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VCE Mathematical Methods ½
Linear & Coordinate Geometry [1.1]
Workbook

Outline:

Section A: Linear Functions and Graphs

Linear Equations

- **Definition:** Equations where the highest power of a variable is 1.
  - **Gradient-intercept form:**

$$y = mx + c$$

$$where\ m = gradient = \frac{rise}{run} =$$

$$and\ c =$$

- No singular solution for a linear equation in two variables.
  - All pairs of coordinates (x, y) that satisfy the equation lie on a **line.** (Hence, *linear* equations).

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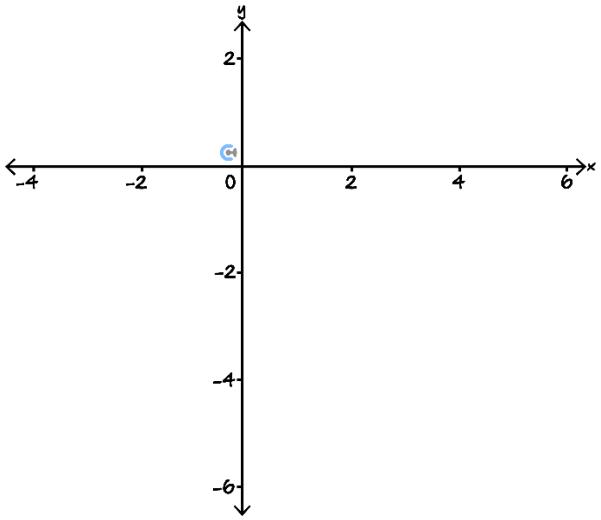
Let's have a look at sketching some of these equations.

Question 1 Walkthrough.

Sketch the graph of the following linear relations, labelling all axes intercepts.

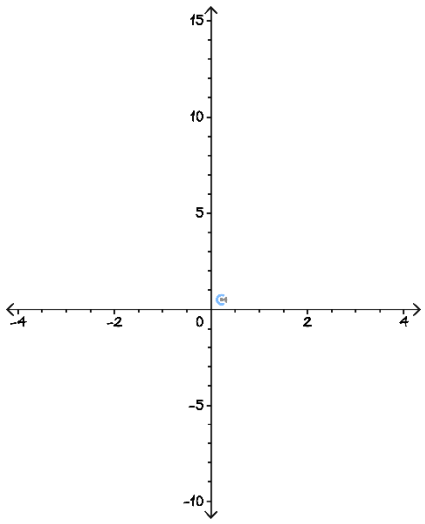
$$y = 2x - 4$$



Question 2

Sketch the graphs of each of the following linear relations, labelling all axes intercepts with their coordinates.

$$y = -3x + 5$$



Sub-Section: Inequality

Inequalities Rule

$$x > \frac{b}{a}, where\ a < 0$$

- Multiplying both sides by a negative number _____ the inequality sign.

Question 3

Solve each of the following for x:

$$9 - 6x \leq 2$$

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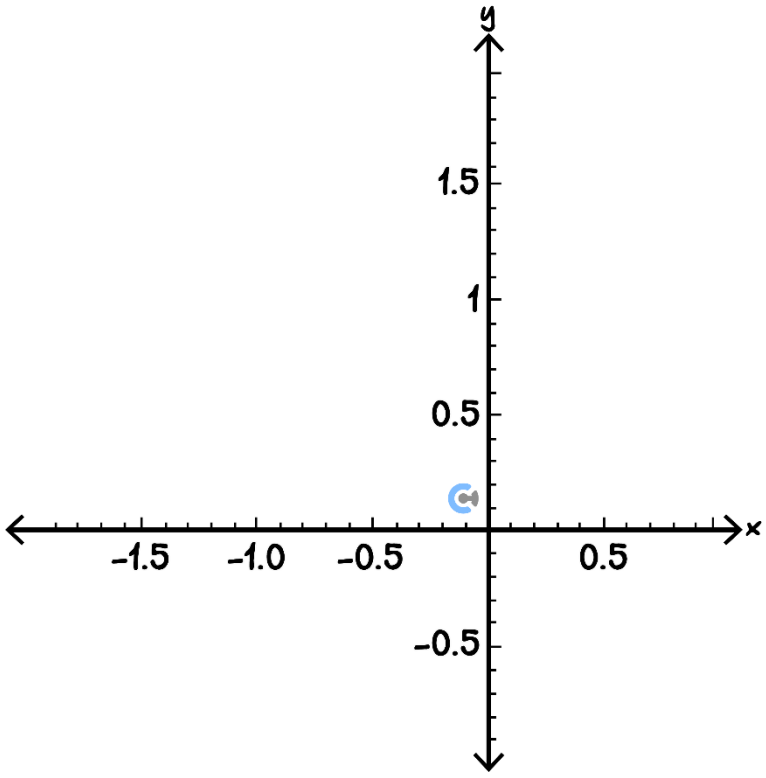
Exploration: Graphs of Linear Inequalities

- Instead of just representing _____ on the Cartesian plane, the graph of a linear inequality represents an entire _____ (x, y) which satisfies the inequality.
- **Step 1:** Put the equation into the form of _____
- **Step 2:** Sketch the linear **equation,** ignoring the inequality sign.
  - If the inequality is inclusive ($\leq or \geq$), we draw a [solid] / [dotted] line.
  - If the inequality is exclusive ($<$ or $>$), we draw a [solid] / [dotted] line
- **Step 3:** Shade the region either **above** or **below** the line
  - **If $y >$ or $y \geq$:** _____
  - **If $y <$ or $y \leq$:** _____

Question 4 Walkthrough.

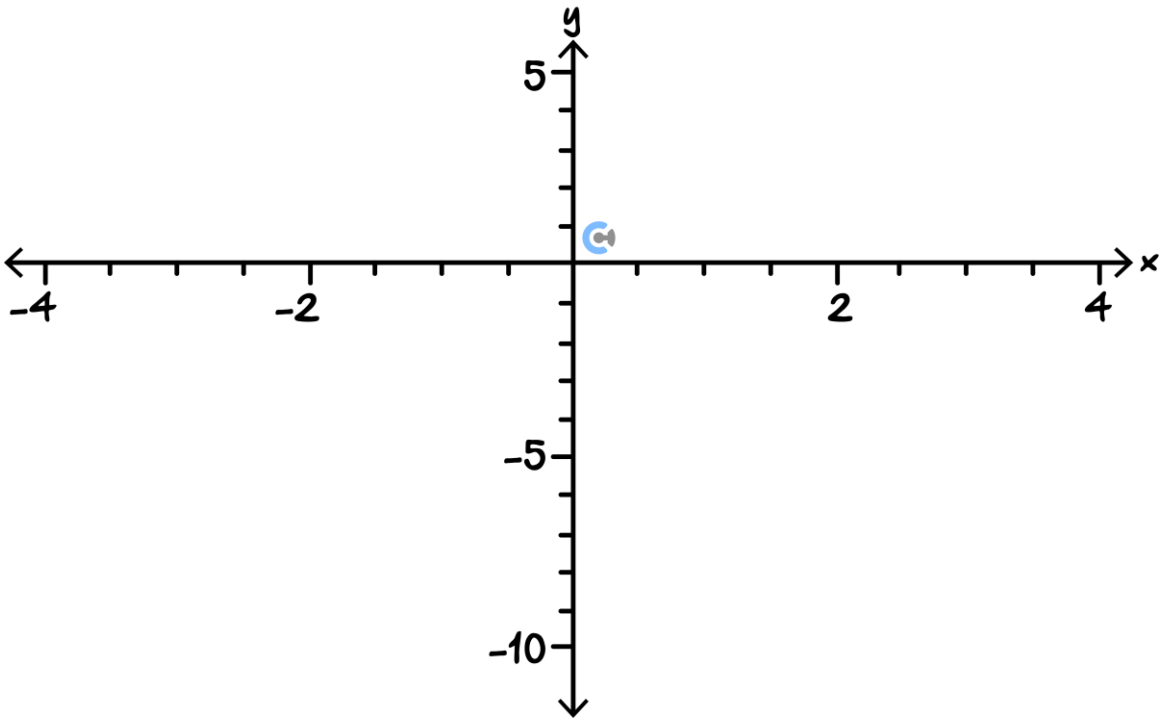
Sketch the linear inequality $y - 1 - x \geq 0$ on the axes below.



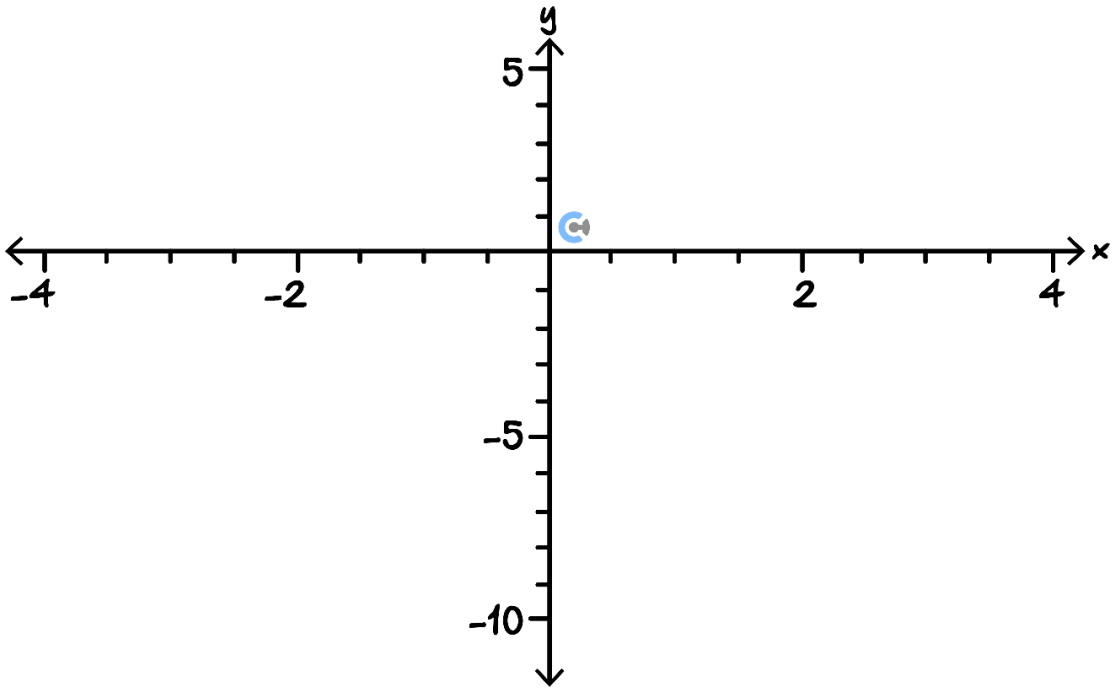
Question 5

Sketch the graphs of each of the following linear inequalities, labelling all axes intercepts with their coordinates, and shading the appropriate regions.

a. $4x + 2y < -6$



b. $5x - 2y - 8 \leq 0$

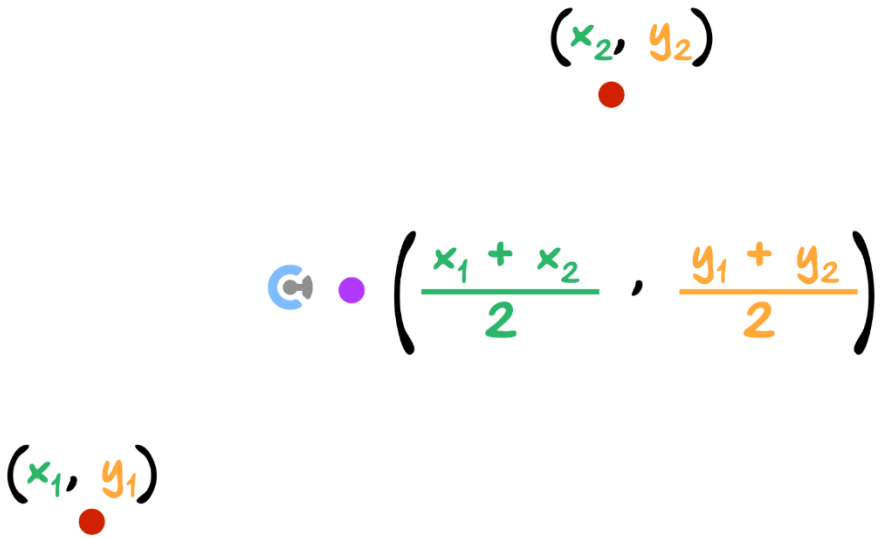


Section B: Midpoint and Distances

Sub-Section: Midpoint

Discussion: How might we find a midpoint between two points?

Midpoint



- **Definition:** The midpoint, M, of two points A and B is the point halfway between A and B.

$$M(x_m, y_m) = \left(\_\_\_\_\_\right)$$

- The midpoint can be found by taking the _____ of the x-coordinate and y-coordinate of the two points.

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Question 6

Find the midpoint between $(3, -5)$ and $(-2, 7)$.

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Sub-Section: Distance Between Two Points

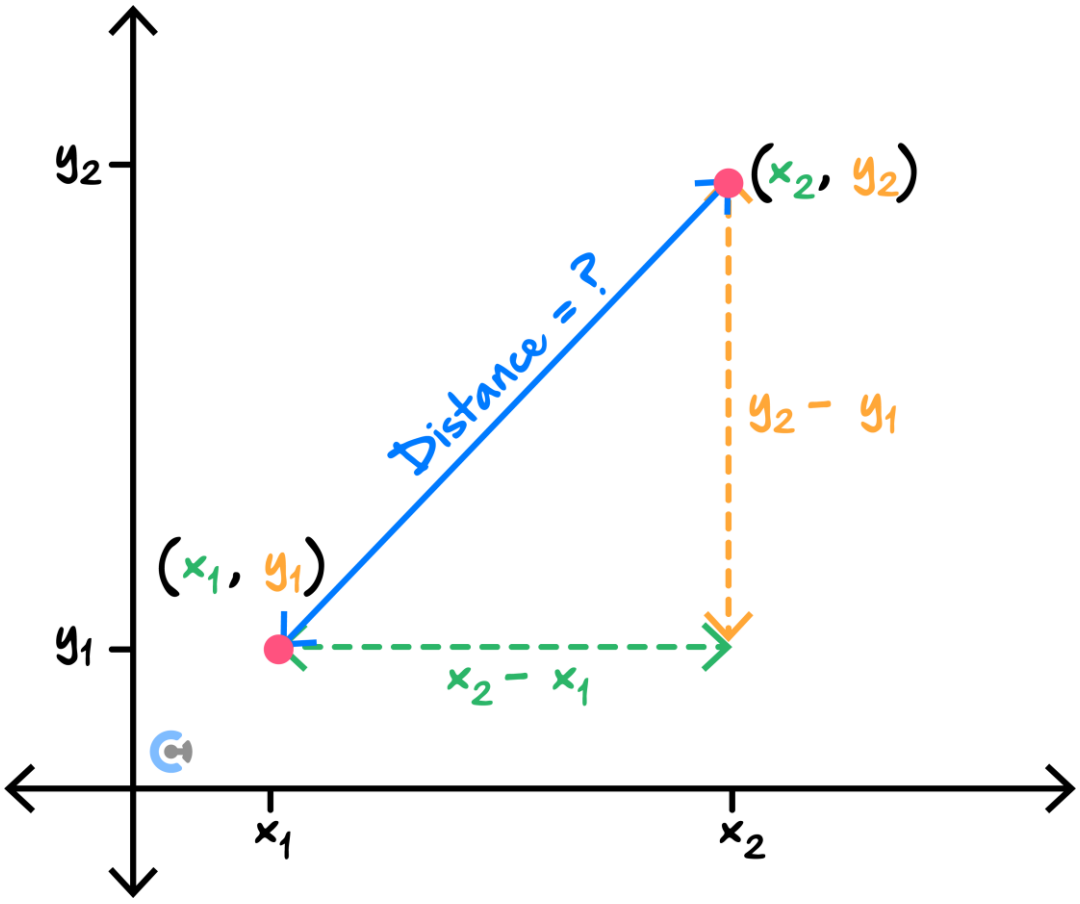
Distance Between Two Points

- **Definition:** The distance between two points (x_1, x_2) and (y_1, y_2) can be found using Pythagoras' theorem:

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

How does this formula work?

Exploration: Distance between two points



- Try to construct a Pythagoras' theorem with the three sides above ☺

Question 7

Find the distance between $\left(4, \frac{5}{2}\right)$ and $(-1, -\frac{1}{2})$.

Question 8 Extension.

a. Find a point(s) on the line $y = x + 3$ which has a distance of 4 from the point $(-2, -3)$.

b. Give a reason as to why there is more than 1 more point found in **part a.**

TIP: Don't hesitate to define a point by letting its y-value be the function (linear in the above question!)

Sub-Section: Vertical Distance VS Horizontal Distance

Discussion: How can we find a horizontal distance between two points?

Horizontal Distance

$$\text{Horizontal Distance} = x_2 - x_1 \text{ where } x_2 > x_1$$

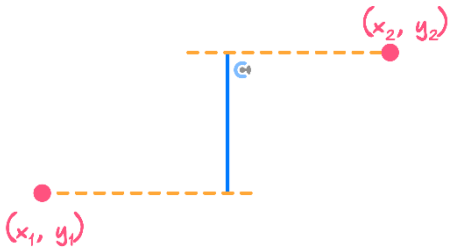
- Find the difference between their x-values.

Question 9

Find the horizontal distance between the two points $(1,9)$ and $(13,-4)$.

What about vertical distance then?

Vertical Distance



Vertical Distance $= y_2 - y_1$ **where** $y_2 > y_1$

- Find the difference between their y values.

Question 10

Find the vertical distance between the two points $(1,9)$ and $(13,-4)$.

Key Takeaways

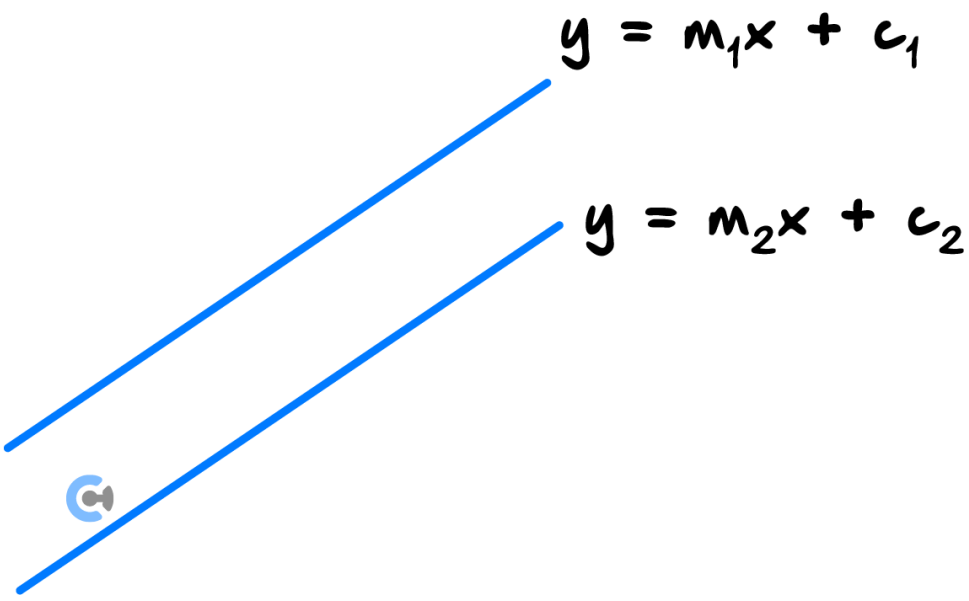
- ☑ A midpoint is simply an average point.
- ☑ The distance between two points is derived from Pythagoras' theorem.
- ☑ Horizontal distance is simply the difference in their x-values
- ☑ Vertical distance is simply the difference in their y-values.

Section C: Line Geometry

Sub-Section: Parallel and Perpendicular Lines

Discussion: What do we need for the two lines to be parallel?

Parallel Lines



- Parallel lines have the _____ gradient.

$$m_1 = m_2$$

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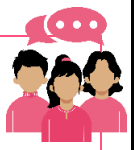
Question 11

Find a line which is parallel to $y = 2x - 1$ passing through the point $(-1,3)$.

TIP: Try to ignore the constant term of the line we must be parallel to. Simply focus on its gradient.



Discussion: What about perpendicular lines?

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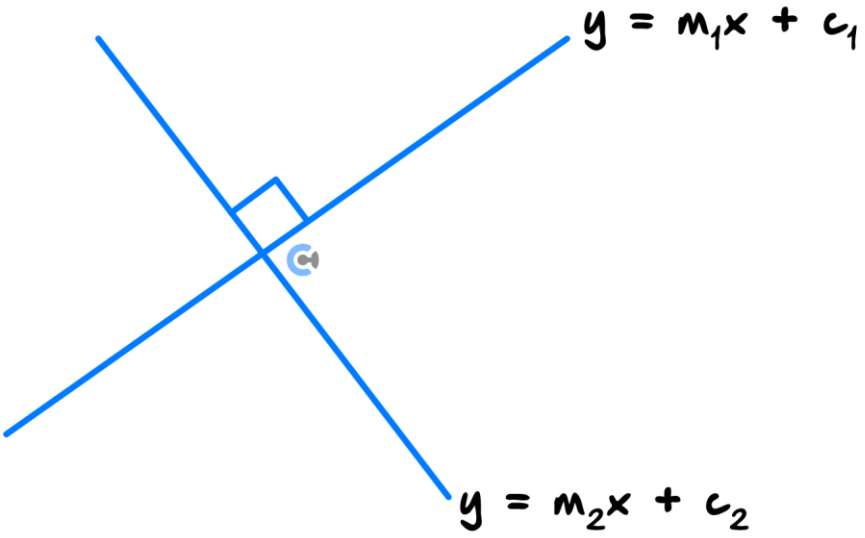
Perpendicular Lines



- A line which is perpendicular to another line has a gradient which is the _____ of the gradient of the first line.

$$m_{\perp} = -\frac{1}{m}$$

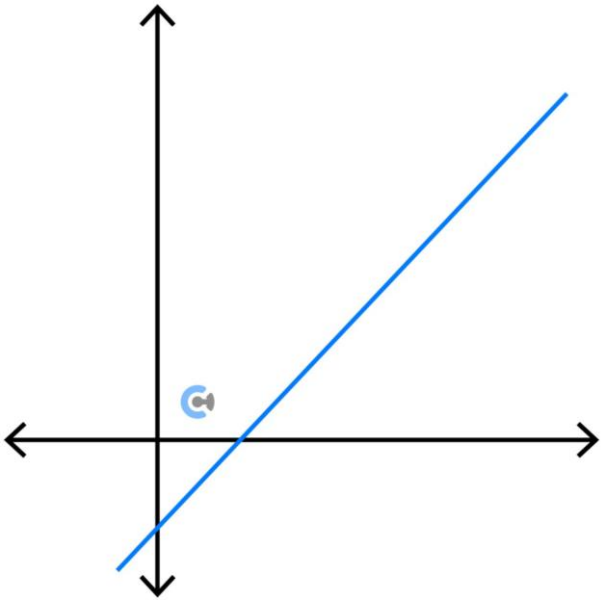
Question 12

Find a line which is perpendicular to $y = -3x - 1$ passing through the point $(5, -1)$.

Sub-Section: Angle Between a Line and the x-axis

How do we find the angle between a line and the x-axis?

Angle between a Line and the x-axis



- The angle between a line and the _____ **direction of the x-axis** (anticlockwise) is given by

$$\tan(\theta) = m$$

Question 13 Tech-Active.

Find the angle made between the line $y = 2x - 6$ and the x-axis measured in the anticlockwise direction. Give your answer in degrees correct to two decimal places.

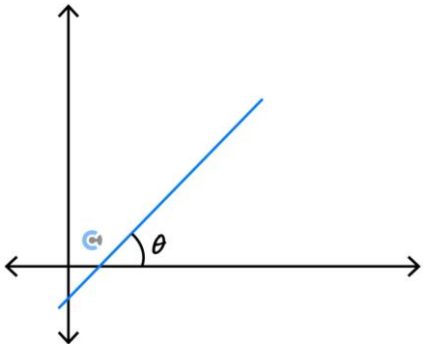
NOTE: Angles from the x-axis measured anticlockwise = _____ angles.

- Don't worry about it too much, it's just convention! (More on this in circular functions.)

How does this formula work?

Exploration: Angle between a line and x-axis.

- **Consider a line in the visual below.**



- Construct a right-angle triangle with the angle θ.

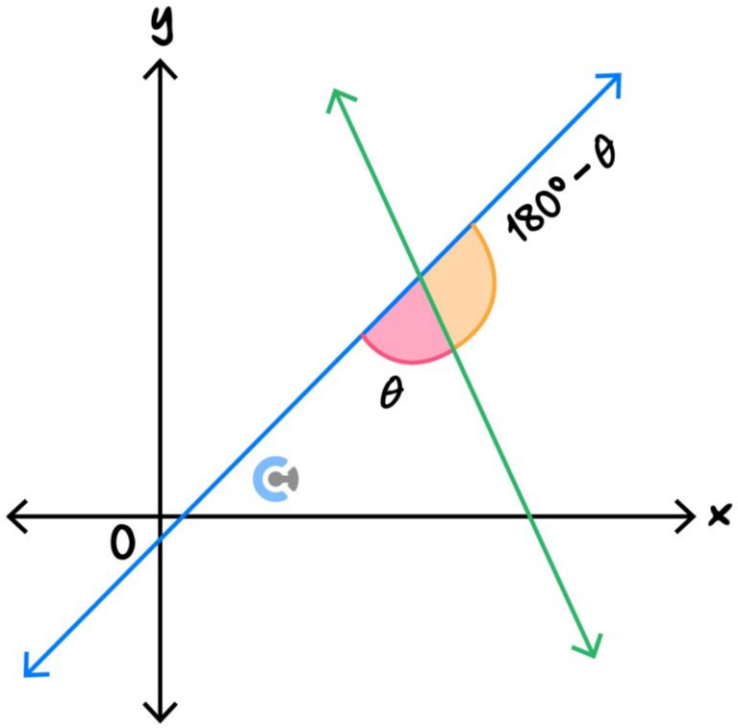
- Consider the opposite and adjacent sides of the right-angle triangle. What can we call them?

- Hence what does $\tan(\theta)$ equal to given that $\tan = \text{opposite}/\text{adjacent}$?

Sub-Section: Angle Between the Two Lines

Slightly more complicated now!
How about an angle between two lines?

Acute Angle Between Two Lines



$$\theta = |\tan^{-1}(m_1) - \tan^{-1}(m_2)|$$

- **Alternatively:**

$$\tan(\theta) = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

- For your understanding, note that this formula is derived from the tan compound angle formula covered in SM12.

NOTE: $|x|$ just takes the positive value of x.

Question 14 Tech-Active.

Find the acute angle between the lines $3x + 2y = 2$ and $y = \frac{4}{7}x + 1$. Give your answer in degrees correct to two decimal places.

TIP: Make sure your CAS is in degrees.

Let's see if it's consistent with parallel lines!

Exploration: Understanding parallel lines using the angle between two lines formula

- When two lines are parallel, what must be the angle θ between them?

$$\tan(\theta) = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

- Let's substitute the value of θ and see what we get!

- This looks rather familiar, doesn't it?

And now perpendicular lines!

Exploration: Understanding perpendicular lines using the angle between two lines formula

- When two lines are perpendicular, what must be the angle θ between them?

$$\tan(\theta) = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

- Let's substitute the value of θ and see what we get! (Note: $\tan(90) = Undef$)

- This looks rather familiar, doesn't it?

Key Takeaways

- ☑ Parallel lines have the same gradient.
- ☑ Perpendicular lines have a negative reciprocal gradient.
- ☑ The angle between a line and x-axis is given by $\tan^{-1}(m)$.
- ☑ Tangent of the angle between two lines is given by $\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$.
- ☑ The parallel lines and perpendicular lines formula is consistent with the angle between the two lines formula.

Section D: Simultaneous Equations

Sub-Section: Finding Simultaneous Equations for Two Variables

Simultaneous Linear Equations

1. Elimination Method:
   - Add or subtract one equation from the other in order to _____ one of the variables. Then have an equation in one variable that can be solved easily.
2. Substitution Method
   - Make one of the variables the subject (generally x or y) and _____ that value into the other equation.

Question 15 Walkthrough.

Solve the following simultaneous linear equations using either elimination or substitution.

$$2x - y = 8 \text{ and } 2y + 5x = 11$$

Question 16

Solve the following equations for x and y.

a. $2x - 5y = 4$ and $2x + y = 16$

b. $-3x + 2y = 2$ and $2 - 2y = x$

Question 17 Extension.

Solve the following:

$$-3x + 2y = 10 \text{ and } -10 + y = \frac{3}{2}x$$

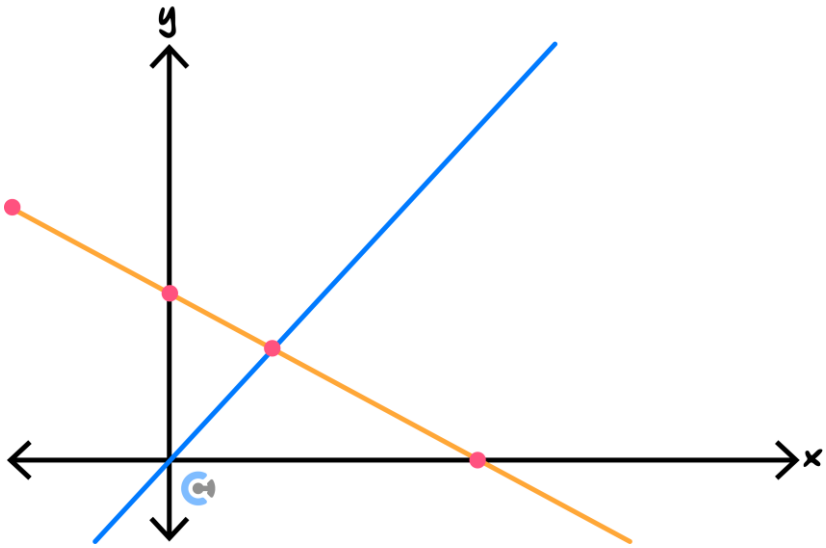
Sub-Section: Number of Solutions For Two Variables

What does the geometry look like for each number of solutions?

Exploration: Geometry of the number of solutions between linear graphs

- Unique Solution

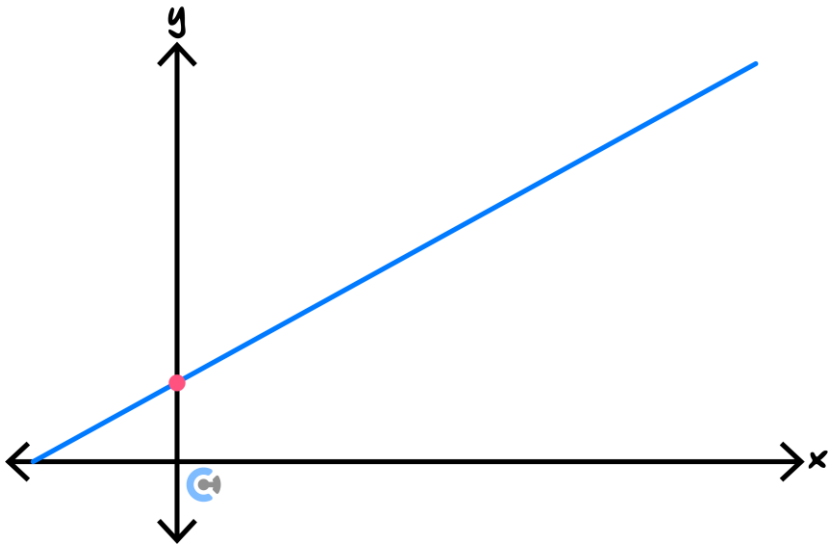
$$m_1 \neq m_2$$



- They just need to have _____.

Infinite Solutions

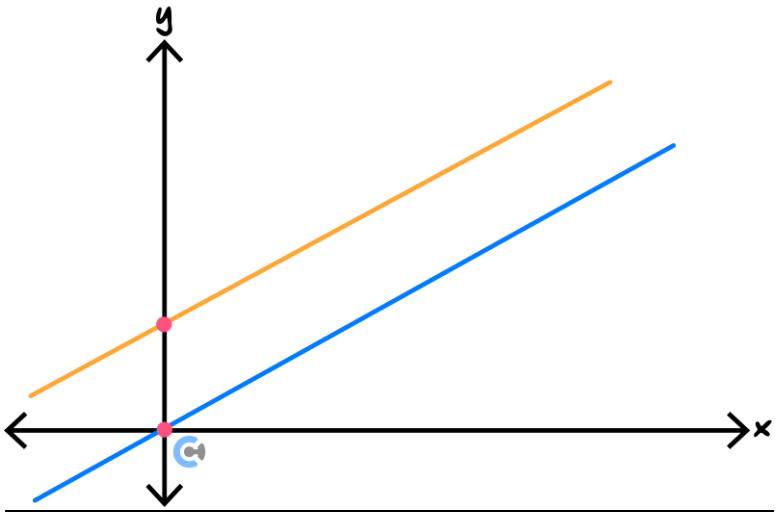
$$m_1 = m_2 \; AND \; c_1 = c_2$$



- They just need to have the same _____ and the same _____.
- In other words, they have to be the _____.

No Solutions

$$m_1 = m_2 \; AND \; c_1 \neq c_2$$



- They need to have the _____ but _____ $+c$.
- They have to be two different _____ lines.

General Solutions of Simultaneous Linear Equations

- Two linear equations are either:
  - The same line is expressed in a different form. In this case, they have _____ solutions.
  - Unique lines which are parallel. In this case, they have _____ solutions.
  - Unique lines which are not parallel. In this case, they have _____ solution.

Question 18 Walkthrough.

Consider the following pair of simultaneous equations in terms of $k \in \mathbb{R}\backslash\{0\}$:

$$y = kx + 5$$

$$y = \frac{2x}{k-1} - 5k$$

a. Find the value of k for which there are no solutions to the simultaneous equations.

b. Find the value(s) of k for which there is a unique solution to the simultaneous equations.

c. Find the value of k for which there are infinite solutions to the simultaneous equations.

TIP: It's a good idea to substitute your answer back into the equations to see if the criteria are met for each part.

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Question 19

Consider the following pair of simultaneous equations in terms of $k \in \mathbb{R}\backslash\{0\}$:

$$y = \frac{x}{1-2k} - 2k$$

$$y = -kx - 2$$

a. Find the value(s) of k for which there is a unique solution to the simultaneous equations.

b. Find the value of k for which there are infinite solutions to the simultaneous equations.

c. Find the value of k for which there are no solutions to the simultaneous equations.

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Question 20 Extension.

Consider the following pair of simultaneous equations in terms of $a \in \mathbb{R}\backslash\{0\}$:

$$ax - 2y = -5$$

$$-3x + (a-1)y = 5$$

a. Find the value(s) of a for which there are no solutions to the simultaneous equations.

b. Find the value(s) of a for which there is a unique solution to the simultaneous equations.

c. Find the value(s) of a for which there are infinite solutions to the simultaneous equations.

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Contour Check

Learning Objective: [1.1.1] – Solve and Graph Linear Equations and Inequalities

Key Takeaways

- ☐ Linear equations are in the form of $y =$ _____ where m is the _____ and c is the _____.
- ☐ The inequality sign _____ when you multiply by a negative.

Learning Objective: [1.1.2] – Find Midpoint, Distance (Horizontal & Vertical) Between Two Points Or Functions

Key Takeaways

- ☐ Midpoint is simply the _____ of 2 points.
- ☐ Distance formula is derived from _____.
- ☐ Horizontal distance is the distance between ____ values.
- ☐ Vertical distance is the distance between _____ values.

Learning Objective: [1.1.3] – Find Parallel and Perpendicular Lines

Key Takeaways

- ☐ Parallel lines have the _____ gradient.
- ☐ Perpendicular lines have _____ gradient.

Learning Objective: [1.1.4] – Find the Angle Between a Line and x axis or Two Lines

Key Takeaways

- To find the angle between a line and the x axis we can use equation $m =$ _____ .
- To find the angle between two lines we can use $\theta =$ _____ or $\tan(\theta) =$ _____.

Learning Objective: [1.1.5] – Find The Unknown Value for Systems of Linear Equations

Key Takeaways

- Two linear equations have unique solution if they have _____ gradients.
- Two linear equations have infinitely many solutions when they have _____ gradient and _____ constant.
- Two linear equations have no solution when they have _____ gradient and _____ constant.

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