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# VCE Mathematical Methods ½ Linear & Coordinate Geometry Exam Skills [1.2]

Workbook

# **Outline:**

Pg 2-21

# Recap of [1.1] - Linear and Coordinate Geometry

Inequality

- Midpoint
- Distance Between Two Points
- Vertical Distance VS Horizontal Distance
- Parallel and Perpendicular Lines
- Angle Between a Line and the x-axis
- Angle Between the Two Lines
- Finding Simultaneous Equations for Two Variables
- Number of Solutions for Two Variables

## **Linear and Coordinate Geometry Exam Skills**

Pg 22-31

- Finding the Equation of the Line
- Applying Midpoint to Find Reflected Points
- Find Vertical Distance Between Two Functions
- Finding Distance Between a Point and a Function

Exam 1 Ouestions Pg 32-36

<u>Tech-Active Exam Skills</u> Pg 37-41

Exam 2 Questions Pg 42-46



# Section A: Recap of [1.1] - Linear and Coordinate Geometry

# Definition

# **Linear equations**

- **Definition:** Equations where the highest power of a variable is 1.
  - Gradient-intercept form:

$$y=mx+c$$
 where  $m=gradient=rac{rise}{run}=$  and  $c=$ 

- No singular solution for a linear equation in two variables.
  - $\bullet$  All pairs of coordinates (x, y) that satisfy the equation lie on a **line**. (Hence, *linear* equations).





# **Sub-Section**: Inequality



# **Inequalities rule**

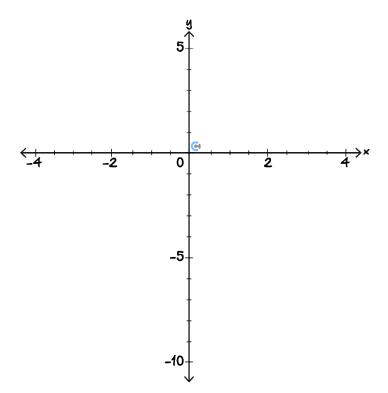


$$x > \frac{b}{a}$$
, where  $a < 0$ 

Multiplying both sides by a negative number \_\_\_\_\_\_ the inequality sign.

### Question 1 Walkthrough.

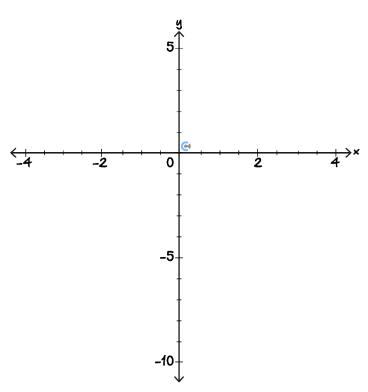
**a.** Sketch the graph of y = 2x - 4 on the axis below.



**b.** Solve the inequality:  $-4x + 3 \ge 7$ 

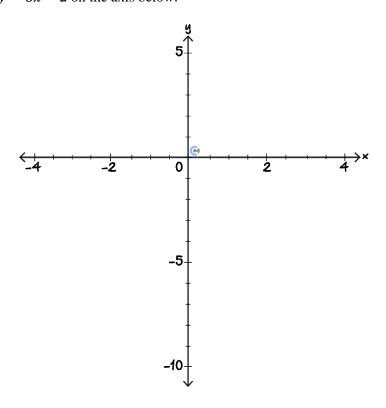


c. Sketch the region 4x - 2y < 4 on the axis below.



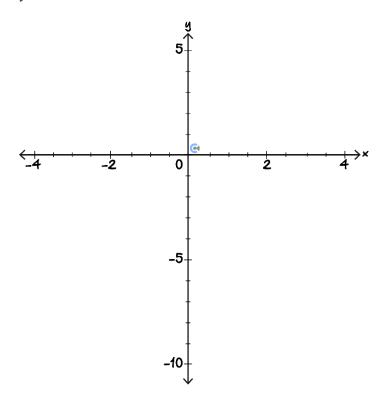
### **Question 2**

**a.** Sketch the graph of y = 3x - 2 on the axis below.





- **b.** Solve the inequality: -2x + 7 > 4
- **c.** Sketch the region x 2y > 4 on the axis below.



# Question 3 Extension.

Solve the inequality:  $\frac{x-4}{2x+3} > 2$ 



# **Sub-Section**: Midpoint



**Midpoint** 



$$\bullet \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

 $(x_2, y_2)$ 

**Definition:** The midpoint, M, of two points A and B is the point halfway between A and B.

$$M(x_m, y_m) = \left( \begin{array}{c} \\ \end{array} \right)$$

The midpoint can be found by taking the \_\_\_\_\_ of the x-coordinate and y-coordinate of the two points.





# **Sub-Section**: Distance Between Two Points



# <u>Distance between two points</u>



**Definition:** The distance between two points  $(x_1, x_2)$  and  $(y_1, y_2)$  can be found using Pythagoras' theorem:

Distance = \_\_\_\_

### **Question 4**

Consider the line segment AB where A(2,4) and B(4,8).

**a.** Find the midpoint of a line segment AB.

**b.** Find the distance between the midpoint of AB and (-1,4).

### **Ouestion 5 Extension.**

Find a point(s) on the line y = 2x + 3 which has a distance of 4 from the point (4,3).





# **Sub-Section: Vertical Distance Vs Horizontal Distance**



## **Horizontal distance**





Horizontal Distance =  $x_2 - x_1$  where \_\_\_\_\_

Find the difference between their *x*-values.

### **Question 6**

Find the horizontal distance between the two points (1,9) and (7,-4).

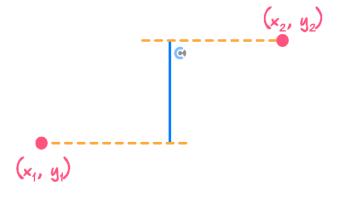


# What about vertical distance then?



# **Vertical distance**





Vertical Distance =  $y_2 - y_1$  where  $y_2 > y_1$ 

Find the difference between their *y*-values.

# **Question 7**

Find the vertical distance between the two points (1,9) and (13,-8).



# **Sub-Section: Parallel and Perpendicular Lines**

# Definition

**Parallel lines** 

$$y = m_1 x + c_1$$

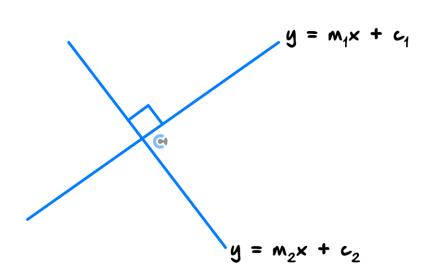
$$y = m_2 x + c_2$$

Parallel lines have the \_\_\_\_\_ gradient.

$$m_1 = m_2$$

# Definition

# Perpendicular lines



A line that is perpendicular to another line has a gradient, which is the \_\_\_\_\_\_ of the gradient of the first line.

$$m_{\perp}=-rac{1}{m}$$



Question	8
Oucsuon	O

**a.** Find a line that is parallel to y = 3x - 1 passing through the point (-1,4).

**b.** Find a line which is perpendicular to y = -3x + 4 passing through the point (3, -1).

### Question 9 Extension.

Find the equation of the line that is a perpendicular bisector of the points A(2,4) and B(8,6).





# <u>Sub-Section</u>: Angle Between a Line and the x-axis

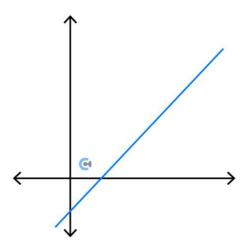


# How do we find the angle between a line and the x-axis?



# Angle between a line and the x-axis





 $\blacktriangleright$  The angle between a line and the \_\_\_\_\_ direction of the x-axis (anticlockwise) is given by:

$$tan(\theta) = m$$

### **Question 10 Tech-Active.**

Find the angle made between the line y = 3x - 6 and the x-axis measured in the anticlockwise direction. Give your answer in degrees correct to two decimal places.

**NOTE:** Angles from the x-axis measured anticlockwise = \_\_\_\_\_ angles.



Don't worry about it too much, it's just convention! (More on this in circular functions).





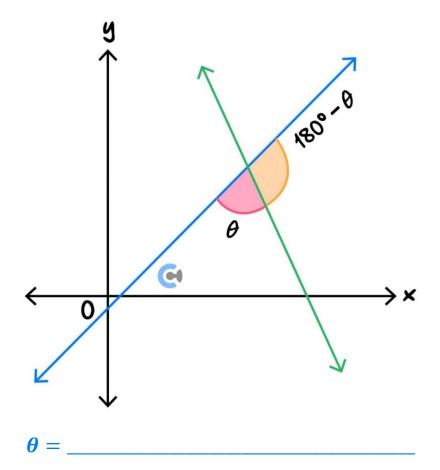
# **Sub-Section**: Angle Between the Two Lines



Slightly more complicated now! How about an angle between two lines?

# Acute angle between two lines





Alternatively:

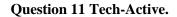
$$tan(\theta) =$$

• For your understanding, note that this formula is derived from the tan compound angle formula covered in SM12.

**NOTE:** |x| just takes the positive value of x.







Find the acute angle between the lines 3x + 4y = 2 and y = x + 1. Give your answer in degrees correct to two decimal places.

**TIP:** Make sure your CAS is in degrees.

**Space for Personal Notes** 



MM12 [1.2] - Linear & Coordinate Geometry Exam Skills - Workbook





# **Sub-Section:** Finding Simultaneous Equations for Two Variables

# Definition

# Simultaneous linear equations

- 1. Elimination method:
  - Add or subtract one equation from the other in order to \_\_\_\_\_\_ one of the variables. Then have an equation in one variable that can be solved easily.
- 2. Substitution method:
  - ullet Make one of the variables the subject (generally x or y) and \_\_\_\_\_\_ that value into the other equation.

# Question 12 Walkthrough.

Solve the following simultaneous linear equations using either elimination or substitution.

$$2x + 3y = 8$$
 and  $4x - 4y = -4$ 



# **Question 13**

Solve the following equation for x and y.

$$2x - 3y = 16$$
 and  $x + y = 3$ 

### **Question 14 Extension.**

Solve the following:

$$-6x + 2y = 10$$
 and  $-10 + y = 3x$ 



# **Sub-Section:** Number of Solutions for Two Variables



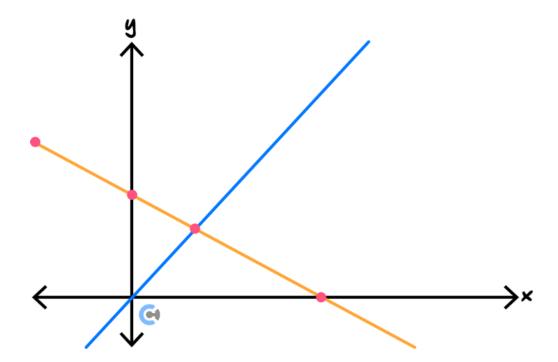


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**Exploration:** Geometry of the number of solutions between linear graphs

Unique solution

$$m_1 \neq m_2$$

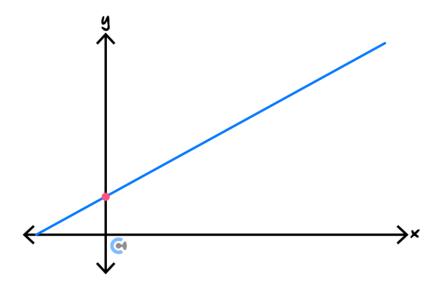


They just need to have \_\_\_\_\_\_



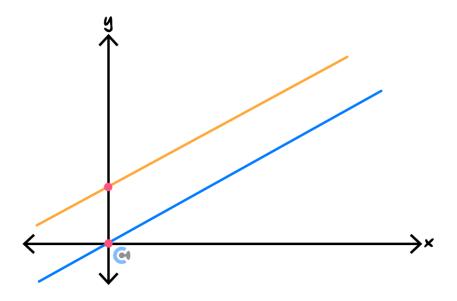
Infinite solutions

$$m_1=m_2$$
 and  $c_1=c_2$ 



- They just need to have the same \_\_\_\_\_ and the same \_\_\_\_\_
- In other words, they have to be the \_\_\_\_\_\_.
- No solutions

$$m_1=m_2$$
 and  $c_1 \neq c_2$ 



- lacktriangledown They need to have the \_\_\_\_\_\_ but \_\_\_\_\_+c.
- They have to be two different \_\_\_\_\_ lines.



# General solutions of simultaneous linear equations



- Two linear equations are either:
  - The same line is expressed in a different form. In this case, they have \_\_\_\_\_ solutions.
  - Unique lines which are parallel. In this case, they have \_\_\_\_\_ solutions.
  - Unique lines which are not parallel. In this case, they have \_\_\_\_\_\_ solution.

# Question 15 Walkthrough.

Consider the following pair of simultaneous equations in terms of  $k \in \mathbb{R} \setminus \{0\}$ :

$$y = kx + 5$$

$$y = 2x - 5k$$

**a.** Find the value of k for which there are no solutions to the simultaneous equations.

**b.** Find the value(s) of k for which there is a unique solution to the simultaneous equations.

 $\mathbf{c}$ . Find the value of k for which there are infinite solutions to the simultaneous equations.



**TIP:** It's a good idea to substitute your answer back into the equations to see if the criteria are met for each part.

### **Question 16**

Consider the following pair of simultaneous equations in terms of  $k \in \mathbb{R} \setminus \{0\}$ :

$$-3x + y = -2k$$

$$-3kx + y = -2$$

**a.** Find the value(s) of k for which there is a unique solution to the simultaneous equations.

**b.** Find the value of k for which there are infinite solutions to the simultaneous equations.

**c.** Find the value of k for which there are no solutions to the simultaneous equations.



Question 17 Extension.

Consider the following pair of simultaneous equations in terms of  $a \in \mathbb{R} \setminus \{0\}$ :

$$-2ax + 4y = 10$$

$$3x + (1-a)y = -5$$

**a.** Find the value(s) of a for which there are no solutions to the simultaneous equations.

**b.** Find the value(s) of a for which there is a unique solution to the simultaneous equations.

**c.** Find the value(s) of  $\alpha$  for which there are infinite solutions to the simultaneous equations.

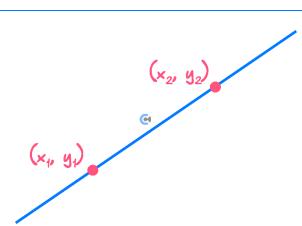


# Section B: Linear and Coordinate Geometry Exam Skills

# Sub-Section: Finding the Equation of the Line







> m: Gradient

$$m=\frac{y_2-y_1}{x_2-x_1}$$

- **→** +*c*: *y*-intercept
  - **G** Substitute in any point to y = mx + c equation.

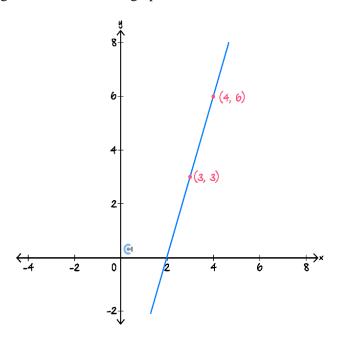
### Question 18 Walkthrough.

Find the equation of the line joining the points (1,3) and (5,5).



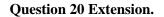


Find the equation of the straight line shown on the graph below.

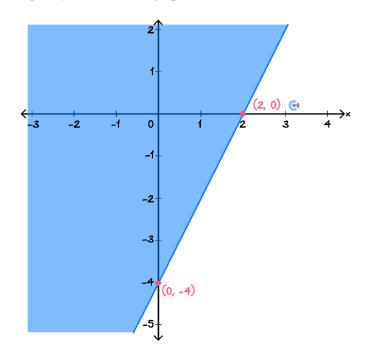


\_\_\_\_\_





Find the expression for the inequality shown on the graph below.



\_\_\_\_\_



# **Sub-Section**: Applying Midpoint to Find Reflected Points



# How can we use the idea of midpoint to find reflections?



# **Exploration:** Finding reflections

Consider a point reflected around y = 3.



- What do you notice about their midpoint?
- Hence, what should the average of two y-values equal to?

$$\frac{y+5}{2} =$$
\_\_\_\_\_

Find the reflected point!



Definition

# Finding reflections around horizontal and vertical axes

Horizontal axis

➤ The \_\_\_\_\_ changes for horizontal reflections.

$$\frac{b+d}{2}=m$$

Vertical axis

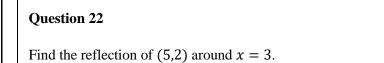
➤ The \_\_\_\_\_ changes for horizontal reflections.

$$\frac{a+c}{2}=m$$

### Question 21 Walkthrough.

Find the reflection of (3,1) around x = 1.





### Question 23 Extension.

Find the reflection of the point (a, b) around the line y = c.





# Sub-Section: Find Vertical Distance between Two Functions



<u>Discussion:</u> What can we call any point on the function, f(x)?

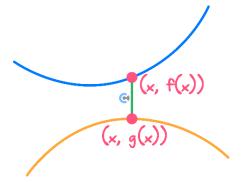


**NOTE:** That's why we say, y = f(x).



# **Vertical Distance between Two Functions**





Find the difference between the two *y*-values.

$$f(x) - g(x)$$
 where  $f$  is above  $g$ 

### Question 24 Walkthrough.

Find the vertical distance between the functions  $f(x) = x^2 + 1$  and g(x) = x - 1 when x = 2.







Find the vertical distance between the functions  $f(x) = x^2 - 4$  and g(x) = x + 1 when x = 2.

### Question 26 Extension.

When x = a, the vertical distance between the functions  $f(x) = x^2 + 5$  and g(x) = x + 4 is 7.

Find the possible values of a.





# Sub-Section: Finding Distance between a Point And a Function

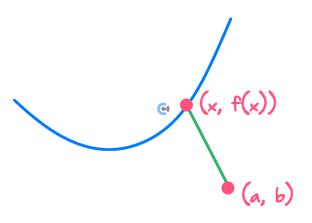
**Active Recall:** Point on a Function



**Point on f**: (x,\_\_\_\_\_)

### Distance between a Function and a Point





Find the distance between the point and (x, function).

**Distance** = 
$$\sqrt{(x-a)^2 + (f(x)-b)^2}$$

### Question 27 Walkthrough.

Find the distance between the point (1,2) and f(x) = 3x - 4 when x = 3.





Find the distance between the point (3,2) and f(x) = 2x - 3 when x = 4.

### Question 29 Extension.

The distance between the point (3,5) and the function f(x) = x + 1 when x = a is 1.

Find the possible value(s) of a.



# Section C: Exam 1 Questions (22 Marks)

INSTRUCTION: 22 Marks. 27.5 Minutes Writing.

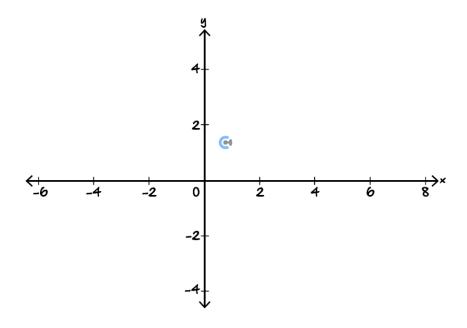


Question 30 (5 marks)

**a.** Solve the linear equation, 2x - 7 = 4x + 8. (1 mark)

**b.** Solve the linear inequality,  $-2x + 3 \le 4x + 5$ . (2 marks)

c. Sketch the inequality 4x - 8 + 6y < 0 on the axis below. (2 marks)

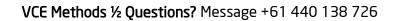




Question 31 (5 marks)  Consider the line segment $AB$ with coordinates $A(2,4)$ and $B(6,6)$ .				
<b>a.</b> Find the midpoint of AB. (1 mark)				
•••				
	·			
b.	Find the equation of the line segment AB. (2 marks)			
c.	Find the perpendicular bisector of $AB$ . (2 marks)			
Sp	ace for Personal Notes			

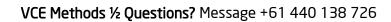


Question 32 (5 marks)				
Consider the points $A(2,4)$ and $B(6,6)$ .				
a.	Find the distance between points $A$ and $B$ . (1 mark)			
b.	The distance between point $A$ and a point $C(3, k)$ is 1. Find the value of $k$ . (2 marks)			
c.	Find the coordinates of the point D obtained by reflecting A in the line $x = -1$ . (1 mark)			
d.	Find the coordinates of the point $E$ obtained by reflecting $B$ in the line $y = 3$ . (1 mark)			
Space for Personal Notes				





uestion 33 (2 marks)	
ichard buys 2 bags of brand $X$ an brand $Y$ chips. Find the cos	chips and 3 bags of brand <i>Y</i> chips for a total of \$12. Brand <i>X</i> chips cost \$1 more tof each chip brand.
	2x + 3y = 12
	x - y = 1
-	
pace for Personal Notes	
pace for Personal Notes	





Question 34 (5 marks)					
Consider the linear equations:					
y - kx = -k $y - 2x = k$					
<b>a.</b> For what value(s) of $k$ , will the system have a unique solution? (2 marks)					
<b>b.</b> For what value of $k$ , will the system have no solution? (2 marks)					
c. Explain why the system can never have infinitely many solutions. (1 mark)					
Space for Personal Notes					



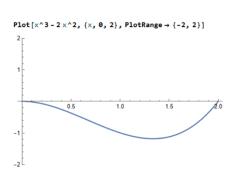
#### Section D: Tech Active Exam Skills

#### **Calculator Commands:** Graphing

# CAS CI

#### Mathematica

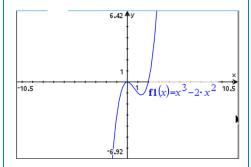
- Plot[function,{x,xmin,xmax}, PlotRange→{ymin, ymax}]
- PlotRange is optional but makes the scale appropriate for the question.



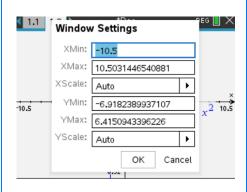
- Menu→ 6 (Analyse) to find min/max x and y intercepts.
- Restrict domain to 0 < x < 2 use the bar can get it from ctrl+ =  $\begin{vmatrix} x & y & y \\ y & y & z \end{vmatrix}$
- $f1(x)=x^3-2x^2|0< x<2$

#### TI-Nspire

• Open a graph page and plot your function.



Zoom settings: Menu $\rightarrow 4$  (window/zoom) $\rightarrow 1$  enter your x and y ranges.



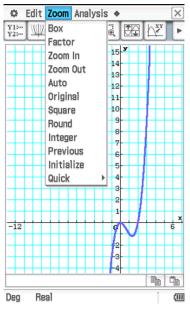
Can also click the axis numbers on the graph and alter them directly.



#### Casio Classpad

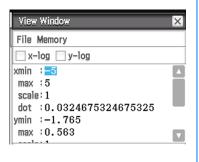
Click Graph & Table, and enter the function.







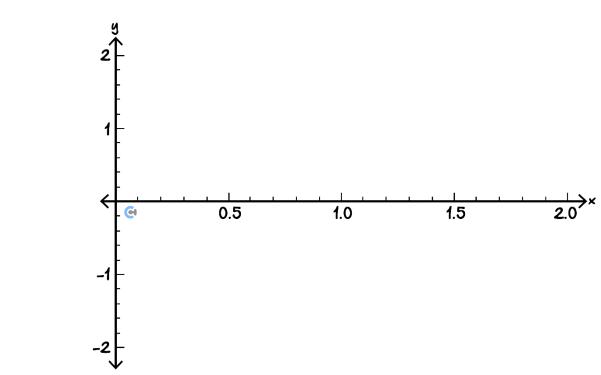
- Analysis→G-Solve to find intercepts.
- Use this button to set the view window.



- Ge Use | to restrict domain → find it in Math 3
- $\sqrt{y_1} = x_3 = 2 \cdot x_1 = x_2 = x_3 = x_3 = x_4 = x_1 = x_1 = x_2 = x_2 = x_3 = x_3 = x_4 = x_4$

#### **Question 35 Tech-Active.**

Sketch the graph of  $y = x^4 - 2x^3$  for  $0 \le x \le \frac{9}{4}$ .





#### **Calculator Commands: Solving Equations**



#### ➤ TI-Nspire

 $\bullet \quad \mathsf{Menu} \rightarrow 3 \rightarrow 1$ 

solve
$$(x^2-4\cdot x-9=0,x)$$
  
 $x=-(\sqrt{13}-2) \text{ or } x=\sqrt{13}+2$ 

#### Casio Classpad

♠ Action→Advanced→Solve

solve(
$$x^2-4x-9=0, x$$
)  
{ $x=-\sqrt{13}+2, x=\sqrt{13}+2$ }

$$\label{eq:continuous} \begin{split} & \ln[122] \text{:= Solve} \left[ \, x^{\,2} - 4 \, x - 9 \, \text{== 0, } x \right] \\ & \text{Out}[122] \text{= } \left\{ \left\{ x \to 2 - \sqrt{13} \, \right\}, \, \left\{ x \to 2 + \sqrt{13} \, \right\} \right\} \end{split}$$

#### **Question 36 Tech-Active.**

Solve  $x^2 - 3x = 2x + 9$ .

#### <u>Calculator Commands:</u> Finding the Angle between a Line and x-axis



#### Mathematica

In[124]:= ArcTan[2] / Degree // N
Out[124]= 63.4349

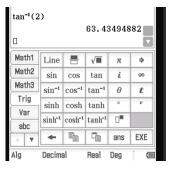
#### ➤ TI-Nspire

etrig button. Check that you are in degrees.

tan<sup>-1</sup>(2) 63.4349

#### Casio Classpad

G Keyboard→Trig. Change to decimals and degrees.





#### Question 37 Tech-Active.

Find the angle between the line y = 3x + 1 and the positive *x*-axis.

#### Calculator Commands: Finding the Angle between Two Lines

- Mathematica
  - Use the Abs[] function.

In[126]:= Abs[ArcTan[2] - ArcTan[1]] / Degree // N
Out[126]:= 18.4349

- ➤ TI-Nspire
  - Find the modulus sign.

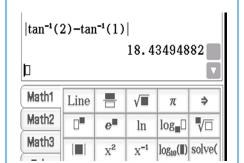




 $|\tan^{-1}(2)-\tan^{-1}(1)|$  18.4349

#### Casio Classpad

Modulus sign under Math1.



#### **Question 38 Tech-Active.**

Find the angle between the lines y = 3x - 2 and y = x + 1.



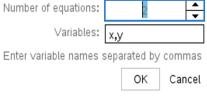
#### **Calculator Commands: Simultaneous Equations**

- Mathematica
  - Just do && between
  - Solve[equation&&equation , {var1, var2}]

In[128]:= Solve[2 x - 3 y = 16 && x + y = 3, {x, y}]
Out[128]:= { $\{x \to 5, y \to -2\}$ }

- TI-Nspire
  - Menu 371

#### Solve a System of Equations



solve 
$$\begin{cases} 2 \cdot x - 3 \cdot y = 16 \\ x + y = 3 \end{cases}, \{x, y\}$$
  $x = 5$  and  $y = -2$ 

#### Casio Classpad

Math1 →click highlighted box→ enter equations and variables you are solving for

#### Question 39 Tech-Active.

Solve simultaneous equations 2x + 3y = 5 and 3x - y = 8. (Give your answer correct to 2 decimal places.)



### Section E: Exam 2 Questions (21 Marks)

#### INSTRUCTION: 21 Marks. 5 Minutes Reading. 26 Minutes Writing.



#### Question 40 (1 mark)

The vertical distance between the function  $f(x) = x^3 + 2$  and g(x) = x - 2 when x = 1 is:

- **A.** 3
- **B.** 4
- **C.** 5
- **D.** 6

#### Question 41 (1 mark)

It is known that the lines y = mx + 4 and y = 2x - 2 make an angle of 45° when they intersect. The possible values for m are:

- **A.** m = 3 only.
- **B.**  $m = -\frac{1}{3}$  only.
- C.  $m = -3, \frac{1}{3}$
- **D.**  $m = -3, -\frac{1}{3}$

#### Question 42 (1 mark)

The angle that the line y = -x + 4 makes with the positive *x*-axis is:

- **A.** 45°
- **B.** 135°
- **C.** 120°
- **D.** 30°

Question 43 (1 mark)

Solve the following simultaneous equations:

$$2x + 3y = 17$$
$$x - y = 1$$

- **A.** x = 2, y = 3
- **B.** x = 4, y = 1
- **C.** x = 4, y = 3
- **D.** x = 3, y = 4

Question 44 (1 mark)

The point (2, k) has a vertical distance of 5 units from the line y = 3x - 4. A possible value of k is:

- **A.** 6
- **B.** 7
- **C.** 8
- **D.** 9

**Space for Personal Notes** 



d.	Hence, find the minimum distance between the line segment AB and the line $y = 3x + 4$ . (3 marks)					
e.						
	i.	Find the coordinates of the point $C$ obtained from reflecting $B$ in the line $y = 4$ . (1 mark)				
	ii.	Find the coordinates of the point D obtained from reflecting A in the line $x = 4$ . (1 mark)				
f.	Co	nsider the triangle BAD.				
	i.	Find the area of the triangle <i>BAD</i> . (2 marks)				



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	_
Find the angle $\angle ABD$ correct to two decimal places. (3 marks)	
for Personal Notes	
	Find the angle ∠ABD correct to two decimal places. (3 marks)  for Personal Notes



## **Contour Check**

<u>Learning Objective</u> : [1.1.1] - Solve and Graph Linear Equations and Inequalities				
Key Takeaways         Linear equations are in the form of $y = $ where $m$ is the and $c$ is the         The inequality sign when you multiply by a negative.				
<u>Learning Objective</u> : [1.1.2] - Find the Midpoint and Distance (Horizontal & Vertical) between Two Points or Functions				
Key Takeaways				
☐ Midpoint is simply the of 2 points.				
□ Distance formula is derived from				
Horizontal distance is the distance between values.				
. □ Vertical distance is the distance between values.				
<u>Learning Objective</u> : [1.1.3] - Find Parallel and Perpendicular Lines				
Key Takeaways				
Parallel lines have the gradient.				
Perpendicular lines have gradient.				



# <u>Learning Objective</u>: [1.1.4] – Find the Angle between a Line and x-axis or Two Lines

Key Takeaways				
To find the angle between a line and the $x$ -axis, we can use equation $m=$				
To find the angle between two lines, we can use $\theta =$ or				
$tan(\theta) = \underline{\hspace{1cm}}$				
<u>Learning Objective</u> : [1.1.5] - Find the Unknown Value for Systems of Linear Equations				
Key Takeaways				

# Key Takeaways Two linear equations have unique solutions if they have \_\_\_\_\_\_ gradients. Two linear equations have infinitely many solutions when they have \_\_\_\_\_\_ gradient and \_\_\_\_\_ constant. Two linear equations have no solution when they have \_\_\_\_\_ gradient and \_\_\_\_\_ constant.

## **Learning Objective**: [1.2.1] - Applying Midpoint to Find Reflected Points

	Key Takeaways
The	changes for reflections about a horizontal line.
The	changes for reflections about a vertical line.





Learning Objective: [1.2.2] - Find the Vertical and Horizontal Distance bet	tween
Functions	

#### **Key Takeaways**

The difference between two y-values is f(x) = g(x) where f is \_\_\_\_\_\_ g.

<u>Learning Objective</u>: [1.2.3] - Finding the Distance between a Point and a Function

#### **Key Takeaways**

- $\square$  A point on the function f is \_\_\_\_\_\_.
- $\square$  The distance between a point (a, b) and a point on the function f is \_\_\_\_\_\_



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## VCE Mathematical Methods ½

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