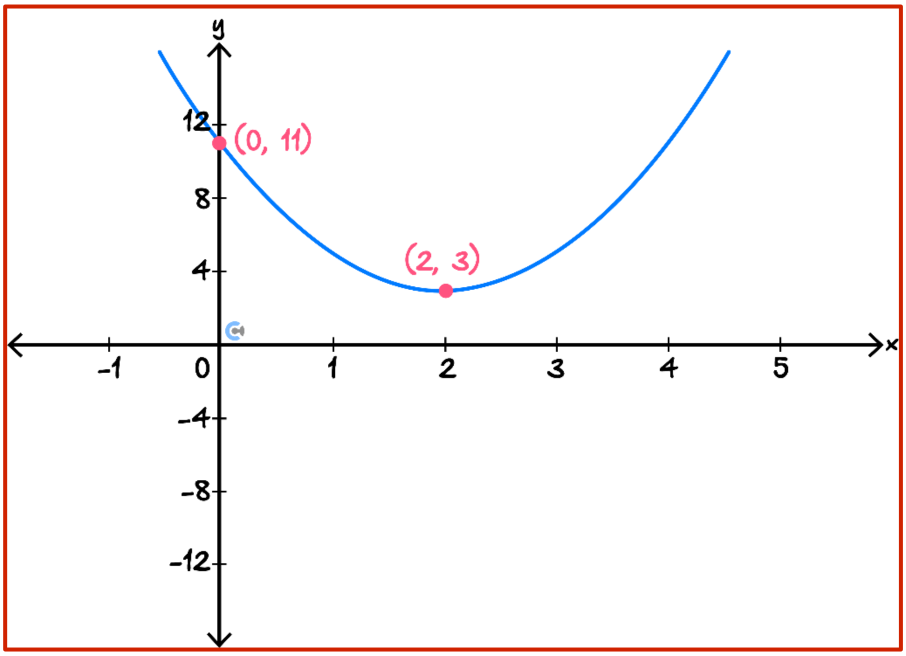
Question 16 (4 marks)

a. Sketch the graph of $y = 2x^2 - 8x + 11$ on the axis below, label all key points with their coordinates. (2 marks)



b. The line $y = k$ has a minimum vertical distance of 2 from the above parabola. Find the value of k. (1 mark)

$k = 1$ observe graph above for intuition.

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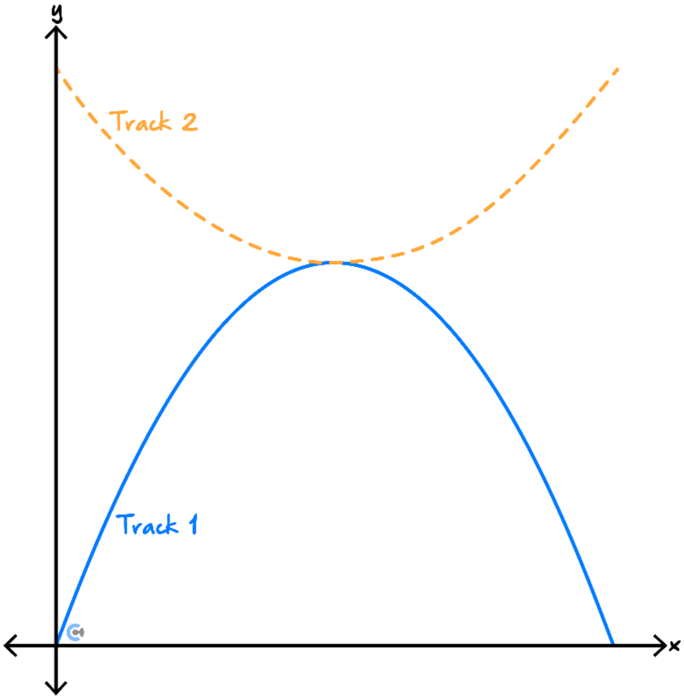
Question 17 (5 marks)

Sam is standing at the start of the walking track 1. Relative to his current position, the coordinates of two walking tracks are drawn below, with the x-ordinate representing kilometres due east and the y-ordinate representing kilometres due north.



Track 1 is described by the function $y = -2x^2 + ax$.
Track 2 is described by the equation $y = b(x-h)^2 + k$
The unit of length is kilometres.

a. Track 1 ends 2 kilometres due east of Sam's current position. Show that $a = 4$. (1 mark)

As track 1 ends 2 kilometers due east of Sam's current position we know that when $x = 0, 2$ that $y = 0$. Thus,

$$y = cx(x-2) = cx^2 - 2cx.$$

Since $c = -2$ we see that $y = -2x^2 + 4x$ thus $a = 4$

b. Track 2 has the same turning point as track one and starts 6 kilometres due north of Sam's current position. Find the values of b, h and k. (1 mark)

The x-value of the turning point of track 1 is 1.
Thus the y-value of the turning point of track 1 is $-2(1)^2 + 4(1) = 2$.
As the turning point of track 2 is also the turning point of track 1 we see that the equation of track 2 is,

$$y = b(x-1)^2 + 2$$

Now we also know that when $x = 0$, $y = 6$, thus $6 = b(-1)^2 + 2 \implies b = 4$.
Hence $b = 4, h = 1$ and $k = 2$.

c. At what point(s) on track 1, is the vertical distance to track 2 equal to 3 kilometres. (2 marks)

The vertical distance between the two tracks is,

$$4(x-1)^2 + 2 - (-2x^2 + 4x) = 6x^2 - 12x + 6$$

We solve this to be equal to 3 for x, yielding,

$$\begin{aligned} & 6x^2 - 12x + 6 = 3 \\ \implies & 2x^2 - 4x + 1 = 0 \\ \implies & x = \frac{4 \pm \sqrt{16-8}}{4} = 1 \pm \frac{1}{\sqrt{2}} \end{aligned}$$

Thus the points on track 1 for which the vertical distance to track 2 is equal to 3 kilometers are,

$$\left(1 + \frac{1}{\sqrt{2}}, -2\left(1 + \frac{1}{\sqrt{2}}\right)^2 + 4\left(1 + \frac{1}{\sqrt{2}}\right)\right) = \left(1 + \frac{1}{\sqrt{2}}, 1\right)$$

and $$\left(1 - \frac{1}{\sqrt{2}}, -2\left(1 - \frac{1}{\sqrt{2}}\right)^2 + 4\left(1 - \frac{1}{\sqrt{2}}\right)\right) = \left(1 - \frac{1}{\sqrt{2}}, 1\right)$$

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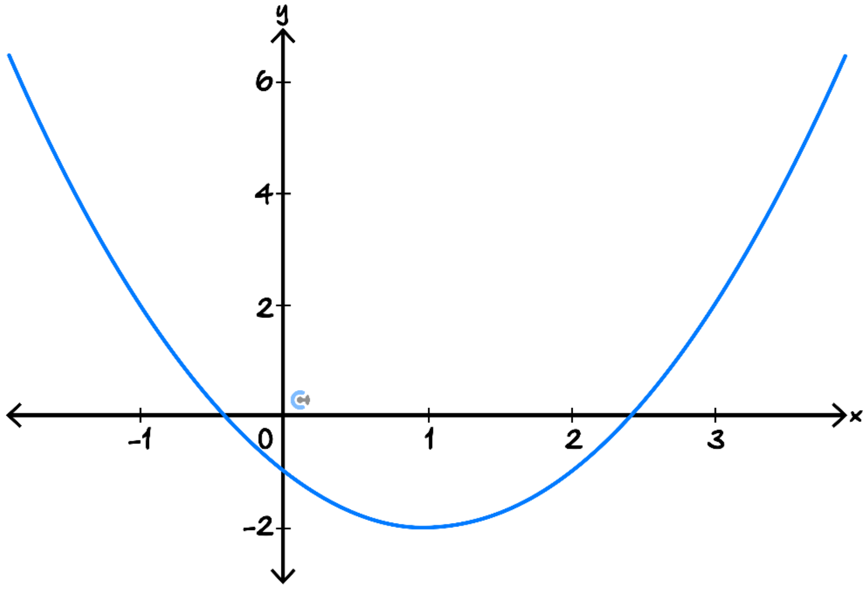
Sub-Section: Exam 2 Questions

Question 18 (1 mark)

The graph of a parabola is drawn below:



Which of the following could be a rule for the parabola?

A. $y = x^2 - 2x - 1$

B. $y = 2x^2 - 2x$

C. $y = -x^2 + 2x + 1$

D. $y = x^2 - 2x$

Question 19 (1 mark)

The number of points required to uniquely determine the equation of any parabola is:

A. 1

B. 2

C. 3

D. 4

Question 20 (1 mark)

The equation $x(x - p) = 1 - p$ has two real solutions when:

A. $p > 2$

B. $-2 < p^2$

C. $p < -2$ or $p > 2$

D. $p^2 + 4p - 4 > 0$

Question 21 (1 mark)

Let $y = 2x^2 - 4x - 3$. If $-1 < x < 4$, the possible values of y are:

A. $-5 < y \leq 13$

B. $-5 \leq y < 13$

C. $3 < y < 13$

D. $-5 < y < 3$

Question 22 (1 mark)

Find all values of k, such that $x^2 + (k-1)x + \frac{k^2 - 4k - 1}{4}$ has two real solutions for x, one positive and one negative.

A. $k < 4$

B. $k > -1$

C. $k \leq -1$ or $k \geq 4$

D. $-1 < k < 4$

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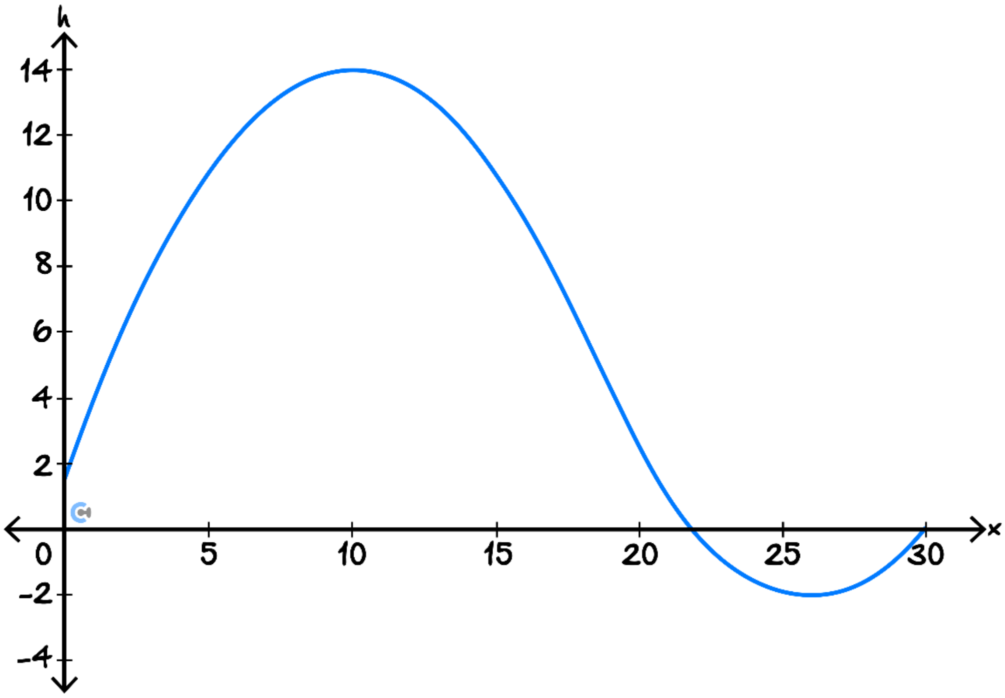
Question 23 (9 marks)

Solar Park is creating a new roller coaster. The cross-section of the first 30 metres is shown below. All distances are in metres.



The height h, of the roller coaster above the ground is related to the horizontal distance travelled along the roller coaster with the following relation.

$$h = \begin{cases} a(x-h)^2 + k & 0 \le x < 18 \\ bx^2 + cx + d & 18 \le x \le 30 \end{cases}$$

a. The roller coaster trip starts 1.5 metres above ground level, and 10 metres into the trip, the roller coaster reaches its peak of 14 metres above ground level. Find a, h and k. (2 marks)

We are given the turning point of $(10, 14)$, thus $h = 10$ and $k = 14$.
Now we solve for a using the fact that when $x = 0$, the height is equal to 1.5. Thus,

$$\frac{3}{2} = a(0-10)^2 + 14 \implies a = -\frac{1}{8}$$

b. 26 metres into the trip, the roller coaster reaches its lowest point of 2 metres below ground level.
Show that $b = \frac{1}{8}, c = -\frac{13}{2}$ and $d = \frac{165}{2}$. (2 marks)

Since 26 meters is greater than 18 meters we are looking at the second part of the roller coaster. This will have an equation of $h = b(x-26)^2 - 2$.
Since the roller coaster joins at $x = 18$, we know that when $x = 18$, h is equal to $-\frac{1}{8}(18-10)^2 + 14 = 6$.
From here we can solve for b as $b(18-26)^2 - 2 = 6 \implies 64b = 8 \implies b = \frac{1}{8}$.
Now we simply expand our expression to get,

$$\frac{1}{8}x^2 - \frac{13}{2}x + \frac{165}{2}$$

Hence $b = \frac{1}{8}$, $c = -\frac{13}{2}$ and $d = \frac{165}{2}$.

c. At what point(s) is the roller coaster exactly 2 metres above ground level? (2 marks)

For the first part of the roller coaster, we solve $-\frac{1}{8}(x-10)^2+14 = 2$ to get $x = 10\pm4\sqrt{6}$. We will only accept the solution $x = 10-4\sqrt{6}$ as the other solution is not in the interval $[0, 18]$. Thus one point is $(10-4\sqrt{6}, 2)$.
For the second part of the roller coaster, we solve $\frac{1}{8}x^2 - \frac{13}{2}x + \frac{165}{2} = 2$ to get $x = 26 \pm 4\sqrt{2}$. We will only accept the solution $x = 26 - 4\sqrt{2}$ as the other solution is not in the interval $[18, 30]$. Thus the other point is $(26-4\sqrt{2}, 2)$.

d. What percentage of the first 30 metres of the roller coaster is above ground? (2 marks)

The roller coaster is only below ground in the second section.
We solve $\frac{1}{8}x^2 - \frac{13}{2}x + \frac{165}{2} = 0$ to get $x = 22, 30$.
From the picture we thus see that the roller coaster spends 8 meters under ground.
Thus $\frac{30-8}{30} \times 100\% = \frac{220}{3}\%$ of the roller coaster is above ground.

e. A section of a roller coaster trip is considered "fully underground" if it is more than 1 metre below ground level.

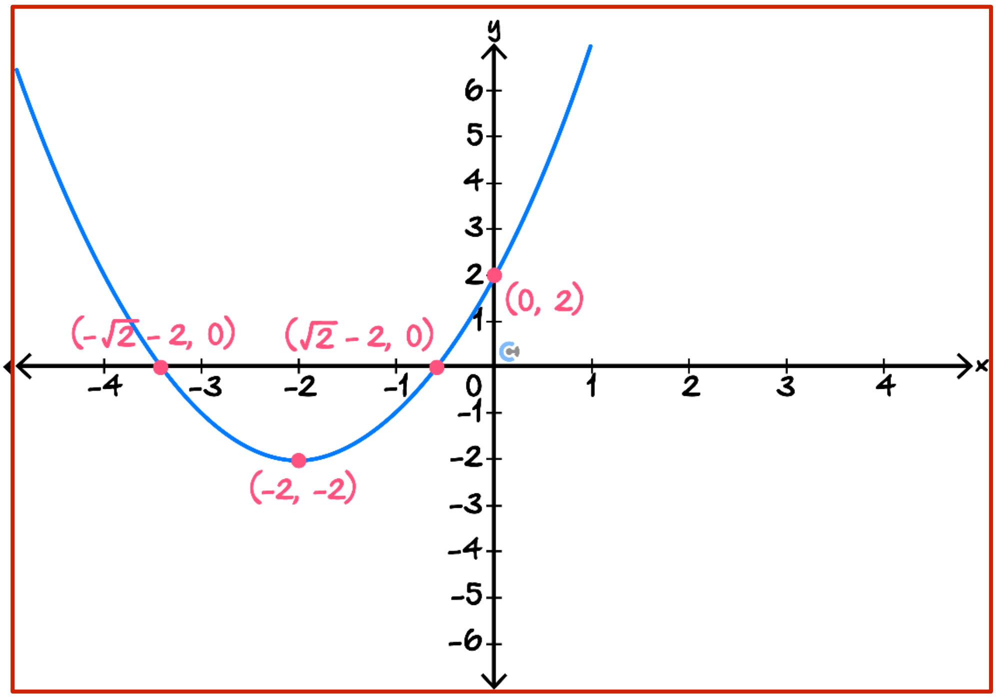
How many metres, correct to 2 decimal places, out of the first 30 metres of the roller coaster trip fully underground? (2 marks)

Solve $\frac{1}{8}x^2 - \frac{13}{2}x + \frac{165}{2} = -1$.
Our two solutions are $x = 23.1716, 28.828$.
Thus the roller coaster spends $28.828 - 23.172 = 5.66$ meters fully under ground.

Question 24 (10 marks)

Consider the family of parabolas $g_a(x) = (x - a)^2 + a$.

a. Sketch the graph of $y = g_a(x)$ when $a = -2$, labelling all key points with their coordinates. (3 marks)



b.

i. For which values of a does the equation $g_a(x) = 0$ have no real solutions? (1 mark)

$g_a(x) = (x-a)^2 + a = 0 \implies (x-a)^2 = -a$.
This is only possible if $a \leq 0$.
Thus the equation $g_a(x)$ has no real solutions if $a > 0$.

ii. Find all solutions to the equation $g_a(x) = 0$ for x. (2 marks)

$$g_a(x) = (x-a)^2 + a = 0$$
$$\implies (x-a)^2 = -a$$
$$\implies x - a = \pm\sqrt{-a}$$
$$\implies x = a \pm \sqrt{-a}$$

c. Let $f(x) = 4x - x^2$.

i. Solve $f(x) > 0$ for x. (1 mark)

f is a negative parabola with roots of 0 and 4. Thus $f(x) > 0$ if $0 < x < 4$.

ii. For what values of a is the solution to $f\big(g_a(x)\big) > 0$ an interval? (1 mark)

From the previous part, $f(g_a(x)) > 0 \implies 0 < g_a(x) < 4$.
For the solution of this to exist we require $a \leq 4$.
For the solution of this to be an interval, we require that $g_a(x)$ never dips to or below 0, hence $a > 0$. Also if $a = 4$ our solution is simply one number, which is not an interval, hence $0 < a < 4$.

iii. For a value of a, the solution to the equation $f\big(g_a(x)\big) > 0$ is $0 < x < b$.
Find the values of a and b correct to 3 decimal places. (2 marks)

The solution $f(g_a(x)) > 0$ is equivalent to $0 < g_a(x) < 4$.
Since our solution must be an interval, the graph of $g_a(x)$ cannot intersect the x-axis.
Since our solution is of the form $0 < x < b$, we require for $g_a(0) = 0$ or $g_a(0) = 4$.
The first case is true if $a = 0, -1$, both of which would make our solution not an interval, thus we must consider $g_a(0) = 4$.
This is true if $a = 1.562$ or $a = -2.562$, of which only the first solution is viable.
Thus $a = -2.562$.
Now we simply solve $g_a(x) = 4$ for $a = -2.562$ to get $x = 3.123$. This is our desired value of b.

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Section B: Supplementary Questions

Sub-Section [1.4.1]: Find Turning Point Form Using Turning Points

Question 25

Find the turning point of the parabola $y = 2(x-1)^2 + 3$.

$(1, 3)$

Question 26

Find the equation of a parabola that has a turning point at $(5, 3)$ and has a y-axis intercept of 8.

From the turning point, $y = a(x-5)^2 + 3$.
From the y-axis intercept, we know that if $x = 0$, then $y = 8$, thus,

$$8 = a(0-5)^2 + 3 \implies 5 = 25a \implies a = \frac{1}{5}$$

Hence the equation of the parabola is $y = \frac{1}{5}(x-5)^2 + 3$

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Question 27

Find the turning point of the parabola $y = 2x^2 - 4x + 5$.

We complete the square to get,

$$y = 2(x-1)^2 + 3$$

Thus the parabola has a turning point of $(1, 3)$.

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Sub-Section [1.4.2]: Apply Quadratics to Model a Scenario

Question 28

A ball is thrown up into the air from a height of 1 metre. It reaches its maximum height of 2 metres after 1 second. The height in metres of the ball h, t seconds after the ball is launched is:

$$h(t) = a(t-1)^2 + 2$$

Find the value of a.

We know that $h(0) = 1$.
Thus $1 = a(0-1)^2 + 2 = a + 2 \implies a = -1$.

Question 29

A parabola-shaped bridge is used to cross a long river. The height of the bridge above the water level in metres, h, is a quadratic function of the horizontal distance of a point of a bridge from the starting river bank, x.

At the starting river bank, the height of the bridge is 2 metres above water level, and 5 metres away from the starting point ($x = 5$), the bridge is at its highest point, 6 metres above the water level ($h = 6$).

Relate x and h.

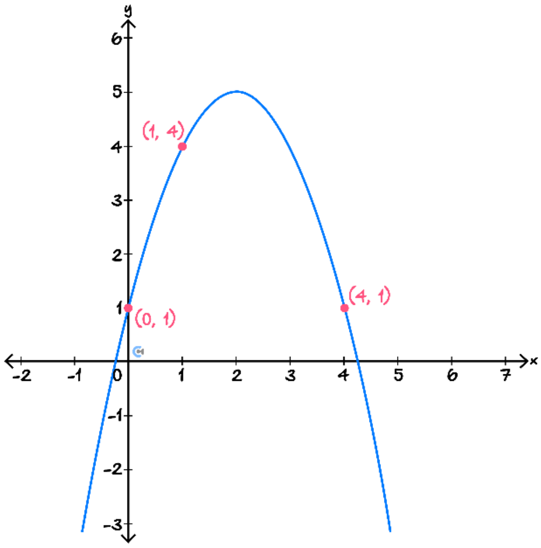
From the highest point we know that $h = a(x-5)^2 + 6$.
We can solve for a by using the fact that $h(0) = 2$, thus,

$$2 = 25a + 6 \implies a = -\frac{4}{25}$$

Thus, $h = -\frac{4}{25}(x-5)^2 + 6$

Question 30

A river passes through 3 points in a park as shown below:



Where the x-axis represents the position due east from the centre of the park, and the y-axis represents the position due north from the centre of the park. We can relate the north position (y) of the river to the east position (x) of the river through the equation:

$$y = ax^2 + bx + c$$

Find the values of a, b and c.

When $x = 0$ we know that $y = 0a + 0b + c = 1$. Hence $c = 1$.
For the other two values we can create a pair of simultaneous equations using the points $(1, 4)$ and $(4, 1)$.

$$4 = a + b + 1 \quad (1)$$
$$1 = 16a + 4b + 1 \quad (2)$$

We subtract $4 \times (1)$ from (2) to get, $-15 = 12a - 3 \implies -12 = 12a \implies a = -1$.
Substituting this back into (1) yields $4 = b \implies b = 4$

Sub-Section [1.4.3]: Apply Family of Functions to Find an Unknown of Function

Question 31

Consider the parabola $y = kx^2 - 6$. Find the value(s) of k such that the horizontal distance between x-axis intercepts of the parabola is less than 4.

We solve $y = 0$ to get $6 = kx^2 \implies x = \pm\sqrt{\frac{6}{k}}$.

Thus the horizontal distance between the x-axis intercepts is $2\sqrt{\frac{6}{k}}$.

We require this quantity to be < 4, thus,

$$\begin{aligned} 2\sqrt{\frac{6}{k}} &< 4 \\ \implies \frac{24}{k} &< 16 \\ \implies k &> \frac{3}{2} \end{aligned}$$

Question 32

Let $y = x^2 + 4kx - 1$. Find the values of k such that $y \geq -2$ for all x.

We complete the square to get, $y = (x + 2k)^2 - 4k^2 - 1$.

Thus $y \geq -4k^2 - 1$.

For $y \geq -2$ for all x we simply require $-4k^2 - 1 \geq -2 \implies 4k^2 \leq 1 \implies \frac{-1}{2} < k < \frac{1}{2}$

Question 33

Find all values of k such that the equation $(x - k - 1)^2 - 4 = k$ has two real solutions for x, one positive and one negative.

The solutions to the equation $(x-k-1)^2 - 4 = k$ are,

$$x = k + 1 \pm \sqrt{4+k}$$

Since $k + 1 + \sqrt{4+k} \geq k + 1 - \sqrt{4+k}$ we require,

$$k + 1 + \sqrt{4+k} > 0 \quad \text{and} \quad k + 1 - \sqrt{4+k} < 0$$

We rearrange the first inequality to become $k + 4 + \sqrt{4+k} - 3 > 0$. Now we substitute $a = \sqrt{4+k}$ to get the quadratic inequality, $a^2 + a - 3 > 0$.

We solve the equality $a^2 + a - 3 = 0$ to get $a = \frac{1 \pm \sqrt{13}}{2}$.

Since $a^2 + a - 3$ is a positive parabola, the solution to our inequality is $a > \frac{-1+\sqrt{13}}{2}$ or $a < \frac{-1-\sqrt{13}}{2}$.

However since $a > 0$ our only solution is $a > \frac{-1+\sqrt{13}}{2}$, hence $4 + k > \frac{14 - 2\sqrt{13}}{4} \implies k > \frac{-\sqrt{13}-1}{2}$.

We do something similar for $k + 1 - \sqrt{4+k} < 0$, again substituting $a = \sqrt{4+k}$ to get the quadratic inequality, $a^2 - a - 3 > 0$.

After taking into account domain restrictions, the solution to this inequality is $0 < a < \frac{1+\sqrt{13}}{2}$.

Hence $-4 < k < \frac{\sqrt{13}-1}{2}$.

From here we combine these two restrictions to get $\frac{-\sqrt{13}-1}{2} k < \frac{\sqrt{13}-1}{2}$.

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Sub-Section [1.4.4]: Harder Quadratic Inequalities

Question 34

Solve $x(x+3) > 4$ for x.

We rearrange the equality to become $x^2+3x-4=(x-1)(x+4)>0$.
Since x^2+3x-4 is a positive parabola with roots of $-4, 1$ we see that it is greater than 0 if $x<-4$ or $x>1$.

Question 35

Solve $1+\frac{2}{x-2} \leq \frac{5}{(x-2)^2}$ for x.

We multiply both sides of our equation by $(x-2)^2$ to get, $(x-2)^2+2(x-2)-5 \leq 0$.
Substituting $a = x-2$ we get the inequality $a^2+2a-5 \leq 0$.
We first solve this as an equality to get $a = \frac{-2 \pm \sqrt{4-20}}{2} = -1 \pm \sqrt{6}$.
As a^2+2a-5 is a positive parabola with roots of $-1 \pm \sqrt{6}$ we see that it is ≤ 0 if $-1-\sqrt{6} \leq a \leq -1+\sqrt{6}$.
In terms of x, $1-\sqrt{6} \leq x \leq 1+\sqrt{6}$.
As in our original equation we divide by $x-2$, x cannot be 2 thus our solution is,

$$1-\sqrt{6} \leq x < 2 \quad \text{or} \quad 2 < x \leq 1+\sqrt{6}$$

Question 36

Solve $(x^2 + 2)^2 - 4 \geq 8x^2$ for x.

First we get everything in terms of $x^2 + 2$ to substitute $a = x^2 + 2$. Thus our inequality becomes,

$$(x^2 + 2)^2 - 8(x^2 + 2) + 12 \geq 0$$

After substituting $a = x^2 + 2$ we need to solve $a^2 - 8a + 12 \geq 0$.
As $a^2 - 8a + 12 = (a - 6)(a - 2)$ is a positive parabola with roots of 2 and 6, our inequality reduces to $a \leq 2$ or $a \geq 6$.
Since $a = x^2 + 2$, the only way $a \leq 2$ is if $x = 0$.
And $x^2 + 2 \geq 6 \implies x^2 \geq 4$, thus $x \leq -2$ or $x \geq 2$. Combining all cases yields,

$$x \leq -2 \quad \text{or} \quad x = 0 \quad \text{or} \quad x \geq 2$$

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Sub-Section: Exam 1 Questions

Question 37 (4 marks)

a. For what values of x is $x^2 - 7x + 12 > 0$? (2 marks)

$x < 3$ or $x > 4$

b. For what values of x is $1 - \frac{1}{x} - \frac{12}{x^2} > 0$? (2 marks)

$x < -3$ or $x > 4$

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Question 38 (3 marks)

The sum of the age of a son and his father is 35 years and the product is 150. Find their ages.

5 and 30

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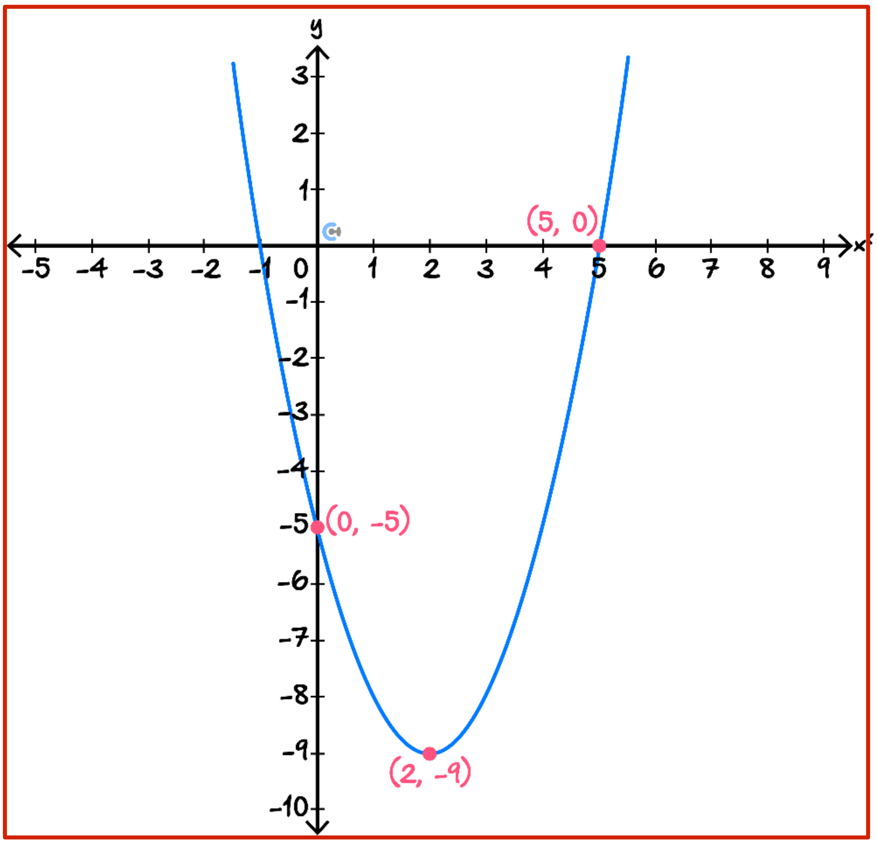
Question 39 (4 marks)

Consider the function $f(x) = x^2 - 4x - 5$.

a. Solve the equation $f(x) = 0$. (1 mark)

$x = -1, 5$

b. Sketch the graph of $y = f(x)$ on the axes below. Label the turning point and all axes intercept with coordinates. (2 marks)



c. Hence, find the value(s) of x such that $f(x) + 5 < 0$. (1 mark)

$0 < x < 4$

Question 40 (2 marks)

Solve the inequality $x^2 - 6x - 7 \leq 0$.

$-1 \leq x \leq 7$

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Question 41 (3 marks)

Consider the function $f(x) = kx^2 - 4x + 6$, where k is a real number. Find all possible values of k if $f(x)$ is always greater than 1.

$$k > \frac{4}{5}$$

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Question 42 (5 marks)

Consider the function $f(x) = x^2 - kx - 4$, where k is a real number.

a. Show that the graph $y = f(x)$ always has two x-intercepts. (1 mark)

Consider the discriminant for $x^2 - kx - 4 = 0$

$\Delta = k^2 + 16 > 0$

Therefore, must have two x-intercepts.

b. Find the values of k such that the distance between the two x-intercepts are less than 6. (3 marks)

$-2\sqrt{5} < k < 2\sqrt{5}$

c. Find the minimum possible distance between the two x-intercepts. (1 mark)

Minimum distance of 4 when $k = 0$.

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Sub-Section: Exam 2 Questions

Question 43 (1 mark)

The equation $2x^2 + 2(p+1)x + p = 0$, where p is real, always has roots that are:

A. Equal.

B. Equal in magnitude but opposite in sign.

C. Irrational.

D. Real.

Question 44 (1 mark)

If $px^2 + 3x + q = 0$ has two roots $x = -1$ and $x = -2$, the value of $q - p$ is:

A. -1

B. 1

C. 2

D. -2

Question 45 (1 mark)

The sum of the areas of two squares is $468\ m^2$. If the difference of their perimeters is $24\ m$, then the sides of the two squares are:

A. $18\ m, 14\ m$

B. $13\ m, 12\ m$

C. $18\ m, 12\ m$

D. None of these.

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Question 46 (1 mark)

The value of p so that the quadratic equation $x^2 + 5px + 16 = 0$ has no real roots:

A. $p > 8$

B. $p < 5$

C. $-\frac{8}{5} < p < \frac{8}{5}$

D. $-\frac{8}{5} \leq p < 0$

Question 47 (1 mark)

The quadratic equation whose roots are $a, \frac{1}{a}$ is:

A. $ax^2 - (a^2 + 1)x + a = 0$

B. $ax^2 - (a^2 - 1)x + a = 0$

C. $ax^2 - (a^2 - 1)x - a = 0$

D. None of these.

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Question 48 (11 marks)

Consider the quadratic function $f(x) = 3x^2 + 5x - 2$.

a.

i. Solve the equation $f(x) = 0$. (2 marks)

$x = -2, \frac{1}{3}$

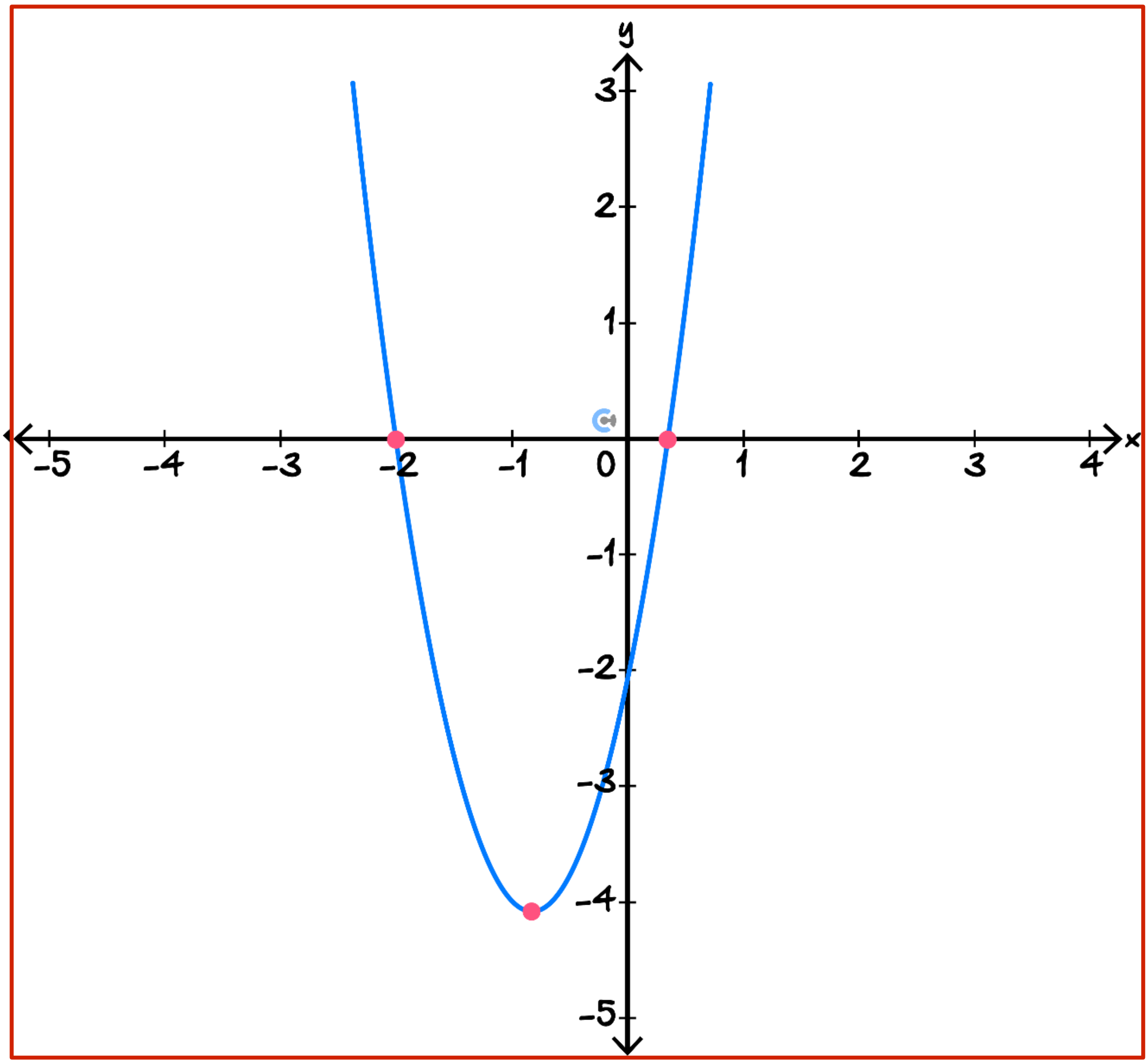
ii. Find the turning point of the graph of $y = f(x)$. (1 mark)

$\left(-\frac{5}{6}, -\frac{49}{12}\right)$

iii. Find the y-intercept of the graph of $y = f(x)$. (1 mark)

$(0, -2)$

b. Sketch the graph of $y = f(x)$ on the axes below.



c. The graph of $y = f(x)$ is translated 1 unit to the left and now has the equation:

$$y = ax^2 + bx + c, \quad a, b, c \in \mathbb{R}$$

Determine the values of a, b, c. (2 marks)

$y = 3x^2 + 11x + 6$; $a = 3, b = 11, c = 6$

d. Consider the graph of the function $g(x) = 3x^2 + kx + 4$. Find the value(s) of k for which the equation $g(x) = 0$ will have:

i. No real root. (1 mark)

$-4\sqrt{3} < k < 4\sqrt{3}$

ii. Equal roots. (1 mark)

$k = \pm 4\sqrt{3}$

iii. Unique real roots. (1 mark)

$k < -4\sqrt{3}$ or $k > 4\sqrt{3}$

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Question 49 (9 marks)

A cricket player hits a ball, and the ball's trajectory is modelled by the quadratic equation:

$$h(x) = ax^2 + bx + c$$

Where $h(x)$ is the height of the ball (in metres) above the ground, and x is the horizontal distance (in metres) from where the ball was hit.

The following conditions are given:

- The ball is hit from a height of 1.5 metres, i.e., $h(0) = 1.5$.
- The ball reaches a height of 20 metres when it has travelled 10 metres horizontally.
- The ball reaches a height of 35 metres when it has travelled 20 metres horizontally.

a. Using the given conditions, set up and solve a system of equations to determine the values of a, b, and c. (3 marks)

$$a = -\frac{7}{400}, b = \frac{81}{40}, c = \frac{3}{2}$$

b. Determine the maximum height that the ball reaches. Give your answer correct to 2 decimal places. (1 mark)

60.08

c. Determine the horizontal distance the ball has travelled when its height is 15 metres. Provide both possible values of x correct to two decimal places. (2 marks)

Solve $h(x) = 15$

$x = 7.10, 108.61$ metres

d. Find the exact height, where the ball has travelled 30 metres horizontally between the two times that it reaches this height. (3 marks)

$\frac{393}{7}$ metres

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