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VCE Mathematical Methods ½
Quadratics Exam Skills [1.4]
Homework

Homework Outline:

Compulsory Questions	Pg 2 – Pg 22
Supplementary Questions	Pg 23 – Pg 43

Section A: Compulsory Questions

Sub-Section [1.4.1]: Find Turning Point Form Using Turning Points

Question 1

Find the turning point of the parabola $y = 3(x - 2)^2 + 5$.

Question 2

Find the equation of a parabola that has a turning point at $(3,4)$ and has a y-axis intercept of -5.

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Question 3

Find the turning point of the parabola, $y = -2x^2 - 4x + 4$.

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Sub-Section [1.4.2]: Apply Quadratics to Model a Scenario

Question 4

A ground-based contraption flings a ball straight up into the air. 4 seconds later, the ball lands on the ground. The height in metres of the ball h, t seconds after the ball is launched is:

$$h(t) = -10t^2 + bt + c$$

Find the values of b and c.

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Question 5

A parabola-shaped bridge is used to cross a long river. The height of the bridge above the water level in metres, h, is a quadratic function of the horizontal distance of a point of a bridge from the starting river bank, x.

At the starting river bank, the height of the bridge is 1 metre above water level, and 3 metres away from the starting point ($x = 3$), the bridge is at its highest point, 4 metres above water level ($h = 4$).

Relate x and h.

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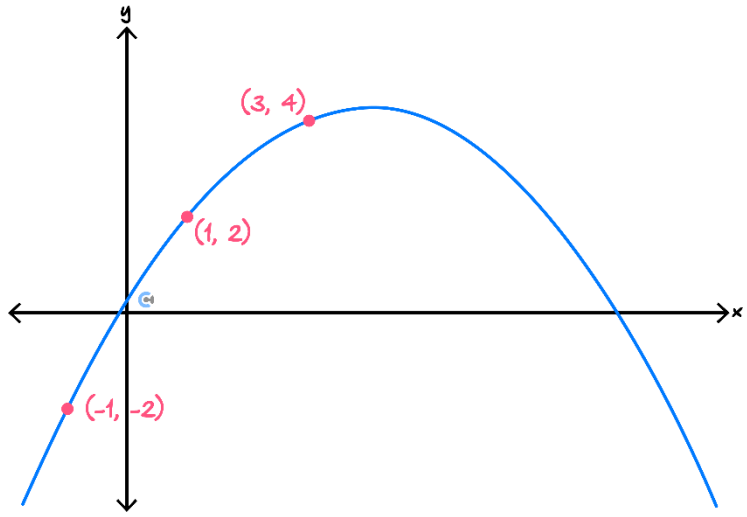
Question 6

A river passes through 3 points in a park as shown below:



Where the x-axis represents the position due east from the centre of the park, and the y-axis represents the position due north from the centre of the park.

We can relate the north position (y) of the river to the east position (x) of the river through the equation:

$$y = ax^2 + bx + c$$

Find the values of a, b and c.

Sub-Section [1.4.3]: Apply Family of Functions to Find an Unknown of Function

Question 7

Consider the parabola $y = x^2 + 2x - 4k^2 + 1$.
Find the value of k such that the horizontal distance between x-axis intercepts of the parabola is less than 8.

Question 8

Let $y = x^2 + 2kx - 2$.
Find the values of k such that $y \geq -6$ for all x.

Question 9

Find all values of k such that the equation $\left(x - k\ -\frac{1}{2}\right)^2 = -k$ has two real solutions for x, one positive and one negative.

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Sub-Section [1.4.4]: Harder Quadratic Inequalities

Question 10

Solve $x(x+1) > 12$ for x.

Question 11

Solve $1 + \frac{3}{x-1} \leq \frac{10}{(x-1)^2}$ for x.

Question 12

Solve $(x^2 + 1)^2 + 3 \geq 7x^2$ for x.

Hint: Break this down into 2 quadratic inequalities.

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Sub-Section: Exam 1 Questions

Question 13 (4 marks)

a. Solve $6 - \frac{5}{x} = x$ for $x \in \mathbb{R}$. (2 marks)

b. Solve $6 - \frac{5}{x} < x$ for $x > 0$. (2 marks)

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Question 14 (3 marks)

Solve the equation $x^4 - 8x^2 - 9 = 0$, for real values of x.

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Question 15 (3 marks)

To abate Alex's loneliness, Contour is building a parabolic bridge from their CBD office to their Box-Hill office.

The distance between the two offices is 14 kilometres.

The height, h on the bridge in metres, x kilometres away from the CBD office satisfies the following equation:

$$h = ax^2 + bx + c$$

Whilst walking on the bridge, when Alex is 2 kilometres away from the CBD office, he is 4 metres above ground level.

a. Find the values of a, b and c. (2 marks)

b. What is the maximum height of the bridge above the ground? (2 marks)

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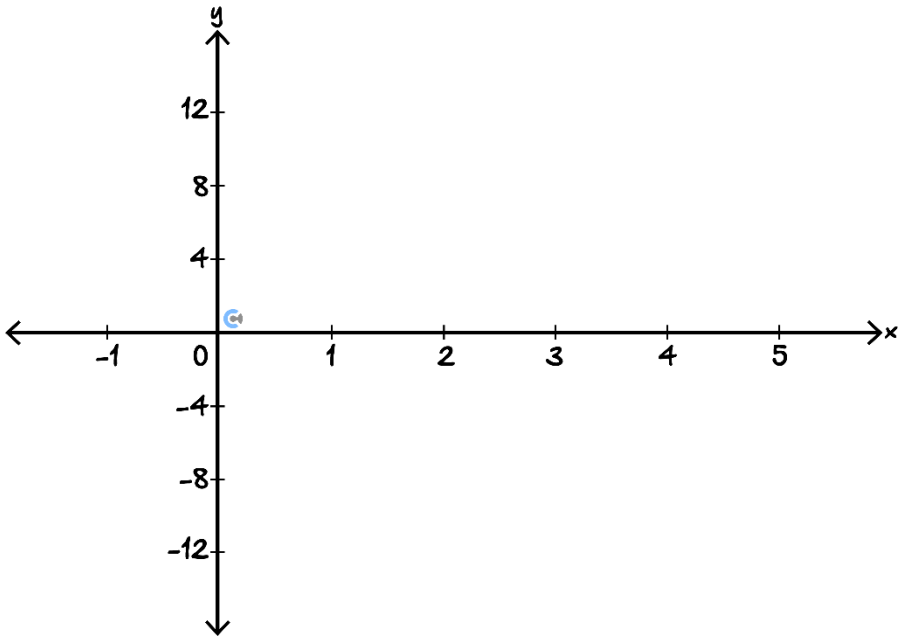
Question 16 (4 marks)

a. Sketch the graph of $y = 2x^2 - 8x + 11$ on the axis below, label all key points with their coordinates. (2 marks)



b. The line $y = k$ has a minimum vertical distance of 2 from the above parabola.
Find the value of k. (1 mark)

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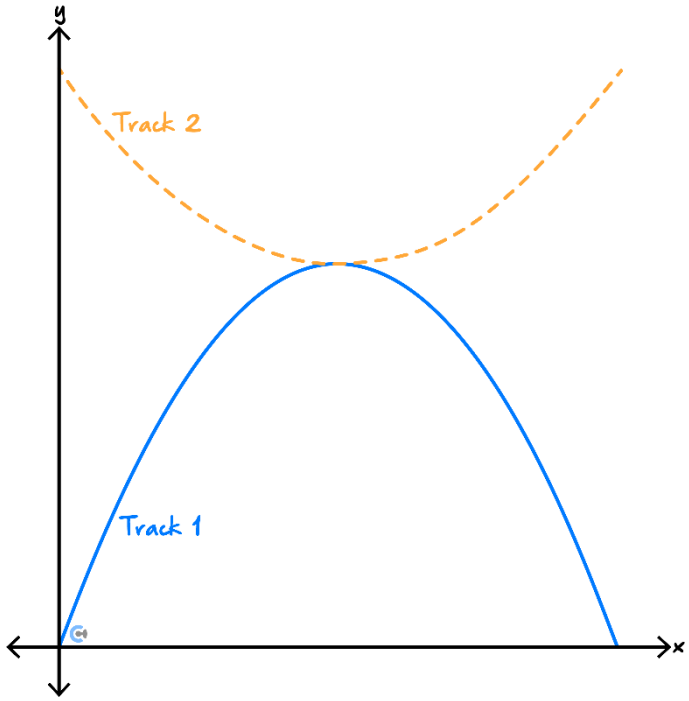
Question 17 (5 marks)

Sam is standing at the start of the walking track 1. Relative to his current position, the coordinates of two walking tracks are drawn below, with the x-ordinate representing kilometres due east and the y-ordinate representing kilometres due north.



Track 1 is described by the function $y = -2x^2 + ax$.
Track 2 is described by the equation $y = b(x-h)^2 + k$
The unit of length is kilometres.

a. Track 1 ends 2 kilometres due east of Sam's current position. Show that $a = 4$. (1 mark)

b. Track 2 has the same turning point as track one and starts 6 kilometres due north of Sam's current position. Find the values of b, h and k. (1 mark)

c. At what point(s) on track 1, is the vertical distance to track 2 equal to 3 kilometres. (2 marks)

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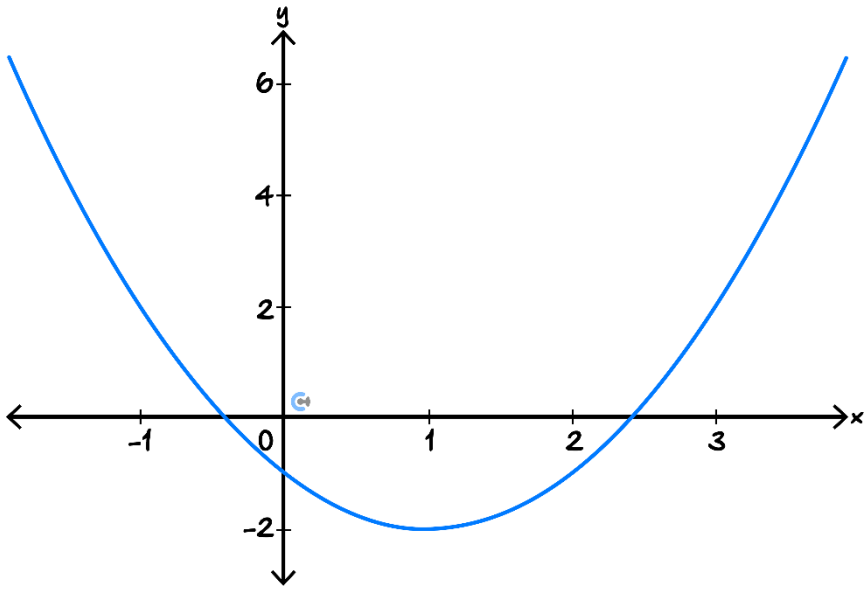
Sub-Section: Exam 2 Questions

Question 18 (1 mark)

The graph of a parabola is drawn below:



Which of the following could be a rule for the parabola?

A. $y = x^2 - 2x - 1$

B. $y = 2x^2 - 2x$

C. $y = -x^2 + 2x + 1$

D. $y = x^2 - 2x$

Question 19 (1 mark)

The number of points required to uniquely determine the equation of any parabola is:

A. 1

B. 2

C. 3

D. 4

Question 20 (1 mark)

The equation $x(x - p) = 1 - p$ has two real solutions when:

A. $p > 2$

B. $-2 < p^2$

C. $p < -2$ or $p > 2$

D. $p^2 + 4p - 4 > 0$

Question 21 (1 mark)

Let $y = 2x^2 - 4x - 3$. If $-1 < x < 4$, the possible values of y are:

A. $-5 < y \leq 13$

B. $-5 \leq y < 13$

C. $3 < y < 13$

D. $-5 < y < 3$

Question 22 (1 mark)

Find all values of k, such that $x^2 + (k - 1)x + \frac{k^2 - 4k - 1}{4}$ has two real solutions for x, one positive and one negative.

A. $k < 4$

B. $k > -1$

C. $k \leq -1$ or $k \geq 4$

D. $-1 < k < 4$

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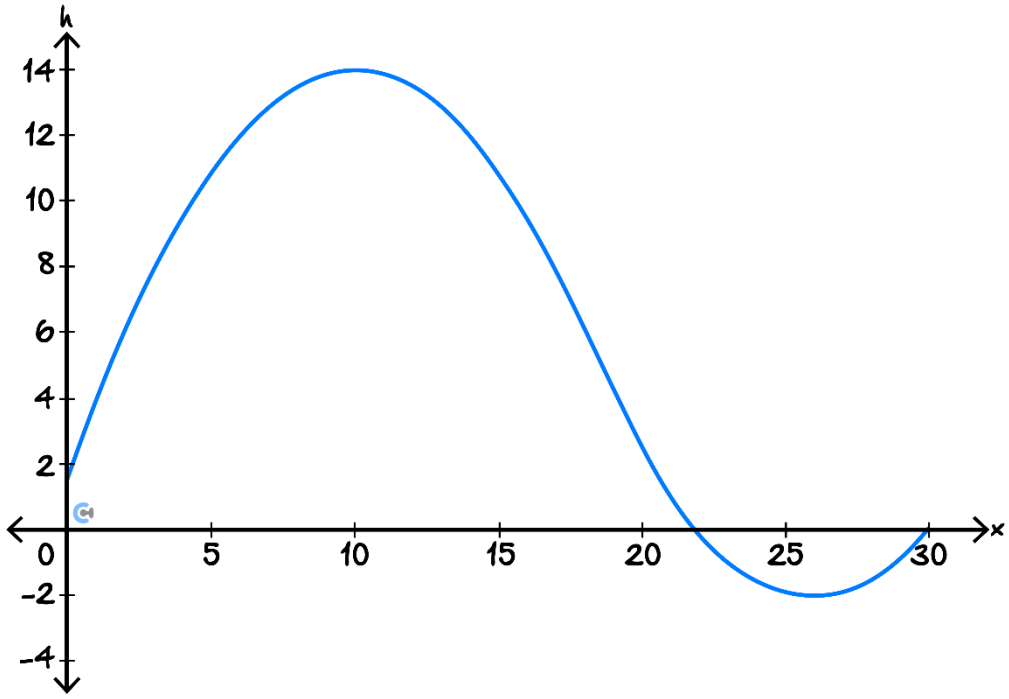
Question 23 (9 marks)

Solar Park is creating a new roller coaster. The cross-section of the first 30 metres is shown below. All distances are in metres.



The height h, of the roller coaster above the ground is related to the horizontal distance travelled along the roller coaster with the following relation.

$$h = \begin{cases} a(x-h)^2 + k & 0 \leq x < 18 \\ bx^2 + cx + d & 18 \leq x \leq 30 \end{cases}$$

a. The roller coaster trip starts 1.5 metres above ground level, and 10 metres into the trip, the roller coaster reaches its peak of 14 metres above ground level. Find a, h and k. (2 marks)

b. 26 metres into the trip, the roller coaster reaches its lowest point of 2 metres below ground level.
Show that $b = \frac{1}{8}$, $c = -\frac{13}{2}$ and $d = \frac{165}{2}$. (2 marks)

c. At what point(s) is the roller coaster exactly 2 metres above ground level? (2 marks)

d. What percentage of the first 30 metres of the roller coaster is above ground? (2 marks)

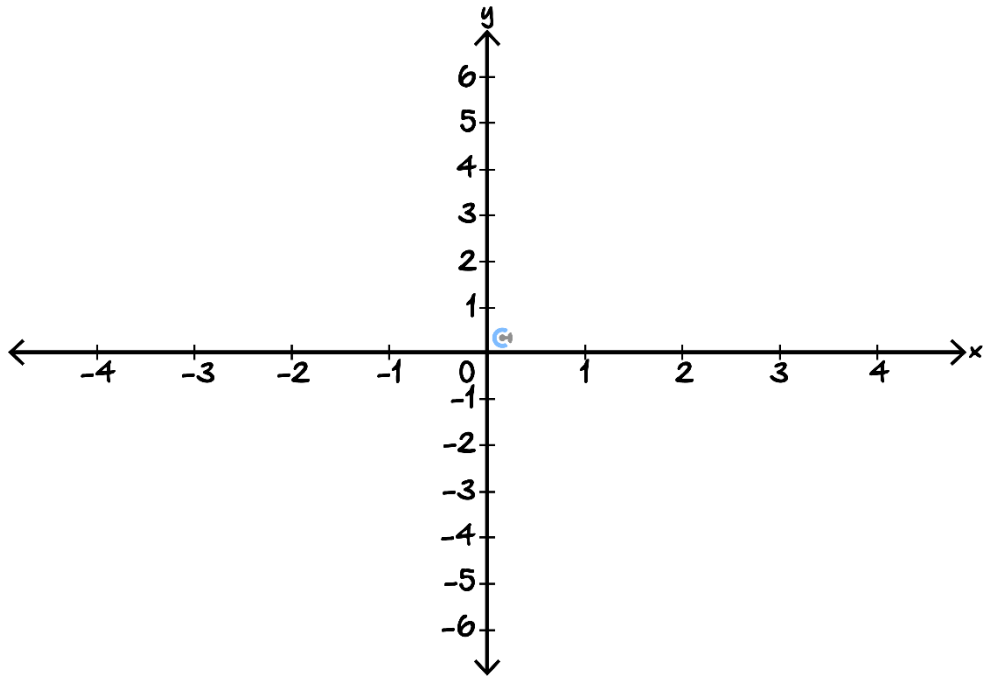
e. A section of a roller coaster trip is considered ”fully underground” if it is more than 1 metre below ground level.

How many metres, correct to 2 decimal places, out of the first 30 metres of the roller coaster trip fully underground? (2 marks)

Question 24 (10 marks)

Consider the family of parabolas $g_a(x) = (x - a)^2 + a$.

a. Sketch the graph of $y = g_a(x)$ when $a = -2$, labelling all key points with their coordinates. (3 marks)



b.

i. For which values of a does the equation $g_a(x) = 0$ have no real solutions? (1 mark)

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ii. Find all solutions to the equation $g_a(x) = 0$ for x. (2 marks)

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c. Let $f(x) = 4x - x^2$.

i. Solve $f(x) > 0$ for x. (1 mark)

ii. For what values of a is the solution to $f(g_a(x)) > 0$ an interval? (1 mark)

iii. For a value of a, the solution to the equation $f(g_a(x)) > 0$ is $0 < x < b$.
Find the values of a and b correct to 3 decimal places. (2 marks)

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Section B: Supplementary Questions

Sub-Section [1.4.1]: Find Turning Point Form Using Turning Points

Question 25

Find the turning point of the parabola $y = 2(x - 1)^2 + 3$.

Question 26

Find the equation of a parabola that has a turning point at $(5, 3)$ and has a y-axis intercept of 8.

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Question 27

Find the turning point of the parabola $y = 2x^2 - 4x + 5$.

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Sub-Section [1.4.2]: Apply Quadratics to Model a Scenario

Question 28

A ball is thrown up into the air from a height of 1 metre. It reaches its maximum height of 2 metres after 1 second. The height in metres of the ball h, t seconds after the ball is launched is:

$$h(t) = a(t-1)^2 + 2$$

Find the value of a.

Question 29

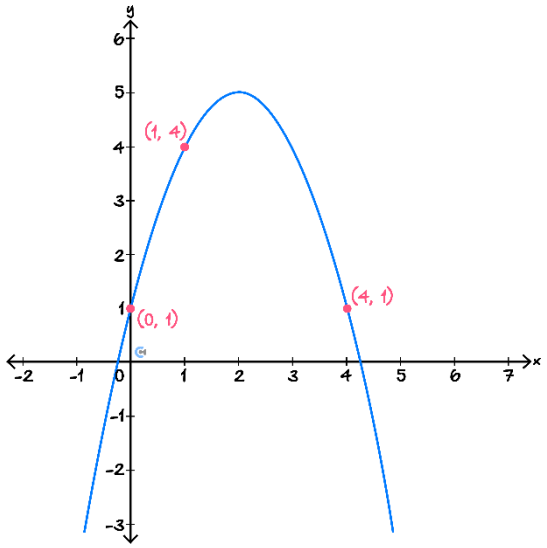
A parabola-shaped bridge is used to cross a long river. The height of the bridge above the water level in metres, h, is a quadratic function of the horizontal distance of a point of a bridge from the starting river bank, x.

At the starting river bank, the height of the bridge is 2 metres above water level, and 5 metres away from the starting point ($x = 5$), the bridge is at its highest point, 6 metres above the water level ($h = 6$).

Relate x and h.

Question 30

A river passes through 3 points in a park as shown below:



Where the x-axis represents the position due east from the centre of the park, and the y-axis represents the position due north from the centre of the park. We can relate the north position (y) of the river to the east position (x) of the river through the equation:

$$y = ax^2 + bx + c$$

Find the values of a, b and c.

Sub-Section [1.4.3]: Apply Family of Functions to Find an Unknown of Function

Question 31

Consider the parabola $y = kx^2 - 6$. Find the value(s) of k such that the horizontal distance between x-axis intercepts of the parabola is less than 4.

Question 32

Let $y = x^2 + 4kx - 1$. Find the values of k such that $y \geq -2$ for all x.

Question 33

Find all values of k such that the equation $(x - k - 1)^2 - 4 = k$ has two real solutions for x, one positive and one negative.

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Sub-Section [1.4.4]: Harder Quadratic Inequalities

Question 34

Solve $x(x + 3) > 4$ for x.

Question 35

Solve $1 + \frac{2}{x-2} \leq \frac{5}{(x-2)^2}$ for x.

Question 36

Solve $(x^2 + 2)^2 - 4 \geq 8x^2$ for x.

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Sub-Section: Exam 1 Questions

Question 37 (4 marks)

a. For what values of x is $x^2 - 7x + 12 > 0$? (2 marks)

b. For what values of x is $1 - \frac{1}{x} - \frac{12}{x^2} > 0$? (2 marks)

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Question 38 (3 marks)

The sum of the age of a son and his father is 35 years and the product is 150. Find their ages.

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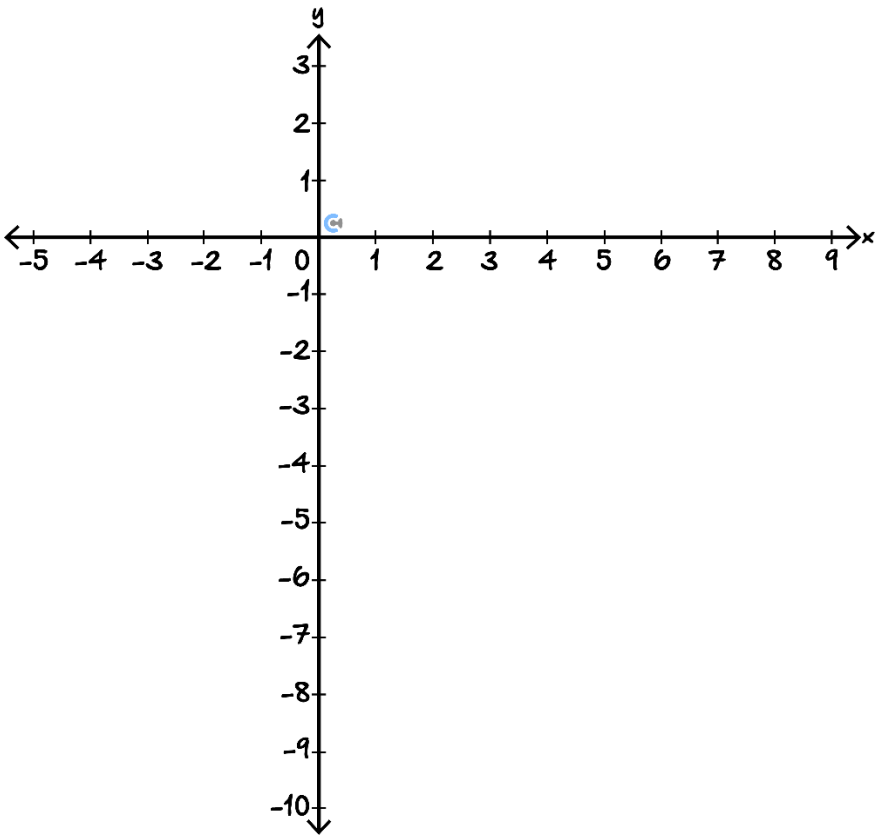
Question 39 (4 marks)

Consider the function $f(x) = x^2 - 4x - 5$.

a. Solve the equation $f(x) = 0$. (1 mark)

b. Sketch the graph of $y = f(x)$ on the axes below. Label the turning point and all axes intercept with coordinates. (2 marks)



c. Hence, find the value(s) of x such that $f(x) + 5 < 0$. (1 mark)

Question 40 (2 marks)

Solve the inequality $x^2 - 6x - 7 \leq 0$.

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Question 41 (3 marks)

Consider the function $f(x) = kx^2 - 4x + 6$, where k is a real number. Find all possible values of k if $f(x)$ is always greater than 1.

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Question 42 (5 marks)

Consider the function $f(x) = x^2 - kx - 4$, where k is a real number.

a. Show that the graph $y = f(x)$ always has two x-intercepts. (1 mark)

b. Find the values of k such that the distance between the two x-intercepts are less than 6. (3 marks)

c. Find the minimum possible distance between the two x-intercepts. (1 mark)

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Sub-Section: Exam 2 Questions

Question 43 (1 mark)

The equation $2x^2 + 2(p+1)x + p = 0$, where p is real, always has roots that are:

A. Equal.

B. Equal in magnitude but opposite in sign.

C. Irrational.

D. Real.

Question 44 (1 mark)

If $px^2 + 3x + q = 0$ has two roots $x = -1$ and $x = -2$, the value of $q - p$ is:

A. -1

B. 1

C. 2

D. -2

Question 45 (1 mark)

The sum of the areas of two squares is $468\ m^2$. If the difference of their perimeters is $24\ m$, then the sides of the two squares are:

A. $18\ m, 14\ m$

B. $13\ m, 12\ m$

C. $18\ m, 12\ m$

D. None of these.

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Question 46 (1 mark)

The value of p so that the quadratic equation $x^2 + 5px + 16 = 0$ has no real roots:

A. $p > 8$

B. $p < 5$

C. $-\frac{8}{5} < p < \frac{8}{5}$

D. $-\frac{8}{5} \leq p < 0$

Question 47 (1 mark)

The quadratic equation whose roots are $a, \frac{1}{a}$ is:

A. $ax^2 - (a^2 + 1)x + a = 0$

B. $ax^2 - (a^2 - 1)x + a = 0$

C. $ax^2 - (a^2 - 1)x - a = 0$

D. None of these.

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Question 48 (11 marks)

Consider the quadratic function $f(x) = 3x^2 + 5x - 2$.

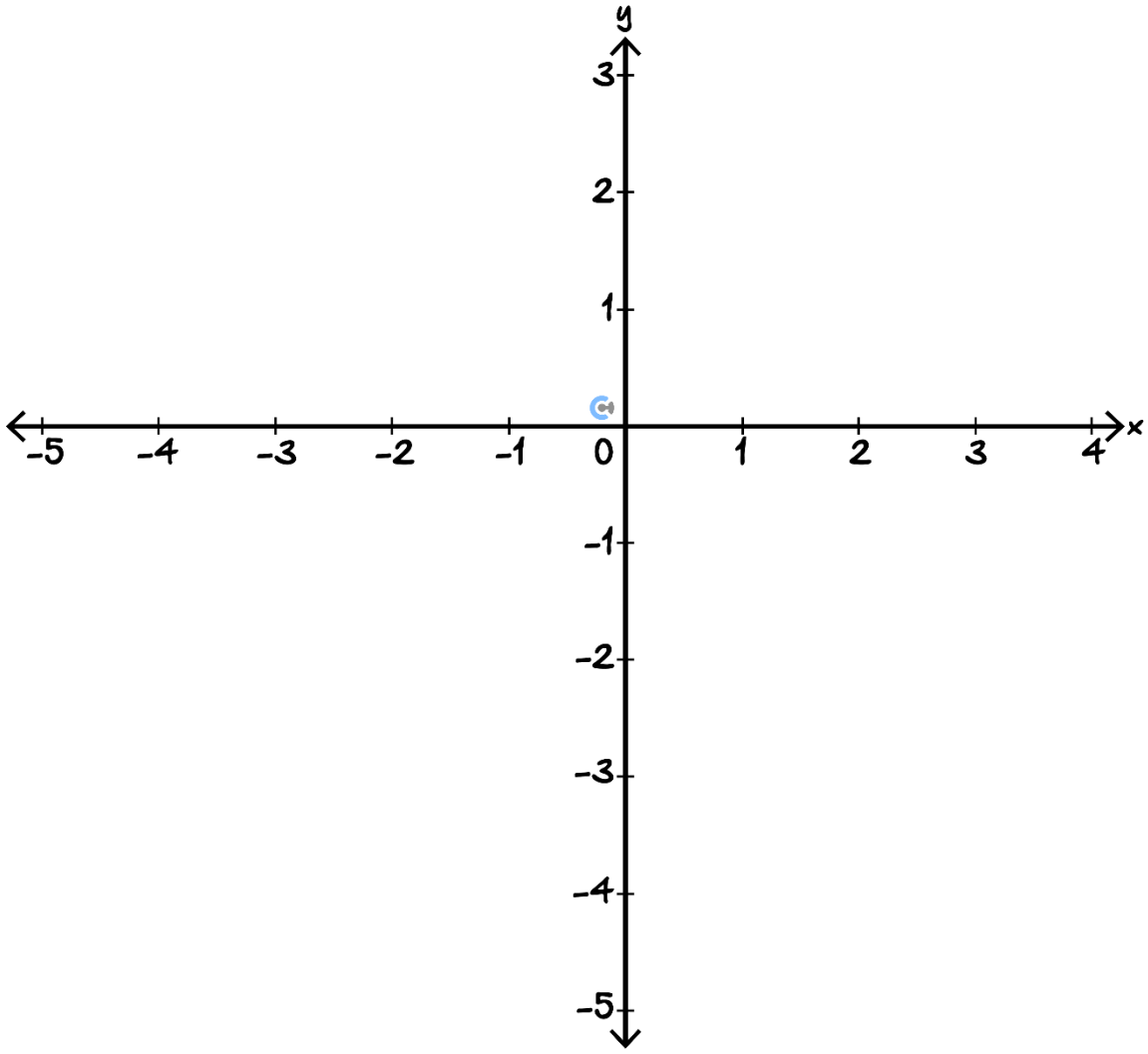
a.

i. Solve the equation $f(x) = 0$. (2 marks)

ii. Find the turning point of the graph of $y = f(x)$. (1 mark)

iii. Find the y-intercept of the graph of $y = f(x)$. (1 mark)

b. Sketch the graph of $y = f(x)$ on the axes below.



c. The graph of $y = f(x)$ is translated 1 unit to the left and now has the equation:

$$y = ax^2 + bx + c, \quad a, b, c \in \mathbb{R}$$

Determine the values of a, b, c. (2 marks)

d. Consider the graph of the function $g(x) = 3x^2 + kx + 4$. Find the value(s) of k for which the equation $g(x) = 0$ will have:

i. No real root. (1 mark)

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ii. Equal roots. (1 mark)

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iii. Unique real roots. (1 mark)

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Question 49 (9 marks)

A cricket player hits a ball, and the ball's trajectory is modelled by the quadratic equation:

$$h(x) = ax^2 + bx + c$$

Where $h(x)$ is the height of the ball (in metres) above the ground, and x is the horizontal distance (in metres) from where the ball was hit.

The following conditions are given:

- The ball is hit from a height of 1.5 metres, i.e., $h(0) = 1.5$.
- The ball reaches a height of 20 metres when it has travelled 10 metres horizontally.
- The ball reaches a height of 35 metres when it has travelled 20 metres horizontally.

a. Using the given conditions, set up and solve a system of equations to determine the values of a, b, and c. (3 marks)

b. Determine the maximum height that the ball reaches. Give your answer correct to 2 decimal places. (1 mark)

c. Determine the horizontal distance the ball has travelled when its height is 15 metres. Provide both possible values of x correct to two decimal places. (2 marks)

d. Find the exact height, where the ball has travelled 30 metres horizontally between the two times that it reaches this height. (3 marks)

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