CONTOUR EDUCATION

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VCE Mathematical Methods ½
AOS 1 Revision [1.0]
Contour Check Part 2 Solutions

Contour Check

Section H: [1.1 - 1.6] – Exam 2 Overall

Question 106

The vertical distance between the function $x^2 + 2$ and the x-axis is 3 when x is equal to:

A. 1

B. 1 and -1

C. 3

D. 3 and -3

Question 107

The distance between points $A(1, 2)$ and $B(4, 6)$ is:

A. 25 units.

B. 16 units.

C. 9 units.

D. 5 units.

Question 108

The image of the point $(a, 3)$ after being reflected about the line $y = 2$ is:

A. $(a, 1)$

B. $(2 - a, 3)$

C. $(4 - a, 3)$

D. $(a, -1)$

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Question 109

The acute angle between the line $3y + \sqrt{3}\,x = 1$ and the x-axis is equal to:

A. $30°$

B. $60°$

C. $150°$

D. $120°$

Question 110

Consider the following pair of simultaneous equations.

$$ay + x = 1$$

$$2y + (3 - a)x = 1$$

For what value(s) of a do the equations have infinitely many solutions?

A. $a = 1$

B. $a = 2$

C. $a = 1, 2$

D. $a \in \mathbb{R}\backslash\{1, 2\}$

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Question 111 (1 mark)

The equation $2x^2 + 2(p+1)x + p = 0$, where p is real, always has roots that are:

A. Equal.

B. Equal in magnitude but opposite in sign.

C. Irrational.

D. Real.

Question 112 (1 mark)

If $px^2 + 3x + q = 0$ has two roots $x = -1$ and $x = -2$, the value of $q - p$ is:

A. -1

B. 1

C. 2

D. -2

Question 113 (1 mark)

The sum of the areas of two squares is $468\ m^2$. If the difference of their perimeters is $24\ m$, then the sides of the two squares are:

A. $18\ m, 14\ m$

B. $13\ m, 12\ m$

C. $18\ m, 12\ m$

D. None of these.

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Question 114 (1 mark)

The value of p so that the quadratic equation $x^2 + 5px + 16 = 0$ has no real roots:

A. $p > 8$

B. $p < 5$

C. $-\frac{8}{5} < p < \frac{8}{5}$

D. $-\frac{8}{5} \leq p < 0$

Question 115 (1 mark)

The quadratic equation whose roots are $a, \frac{1}{a}$ is:

A. $ax^2 - (a^2 + 1)x + a = 0$

B. $ax^2 - (a^2 - 1)x + a = 0$

C. $ax^2 - (a^2 - 1)x - a = 0$

D. None of these.

Question 116

The equation $x^2(x - 2k) = -2x$ has exactly two solutions when:

A. $k < -\sqrt{2}$ or $k > \sqrt{2}$

B. $k = \pm\sqrt{2}$

C. $-\sqrt{2} < k < 0$ or $0 < k < \sqrt{2}$

D. $-\sqrt{2} < k < \sqrt{2}$

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Question 117

The polynomial $x^3 + ax^2 - 2x + b$ has a factor of $x + 1$, and has a remainder of 12 when divided by $x - 2$. The values of a and b are:

A. $a = 3$ and $b = -4$

B. $a = \frac{7}{3}$ and $b = -\frac{4}{3}$

C. $a = \frac{17}{3}$ and $b = -\frac{20}{3}$

D. $a = 5$ and $b = -4$

Question 118

A bisection method is used to solve the equation $x^3 = 7$. The initial interval is $[1,2]$. The bisection reduces this interval down four times and then takes the midpoint of the final interval. The result of this method is closest to:

A. 1.94

B. 1.92

C. 1.91

D. 1.88

Question 119

The equation $kx^3 - 3kx = 1$ has exactly one solution.

The possible values of k are:

A. $k < -2$ or $k > 2$

B. $-2 < k < 2$

C. $k < -\frac{1}{2}$ or $k > \frac{1}{2}$

D. $-\frac{1}{2} < k < \frac{1}{2}$

Question 120

The maximum number of x-intercepts a quartic can have is:

A. 2

B. 3

C. 4

D. 5

Question 121 (1 mark)

The midpoint of the line segment that joins $(1, -5)$ to $(d, 2)$ is:

A. $\left(\frac{d+1}{2}, -\frac{3}{2}\right)$

B. $\left(\frac{1-d}{2}, -\frac{7}{2}\right)$

C. $\left(\frac{d-4}{2}, 0\right)$

D. $\left(0, \frac{1-d}{3}\right)$

E. $\left(\frac{5+d}{2}, 2\right)$

Question 122 (1 mark)

The midpoint of the line segment joining $(0, -5)$ to $(d, 0)$ is:

A. $\left(\frac{d}{2}, -\frac{5}{2}\right)$

B. $(0, 0)$

C. $\left(\frac{d-5}{2}, 0\right)$

D. $\left(0, \frac{5-d}{2}\right)$

E. $\left(\frac{5+d}{2}, 0\right)$

Question 123 (1 mark)

The gradient of a line **perpendicular** to the line that passes through $(-2, 0)$ and $(0, -4)$ is:

A. $\frac{1}{2}$

B. -2

C. $-\frac{1}{2}$

D. 4

E. 2

$$m_{tangent} = \frac{-4-0}{0+2} = -2, \; m_{normal} = \frac{-1}{-2} = \frac{1}{2}$$

Question 124 (1 mark)

The set of values of p for which $x^3 - px + 2 = 0$ has three distinct, real solutions is:

A. $(3, \infty)$

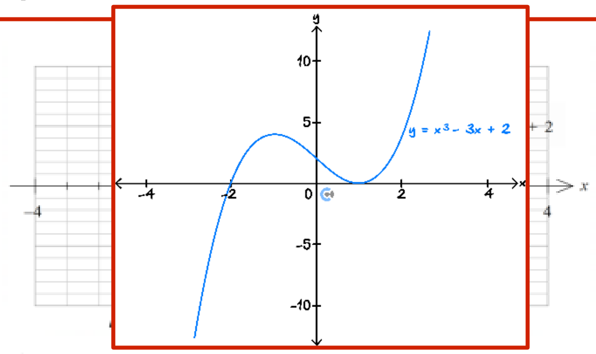
B. $(-\infty, -3)$

C. $(-3, 3)$

D. $(-\infty, 3]$

E. $[3, \infty)$

A



$x^3 - px + 2 = 0$ has three distinct real solutions for $p \in (3, \infty)$. When $x = 3$, there are two distinct real solutions as shown. For values of p greater than three, the y-coordinate of the local maximum turning point is positive and the y-coordinate of the local minimum turning point is negative, which means there will be three distinct real solutions.

Question 125 (1 mark)

The simultaneous linear equations $2y + (m - 1)x = 2$ and $my + 3x = k$ have infinitely many solutions for:

A. $m = 3$ and $k = -2$

B. $m = 3$ and $k = 2$

C. $m = 3$ and $k = 4$

D. $m = -2$ and $k = -2$

E. $m = -2$ and $k = 3$

D

Solve $\frac{m-1}{3} = \frac{2}{m}$ for m for the lines to be parallel, $m = -2$ or $m = 3$,

$\frac{2}{m} = \frac{2}{k}, m = k$ for an infinite number of solutions, $m = -2$ and $k = -2$

Question 126 (1 mark)

The gradient of a line perpendicular to the line that passes through $(3, 0)$ and $(0, -6)$ is:

A. $-\frac{1}{2}$

B. -2

C. $\frac{1}{2}$

D. 4

E. 2

Question 127 (1 mark)

The simultaneous linear equations $mx + 7y = 12$ and $7x + my = m$ have a unique solution only for:

A. $m = 7$ or $m = -7$

B. $m = 12$ or $m = 3$

C. $m \in R \setminus \{-7, 7\}$

D. $m = 4$ or $m = 3$

E. $m \in R \setminus \{12, 1\}$

Question 128 (1 mark)

The graph of $y = kx - 2$ will not intersect or touch the graph of $y = x^2 + 3x$ when:

A. $3 - 2\sqrt{2} < k < 3 + 2\sqrt{2}$

B. $\{k: k < 3 - 2\sqrt{2}\} \cup \{k: k > 3 + 2\sqrt{2}\}$

C. $-5 < k < 11$

D. $3 - 2\sqrt{2} \leq k \leq 3 + 2\sqrt{2}$

E. $k \in R^+$

Question 129 (1 mark)

The simultaneous linear equations $ax - 3y = 5$ and $3x - ay = 8 - a$ have **no solution** for:

A. $a = 3$

B. $a = -3$

C. Both $a = 3$ and $a = -3$.

D. $a \in R \setminus \{3\}$

E. $a \in R \setminus [-3, 3]$

Question 130 (1 mark)

Let $p(x) = x^3 - 2ax^2 + x - 1$, where $a \in R$. When p is divided by $x + 2$, the remainder is 5.

The value of a is:

A. 2

B. $-\frac{7}{4}$

C. $\frac{1}{2}$

D. $-\frac{3}{2}$

E. -2

Question 131 (1 mark)

If $x + a$ is a factor of $8x^3 - 14x^2 - a^2x$, where $a \in R \setminus \{0\}$, then the value of a is:

A. 7

B. 4

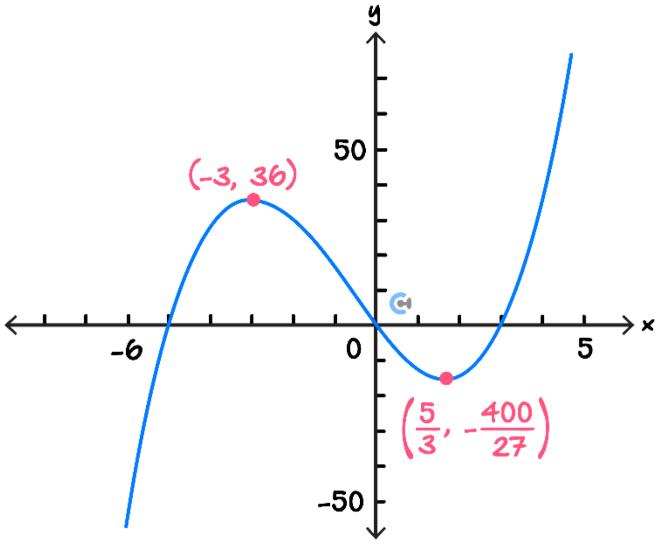
C. 1

D. -2

E. -1

Question 132 (1 mark)

Part of the graph of a cubic polynomial function f and the coordinates of its stationary points are shown below.



$f'(x) < 0$ for the interval:

A. $(0, 3)$

B. $(-\infty, -5) \cup (0, 3)$

C. $(-\infty, -3) \cup \left(\frac{5}{3}, \infty\right)$

D. $\left(-3, \frac{5}{3}\right)$

E. $\left(\frac{-400}{27}, 36\right)$

Question 133 (1 mark)

The equation $(p-1)x^2 + 4x = 5 - p$ has no real roots when:

A. $p^2 - 6p + 6 < 0$

B. $p^2 - 6p + 1 > 0$

C. $p^2 - 6p - 6 < 0$

D. $p^2 - 6p + 1 < 0$

E. $p^2 - 6p + 6 > 0$

$(p-1)x^2 + 4x = 5 - p$

$(p-1)x^2 + 4x - 5 + p = 0$

The discriminant is negative for no real solutions.

$16 - 4(p-1)(p-5) < 0$

$-4p^2 + 24p - 4 < 0$

Divide by -4 and change the inequality.

$p^2 - 6p + 1 > 0$

Question 134 (1 mark)

The simultaneous linear equations $(m-1)x+5y=7$ and $3x+(m-3)y=0.7m$ have infinitely many solutions for:

A. $m \in R \setminus \{0,-2\}$

B. $m \in R \setminus \{0\}$

C. $m \in R \setminus \{6\}$

D. $m = 6$

E. $m = -2$

$(m-1)x+5y=7$ and $3x+(m-3)y=0.7m$
have to represent the same answer line for infinitely many solutions. For this to be the case
$\frac{m-1}{3}=\frac{5}{m-3}=\frac{7}{0.7m}$. Hence $m=6$.

Question 135 (1 mark)

The simultaneous linear equations,

$$kx-3y=0$$

$$5x-(k+2)y=0$$

Where k is a real constant with a unique solution provided:

A. $k \in \{-5,3\}$

B. $k \in R \setminus \{-5,3\}$

C. $k \in \{-3,5\}$

D. $k \in R \setminus \{-3,5\}$

E. $k \in R \setminus \{0\}$

Using multiples of coefficients and combining equations, a unique solution exists when $k(k+2)-15$ is non zero, that is, $k \in R \setminus \{-5,3\}$

Alternatively, solve $\begin{vmatrix} k & -3 \\ 5 & -k-2 \end{vmatrix} = 0$

$k=-5$ or $k=3$ (infinite number of solutions).
A unique solution will occur if $k \in R \setminus \{-5,3\}$.

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Question 136 (1 mark)

The simultaneous linear equations,

$$ax + 3y = 0$$

$$2x + (a + 1)y = 0$$

Where a is a real constant, have infinitely many solutions for:

A. $a \in R$

B. $a \in \{-3, 2\}$

C. $a \in R \setminus \{-3, 2\}$

D. $a \in \{-2, 3\}$

E. $a \in R \setminus \{-2, 3\}$

Question 137 (1 mark)

The simultaneous linear equations,

$$mx + 12y = 24$$

$$3x + my = m$$

Have a unique solution only for:

A. $m = 6$ or $m = -6$

B. $m = 12$ or $m = 3$

C. $m \in R \setminus \{-6, 6\}$

D. $m = 2$ or $m = 1$

E. $m \in R \setminus \{-12, -3\}$

$mx + 12y = 24$

$3x + my = m$

There will be no solution if $\begin{vmatrix} m & 12 \\ 3 & m \end{vmatrix} = 0$, or equivalent. A unique solution exists for $m \in R \setminus \{-6, 6\}$.

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Question 138 (1 mark)

The graph of $y = kx - 3$ intersects the graph of $y = x^2 + 8x$ at two distinct points for:

A. $k = 11$

B. $k > 8 + 2\sqrt{3}$ or $k < 8 - 2\sqrt{3}$

C. $5 \le k \le 6$

D. $8 - 2\sqrt{3} \le k \le 8 + 2\sqrt{3}$

E. $k = 5$

At the point(s) of intersection, $x^2 + 8x = kx - 3$ or $x^2 + (8-k)x + 3 = 0$

there will be two distinct solutions when the discriminant is greater than zero.

$b^2 - 4ac > 0$

$(8-k)^2 - 4 \times 1 \times 3 > 0$

$(8-k)^2 - 12 > 0$

Solving for k gives the result

$k < 8 - 2\sqrt{3}$ or $k > 8 + 2\sqrt{3}$

Question 139 (1 mark)

The solution set of the equation $e^{4x} - 5e^{2x} + 4 = 0$ over R is:

A. $\{1, 4\}$

B. $\{-4, -1\}$

C. $\{-2, -1, 1, 2\}$

D. $\{-\log_e(2), 0, \log_e(2)\}$

E. $\{0, \log_e(2)\}$

Question 140 (1 mark)

The simultaneous linear equations $(m-2)x + 3y = 6$ and $2x + (m-3)y = m - 1$ have **no solution** for:

A. $m \in R \setminus \{0, 5\}$

B. $m \in R \setminus \{0\}$

C. $m \in R \setminus \{6\}$

D. $m = 5$

E. $m = 0$

To have either no solutions, or infinitely many solutions, the ratio of the coefficients of the x and the y terms must be equal,

hence $\frac{m-2}{2} = \frac{3}{m-3}, m \ne 3$. (If $m = 3$, then the equations will have a unique solution $x = 1$ and $y = \frac{5}{3}$.) This can be rearranged to form the quadratic equation $(m-2)(m-3) = 6$, or $m^2 - 5m = 0$, which has solutions $m = 0$ or $m = 5$. If $m = 0$, the simultaneous equations are $-2x + 3y = 6$ and $2x - 3y = -1$ and they have no solution as the equations correspond to distinct parallel lines. If $m = 5$, the simultaneous equations are $3x + 3y = 6$ and $2x + 2y = 4$, and they have many solutions since each equation represents the same line.

Question 141 (1 mark)

The set of values of k for which $x^2 + 2x - k = 0$ has two real solutions is:

A. $\{-1, 1\}$

B. $(-1, \infty)$

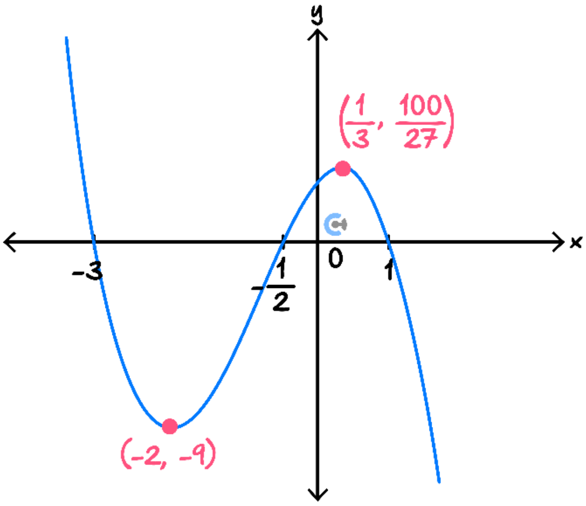
C. $(-\infty, -1)$

D. $\{-1\}$

E. $[-1, \infty)$

Question 142 (1 mark)

Part of the graph $y = f(x)$ of the polynomial function f is shown below.



$f'(x) < 0$ for:

A. $x \in (-2,0) \cup \left(\frac{1}{3}, \infty\right)$

B. $x \in \left(-9, \frac{100}{27}\right)$

C. $x \in (-\infty, -2) \cup \left(\frac{1}{3}, \infty\right)$

D. $x \in \left(-2, \frac{1}{3}\right)$

E. $x \in (-\infty, -2] \cup (1, \infty)$

Question 143 (1 mark)

The line with equation $y = mx + 1$ and the curve with equation $y = 3x^2 + 2x + 4$ intersect at two distinct points. The values of m are:

A. $-4 < m < 8$

B. $m < -4$

C. $m > 8$

D. $m < -4$ or $m > 8$

E. $m = -4$ or $m = 8$

Question 144 (1 mark)

The simultaneous linear equations $2y + (m - 1)x = 2$ and $my + 3x = k$ have infinitely many solutions for:

A. $m = 3$ and $k = -2$

B. $m = 3$ and $k = 2$

C. $m = 3$ and $k = 4$

D. $m = -2$ and $k = -2$

E. $m = -2$ and $k = 3$

Solve $\frac{m-1}{3} = \frac{2}{m}$ for m for the lines to be parallel, $m = -2$ or $m = 3$,

$\frac{2}{m} = \frac{2}{k}, m = k$ for an infinite number of solutions, $m = -2$ and $k = -2$

Question 145 (1 mark)

The simultaneous linear equations $mx + 7y = 12$ and $7x + my = m$ have a unique solution only for:

A. $m = 7$ and $m = -7$

B. $m = 12$ and $m = 3$

C. $m \in R \setminus \{-7, 7\}$

D. $m = 4$ and $m = 3$

E. $m \in R \setminus \{12, 1\}$

Question 146 (1 mark)

Let $f: [0, \infty) \to R, f(x) = x^2 + 1$.

The equation $f(f(x)) = \frac{185}{16}$ has real solution(s):

A. $x = \pm\frac{\sqrt{13}}{4}$

B. $x = \frac{\sqrt{13}}{4}$

C. $x = \pm\frac{\sqrt{13}}{2}$

D. $x = \frac{3}{2}$

E. $x = \pm\frac{3}{2}$

D	$f(x) = x^2 + 1$, solve $f(f(x)) = \frac{185}{16}$ for x, $x = \pm\frac{3}{2}$, domain $x \geq 0$, $x = \frac{3}{2}$

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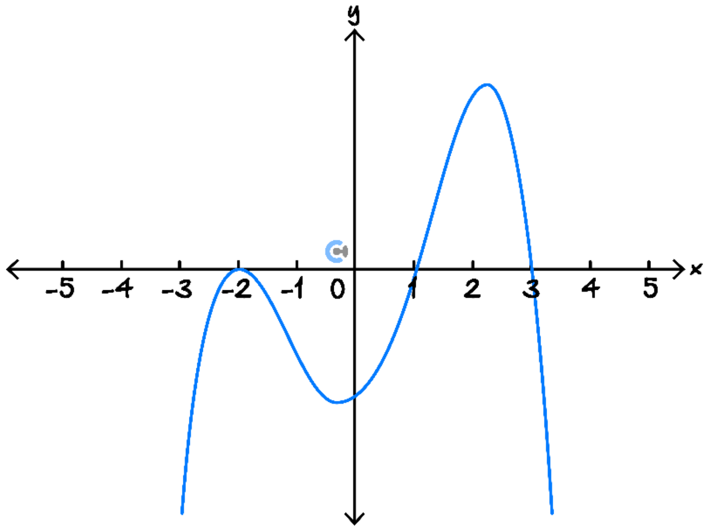
Question 147 (1 mark)

The diagram below shows part of the graph of a polynomial function.



A possible rule for this function is:

A. $y = (x+2)(x-1)(x-3)$

B. $y = (x+2)^2(x-1)(x-3)$

C. $y = (x+2)^2(x-1)(3-x)$

D. $y = -(x-2)^2(x-1)(3-x)$

E. $y = -(x+2)(x-1)(x-3)$

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Question 148 (1 mark)

A set of three numbers that could be the solutions of $x^3 + ax^2 + 16x + 84 = 0$ is:

A. $\{3, 4, 7\}$

B. $\{-4, -3, 7\}$

C. $\{-2, -1, 21\}$

D. $\{-2, 6, 7\}$

E. $\{2, 6, 7\}$

Question 149

Consider the line $l : y = 2x + 3$ and the point $p(1, 0)$.

The shortest distance between p and l is the distance between p, and a point q on the line l for which the line segment pq is perpendicular to l.

a.

i. Find the vertical distance between l and p.

$2(1) + 3 - 0 = 5$

ii. Find the horizontal distance between l and p.

For a point (x, y) on l. When $y = 0$ we see that $x = -\frac{3}{2}$.
Hence the horizontal distance between l and p is $1 + \frac{3}{2} = \frac{5}{2}$

b. The line m is perpendicular to l and goes through the point p. Find the equation of m.

The gradient of l is 2. Hence the gradient of m is $-\frac{1}{2}$.
Thus the equation of m is,

$$y = -\frac{1}{2}(x - 1) + 0 = \frac{1 - x}{2}$$

c. The point q is the point of intersection between lines l and m.
Show, by solving simultaneous equations, that the coordinates of q are $(-1, 1)$.

Assume the co-ordinates of q are (a, b). Since (a, b) lie on l, we see that $b = 2a + 3$
Since (a, b) lie on m, we see that $b = \frac{1-a}{2}$. Since both equations' left hand side is equal to b, we see that,

$$2a + 3 = \frac{1-a}{2} \implies 4a + 6 = 1 - a \implies 5a = -5 \implies a = -1$$

Thus $b = 2(-1) + 3 = 1$, hence the co-ordinates of q are $(-1, 1)$.

d. Hence, find the shortest distance between the point p and the line l.

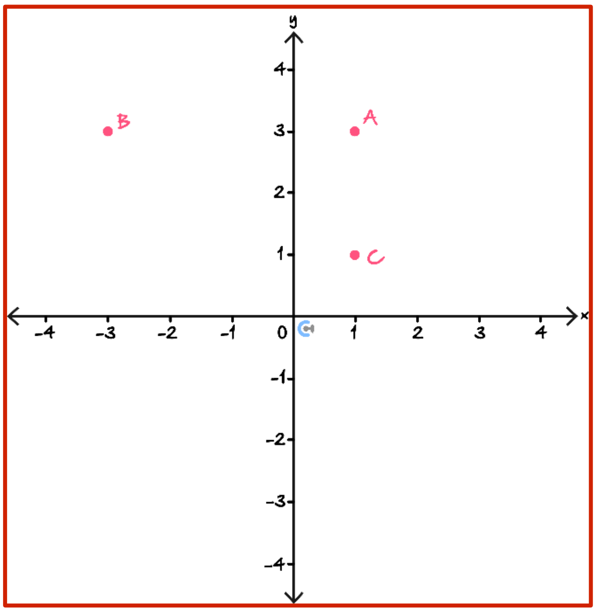
The shortest distance between p and l is the distance between p and q, which is $\sqrt{(1-(-1))^2 + (0-1)^2} = \sqrt{4+1} = \sqrt{5}$

e. Find the image of the point p after being reflected by the line l.

Denote by r, the image of p after being reflected by l.
Thus q is the midpoint of p and r.
Hence the co-ordinates of r are $(-3, 2)$.

Question 150

Consider the point $A(1, 3)$, drawn on the axis below.



i. The point B is the image of A, when reflected in the line $x = -1$.

ii. The point C is the image of A, when reflected in the line $y = 2$.

a. Label the points B and C on the axis above.

b. Find the equation of the line going through B and C.

The gradient of the line going through B and C is, $\dfrac{3-1}{-3-1} = \dfrac{-1}{2}$.
Hence the equation of the line going through B and C is,

$$y = -\frac{1}{2}(x-1) + 1 = \frac{3-x}{2}$$

c. The line $l : y = -\frac{1}{2}x + 2$ is parallel to the line segment BC.

i. Find the angle l makes with the positive direction of the x-axis, correct to the 2 decimal places.

$$180 + \tan\left(\frac{-1}{2}\right) = 148.70^{\circ}$$

ii. Find the acute angle between the line segments BC and AC correct to the 2 decimal places.

$$90 + \tan\left(\frac{-1}{2}\right) = 58.70^{\circ}$$

d. The line m is the image of l after it is reflected along the line going through A and B. Find the equation of m.

The line going through A and B is $y = 3$.
Thus the point of intersection of l and $y = 3$ is also a point on m.
The x-value of this point of intersection can be obtained by solving, $-\frac{1}{2}x + 2 = 3 \implies x = -2$.
Thus the point $(-2, 3)$ is on the line m.
The gradient of m is equal to negative of the gradient of l, $\frac{1}{2}$.
Hence the equation of m is,

$$y = \frac{1}{2}(x + 2) + 3 = \frac{x}{2} + 4$$

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Question 151 (11 marks)

Consider the quadratic function $f(x) = 3x^2 + 5x - 2$.

a.

i. Solve the equation $f(x) = 0$. (2 marks)

$x = -2, \frac{1}{3}$

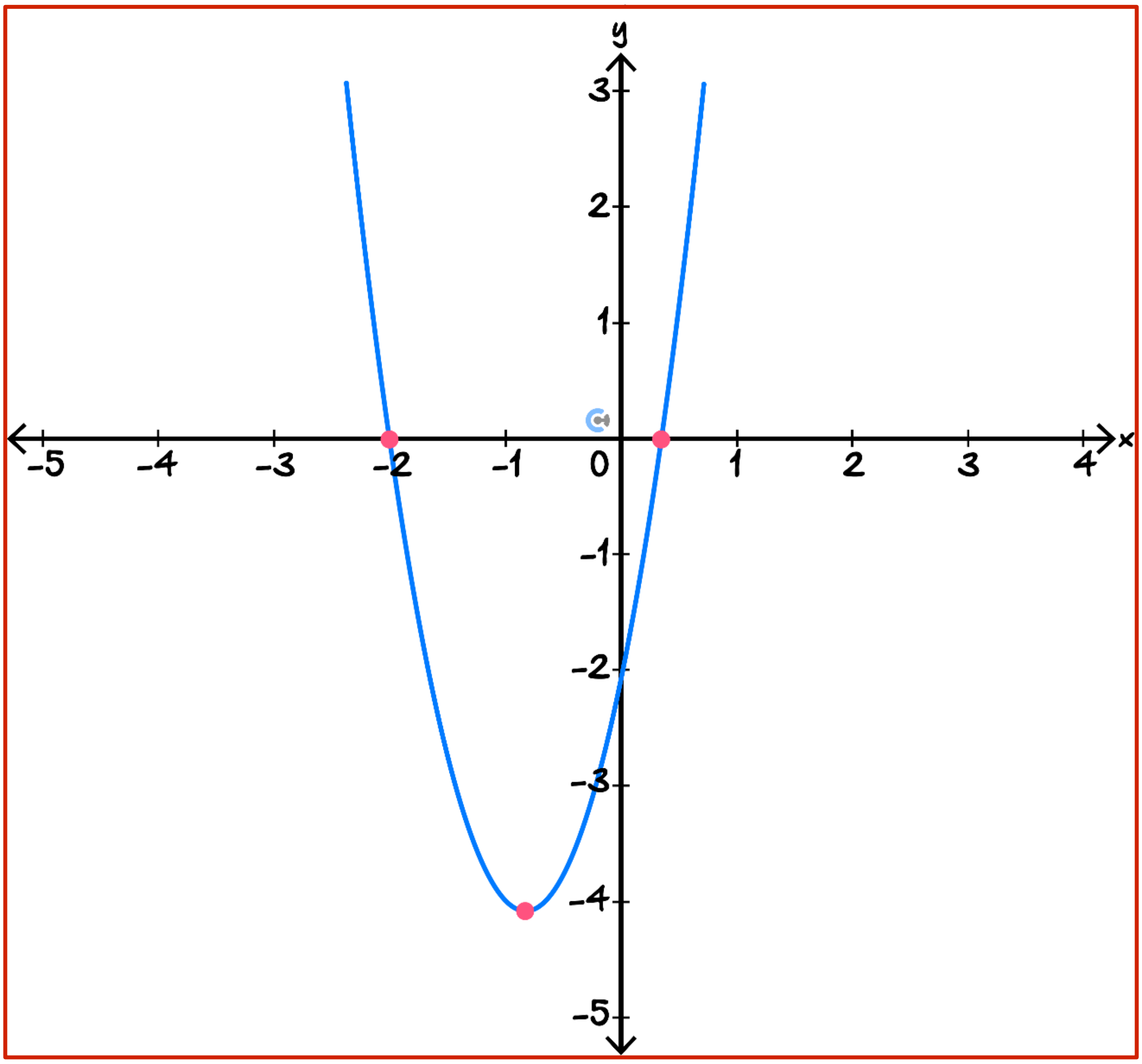
ii. Find the turning point of the graph of $y = f(x)$. (1 mark)

$\left(-\frac{5}{6}, -\frac{49}{12}\right)$

iii. Find the y-intercept of the graph of $y = f(x)$. (1 mark)

$(0, -2)$

b. Sketch the graph of $y = f(x)$ on the axes below.



c. The graph of $y = f(x)$ is translated 1 unit to the left and now has the equation:

$$y = ax^2 + bx + c, \quad a, b, c \in \mathbb{R}$$

Determine the values of a, b, c. (2 marks)

$y = 3x^2 + 11x + 6$; $a = 3, b = 11, c = 6$

d. Consider the graph of the function $g(x) = 3x^2 + kx + 4$. Find the value(s) of k for which the equation $g(x) = 0$ will have:

i. No real root. (1 mark)

$-4\sqrt{3} < k < 4\sqrt{3}$

ii. Equal roots. (1 mark)

$k = \pm 4\sqrt{3}$

iii. Unique real roots. (1 mark)

$k < -4\sqrt{3}$ or $k > 4\sqrt{3}$

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Question 152 (9 marks)

A cricket player hits a ball, and the ball's trajectory is modelled by the quadratic equation:

$$h(x) = ax^2 + bx + c$$

Where $h(x)$ is the height of the ball (in metres) above the ground, and x is the horizontal distance (in metres) from where the ball was hit.

The following conditions are given:

- The ball is hit from a height of 1.5 metres, i.e., $h(0) = 1.5$.
- The ball reaches a height of 20 metres when it has travelled 10 metres horizontally.
- The ball reaches a height of 35 metres when it has travelled 20 metres horizontally.

a. Using the given conditions, set up and solve a system of equations to determine the values of a, b, and c. (3 marks)

$$a = -\frac{7}{400}, b = \frac{81}{40}, c = \frac{3}{2}$$

b. Determine the maximum height that the ball reaches. Give your answer correct to 2 decimal places. (1 mark)

60.08

c. Determine the horizontal distance the ball has travelled when its height is 15 metres. Provide both possible values of x correct to two decimal places. (2 marks)

Solve $h(x) = 15$

$x = 7.10, 108.61$ metres

d. Find the exact height, where the ball has travelled 30 metres horizontally between the two times that it reaches this height. (3 marks)

$\frac{393}{7}$ metres

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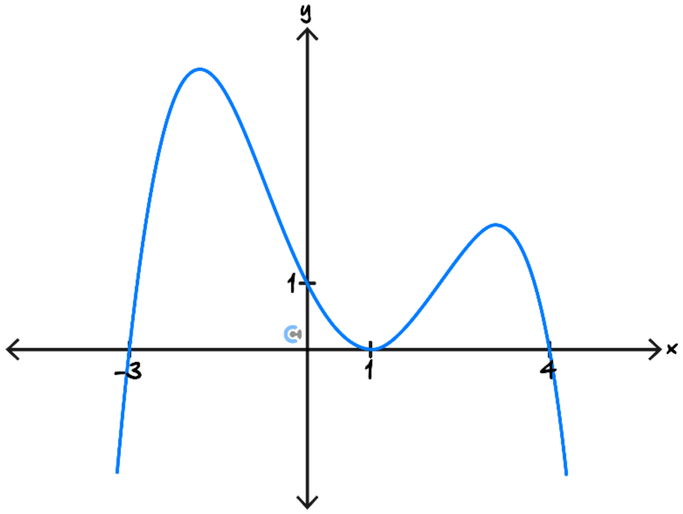
Question 153

The graph of $f(x) = ax^4 + bx^3 + cx^2 + dx + 1$ is drawn below.



a. Find the values of a, b, c and d.

$f(x) = (x-1)^2(x+3)(x-4) = -\frac{1}{12}x^4 + \frac{1}{4}x^3 + \frac{3}{4}x^2 - \frac{23}{12}x + 1.$

Thus $a = -\frac{1}{12}, b = \frac{1}{4}, c = \frac{3}{4}$ and $d = -\frac{23}{12}$

b. Hence or otherwise, solve $f(x) > 1$. Give your answers correct to 2 decimal places.

$-2.88 < x < 0$ or $2.12 < x < 3.77$

c. Find all values of a correct to 3 decimal places such that $f(x) = a$ has exactly three solutions.

$a = 0$ or $a = 2.018$

d. Consider the polynomial $g(x) = (x - a)^2(x + 3)(x - 4)$.

i. For what values of a are the solution to $g(x) \leq 0$ an interval?

$-3 \leq a \leq 4$

ii. For what values of a is the solution to $g(x) \geq 0$ an interval?

$a \leq -3$ or $a \geq 4$

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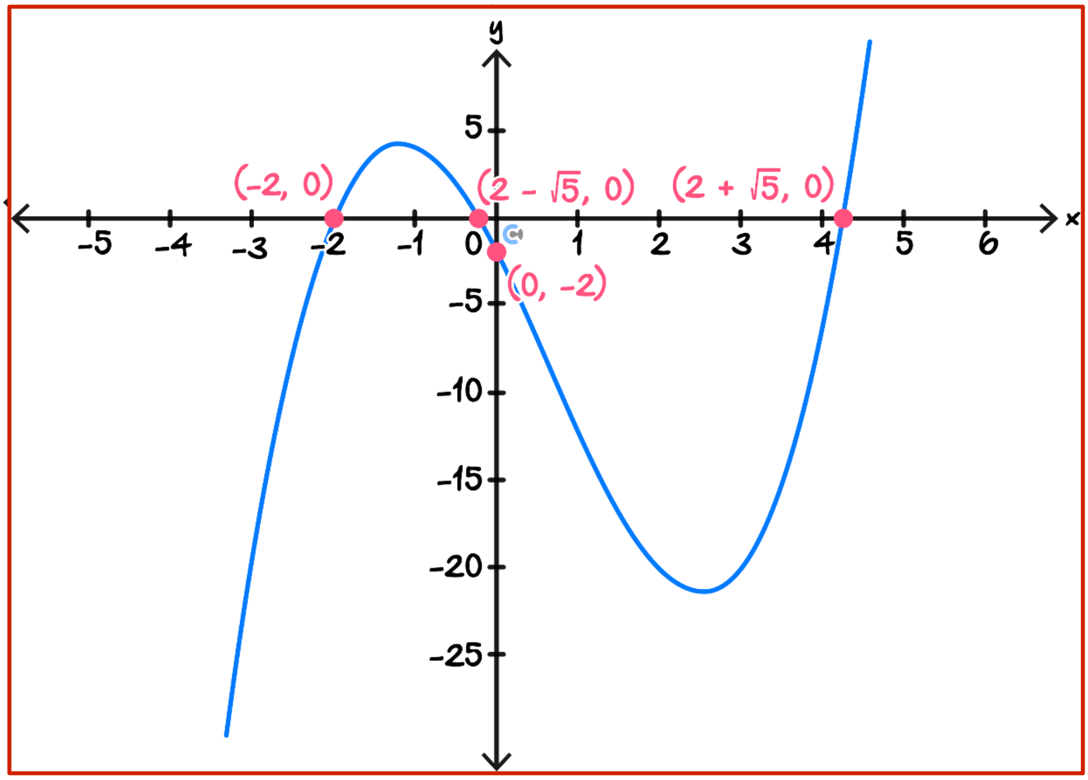
Question 154

Consider the polynomial $f(x) = x^3 - 2x^2 - 9x - 2$.

a. State the coordinates of the axis intercepts of f.

$(-2, 0), \left(2 - \sqrt{5}, 0\right), \left(2 + \sqrt{5}, 0\right)$, and $(0, -2)$.

b. Hence, sketch the graph of f, labelling all axis intercepts with their coordinates.



c. A bisection method with an initial interval of $[3, 5]$ is used to approximate the solution to $f(x) = 0$.

First, the interval is refined n times, before the midpoint of the last interval is taken as an answer.

i. If $n = 3$, what answer will this approach yield?

We refine the interval to $[4, 5]$ then to $[4, 4.5]$ and lastly to $[4, 4.25]$.
Thus our answer is 4.125.

ii. What is the smallest value of $n > 2$ which gives a better approximation to the actual solution than $n = 2$ does?

$n = 6$.

d. If the bisection method is instead applied with an initial interval of $[-11, 5]$, what root will be approximated?

Justify your answer.

The interval will be refined to $[-3, 5]$ since $f(-3)f(-11) > 0$ and then to $[1, 5]$ since $f(1)f(-3) > 0$.
The only root remaining in $[1, 5]$ is $2 + \sqrt{5}$, which is the root our method will approximate.

e. Use the rational root theorem to show that $\sqrt{7}$ cannot be rational.

We know that $\sqrt{7}$ is a solution to the polynomial equation $x^2 - 7$.
By the rational root theorem, the only possible rational solutions to this equation are $\pm 1, \pm 7$.
However as $(\pm 1)^2 - 7 = -6$ and $(\pm 7)^2 - 7 = 42$ we see $x^2 - 7$ has no rational roots.
Hence $\sqrt{7}$ is not rational.

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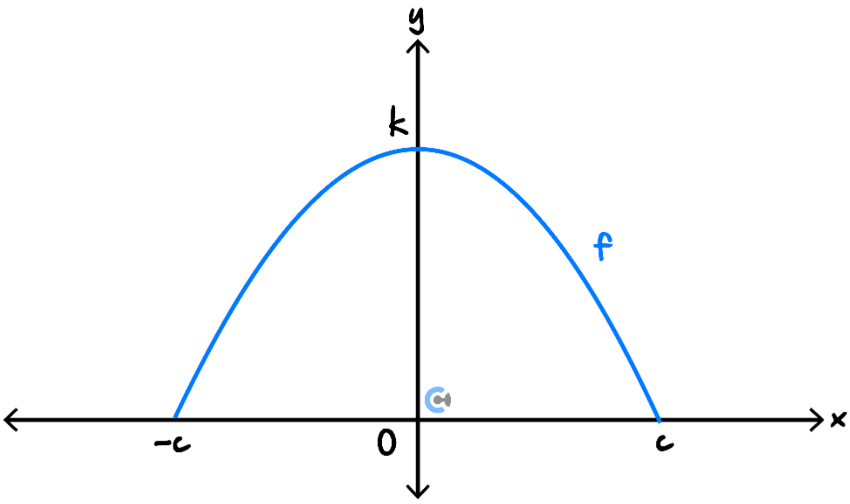
Question 155 (10 marks)

The parabolic arch of a tunnel is modelled by the function $f: [-c, c] \to R, f(x) = ax^2 + b$, where $a < 0$, $b \in R$ and $c > 0$.

Let x be the horizontal distance, in metres, from the origin and let y be the vertical distance, in metres, above the base of the arch.

The graph of f is shown below, where the coordinates of the y-intercept are $(0, k)$ and the coordinates of the x-intercepts are $(-c, 0)$ and $(c, 0)$.



a. Express a and b in terms of c and k. (2 marks)

$$a = -\frac{k}{c^2} \text{ and } b = k$$

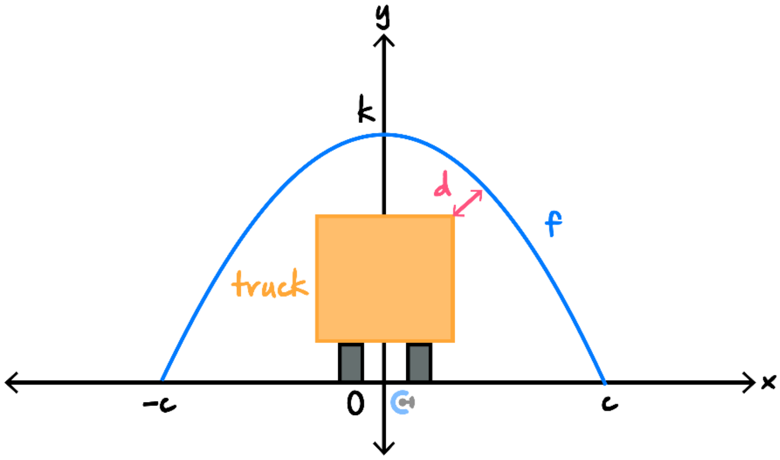
A particular tunnel has an arch modelled by f. It has a height of 6 m at the centre and a width of 8 m at the base.

b.

i. Find the rule for this arch. (1 mark)

$$f(x) = -\frac{3}{8}x^2 + 6$$

ii. A truck that has a height of 3.7 m and a width of 2.7 m will fit through the arch with the function f found in **part b.i.**



Assuming that the truck drives directly through the middle of the arch, let d be the minimum distance between the arch and the top corner of the truck.

Find d and the value of x for which this occurs, correct to three decimal places. (3 marks)

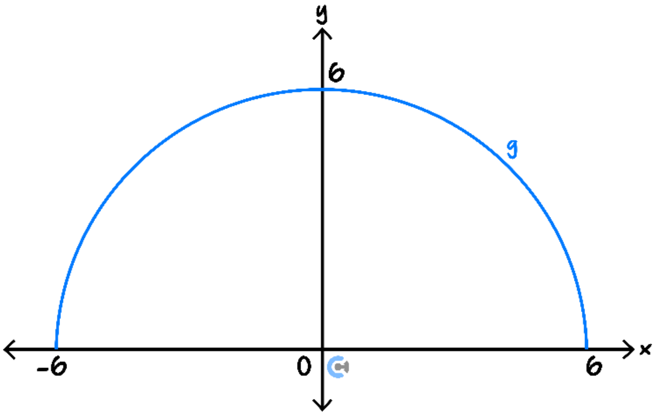
$$\text{Distance} = \sqrt{(x - 1.35)^2 + \left(-\frac{3}{8}x^2 + 6 - 3.7\right)^2}$$

$x = 2.185$ (2.18506 …)

distance = 0.978 (0.9782556 …)

A different tunnel has a semicircular arch. This arch can be modelled by the function $g: [-6, 6] \to R$, $g(x) = \sqrt{r^2 - x^2}$, where $r > 0$.

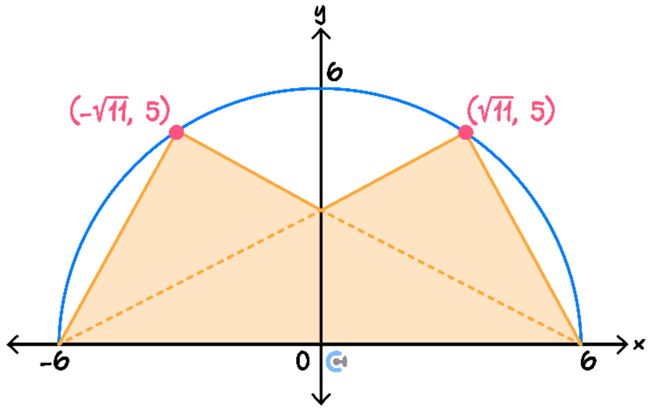
The graph of g is shown below.



c. State the value of r. (1 mark)

$r = 6$

d. Two lights have been placed on the arch to light the entrance of the tunnel. The positions of the lights are $(-\sqrt{11}, 5)$ and $(\sqrt{11}, 5)$. The area that is lit by these lights is shaded in the diagram below.



Determine the proportion of the cross-section of the tunnel entrance that is lit by the lights. Give your answer as a percentage, correct to the nearest integer. (3 marks)

y intercept of the light lines $= \dfrac{36 - 6\sqrt{11}}{5} = \dfrac{30}{6 + \sqrt{11}} \approx 3.22 \ldots$

Shaded area (using sum of 2 triangles subtract a third)

$= 2\dfrac{12 \times 5}{2} - \dfrac{1}{2} \times 12 \times \dfrac{36 - 6\sqrt{11}}{5} = \dfrac{36\sqrt{11} + 84}{5} \approx 40.679698 \ldots$

OR

Shaded area (using trapizum and triangle)

$= 2\left(\dfrac{1}{2}\left(\dfrac{36 - 6\sqrt{11}}{5} + 5\right)\sqrt{11} + \dfrac{1}{2}(6 - \sqrt{11})5\right) = \dfrac{12(3\sqrt{11} + 7)}{5}$

$\approx 40.679698 \ldots$

% of area $= \dfrac{\frac{(36\sqrt{11} + 84)}{5}}{\frac{36\pi}{2}} \times 100 = \dfrac{2(3\sqrt{11} + 7)}{15\pi} \times 100 \approx 72\%$

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Question 156 (16 marks)

Let $g: R \to R, g(x) = (x+2)^2 - 1$.

a. Express the rule for g in the form $g(x) = ax^2 + bx + c$, where $a, b, c \in R$. (1 mark)

$g(x) = x^2 + 4x + 3$

b. The function g can also be written in the form $g(x) = (x-p)(x-q)$, where $p, q \in Z$. Give the values of p and q. (1 mark)

$p = -1,\ q = -3$ or $p = -3,\ q = -1$

c. Find the value of k for which the graph of $y = g(x) + k$ passes through the origin. (2 marks)

Method 1:

Solving $g(0) + k = 0$ for k

$k = -3$

Method 2:

$g(x)$ has a y-intercept at (0, 3)

k is a vertical translation, so for $y = g(x) + k$ to pass through the origin $k = -3$

d. Using algebra, find the value(s) of d such that the graph of $y = g(x-d)$ will pass through the origin. (2 marks)

Method 1:

$g(x)$ has x-intercepts at $(-1, 0)$ and $(-3, 0)$

d is a horizontal translation, so for $y = g(x-d)$ to pass through the origin

$d = 1$ or $d = 3$

Method 2:

Solving $g(0-d) = 0$ for d gives

$d = 1$ or $d = 3$

e. Describe the transformation from the graph of $y = g(x)$ to the graph of $y = g(3x)$. (1 mark)

Dilation by a factor of $\frac{1}{3}$ from the y-axis (in the direction of the x-axis)

Let $h: R \to R, h(x) = mx + n$, where m and n are real numbers.

f. Find the value of m, such that the graph of the sum function $y = g(x) + h(x)$ has a turning point on the y-axis. (2 marks)

$y = g(x) + h(x) = x^2 + 4x + 3 + mx + n = x^2 + (4 + m)x + 3 + n$

At $x = 0$, $\frac{dy}{dx} = 2x + 4 + m = 0$

$m = -4$

g. Find n in terms of m, such that the graph of the sum function $y = g(x) + h(x)$ has a turning point on the x-axis. (2 marks)

Method 1:

Using the discriminant condition $\Delta = 0$

$(m+4)^2 - 4(1)(n+3) = 0$

$n = \frac{m^2 + 8m + 4}{4}$

Method 2:

$x_{TP} = \frac{-4 - m}{2}$, Solving $y(x_{TP}) = 0$

$n = \frac{m^2 + 8m + 4}{4}$

h. Find **two** pairs of values for m and n, such that the graph of the product function $y = g(x)h(x)$ has exactly two x-intercepts. (3 marks)

$y = g(x)h(x) = (x+3)(x+1)(mx+n)$

$x = -3$, $x = -1$, $x = -\frac{n}{m}$

Solving $-1 = -\frac{n}{m}$ or $-3 = -\frac{n}{m}$

$m = n$ or $n = 3m$

Two pairs (examples)

$m = 1$, $n = 1$

$m = 1$, $n = 3$

$m = 0$, $n \in R \setminus \{0\}$

i. Find the coordinates of the turning point of the graph of $y = g(h(x))$, giving your answer in terms of m and n. (2 marks)

$g(h(x)) = (mx + n + 2)^2 - 1$

$\left(-\frac{n+2}{m}, -1\right)$

Question 157 (9 marks)

Let $f: R \to R, f(x) = x^4 - 4x - 8$.

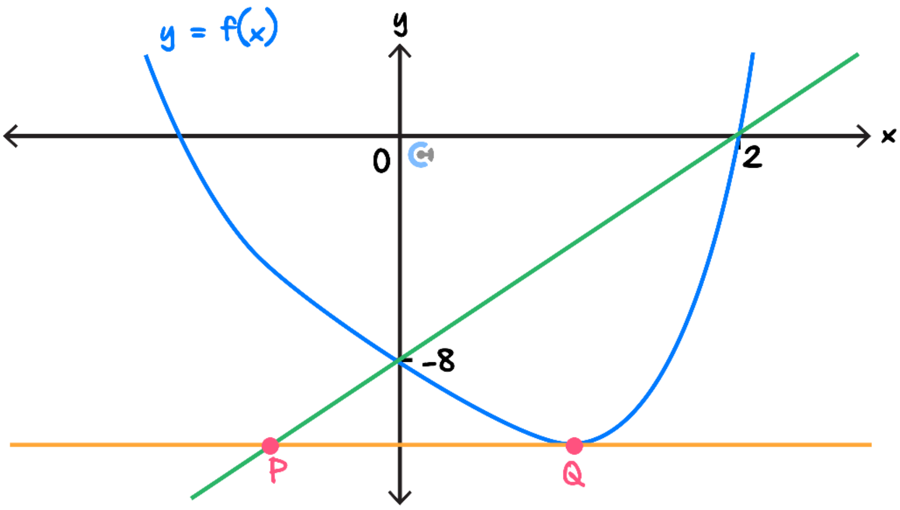
a. Given $f(x) = (x-2)(x^3 + ax^2 + bx + c)$, find a, b and c. (1 mark)

$a = 2, \ b = 4, \ c = 4$

b. Find two consecutive integers m and n such that a solution to $f(x) = 0$ is in the interval (m, n), where $m < n < 0$. (2 marks)

$x = -1.29..., \ m = -2, \ n = -1$

The diagram below shows part of the graph of f and a straight line drawn through the points $(0,-8)$ and $(2,0)$. A second straight line is drawn parallel to the horizontal axis and it touches the graph off at the point Q. The two straight lines intersect at the point P.



c.

i. Find the equation of the line through $(0,-8)$ and $(2,0)$. (1 mark)

$y = 4x - 8$

ii. State the equation of the line through the points P and Q. (1 mark)

$y = -11$

iii. State the coordinates of the points P and Q. (2 marks)

$P\left(-\frac{3}{4}, -11\right)$, $Q(1, -11)$

d. A transformation $T: R^2 \rightarrow R^2, T(x, y) = (x + d, y)$ is applied to the graph of f.

i. Find the value of d for which P is the image of Q. (1 mark)

$d = -\frac{7}{4}$

ii. Let $(m', 0)$ and $(n', 0)$ be the images of $(m, 0)$ and $(1, 0)$ respectively, under the transformation T, where m and n are defined in **part b**.

Find the values of m' and n'. (1 mark)

$m' = -\frac{15}{4}, n' = -\frac{11}{4}$

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