



Website: [contoureducation.com.au](http://contoureducation.com.au) | Phone: 1800 888 300

Email: [hello@contoureducation.com.au](mailto:hello@contoureducation.com.au)

**VCE Mathematical Methods ½**  
**Linear & Coordinate Geometry [1.1]**  
**Homework Solutions**

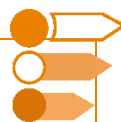
**Homework Outline:**

|                      |               |
|----------------------|---------------|
| Compulsory Knowledge | Pg 2 – Pg 16  |
| Extension Work       | Pg 17 – Pg 29 |



## Section A: Compulsory Questions

### Sub-Section: Solve and Graph Linear Equations and Inequalities



#### Question 1



Solve the following linear equations and inequalities for  $x$ .

a.  $3x + 2 = 20$

$$3x = 18 \implies x = 6$$

b.  $2x + 6 = 3(x - 4)$

$$2x + 6 = 3x - 12 \implies x = 18$$

c.  $5x + 2 < 4x + 7$

$$x < 5$$

#### Question 2



Solve the following linear equations and inequalities for  $x$ .

a.  $3x + 2 = 9x + 3$

$$6x = -1 \implies x = -\frac{1}{6}$$

b.  $\frac{2x+3}{3} > 3(x-4)$

$$2x + 3 > 9x - 36 \implies 39 > 7x \implies x < \frac{39}{7}$$

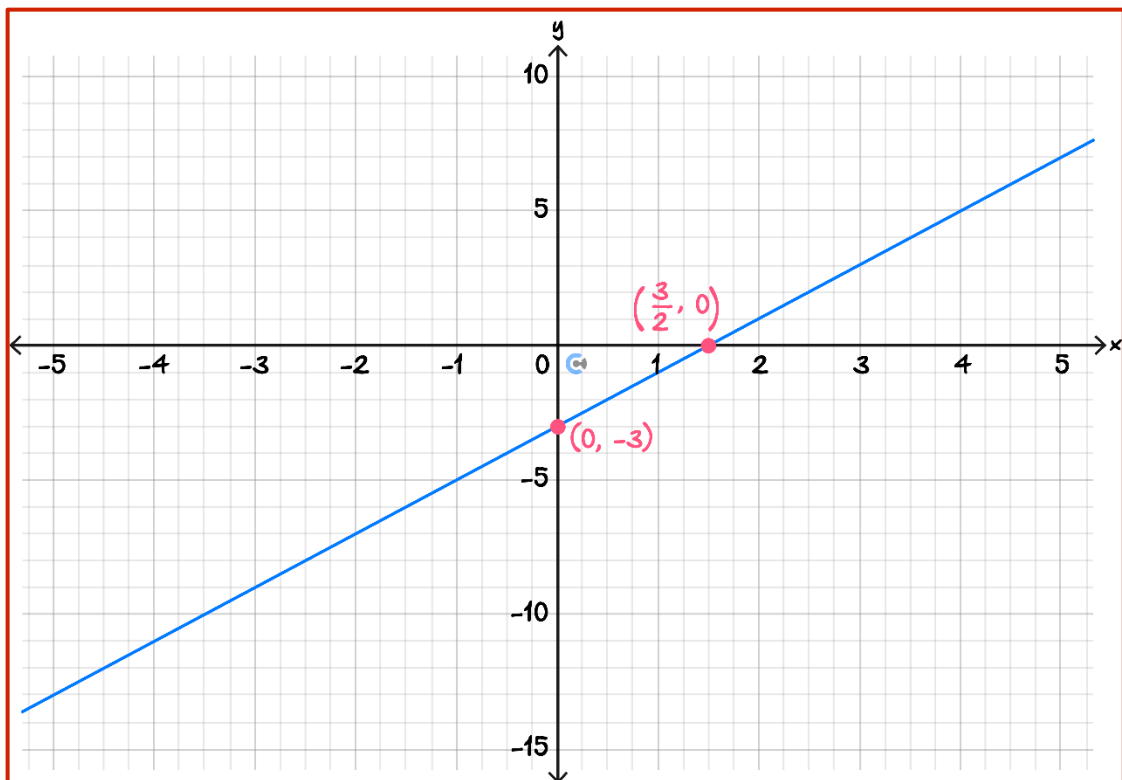
c.  $\frac{5x+3}{4} \leq 8x + 7$

$$5x + 3 \geq 32x + 28 \implies -25 \geq 27x \implies x \geq -\frac{25}{27}$$

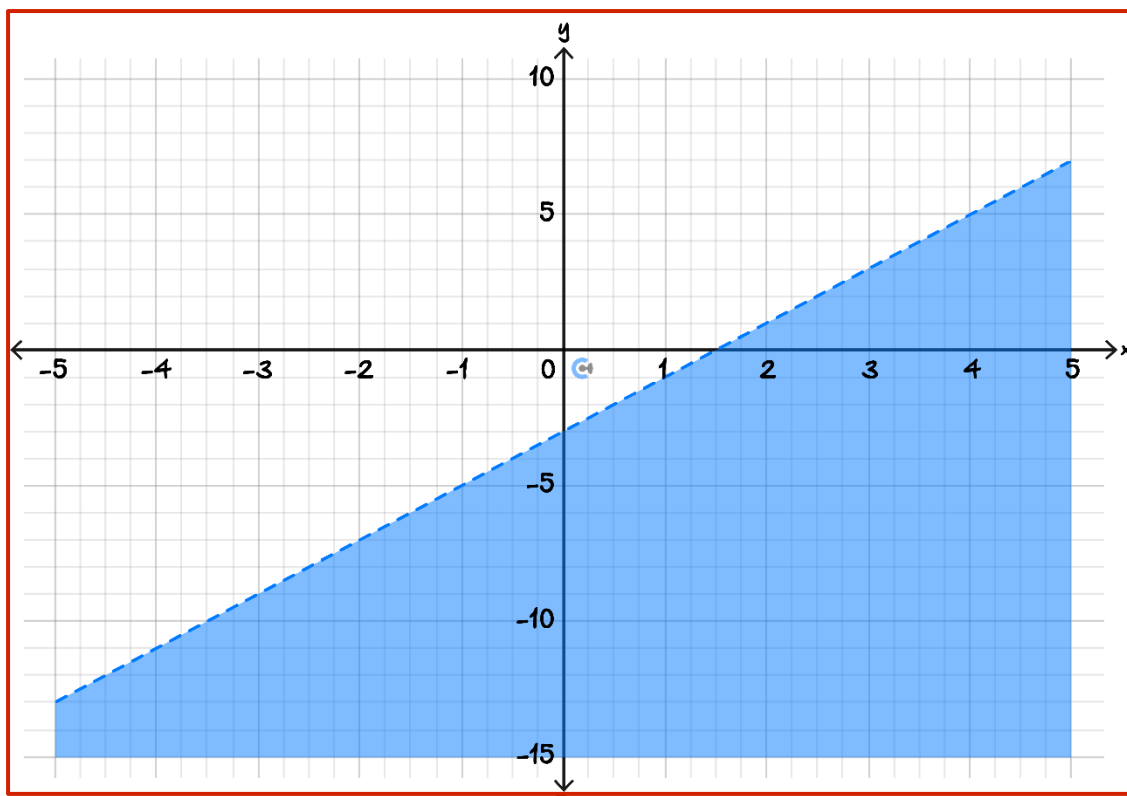
### Question 3



- a. Sketch the line governed by the equation  $2y - 4x = -6$  on the axis below. Label all axes intercepts.



- b. Shade the region governed by the equation  $2y - 4x < -6$  on the axis below.



#### Question 4 Tech-Active.

Solve the inequality  $\frac{1}{4}(5x - 3) \geq 2x + 8$  for  $x$ .

$$x \leq -\frac{35}{3}.$$

TI:

$$\text{solve}\left(\frac{1}{4} \cdot (5 \cdot x - 3) \geq 2 \cdot x + 8, x\right) \quad x \leq -\frac{35}{3}$$

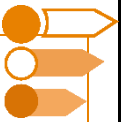
Mathematica:

$$\text{In[1]:= Reduce}\left[\frac{1}{4} (5 x - 3) \geq 2 x + 8, x\right]$$

$$\text{Out[1]= } x \leq -\frac{35}{3}$$

Casio:

$$\text{solve}(1/4(5x-3) \geq 2x+8, x) \quad \left\{x \leq -\frac{35}{3}\right\}$$



## Sub-Section: Find the midpoint and distance between two points or functions

### Question 5



- a. Find the midpoint of  $(1, -3)$  and  $(5, -9)$ .

$$\left( \frac{1+5}{2}, \frac{-3-9}{2} \right) = (3, -6)$$

- b. The points  $(a, b)$  and  $(1, 3)$  have a midpoint  $(2, 4)$ . Find the values of  $a$  and  $b$ .

$$\frac{a+1}{2} = 2 \implies a = 3 \text{ and } \frac{b+3}{2} = 4 \implies b = 5.$$

### Question 6



- a. Find the distance between points  $(2, 3)$  and  $(5, 2)$ .

$$d = \sqrt{9+1} = \sqrt{10}.$$

- b. The curve  $y = (x - 1)^2 + k$  and the line  $y = 1$  has a minimum vertical distance of 5. Find the value of  $k$ .

$$\text{The parabola has a minimum at } (1, k). \text{ Therefore } 5 = k - 1 \implies k = 6.$$

Space for Personal Notes


**Question 7**

The distance between the point  $(1, 1)$  and a point  $P$  on the line  $y = 2x + 1$  is 5 units. Find all possible coordinates for  $P$ .

$$d = \sqrt{(x-1)^2 + (2x+1-1)^2} = 5$$

$$\Rightarrow x^2 - 2x + 1 + 4x^2 = 25$$

$$5x^2 - 2x - 24 = 0$$

$$x = \frac{2 \pm \sqrt{4 + 480}}{10}$$

$$= \frac{2 \pm \sqrt{4 \times 121}}{10}$$

$$= \frac{2 \pm 22}{10}$$

$$= -2, \frac{12}{5}$$

Substitute these  $x$ -values into the line  $y = 2x + 1$  to get possible coordinates for  $P$  as

$$(-2, -3) \quad \text{and} \quad \left(\frac{12}{5}, \frac{29}{5}\right).$$

**Question 8 Tech-Active.**

The distance between the point  $(1, 2)$  and a point  $P$  on the line  $y = x - 1$  is 5 units. Find all possible coordinates for  $P$ .

$$d = \sqrt{(x-1)^2 + (x-1-2)^2} = 4 \Rightarrow x = 2 \pm \sqrt{7}. \text{ Therefore possible coordinates are}$$

$$(2 - \sqrt{7}, 1 - \sqrt{7}) \quad \text{and} \quad (2 + \sqrt{7}, 1 + \sqrt{7}).$$

Space for Personal Notes



## Sub-Section: Find parallel and Perpendicular Lines

### Question 9



State whether the following lines are parallel or perpendicular.

a.  $y = 2x + 1$  and  $y = 2x + 3$

$$m_1 = m_2 \implies \text{parallel.}$$

b.  $y = 3x + 3$  and  $y = -\frac{1}{3}x + 2$

$$m_1 \times m_2 = -1 \implies \text{perpendicular.}$$

### Question 10



Find the equation of the line that is parallel to the line  $y = 2x + 1$  and passes through the point  $(2,3)$ .

Since the line is parallel to  $y = 2x + 1$  it must have the form  $y = 2x + c$ .  
 Sub in the point  $(2, 3) \implies 3 = 4 + c \implies c = -1$   
 The line is  $y = 2x - 1$ .

Space for Personal Notes


**Question 11**

Find the equation of the line that is perpendicular to  $y = 3x + 6$  and passes through the point  $(3, 2)$ .

The line has gradient  $-\frac{1}{3}$  and passes through  $(3, 2)$  therefore,

$$y - 2 = -\frac{1}{3}(x - 3)$$

$$y = -\frac{1}{3}x + 3.$$

**Question 12 Tech-Active.**

Find the equation of the line that is perpendicular to  $y = 2x + 1$  and passes through the point  $(1, 2)$ .

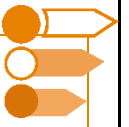
The line has gradient  $-\frac{1}{2}$  and passes through  $(1, 2)$  therefore,

$$y - 2 = -\frac{1}{2}(x - 1)$$

$$y = -\frac{1}{2}x + \frac{5}{2}.$$

Space for Personal Notes





## Sub-Section: Finding the angle between a line and the $x$ -axis or between two lines

### Question 13



Find the angle that  $y = x + 1$  makes with the positive direction of the  $x$ -axis.

$$\theta = \tan^{-1}(1) = 45^\circ.$$

### Question 14



A line that makes an angle  $60^\circ$  with the positive  $x$ -axis passes through the point  $(1,1)$ . Find the equation of the line.

We know that the gradient  $m = \tan(60) = \sqrt{3}$ . Now since we know the gradient and a point we can find the equation of the line.

$$y - 1 = \sqrt{3}(x - 1)$$

$$y = \sqrt{3}x + 1 - \sqrt{3}.$$

Space for Personal Notes



### Question 15

It is known that the lines  $y = mx + 1$  and  $y = 3x - 1$  make an angle of  $45^\circ$  when they intersect. Find all possible values of  $m$ .

The formula for the angle  $\theta$  between two lines with gradients  $m_1$  and  $m_2$  is given by

$$\tan(\theta) = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

here we have  $m_1 = m$  and  $m_2 = 3$  and  $\theta = 45^\circ \Rightarrow \tan(\theta) = 1$ . Therefore,

$$\left| \frac{m - 3}{1 + 3m} \right| = 1$$

$$\frac{m - 3}{1 + 3m} = \pm 1$$

solve the two cases separately.

$$m - 3 = 1 + 3m \Rightarrow m = -2$$

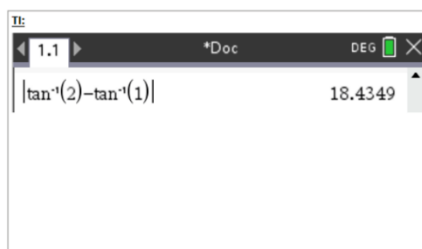
$$m - 3 = -1 - 3m \Rightarrow m = \frac{1}{2}$$

Conclude that  $m = -2$  or  $m = \frac{1}{2}$ .

### Question 16 Tech-Active.

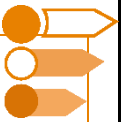
Find the acute angle made between the lines  $y = 2x + 1$  and  $y = x - 1$ . Give your answer in degrees correct to two decimal places.

$$\theta = |\tan^{-1}(2) - \tan^{-1}(1)| = 18.43^\circ$$



Mathematica:  
In[3]:= Abs[ArcTan[2] - ArcTan[1]] / Degree // N  
Out[3]:= 18.4349

Casio:  
abs(tan<sup>-1</sup>(2)-tan<sup>-1</sup>(1))  
18.43494882  
Alg Decimal Real Deg



## Sub-Section: Find the unknown value for a system of linear equations

### Question 17



Consider the simultaneous linear equations

$$y = 2kx + k$$

$$y = 2x + 3$$

where  $x, y \in R$  and  $k$  is a real constant.

- a. Find the value(s) of  $k$  for which the system of equations has no solution.

Lines must be parallel for no solution  $\implies k = 1$ . Then we have  $y = 2x + 1$  and  $y = 2x + 3$  which never intersect. Hence no solution if  $k = 1$ .

- b. Find the value(s) of  $k$  for which the system of equations has infinitely many solutions.

No value of  $k$  will result in the system having infinitely many solutions. This is because there can only be infinitely many solutions if the lines are identical. No value of  $k$  will make the lines identical.

- c. Find the value(s) of  $k$  for which the system of equations has a unique solution.

$$k \neq 1.$$

Space for Personal Notes


**Question 18**

Consider the simultaneous linear equations

$$-2x - ky = -4$$

$$(k - 1)x + 6y = 2(k - 1)$$

where  $x, y \in \mathbb{R}$  and  $k$  is a real constant.

- a. Find the value(s) of  $k$  for which the system of equations has no real solution.

The gradients must be equal for a possibility of there being no real solutions.

$$\begin{aligned} \frac{-2}{-k} &= \frac{k-1}{6} \\ \Rightarrow k(k-1) &= 12 \\ k^2 - k - 12 &= 0 \\ (k-4)(k+3) &= 0 \end{aligned}$$

If  $k = 4$  the lines are  $-2x - 4y = -4$  and  $3x + 6y = 6$ . These lines are identical therefore infinitely many solutions if  $k = 4$ .

If  $k = -3$  the lines are  $-2x + 3y = -4$  and  $-4x + 6y = -8 \Rightarrow 3x + y = 1$ . These lines are identical therefore infinitely many solutions if  $k = -3$ .

Therefore, there is no value of  $k$  for which the system will have no real solutions.

- b. Find the value(s) of  $k$  for which the system of equations has infinitely many solutions.

From above. The system will have infinitely many solutions for  $k = -3, 4$ .

- c. Find the value(s) of  $k$  for which the system of equations has a unique solution.

$$k \in \mathbb{R} \setminus \{-3, 4\}$$

Space for Personal Notes


**Question 19**

Consider the simultaneous linear equations

$$kx + 3y = 6$$

$$x + (7 - 2k)y = 2$$

where  $x, y \in \mathbb{R}$  and  $k$  is a real constant.

- a. Find the value(s) of  $k$  for which the system of equations has no real solution.

Equate the gradients.

$$\frac{k}{3} = \frac{1}{7 - 2k}$$

$$7k - 2k^2 = 3$$

$$2k^2 - 7k + 3 = 0$$

$$(2k - 1)(k - 3) = 0$$

$$k = \frac{1}{2}, 3$$

Now equate the  $y$ -intercepts.

$$\frac{6}{3} = \frac{2}{7 - 2k} \implies k = 3$$

Therefore if  $k = 3$  the lines are the same. So there is no real solution for  $k = \frac{1}{2}$  only.

- b. Find the value(s) of  $k$  for which the system of equations has infinitely many solutions.

From above there are infinitely many solutions if  $k = 3$ .

- c. Find the value(s) of  $k$  for which the system of equations has a unique solution.

$$k \in \mathbb{R} \setminus \left\{ \frac{1}{2}, 3 \right\}$$

Space for Personal Notes

**Question 20 Tech-Active.**

Consider the simultaneous linear equations

$$kx + 2y = 6$$

$$2x + (k - 1)y = 3$$

where  $x, y \in \mathbb{R}$  and  $k$  is a real constant.

Find the value(s) of  $k$  for which the system has no real solution.

Solve  $\frac{k}{2} = \frac{2}{k-1} \Rightarrow k = \frac{1 \pm \sqrt{17}}{2}$ . Now equate the  $y$ -intercepts

$$\frac{6}{2} = \frac{3}{k-1} \Rightarrow k = 2.$$

Therefore, it is not possible for both lines to have the same gradient and  $y$ -intercept. The system has no real solution when

$$k = \frac{1 \pm \sqrt{17}}{2}.$$

Spac

| TI   | Mathematica   | Casio  |
|--|---|--|
| $\text{solve}\left(\frac{k}{2} = \frac{2}{k-1}, k\right)$ $k = \frac{-(\sqrt{17}-1)}{2} \text{ or } k = \frac{\sqrt{17}+1}{2}$ $\text{solve}\left(\frac{6}{2} = \frac{3}{k-1}, k\right)$ $k=2$ | $\text{In}[4] = \text{Solve}\left[k/2 == \frac{2}{k-1}, k\right]$ $\text{Out}[4] = \left\{\left\{k \rightarrow \frac{1}{2}(1 - \sqrt{17})\right\}, \left\{k \rightarrow \frac{1}{2}(1 + \sqrt{17})\right\}\right\}$ $\text{In}[5] = \text{Solve}\left[\frac{6}{2} == \frac{3}{k-1}, k\right]$ $\text{Out}[5] = \left\{\left\{k \rightarrow 2\right\}\right\}$ | $\text{solve}(k/2 = 2/(k-1), k)$ $\left\{k = \frac{-\sqrt{17}}{2} + \frac{1}{2}, k = \frac{\sqrt{17}}{2} + \frac{1}{2}\right\}$ $\text{solve}(6/2 = \frac{3}{k-1}, k)$ $\{k=2\}$ |



## Sub-Section: Final Boss

### Question 21

Consider the points  $A(1, 2)$  and  $B(3, 6)$ .

- a. Find the equation of the line segment  $AB$ .

The gradient of  $AB$  is  $\frac{6-2}{3-1} = 2$  and the point  $(1, 2)$  is on the line. Therefore the equation of  $AB$  is

$$y - 2 = 2(x - 1)$$

$$y = 2x$$

There is another point  $C$  such that  $A$  is the midpoint of the line segment  $CB$ .

- b.

- i. Find the coordinates of  $C$ .

Let  $(x, y)$  be the coordinates of  $C$ . Then,

$$\frac{x+3}{2} = 1 \implies x = -1 \quad \text{and} \quad \frac{y+6}{2} = 2 \implies y = -2.$$

Therefore  $C(-1, -2)$ .

- ii. Find the length of  $CB$ .

$$d = \sqrt{(-1-3)^2 + (-2-6)^2} = \sqrt{4^2 + 8^2} = \sqrt{80} = 4\sqrt{5}.$$

- c. Find the equation of the perpendicular bisector of the line segment  $CB$ .

Perpendicular to  $BC \implies$  gradient  $= -\frac{1}{2}$ , and the line goes through  $A(1, 2)$  since it is a bisector. Therefore,

$$y - 2 = -\frac{1}{2}(x - 1)$$

$$y = -\frac{1}{2}x + \frac{5}{2}.$$

- d. Hence, find the minimum distance between the line segment  $CB$  and the line  $y = 2x - 4$ .

$CB$  is on the line  $y = 2x$  and is parallel to  $y = 2x - 4$ . The minimum distance will be on a perpendicular line joining the two parallel lines. Take  $y = -\frac{1}{2}x + \frac{5}{2}$  to be our perpendicular line. Solve,

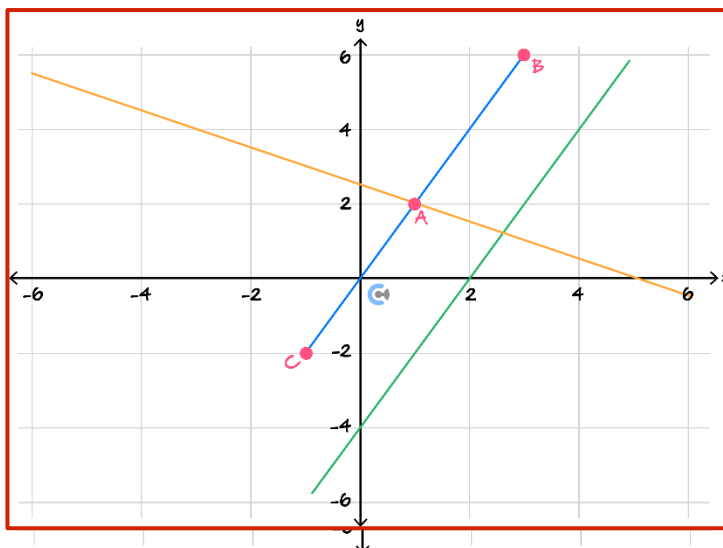
$$-\frac{1}{2}x + \frac{5}{2} = 2x \implies x = 1$$

$$-\frac{1}{2}x + \frac{5}{2} = 2x - 4 \implies \frac{5}{2}x = \frac{13}{2} \implies x = \frac{13}{5}.$$

Therefore the shortest distance is the distance between  $(1, 2)$  and  $(\frac{13}{5}, \frac{6}{5})$ .

$$d = \sqrt{\left(\frac{8}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = \sqrt{\frac{80}{25}} = \frac{4\sqrt{5}}{5}.$$

- e. Sketch the line segment  $BC$ , the line  $y = 2x - 4$  and the perpendicular bisector of  $BC$  on the axes below. Label the points  $A$ ,  $B$  and  $C$



- f. It is known that the lines  $y = mx + 1$  and  $y = 2x - 4$  make an angle of  $45^\circ$  when they intersect. Find all possible values of  $m$ .

The formula for the angle  $\theta$  between two lines with gradients  $m_1$  and  $m_2$  is given by

$$\tan(\theta) = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

here we have  $m_1 = m$  and  $m_2 = 2$  and  $\theta = 45^\circ \implies \tan(\theta) = 1$ . Therefore,

$$\left| \frac{m - 2}{1 + 2m} \right| = 1$$

$$\frac{m - 2}{1 + 2m} = \pm 1$$

solve the two cases separately.

$$m - 2 = 1 + 2m \implies m = -3$$

$$m - 2 = -1 - 2m \implies m = \frac{1}{3}.$$

Conclude that  $m = -3$  or  $m = \frac{1}{3}$ .



## Section B: Supplementary Questions

### Sub-Section: Solve and Graph Linear Equations and Inequalities



#### Question 22



Solve the following linear equations and inequalities for  $x$ .

a.  $3x + 8 = 20$

$$3x = 12 \implies x = 4$$

b.  $2x + 6 = 3(x - 2)$

$$2x + 6 = 3x - 6 \implies x = 12$$

c.  $5x + 2 < 4x + 10$

$$x < 8$$

#### Question 23



Solve the following linear equations and inequalities for  $x$ .

a.  $3x + 2 = 12x + 3$

$$9x = -1 \implies x = -\frac{1}{9}$$

b.  $\frac{2x+3}{3} > 3(x-5)$

$$2x + 3 > 9x - 45 \implies 48 > 7x \implies x < \frac{48}{7}$$

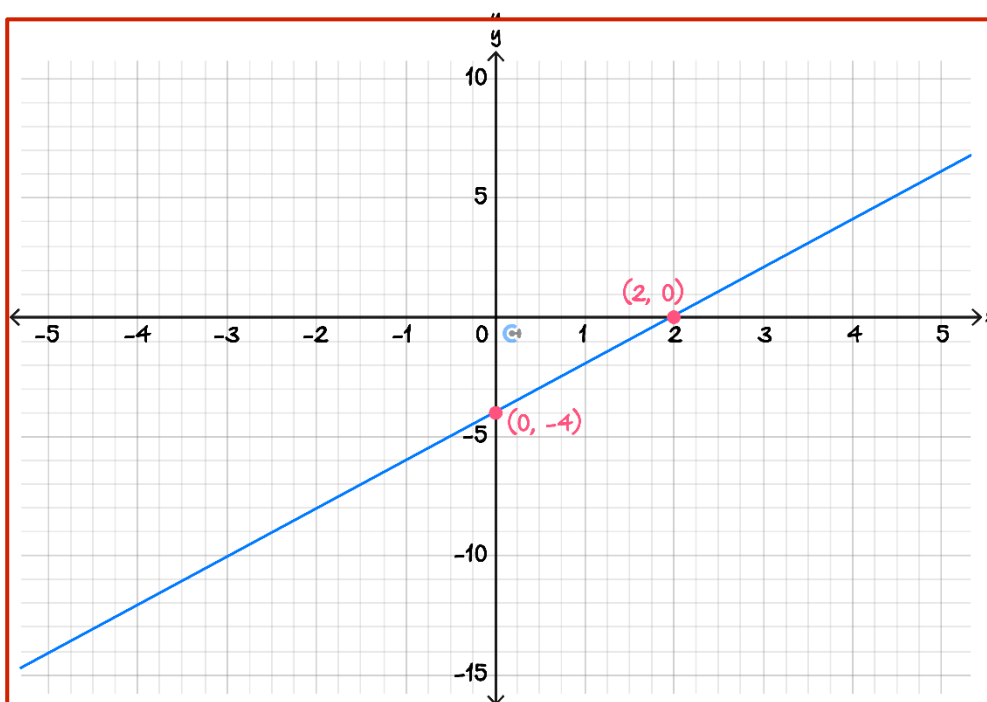
c.  $\frac{5x+3}{4} \leq 10x + 8$

$$5x + 3 \geq 40x + 32 \implies -29 \geq 35x \implies x \geq -\frac{29}{35}$$

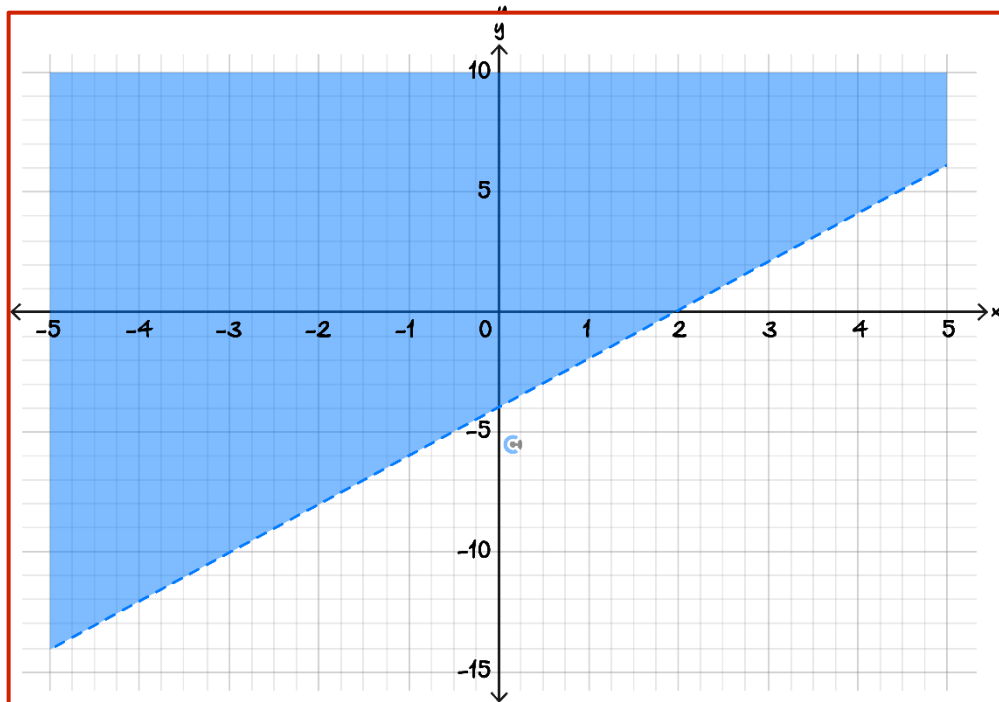
### Question 24



- a. Sketch the line governed by the equation  $2y - 4x = -8$  on the axis below. Label all axes intercepts.



b. Shade the region governed by the equation  $2y - 4x > -8$  on the axis below



### Question 25



Solve the inequality  $\frac{1}{4}(5x - 3) \geq 2x + 8$  for  $x$ .

---



---

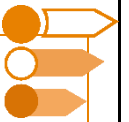
$x \leq -\frac{35}{3}$

---



---

Space for Personal Notes



## Sub-Section: Find the midpoint and distance between two points or functions

### Question 26



- a. Find the midpoint of  $(1, -3)$  and  $(6, -10)$ .

$$\left( \frac{1+6}{2}, \frac{-3-10}{2} \right) = \left( \frac{7}{2}, -\frac{13}{2} \right)$$

- b. The points  $(a, b)$  and  $(3, 4)$  have a midpoint  $(2, 3)$ . Find the values of  $a$  and  $b$ .

$$\frac{a+3}{2} = 2 \implies a = 1 \text{ and } \frac{b+4}{2} = 3 \implies b = 2.$$

### Question 27



- a. Find the distance between points  $(2, 5)$  and  $(5, 2)$ .

$$d = \sqrt{9+9} = 3\sqrt{2}.$$

- b. The curve  $y = (x - 1)^2 + k$  and the line  $y = 3$  has a minimum vertical distance of 4. Find the value of  $k$ .

$$\text{The parabola has a minimum at } (1, k). \text{ Therefore } 4 = k - 3 \implies k = 7.$$

Space for Personal Notes


**Question 28**

The distance between the point  $(2,2)$  and a point  $P$  on the line  $y = 2x + 2$  is 4 units. Find all possible coordinates for  $P$ .

$$\begin{aligned} d &= \sqrt{(x-2)^2 + (2x+2-2)^2} = 4 \\ \Rightarrow x^2 - 4x + 4 + 4x^2 &= 16 \\ 5x^2 - 4x - 12 &= 0 \\ x &= \frac{4 \pm \sqrt{256}}{10} \\ &= -\frac{6}{5}, 2 \end{aligned}$$

Substitute these  $x$ -values into the line  $y = 2x + 2$  to get possible coordinates for  $P$  as

$$\left(-\frac{6}{5}, -\frac{2}{5}\right) \quad \text{and} \quad (2, 6).$$

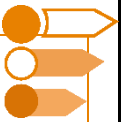
**Question 29**


The distance between the point  $(1,2)$  and a point  $P$  on the line  $y = 3x - 1$  is 4 units. Find all possible coordinates for  $P$ .

$$d^2 = (x-1)^2 + (3x-3)^2 = 16 \Rightarrow x = \frac{5 \pm 2\sqrt{10}}{5}. \text{ Therefore points are}$$

$$\left(\frac{5-2\sqrt{10}}{5}, -\frac{2}{5}(3\sqrt{10}-5)\right) \quad \text{and} \quad \left(\frac{5+2\sqrt{10}}{5}, \frac{2}{5}(3\sqrt{10}+5)\right)$$

Space for Personal Notes



## Sub-Section: Find parallel and Perpendicular Lines

### Question 30



State whether the following lines are parallel or perpendicular.

a.  $y = 3x + 1$  and  $y = 3x + 3$

$$m_1 = m_2 \implies \text{parallel.}$$

b.  $y = 2x + 3$  and  $y = -\frac{1}{2}x + 2$

$$m_1 \times m_2 = -1 \implies \text{perpendicular.}$$

### Question 31



Find the equation of the line that is parallel to the line  $y = 2x + 1$  and passes through the point  $(5, 2)$ .

Since the line is parallel to  $y = 2x + 1$  it must have the form  $y = 2x + c$ .  
 Sub in the point  $(5, 2) \implies 2 = 10 + c \implies c = -8$   
 The line is  $y = 2x - 8$ .

Space for Personal Notes

**Question 32**


Find the equation of the line that is perpendicular to  $y = 3x + 6$  and passes through the point  $(6,3)$ .

The line has gradient  $-\frac{1}{3}$  and passes through  $(6,3)$  therefore,

$$y - 3 = -\frac{1}{3}(x - 6)$$

$$y = -\frac{1}{3}x + 5.$$

**Question 33**


Find the equation of the line that is perpendicular to  $y = \sqrt{3}x + 1$  and passes through the point  $(2,4)$ .

The line has gradient  $-\frac{1}{\sqrt{3}}$  and passes through  $(2,4)$  therefore,

$$y - 4 = -\frac{1}{\sqrt{3}}(x - 2)$$

$$y = -\frac{1}{\sqrt{3}}x + \frac{2}{\sqrt{3}} + 4.$$

Space for Personal Notes

## Sub-Section: Finding the angle between a line and the $x$ -axis or between two lines

### Question 34



Find the angle that  $y = -x + 1$  makes with the positive direction of the  $x$ -axis.

$$\theta = \tan^{-1}(-1) = 135^\circ$$

### Question 35



A line that makes an angle  $30^\circ$  with the positive  $x$ -axis passes through the point  $(1,1)$ . Find the equation of the line.

We know that the gradient  $m = \tan(30) = \frac{1}{\sqrt{3}}$ . Now since we know the gradient and a point we can find the equation of the line.

$$y - 1 = \frac{1}{\sqrt{3}}(x - 1)$$

$$y = \frac{1}{\sqrt{3}}x + 1 - \frac{1}{\sqrt{3}}$$

Space for Personal Notes



**Question 36**


It is known that the lines  $y = mx + 3$  and  $y = 4x - 2$  make an angle of  $45^\circ$  when they intersect. Find all possible values of  $m$ .

The formula for the angle  $\theta$  between two lines with gradients  $m_1$  and  $m_2$  is given by

$$\tan(\theta) = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

here we have  $m_1 = m$  and  $m_2 = 4$  and  $\theta = 45^\circ \implies \tan(\theta) = 1$ . Therefore,

$$\left| \frac{m - 4}{1 + 4m} \right| = 1$$

$$\frac{m - 4}{1 + 4m} = \pm 1$$

solve the two cases separately.

$$m - 4 = 1 + 4m \implies m = -2$$

$$m - 4 = -1 - 4m \implies m = -\frac{5}{3}$$

Conclude that  $m = -\frac{5}{3}$  or  $m = \frac{3}{5}$ .

**Question 37**


Find the acute angle made between the lines  $y = \sqrt{3}x + 1$  and  $y = \frac{x}{\sqrt{3}} - 1$ . Give your answer in degrees correct to two decimal places.

$$\theta = \left| \tan^{-1}(\sqrt{3}) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \right| = 60 - 30 = 30^\circ$$

Space for Personal Notes



## Sub-Section: Find the unknown value for a system of linear equations

### Question 38



Consider the simultaneous linear equations

$$y = kx + 6$$

$$y = 2x + 5$$

where  $x, y \in R$  and  $k$  is a real constant.

- a. Find the value(s) of  $k$  for which the system of equations has no solution.

Lines must be parallel for no solution  $\implies k = 2$ . Then we have  $y = 2x + 6$  and  $y = 2x + 5$  which never intersect. Hence no solution if  $k = 2$ .

- b. Find the value(s) of  $k$  for which the system of equations has infinitely many solutions.

No value of  $k$  will result in the system having infinitely many solutions. This is because there can only be infinitely many solutions if the lines are identical. No value of  $k$  will make the lines identical.

- c. Find the value(s) of  $k$  for which the system of equations has a unique solution.

$$k \neq 2.$$

Space for Personal Notes


**Question 39**

Consider the simultaneous linear equations

$$-3kx + y = k$$

$$-3x + ky = -1$$

where  $x, y \in \mathbb{R}$  and  $k$  is a real constant.

- a. Find the value(s) of  $k$  for which the system of equations has no real solution.

The gradients must be equal for a possibility of there being no real solutions.

$$3k = \frac{3}{k} \implies 3k^2 = 3 \implies k = \pm 1.$$

If  $k = 1$  the lines are  $-3x + y = 1$  and  $-3x + y = -1$ . Therefore no solution if  $k = 1$ .

If  $k = -1$  the lines are  $3x + y = -1$  and  $-3x - y = -1 \implies 3x + y = 1$ . Therefore no solution if  $k = -1$ .

- b. Find the value(s) of  $k$  for which the system of equations has infinitely many solutions.

From above. The system will never have infinitely many solutions.

- c. Find the value(s) of  $k$  for which the system of equations has a unique solution.

$$k \in \mathbb{R} \setminus \{-1, 1\}.$$

Space for Personal Notes


**Question 40**

Consider the simultaneous linear equations

$$kx + y = 2$$

$$2x + (k - 2)y = 4$$

where  $x, y \in \mathbb{R}$  and  $k$  is a real constant.

- a. Find the value(s) of  $k$  for which the system of equations has no real solution.

The gradients must be equal for a possibility of there being no real solutions.

$$k = \frac{2}{k-2} \implies k = 1 \pm \sqrt{3}.$$

The  $y$ -intercepts are the same if  $2 = \frac{4}{k-2} \implies k = 4.$

Therefore no real solution if  $k = 1 \pm \sqrt{3}$

- b. Find the value(s) of  $k$  for which the system of equations has infinitely many solutions.

From above. The system will never have infinitely many solutions.

- c. Find the value(s) of  $k$  for which the system of equations has a unique solution.

$$k \in \mathbb{R} \setminus \{1 - \sqrt{3}, 1 + \sqrt{3}\}.$$

Space for Personal Notes


**Question 41**

Consider the simultaneous linear equations

$$(k - 2)x + 3y = 5$$

$$4x + (k + 1)y = k + 7$$

where  $x, y \in \mathbb{R}$  and  $k$  is a real constant.

Find the value(s) of  $k$  for which the system has no real solution.

Solve  $\frac{k-2}{3} = \frac{4}{k+1} \implies k = \frac{1 \pm \sqrt{57}}{2}$ . We check and see that both these values of  $k$  result in no solution.

$$k = \frac{1 \pm \sqrt{57}}{2}.$$

Space for Personal Notes

## VCE Mathematical Methods ½

# Free 1-on-1 Support



### Be Sure to Make The Most of These (Free) Services!

- Experienced Contour tutors (45+ raw scores, 99+ ATARs).
- For fully enrolled Contour students with up-to-date fees.
- After school weekdays and all-day weekends.

| <u>1-on-1 Video Consults</u>   | <u>Text-Based Support</u>   |
|--|---|
| <ul style="list-style-type: none"><li>➤ Book via <a href="https://bit.ly/contour-methods-consult-2025">bit.ly/contour-methods-consult-2025</a> (or QR code below).</li><li>➤ One active booking at a time (must attend before booking the next).</li></ul> | <ul style="list-style-type: none"><li>➤ Message <a href="tel:+61440138726">+61 440 138 726</a> with questions.</li><li>➤ Save the contact as "Contour Methods".</li></ul> |

[Booking Link for Consults](https://bit.ly/contour-methods-consult-2025)  
[bit.ly/contour-methods-consult-2025](https://bit.ly/contour-methods-consult-2025)



[Number for Text-Based Support](tel:+61440138726)  
[+61 440 138 726](tel:+61440138726)