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VCE Mathematical Methods ½
Quadratics [1.3]
Homework Solutions

Homework Outline:

Compulsory	Pg 2 – Pg 16
Supplementary	Pg 17 – Pg 31

Section A: Compulsory

Sub-Section [1.3.1]: Rewriting quadratics in different forms

Question 1

Find the factorised forms of these quadratics.

a. $x^2 - 9$

We apply difference of squares (recall $a^2 - b^2 = (a-b)(a+b)$) to get,

$$x^2 - 9 = (x-3)(x+3)$$

b. $x^2 + 7x$

We can simply factorise an x out from our expression, hence,

$$x^2 + 7x = x(x+7)$$

c. $4 - 4x^2$

We can first factorise out a 4 from our expression to get,

$$4 - 4x^2 = 4(1 - x^2)$$

and then apply difference of squares to get,

$$4 - 4x^2 = 4(1-x)(1+x)$$

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Question 2

a. Factorise $x^2 + 4x + 4$.

We observe that $x^2+4x+4 = x^2+2x+2x+4 = x(x+2)+2(x+2) = (x+2)^2$.

b. Express $x^2 + 6x + 8$ in intercept form, $(a(x - b)(x - c))$.

$x^2+6x+8 = x^2+2x+4x+8 = x(x+2)+4(x+2) = (x+2)(x+4)$

c. Express $x^2 + 6x + 8$ in turning point form, $(a(x - h)^2 + k)$.

If $x^2+6x+8 = a(x-h)^2+k = ax^2-2ahx+ah^2+k$ for some a, h, k we should be able to compare coefficients to solve for a, h and k.
From the x^2 coefficient we see that $a = 1$.
From the x coefficient we see that $-2ah = -2h = 6 \implies h = -3$.
From the constant term we see that $ah^2+k = 1(3)^2+k = 8 \implies k = -1$.
Hence in turning point form, $x^2+6x+8 = (x+3)^2-1$.

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Question 3

a. Factorise: $4x^2 - 8x - 12$.

Since 4 is a factor of $4, -8, -12$ we see that,

$$4x^2 - 8x - 12 = 4(x^2 - 2x - 3)$$

Thus we need to factorise $x^2 - 2x - 3$.
Since $x^2 - 2x - 3 = x^2 + x - 3x - 3 = x(x+1) - 3(x+1) = (x-3)(x+1)$, we see that,

$$4x^2 - 8x - 12 = 4(x-3)(x+1).$$

b. Express $3x^2 - 6x + 5$ in turning point form.

We will compare coefficients with $3x^2 - 6x + 5 = a(x-h)^2 + k = ax^2 - 2ahx + ah^2 + k$.
From the x^2 coefficient we see that $a = 3$.
From the x coefficient we see that $-2ah = -6h = -6 \implies h = 1$.
From the constant term we see that $ah^2 + k = 3 + k = 5 \implies k = 2$.
Hence in turning point form, $3x^2 - 6x + 5 = 3(x-1)^2 + 2$

c. Factorise $2x^2 - 7x - 15$.

We cannot take out a factor of 2 from our expression.
We observe that,

$$\begin{aligned} 2x^2 - 7x - 15 &= 2x^2 - 10x + 3x - 15 \\ &= 2x(x-5) + 3(x-5) \\ &= (2x+3)(x-5) \end{aligned}$$

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Question 4 Tech-Active.

a. Factorise: $12x^2 + 4x - 40$.

Use the factorise function on the CAS to get,

$$12x^2 + 4x - 40 = 4(x + 2)(3x - 5)$$

b. Express $12x^2 - 120x + 337$ in turning point form.

Recall that $a(x - h)^2 + k = ax^2 - 2ahx + ah^2 + k$.
By comparing coefficients we get 3 equations,

$$a = 12$$
$$-2ah = -120$$
$$ah^2 + k = 337$$

Solving these simultaneously on the CAS yields, $a = 12, h = 5$ and $k = 37$. Hence,

$$12x^2 - 120x + 337 = 12(x - 5)^2 + 37$$

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Sub-Section [1.3.2]: Find solutions and number of solutions to quadratic equations.

Question 5

Find all real solutions to the following equations:

a. $x^2 - 25 = 0$

We factorise our expression and apply the null factor law.

$$x^2 - 25 = (x+5)(x-5) = 0$$
$$\implies x+5 = 0 \text{ or } x-5 = 0$$
$$\implies x = -5, 5$$

b. $3x^2 = 6x$

We factorise our expression and apply the null factor law.

$$3x^2 = 6x$$
$$\implies 3x^2 - 6x = 3x(x-2) = 0$$
$$\implies x = 0, 2$$

c. $3x^2 - 27 = 0$

We factorise our expression and apply the null factor law.

$$3x^2 - 27 = 3(x^2 - 9) = 3(x-3)(x+3) = 0$$
$$\implies x = -3, 3$$

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Question 6

a. Find all real solutions to the equation $4x^2 + 16x + 16 = 0$.

$$\begin{aligned} & 4x^2 + 16x + 16 = 0 \\ \implies & 4(x^2 + 4x + 4) = 0 \\ \implies & 4(x+2)^2 = 0 \\ \implies & x = -2 \end{aligned}$$

b. How many solutions does the equation $x^2 - 4x + 7 = 0$ have?

Recall that for a quadratic equation $ax^2 + bx + c$, the discriminant, $\Delta = b^2 - 4ac$ determines the number of real solutions, with

- $\Delta > 0$ implying two real solutions.
- $\Delta = 0$ implying one real solution.
- $\Delta < 0$ implying no real solutions.

Since our discriminant is equal to $(-4)^2 - 4(1)(7) = 16 - 28 = -12 < 0$ our equation has no real solutions.

c. Find all real solutions to the equation $2x^2 + 2x = 24$.

$$\begin{aligned} & 2x^2 + 2x = 24 \\ \implies & 2(x^2 + x - 12) = 0 \\ \implies & 2(x^2 - 3x + 4x - 12) = 0 \\ \implies & 2(x(x-3) + 4(x-3)) = 0 \\ \implies & 2(x+4)(x-3) = 0 \\ \implies & x = -4, 3 \end{aligned}$$

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Question 7

a. Find all real solutions to the equation $x(x - 4) = 1$.

We rearrange our to be in the form $ax^2 + bx + c$, getting,

$$x^2 - 4x - 1 = 0$$

From here we apply the quadratic formula $\left(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right)$ to our equation to get,

$$x = \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2} = \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$$

b. For what values of c does the equation $x^2 - 2x = c$ have two real solutions?

We rearrange our equation to be,

$$x^2 - 2x - c = 0$$

The discriminant of such an equation is,

$$\Delta = (-2)^2 - 4(1)(-c) = 4 + 4c$$

This is greater than 0 for $c > -1$.

c. Find all real solutions to the equation $9x^2 - 12x - 3 = 0$.

Before we apply the quadratic formula, we can divide our equation by 3 from both sides to get,

$$3x^2 - 4x - 1 = 0$$

Then from the quadratic formula we get,

$$x = \frac{4 \pm \sqrt{16 - 4(3)(-1)}}{2(3)} = \frac{4 \pm \sqrt{4(4+3)}}{6} = \frac{2 \pm \sqrt{7}}{3}$$

Question 8 Tech-Active.

Find all real solutions to the equation $2x^2 - 20x + 37 = 0$.

Use the solve function on the CAS to get,

$$x = 5 \pm \sqrt{\frac{13}{2}}$$

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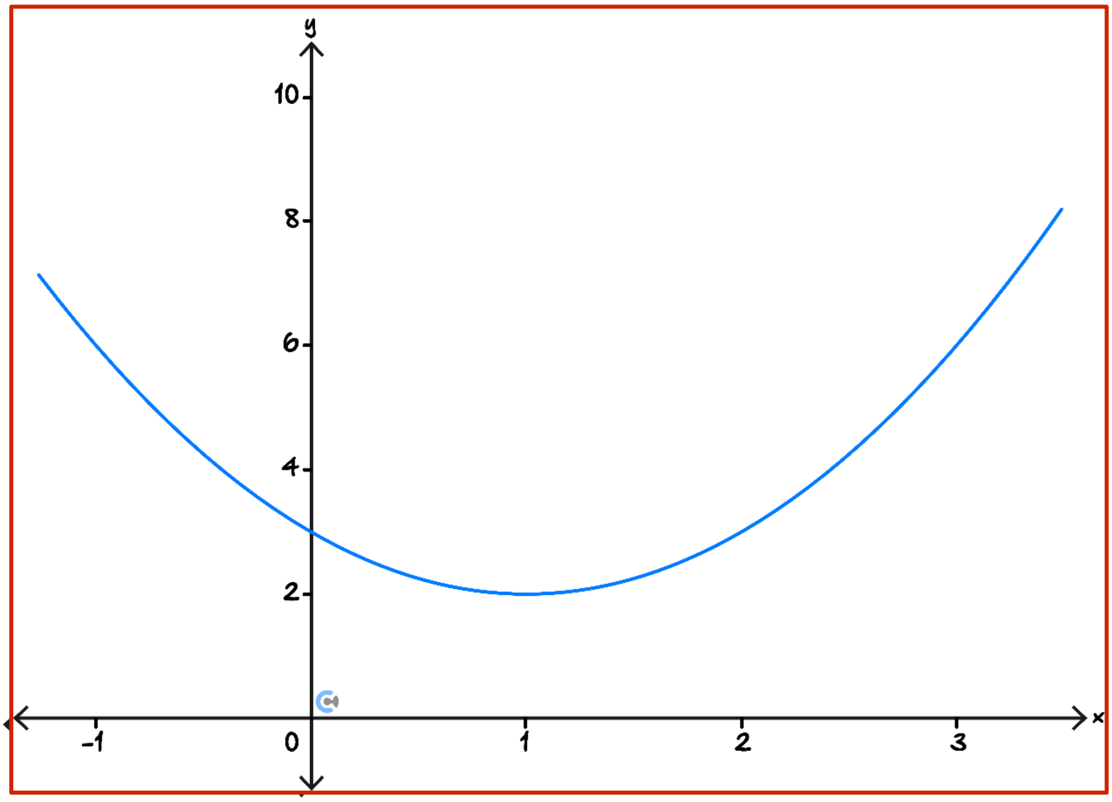
Sub-Section [1.3.3]: Graph and find rules from the graph of quadratic equations.

Question 9

Sketch the graph of $y = (x - 1)^2 + 2$ on the axis below.



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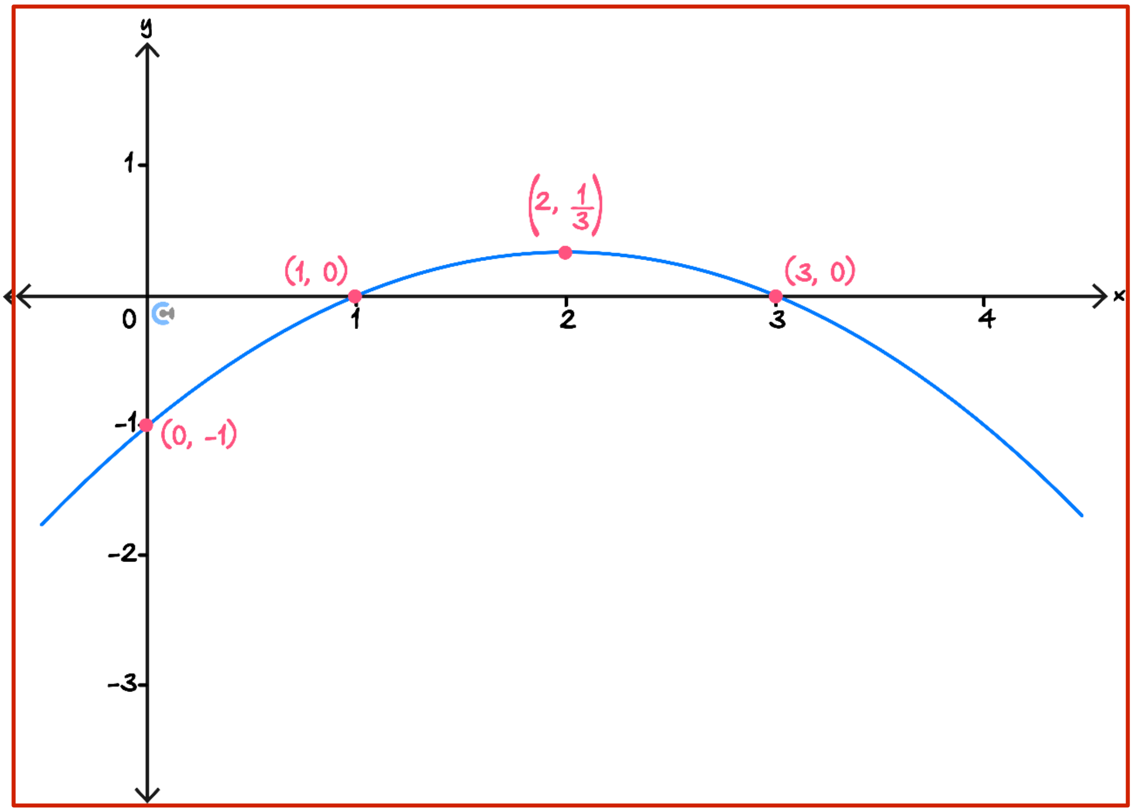
Question 10

Sketch the graph of $y = -\frac{1}{3}(x-1)(x-3)$ on the axis below, labelling axis intercepts and turning points with their coordinates.



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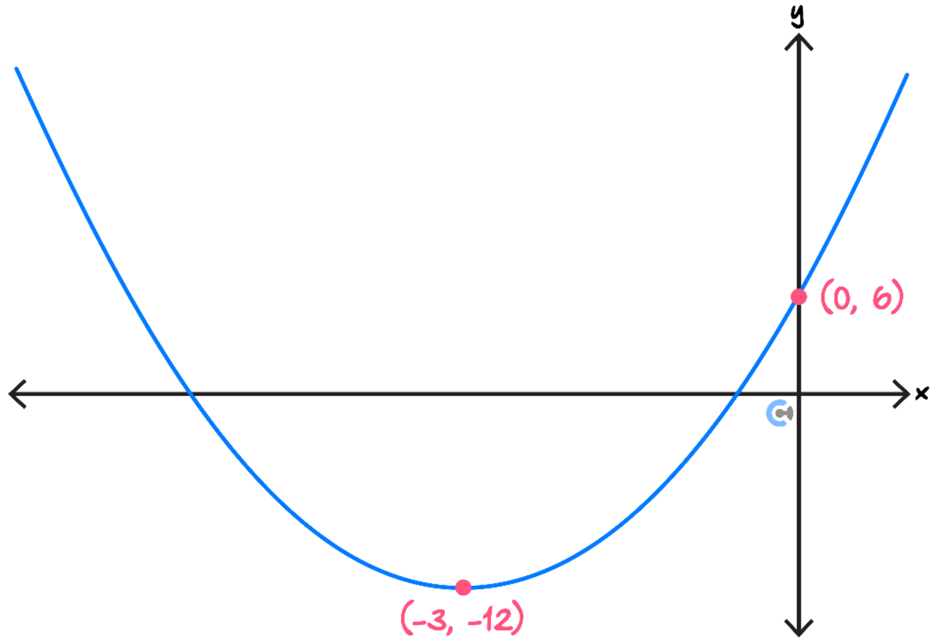
Question 11

The graph of a parabola is shown below.



Find the rule of this parabola.

Since we have the turning point of our parabola, we can express the equation in turning point form as,

$$y = a(x+3)^2 - 12$$

We can solve for a since when x is equal to 0, y is equal to 6. Hence,

$$6 = a(0+3)^2 - 12 = 9a - 12 \implies 9a = 18 \implies a = 2$$

Hence our parabola has equation $y = 2(x+3)^2 - 12$.

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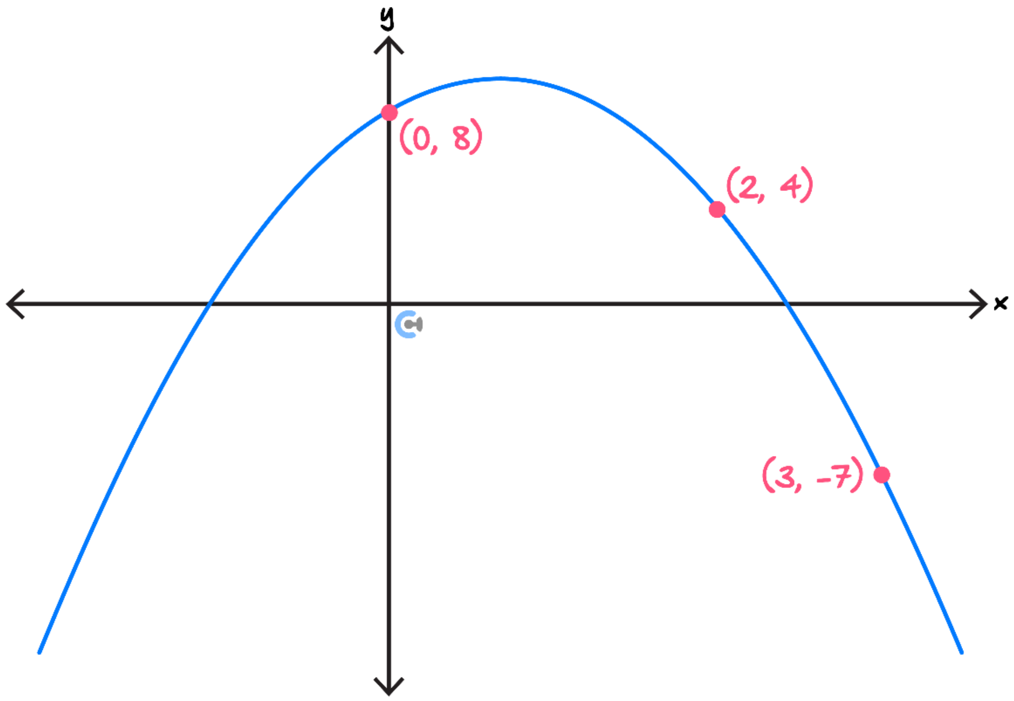
Question 12 Tech-Active.

The graph of a parabola is shown below.



Find the rule of this parabola.

We do not have any key points, hence we will use the form $y = ax^2 + bx + c$.
When $x = 0$ we see that $y = 8$, hence $c = 8$.
When $x = 2$ we see that $y = 4$, hence $4 = 4a + 2b + 8$.
When $x = 3$ we see that $y = -7$, hence $-7 = 9a + 3b + 8$.
We can solve the last two equation simultaneously (on the CAS) to find a and b.
Thus $a = -3$ and $b = 4$.
Hence the rule for our parabola is $y = -3x^2 + 4x + 8$

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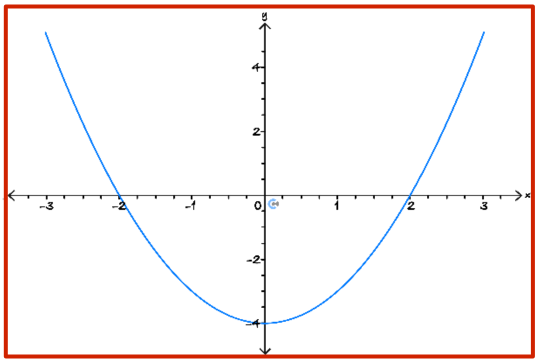
Sub-Section [1.3.4]: Solving Quadratic Inequalities and Hidden Quadratics.

Question 13

a. Solve $x^2 - 4 < 0$ for x.

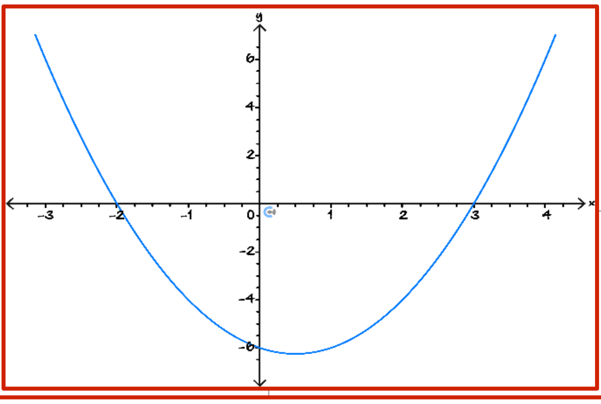
We sketch the graph of $x^2 - 4$,



From this graph it is clear that $x^2 - 4 < 0$ if $x \in (-2, 2)$.
Alternatively you can solve $x^2 - 4 = 0 \implies x = \pm 2$, and infer the result from the fact that $x^2 - 4$ is a positive parabola (the x^2 coefficient is > 0).

b. Solve $(x - 3)(x + 2) \geq 0$ for x.

We notice that $(x - 3)(x + 2)$ is a positive parabola with x-intercepts of -2 and 3. Thus imagining the graph of $(x - 3)(x + 2)$ we see that $(x - 3)(x + 2) \geq 0$ if $x \leq -2$ or if $x \geq 3$.



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Question 14

Solve $x^4 - 5x^2 + 4 = 0$.

We let $a = x^2$. After substituting this value into our equation we get the quadratic,

$$a^2 - 5a + 4 = 0$$

which we can solve as usual.
As $a^2 - 5a + 4 = a^2 - 4a - a + 4 = (a-4)(a-1) = 0$, we see that $a = 1, 4$.
Since $x^2 = a$, we see that $x = -2, -1, 1, 2$

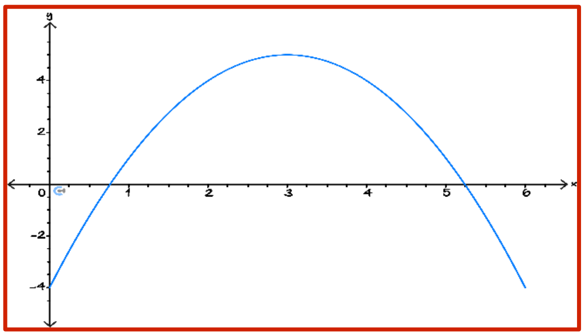
Question 15

Solve $x(6 - x) < 4$ for x.

We first rearrange our inequality to get everything on one side. Thus,

$$-x^2 + 6x - 4 < 0$$

We want to imagine the graph of $-x^2 + 6x - 4$, specifically it's shape and x-axis intercepts. Since the x^2 coefficient is negative it is "upside-down".



Now to get the x-axis intercepts we apply the quadratic formula to, $x^2 - 6x + 4 = 0$. Hence,

$$x = \frac{6 \pm \sqrt{36 - 16}}{2} = 3 \pm \sqrt{5}$$

From these two facts we see that $-x^2 + 6x - 4 < 0$ if $x < 3 - \sqrt{5}$ or if $x > 3 + \sqrt{5}$.

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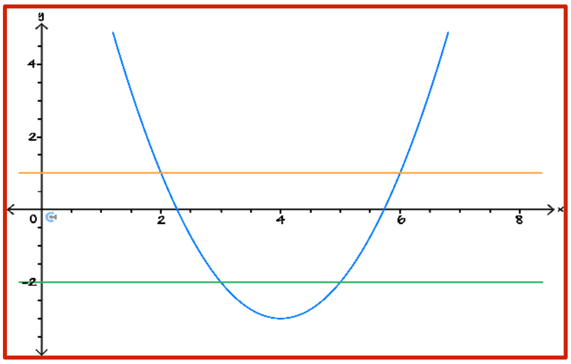
Question 16 Tech-Active.

Solve $-2 < x^2 - 8x + 13 \leq 1$ for x.

We sketch the graphs of $y = -2$, $y = x^2 - 8x + 13$ and $y = 1$ on the same axis.



Solving $x^2 - 8x + 13 = -2$ yields $x = 3, 5$, and solving $x^2 - 8x + 13$ yields $x = 2, 6$. Hence from the graph we see that our inequality implies that $x \in [2, 3) \cup (5, 6]$.

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Section B: Supplementary

Sub-Section [1.3.1]: Rewriting quadratics in different forms

Question 17

Find the factorised forms of these quadratics.

a. $x^2 - 4$

We apply difference of squares (recall $a^2 - b^2 = (a-b)(a+b)$) to get,

$$x^2 - 4 = (x-2)(x+2)$$

b. $x^2 - 3x$

We can simply factorise an x out from our expression, hence,

$$x^2 - 3x = x(x-3)$$

c. $5x^2 + 10x$

We factorise a factor of $5x$ out of our expression to get,

$$5x^2 + 10x = 5x(x+2)$$

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Question 18

a. Express $x^2 + 4x + 3$ in intercept form, $(a(x - b)(x - c))$.

$x^2 + 4x + 3 = x^2 + 3x + x + 3 = x(x + 3) + (x + 3) = (x + 3)(x + 1)$

b. Express $x^2 - 2x + 3$ in turning point form, $(a(x - h)^2 + k)$.

Recall that $x^2 - 2x + 1 = (x - 1)^2$.
Thus $x^2 - 2x + 3 = x^2 - 2x + 1 + 2 = (x - 1)^2 + 2$.

c. Factorise: $x^2 + 6x + 9$.

We observe that $x^2 + 6x + 9 = x^2 + 3x + 3x + 9 = x(x + 3) + 3(x + 3) = (x + 3)^2$.

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Question 19

a. Factorise: $3x^2 - 12x - 15$.

Since 3 is a factor of $3, -12, -15$ we see that,

$$3x^2 - 12x - 15 = 3(x^2 - 4x - 5)$$

Thus we need to factorise $x^2 - 4x - 5$.
Since $x^2 - 4x - 5 = x^2 - 5x + x - 5 = x(x-5) + (x-5) = (x+1)(x-5)$, we see that,

$$3x^2 - 12x - 15 = 3(x-5)(x+1).$$

b. Express $2x^2 - 12x + 9$ in turning point form.

Observe that $2(x-3)^2 = 2x^2 - 12x + 18$.
Hence $2x^2 - 12x + 9 = 2x^2 - 12x + 18 - 9 = 2(x-3)^2 - 9$.

c. Express $2(x - 1)(x + 3)$ in turning point form.

We first expand $2(x-1)(x+3)$ out to get,

$$2(x-1)(x+3) = 2(x^2 + 2x - 3) = 2x^2 + 4x - 6$$

Since $2(x+1)^2 = 2x^2 + 4x + 2$ we see that,

$$2(x-1)(x+3) = 2x^2 + 4x + 2 - 8 = 2(x+1)^2 - 8$$

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Question 20

Factorise $6x^2 - \sqrt{5}x - 5$.

We apply the quadratic formula to find the roots of the equation $6x^2 - \sqrt{5}x - 5 = 0$. Thus

$$x = \frac{\sqrt{5} \pm \sqrt{5+120}}{12} = \frac{\sqrt{5} \pm 5\sqrt{5}}{12} = -\frac{\sqrt{5}}{3}, \frac{\sqrt{5}}{2}$$

Hence our expression can be factorised as,

$$6x^2 - \sqrt{5}x - 5 = 6\left(x - \frac{\sqrt{5}}{2}\right)\left(x + \frac{\sqrt{5}}{3}\right) = (2x - \sqrt{5})(3x + \sqrt{5})$$

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Sub-Section [1.3.2]: Find solutions and number of solutions to quadratic equations.

Question 21

Find all real solutions to the following equations:

a. $x^2 = -5x$

We factorise our expression and apply the null factor law.

$$x^2 = -5x$$
$$\implies x^2 + 5x = x(x+5) = 0$$
$$\implies x = 0, -5$$

b. $4x^2 - 16 = 0$

We factorise our expression and apply the null factor law.

$$4x^2 - 16 = 4(x+2)(x-2) = 0$$
$$\implies x + 2 = 0 \text{ or } x - 2 = 0$$
$$\implies x = -2, 2$$

c. $2x^2 - 18x = 0$

We factorise our expression and apply the null factor law.

$$2x^2 - 18x = 2x(x-9)$$
$$\implies x = 0, 3$$

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Question 22

a. Find all real solutions to the equation $x^2 - 10x + 25 = 0$.

$$x^2 - 10x + 25 = 0$$
$$\implies (x-5)^2 = 0$$
$$\implies x = 5$$

b. How many solutions does the equation $x^2 + 2x - 15$ have?

Recall that for a quadratic equation $ax^2 + bx + c$, the discriminant, $\Delta = b^2 - 4ac$ determines the number of real solutions, with

- $\Delta > 0$ implying two real solutions.
- $\Delta = 0$ implying one real solution.
- $\Delta < 0$ implying no real solutions.

Since our discriminant is equal to $4 - 4(-15) = 64 > 0$ our equation has two real solutions.

c. Find all real solutions to the equation $3(x + 1)^2 = 12$.

$$3(x+1)^2 = 12$$
$$\implies (x+1)^2 = 4$$
$$\implies x + 1 = \pm 2$$
$$\implies x = -3, 1$$

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Question 23

a. Find all real solutions to the equation $x^2 - 6x = 4$.

We rearrange our to be in the form $ax^2 + bx + c$, getting,

$$x^2 - 6x - 4 = 0$$

From here we apply the quadratic formula $\left(x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right)$ to our equation to get,

$$x = \frac{6 \pm \sqrt{36 - 4(-4)(1)}}{2} = \frac{6 \pm 2\sqrt{13}}{2} = 3 \pm \sqrt{13}$$

b. For what values of a does the equation $ax^2 - 6x = 18$ have no real solutions?

We rearrange our to be in the form $ax^2 + bx + c$, getting,

$$ax^2 + 6x - 18 = 0$$

Since we have no solutions our discriminant is $\Delta = 36 + 72a$ is less than 0. Hence $a < -\frac{1}{2}$.

c. Find all real solutions to the equation $5x^2 + 20x = 15$.

We observe that $5(x+2)^2 = 5x^2 + 20x + 20$, hence,

$$\begin{aligned} 5x^2 + 20x = 15 &\implies 5x^2 + 20x + 20 = 35 \\ &\implies 5(x+2)^2 = 35 \\ &\implies (x+2)^2 = 7 \\ &\implies x + 2 = \pm\sqrt{7} \\ &\implies x = -2 \pm \sqrt{7} \end{aligned}$$

Question 24

For what values of b does the equation $2x(b - x) = 5$ have no real solutions?

We rearrange our equation to be in the form $ax^2 + bx + c = 0$, getting,

$$2x^2 - 2bx + 5 = 0$$

Since we desire no real solutions, we require that

$$\Delta = 4b^2 - 4(2)(5) = 4(b^2 - 10) < 0.$$

From the graph of $x^2 - 10$, we see that this occurs if $b \in (-\sqrt{10}, \sqrt{10})$.

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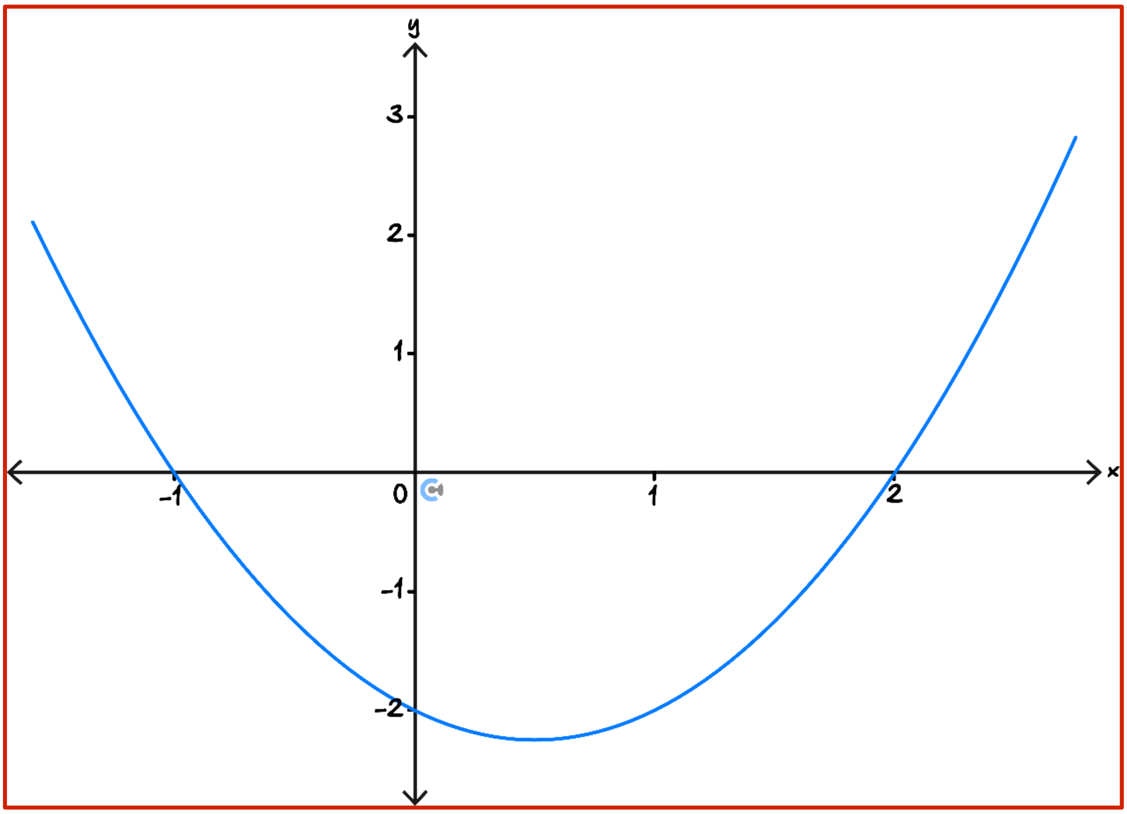
Sub-Section [1.3.3]: Graph and find rules from the graph of quadratic equations.

Question 25

Sketch the graph of $y = (x+1)(x-2)$ on the axis below.



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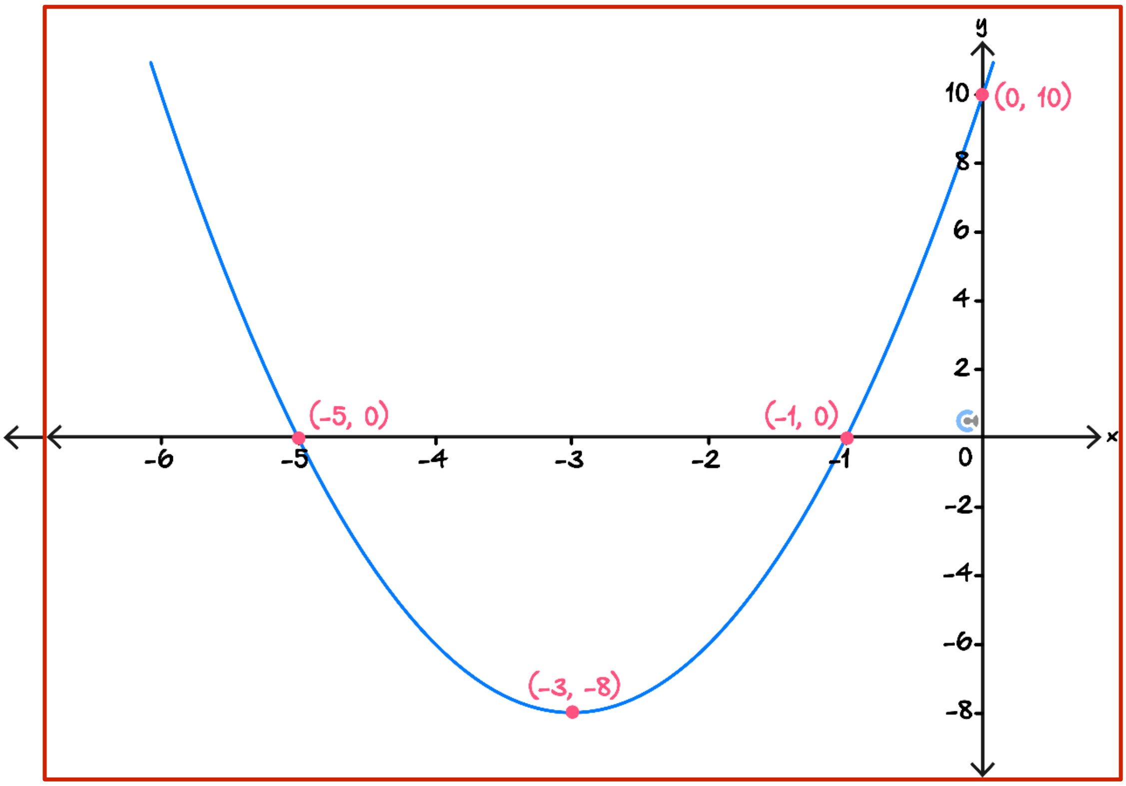
Question 26

Sketch the graph of $y = 2(x + 3)^2 - 8$ on the axis below, labelling axis intercepts and turning points with their coordinates.



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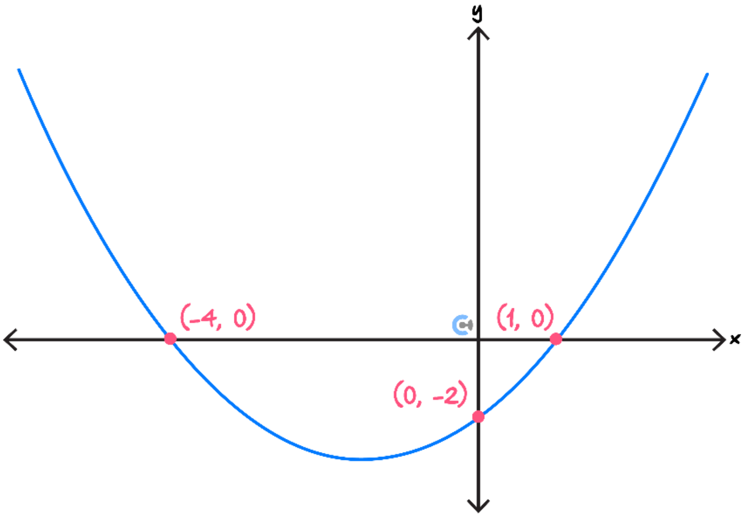
Question 27

The graph of a parabola is shown below.



Find the rule of this parabola.

Since we have the axis intercepts of this parabola, we can express the equation in intercept form as,

$$y = a(x-1)(x+4)$$

We can solve for a since when x is equal to 0, y is equal to -2. Hence,

$$-2 = a(0-1)(0+4) \implies -2 = -4a \implies a = \frac{1}{2}$$

Hence our parabola has equation, $y = \frac{1}{2}(x-1)(x+4)$

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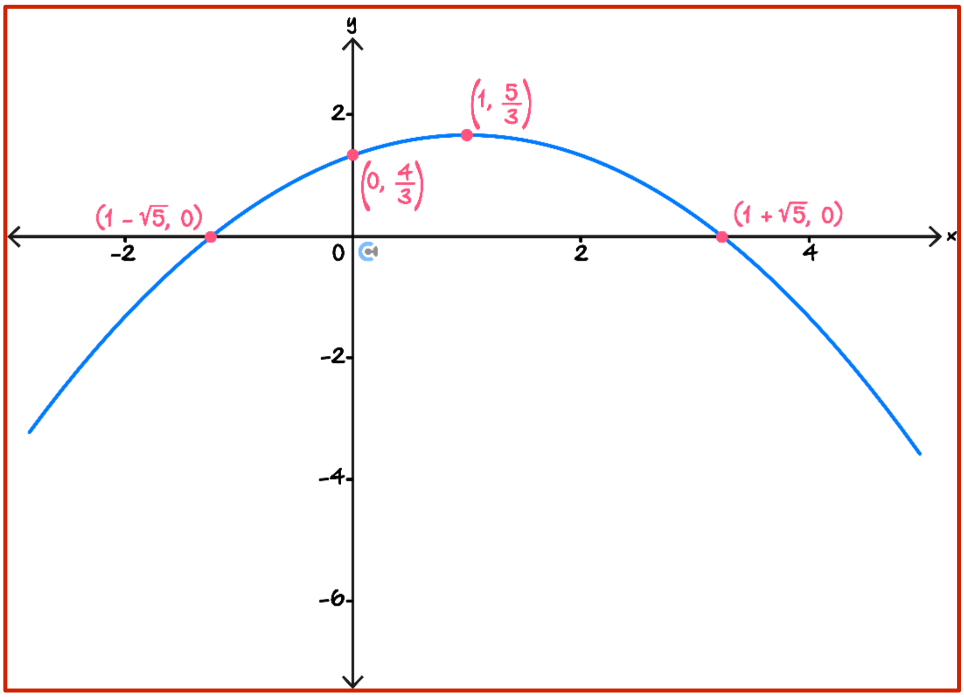
Question 28

Sketch the graph of $3y = 5 - (x - 1)^2$ on the axis below, labelling axis intercepts and turning points with their coordinates.



The x-axis intercepts can be obtained by solving

$$5 - (x-1)^2 = 0 \implies x - 1 = \pm\sqrt{5} \implies x = 1 \pm \sqrt{5}.$$

The y-axis intercept can be obtained by evaluating y when $x = 0$. Hence,

$$3y = 5 - (0-1)^2 = 4 \implies y = \frac{4}{3}$$

The turning point can be read off, since,

$$y = \frac{-1}{3}(x-1)^2 + \frac{5}{3}$$

Hence our turning point is $\left(1, \frac{5}{3}\right)$.

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Sub-Section [1.3.4]: Solving Quadratic Inequalities and Hidden Quadratics.

Question 29

a. Solve $x^2 > 1$ for x.

$x > 1$ or $x < -1$.

b. Solve $x(x-2) \leq 3$ for x.

We rearrange our inequality to get everything on one side. Thus,

$$x^2 - 2x - 3 \leq 0$$

We consider the graph of $x^2 - 2x - 3$. It will be a positive parabola, and have x-axis intercepts when,

$$x^2 - 2x - 3 = x^2 - 3x + x - 3 = (x-3)(x+1) = 0 \implies x = -1, 3$$

From here we see that $x^2 - 2x - 3 \leq 0$ if $x \in [-1, 3]$.

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Question 30

Solve $(x - 1)^4 - (x - 1)^2 = 12$ for x.

We let $a = (x-1)^2$. After substituting this value into our equation we get the quadratic,

$$a^2 - a - 12 = 0$$

which we can solve as usual.
As $a^2 - a - 12 = a^2 - 4a + 3a - 12 = a(a-4) + 3(a-4) = (a-4)(a+3) = 0$, we see that $a = -3, 4$.
Since $a = (x-1)^2 \geq 0$, we reject $a = -3$, leaving us with,

$$(x-1)^2 = 4 \implies x - 1 = \pm 2 \implies x = -1, 3$$

Question 31

Solve $x^2 + 6x + 8 \geq 2$ for x.

We first rearrange our inequality to get everything on one side. Thus,

$$x^2 + 6x + 6 \geq 0$$

We consider the graph of $x^2 + 6x + 6$. It will be a positive parabola, and have x-axis intercepts when,

$$x^2 + 6x + 6 = 0 \implies x = \frac{-6 \pm \sqrt{36 - 24}}{2} = -3 \pm \sqrt{3}$$

From here we see that $x^2 + 6x + 8 \geq 2$ if $x \leq -3 - \sqrt{3}$ or if $x \geq -3 + \sqrt{3}$.

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Question 32

For what values of x is $ax^2 + bx + c < d$, where $a, b, c, d \in R$, $a < 0$ and $c > d$?

We first rearrange our inequality to get everything on one side. Thus,

$$ax^2 + bx + c - d < 0$$

We consider the graph of $ax^2 + bx + c - d < 0$. It will be a negative parabola, and have x-axis intercepts when,

$$ax^2 + bx + c - d \implies x = \frac{-b \pm \sqrt{b^2 - 4a(c-d)}}{2a}$$

Since $c > d$ and $a < 0$, we know that $b^2 - 4a(c-d) > 0$, thus the above values of x are real numbers.
By the properties of a negative parabola, we see that $ax^2 + bx + c < d$ if,

$$x < \frac{-b - \sqrt{b^2 - 4a(c-d)}}{2a} \quad \text{or} \quad x > \frac{-b + \sqrt{b^2 - 4a(c-d)}}{2a}$$

Space for Personal Notes

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VCE Mathematical Methods ½

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