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# VCE Mathematical Methods ½ Quadratics [1.3]

**Test Solutions** 

#### **Results**:

Test Questions	/21	
Extension Test Question	/5	





### Section A: Test Questions (21 Marks)

INSTRUCTION: 21 Marks. Y Minutes Reading. Z Minutes Writing.



Question 1 (4 marks)

Tick whether the following statements are **true** or **false**.

	Statement	True	False
a.	Every quadratic can be factorised as the product of two real linear factors.		<b>✓</b>
b.	If the discriminant of a quadratic is negative, then the quadratic has two real solutions.		<b>✓</b>
c.	We can find the turning point of a quadratic if we know only the coordinates of two $x$ -intercepts.		<b>✓</b>
d.	All quadratics have a turning point form.	✓	
e.	The solution to the quadratic inequality $x^2 > 4$ is $x \le -2$ or $x \ge 2$ .		<b>✓</b>
f.	The axis of symmetry of $y = 3x^2 - 12x + 13$ is at $x = 2$ .	<b>✓</b>	
g.	The equation $x^4 - 2x^2 + 1 = 0$ has two distinct real solutions.	<b>✓</b>	<b>✓</b>
h.	The graph of $y = ax^2 + bx + c$ is symmetric about the line $x = \frac{b}{2a}$ .		

Question 2 (3 marks)

The sum of two numbers is 8 and the product of the two numbers is 15.

**a.** Write down a quadratic equation in the form  $ax^2 + bx + c = 0$  that can be solved to find the numbers. (1 mark)

$$x(8 - x) = 15$$

$$8x - x^{2} = 15$$

$$x^{2} - 8x + 15 = 0$$

**b.** Find the two numbers. (2 marks)

Solution:  $(x-3)(x-5) = 0 \implies x = 3, 5$ The two numbers are 3 and 5.



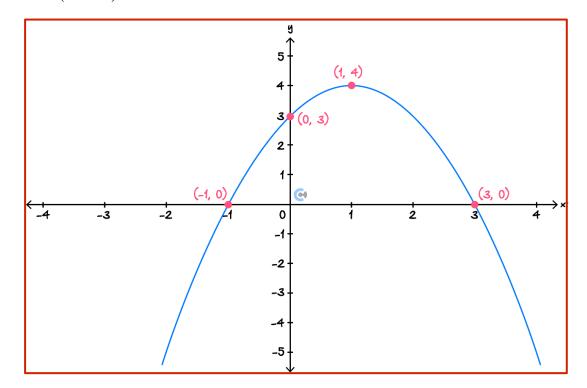
Question 3 (5 marks)

Consider the function  $f(x) = -x^2 + 2x + 3$ .

**a.** Write f(x) in the form  $a(x-h)^2 + k$ . (1 mark)

**Solution:**  $f(x) = -(x-1)^2 + 4$ 

**b.** Sketch the graph of y = f(x) on the axes below. Label the turning point and all axes intercepts with coordinates. (2 marks)



**c.** Hence, find the value(s) of x, such that f(x) > 3. (2 marks)

Solution: 0 < x < 2



Question 4 (4 marks)

Sam is competing in a shotput competition. The trajectory of the shot (name of the spherical ball used), is modelled by a quadratic equation  $y = ax^2 + bx + c$ , where  $y \ge 0$  is the height of the shot above the ground in metres and  $x \ge 0$  is the horizontal distance of the shot in metres.

The shot reaches a maximum height of  $\frac{7}{2}$  metres when it has travelled two metres horizontally, and it has a height of  $\frac{3}{2}$  when it is released (x = 0).

**a.** Write down the trajectory of the shot in turning point form. (2 marks)

**Solution:**  $y = a(x-2)^2 + \frac{7}{2}$  from the description. We sub in the point  $\left(0, \frac{3}{2}\right)$  to find a

$$\frac{3}{2} = 4a + \frac{7}{2}$$

$$4a = -2$$

$$a = -\frac{1}{2}$$

Therefore,  $y = -\frac{1}{2}(x-2)^2 + \frac{7}{2}$ 

**b.** Find the horizontal distance that Sam's shot travels. (2 marks)

**Solution:** Find when y = 0. Solve

$$-\frac{1}{2}(x-2)^2 + \frac{7}{2} = 0$$
$$(x-2)^2 = 7$$
$$x = 2 \pm \sqrt{7}$$

Since x > 0, the horizontal distance that the shot travels is  $2 + \sqrt{7}$  metres.



Question 5 (5 marks)

Consider the function  $f(x) = x^4 - 6x^2 + 8$ .

**a.** Solve the equation f(x) = 0. (3 marks)

Solution: Let  $x^2 = a$ 

$$a^{2} - 6a + 8 = 0$$
  
 $(a - 3)^{2} = 1$   
 $a = 3 \pm 1$ 

a = 2, 4

Therefore,  $x = \pm \sqrt{2}, \pm 2$ 

**b.** Use the discriminant to determine the value(s) of k, such that f(x) + k = 0 has no real solutions. (2 marks)

**Solution:** Let  $x^2 = a$ , now consider the quadratic

$$a^2 - 6a + 8 + k = 0$$

If this quadratic has no real solutions then f(x) = k = 0 will have no real solutions.

$$\Delta < 0 \implies 36 - 4(8 + k) < 0$$
  
 $4 - 4k < 0$ 

$$4-4k<0$$

k > 1

Our solution is k > 1.



#### Section B: Extension Test Question (5 Marks)

INSTRUCTION: 5 Marks. Y Minutes Reading. Z Minutes Writing.



Question 6 (5 marks)

**a.** Solve  $x^4 - 2kx^2 + 4 = 0$  for x, in terms of k, where  $k \in R$ . (3 marks)

Solution: Let  $x^2 = a^2$ 

$$a^{2} - 2ak + 4 = 0$$
  
 $(a - k)^{2} = k^{2} - 4$   
 $a = k \pm \sqrt{k^{2} - 4}$ 

Therefore,

$$x = \sqrt{k + \sqrt{k^2 - 4}}, \sqrt{k - \sqrt{k^2 - 4}}, -\sqrt{k + \sqrt{k^2 - 4}}, -\sqrt{k - \sqrt{k^2 - 4}}$$

**b.** Hence, determine the values of k for which  $x^4 - 2kx^2 + 4 = 0$  has 4 real solutions. (2 marks)

**Solution:** Firstly, require  $k^2-4>0 \implies k>2$  or k<-2. Then we require that  $k-\sqrt{k^2-4}>0 \implies k>2$  Therefore, k>2.



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#### VCE Mathematical Methods ½

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