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VCE Mathematical Methods ½
Functions & Relations Exam Skills [0.9]
Workshop Solutions

Error Logbook:

New Ideas/Concepts	Didn't Read Question
Pg / Q #: ____________ Notes:	Pg / Q #: ____________ Notes:
Algebraic/Arithmetic/ Calculator Input Mistake	**Working Out Not Detailed Enough**
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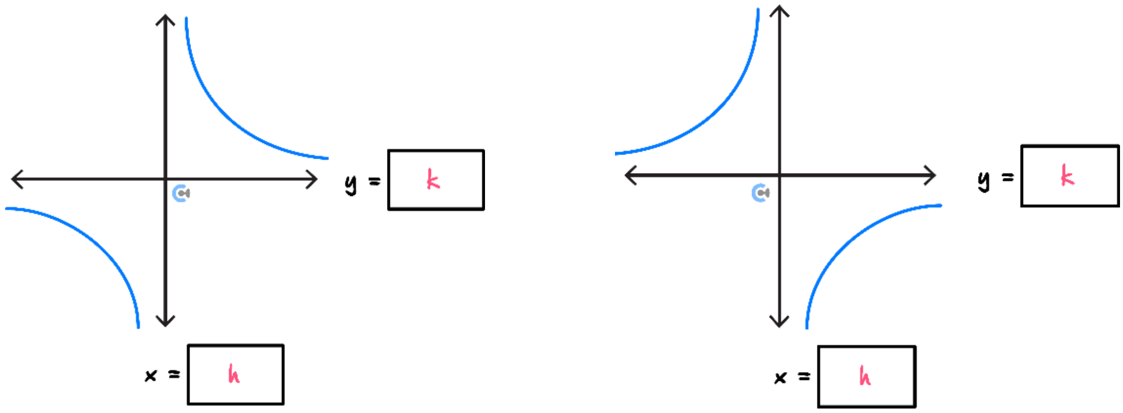
Section A: Recap

Definition

Rectangular Hyperbola

$$y = \frac{a}{x-h} + k$$



Where, $a > 0$ Where, $a < 0$

- **Steps:**
  1. Find the horizontal and vertical asymptotes and plot them on the axis.
  2. Find the x- and y-intercepts and plot on the axes (if they exist).
  3. Identify the shape of the graph by considering any reflections, and sketch the curve.

Finding the Equation of a Hyperbola from its Graph

- We generally need three facts (h, k, and a) about the hyperbola.

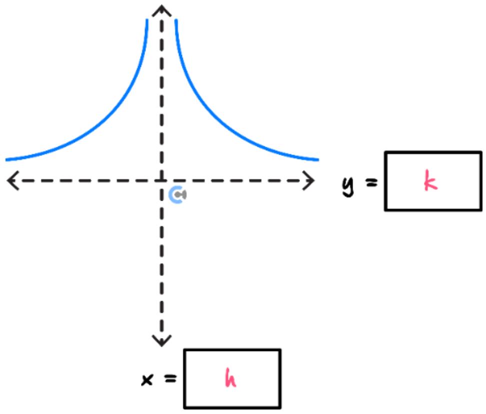
$$y = \frac{a}{x-h} + k$$

- **Steps:**
  1. Look for the asymptotes.
  2. Sub in a point to find the value of a.

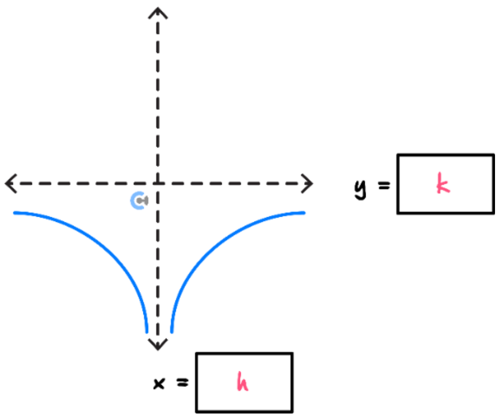
Definition

Truncus

$$y = \frac{a}{(x-h)^2} + k$$



Where, $a > 0$



Where, $a < 0$

- Steps:
  1. Find the horizontal and vertical asymptotes and plot them on the axis.
  2. Find the x- and y-intercepts and plot on the axes (if they exist).
  3. Identify the shape of the graph by considering any reflections and sketch the curve.

Finding the Equation of a Truncus from its Graph

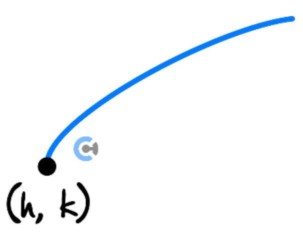
- We generally need three facts (h, k, and a) about the truncus.

$$y = \frac{a}{(x-h)^2} + k$$

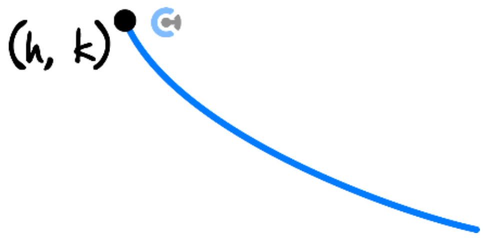
- Steps:
  - Look for the asymptotes.
  - Sub in a point to solve the value of a.

Square Root Functions

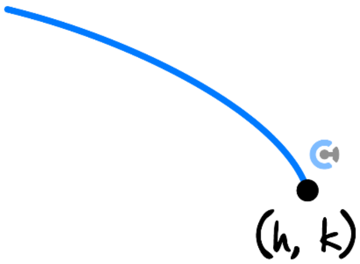
$$y = a\sqrt{b(x-h)} + k$$



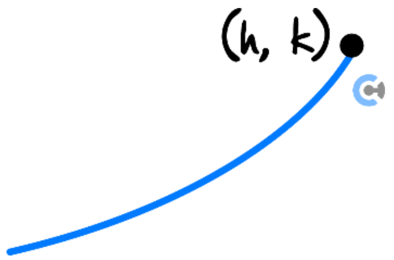
Where, $a > 0$ and $b > 0$.



Where, $a < 0$ and $b > 0$.



Where, $a > 0$ and $b < 0$.



Where, $a < 0$ and $b < 0$.

- **Steps for sketching roots:**

  1. Find the starting point (h, k).
  2. Find the x- and y-intercepts and plot on the axes (if they exist).
  3. Identify the shape of the graph by considering any reflections and sketch the curve.

Space for Personal Notes

Finding the Equation of a Root Function from its Graph

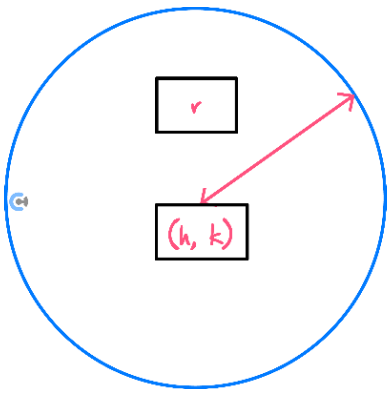
- We generally need three facts about the root function.

$$y = a\sqrt{\pm(x-h)} + k$$

- **Steps:**

1. Look for the starting point (h, k).
2. Sub in a point to solve the value of a.

Circles



$$(x-h)^2 + (y-k)^2 = r^2$$

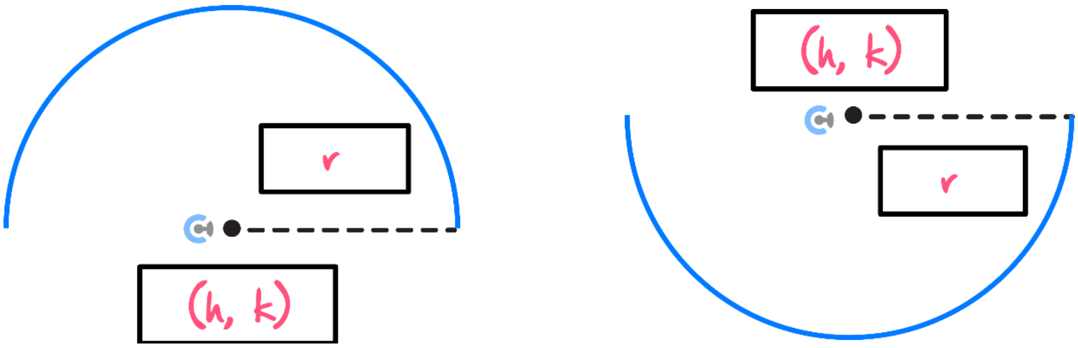
Where, $r > 0$

- **Centre:** (h, k)
- **Radius:** r
- **Steps:**

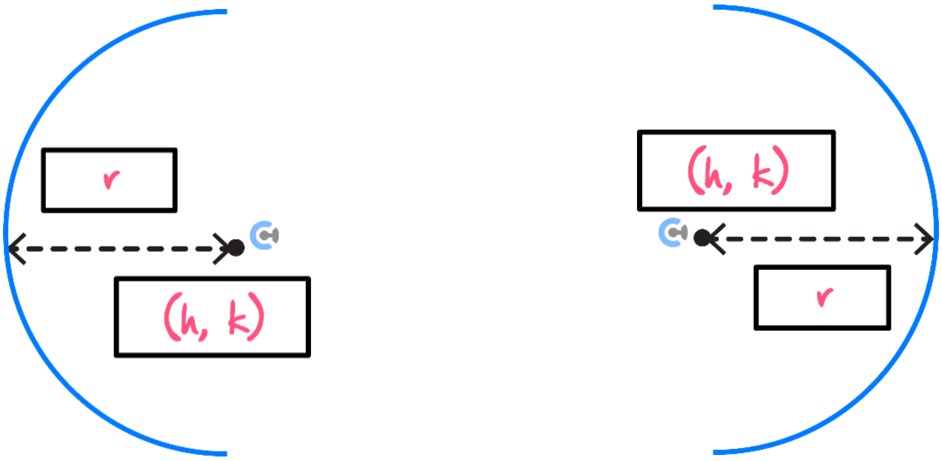
1. Find the centre of the circle.
2. Find the radius of the circle.
3. Find axes intercepts (if they exist).
4. Identify the shape of the graph and sketch the curve.

Definition

Semicircles



$$y = \pm\sqrt{r^2 - (x-h)^2} + k$$



$$x = \pm\sqrt{r^2 - (y-k)^2} + h$$

- **Steps:**

1. Find the centre of the semicircle.
2. Find the radius of the circle.
3. Find axes intercepts if they exist.
4. Identify the shape of the graph and sketch the curve.

Space for Personal Notes

Finding the Equation of a Root Function from its Graph

- We generally need three facts about circles/semicircles.

$$(x-h)^2+(y-k)^2=r^2$$

$$y=\pm\sqrt{r^2-(x-h)^2}+k$$

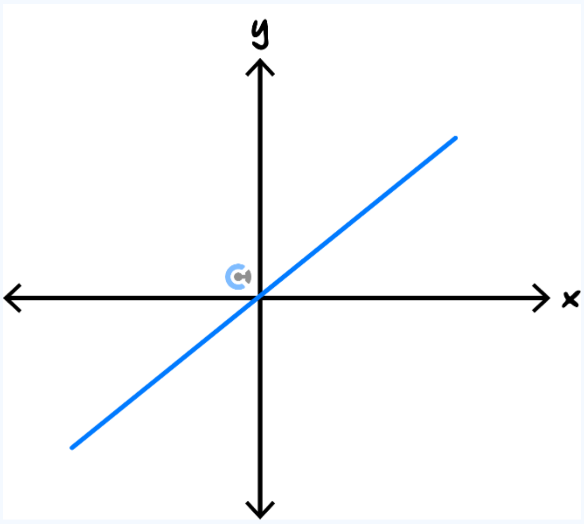
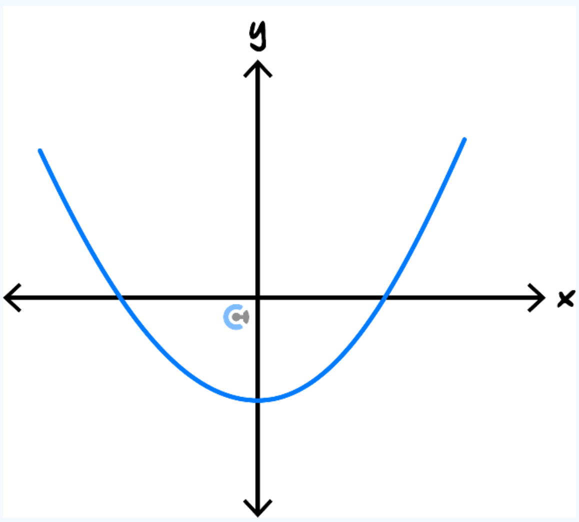
$$x=\pm\sqrt{r^2-(y-k)^2}+h$$

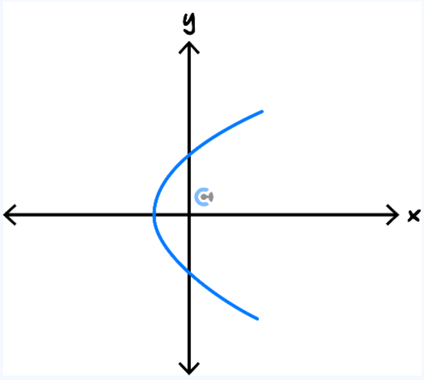
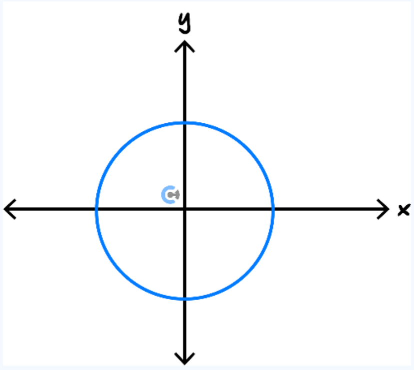
- **Steps:**

1. Identify the centre, (h, k).
2. Identify the radius, r.

Types of Relations

- There are four types of relations:

One to One	Many to One
	
One x to One y.	Many x's to One y.

One to Many	Many to Many
	
One x to Many y's.	Many x's to Many y's.

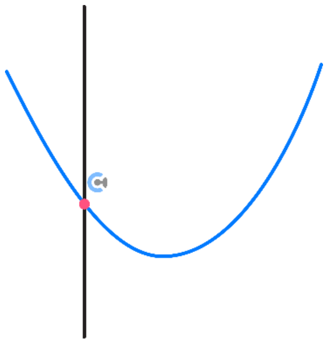
Functions

$$y = f(x)$$

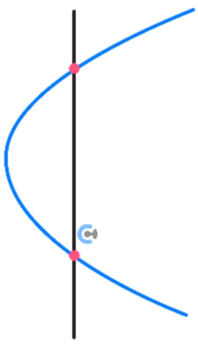
- Functions are relations which make one y-value at any given x-value.

Vertical Line Test

- **Definition**: Tells apart between functions and non-function relations.



Passes : Function

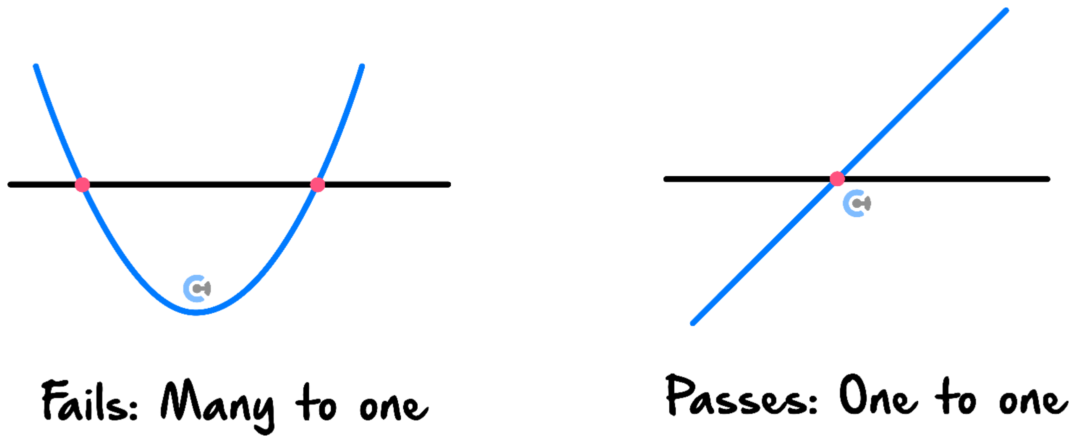


Fails : Not Function

Every function only intersects a vertical line once.

Horizontal Line Test

- **Definition**: Tells apart between many-to-one and one-to-one functions. (And relations.)



Set Operators

- Intersection: "AND"

$$A \cap B = \text{What values are in set } A \text{ AND in set } B.$$

- Union: "OR"

$$A \cup B = \text{What values are in set } A \text{ OR in set } B.$$

- Set difference: "Except"

$$A \setminus B = \text{What values are in set } A\text{, except those also in set } B.$$

Interval Notation

- Parentheses (non-inclusive):

$$x \in (a, b) \Rightarrow a < x < b$$

- Square brackets [inclusive]:

$$x \in [a, b] \Rightarrow a \leq x \leq b$$

Definition

Maximal Domain

- The maximal domain is the biggest possible domain for a rule without committing a mathematical crime.
- In Methods, we need to consider 3 important rules:

$$\sqrt{z}, \qquad z \geq 0$$

$$\log(z), \qquad z > 0$$

$$\frac{1}{z}, \qquad z \neq 0$$

Range

- The range is the possible value for the output of a function.

Definition

Functional Notation

$$f: Domain \to Codomain, f(x) = Rule$$

- Codomain is simply all the values the function works within.
- Codomain is **not** the same as range.

Definition

Piecewise (Hybrid) Functions

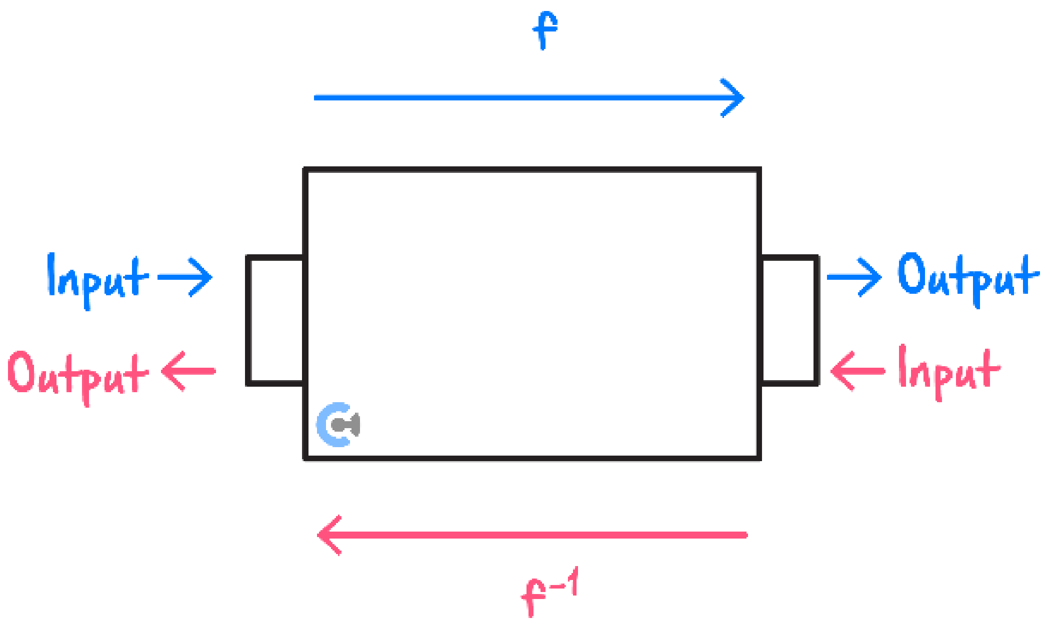
- Series of functions.

$$h(x) = \begin{cases} f(x), & Domain_1 \\ g(x), & Domain_2 \end{cases}$$

- $Domain_1$ and $Domain_2$ represent the x-values for which the two functions are defined.
- The two domains do not have to join!

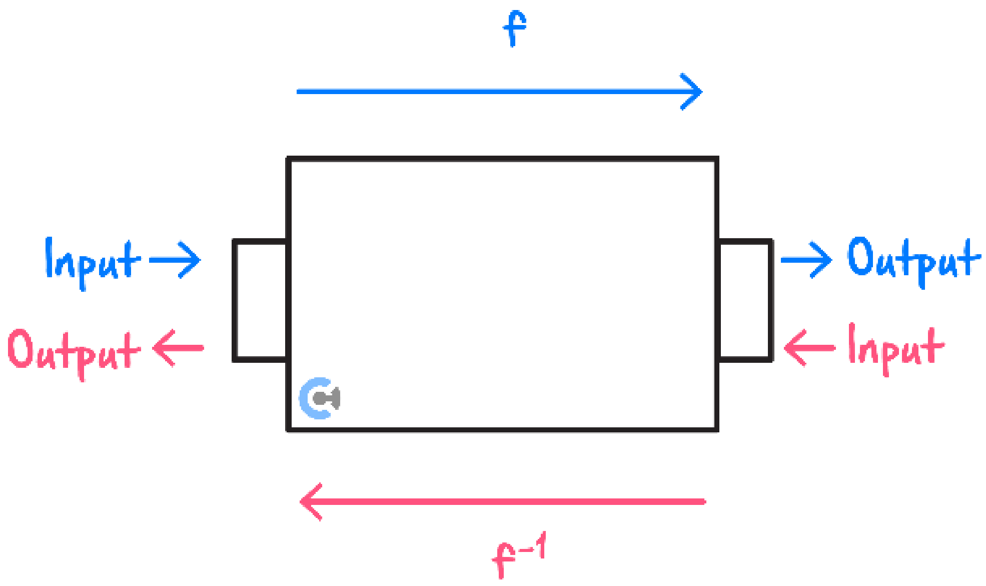
Inverse Relation

- **Definition**: Inverse is a relation that does the opposite.

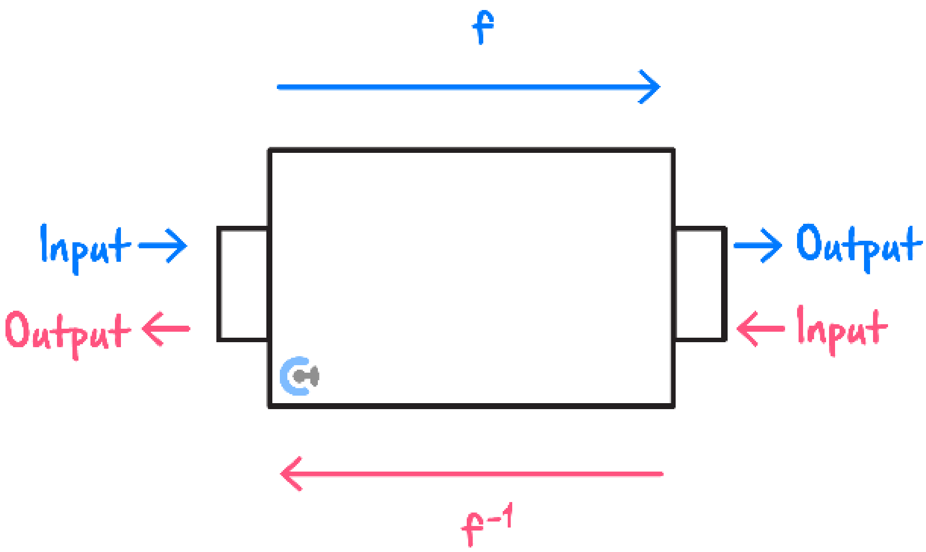


Solving for an Inverse Relation

- Swap x and y.



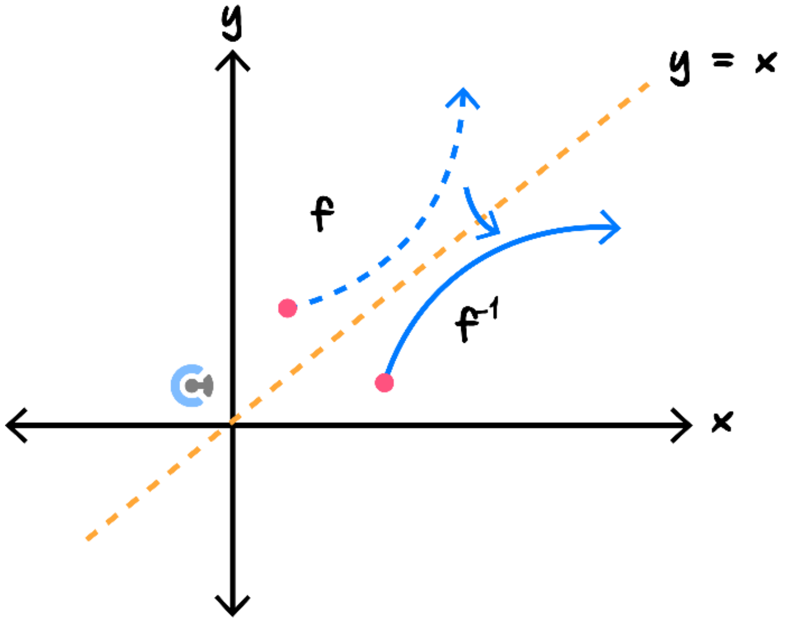
Domain and Range of Inverse Functions



$$\textbf{Dom } f^{-1} = \textbf{Ran } f$$

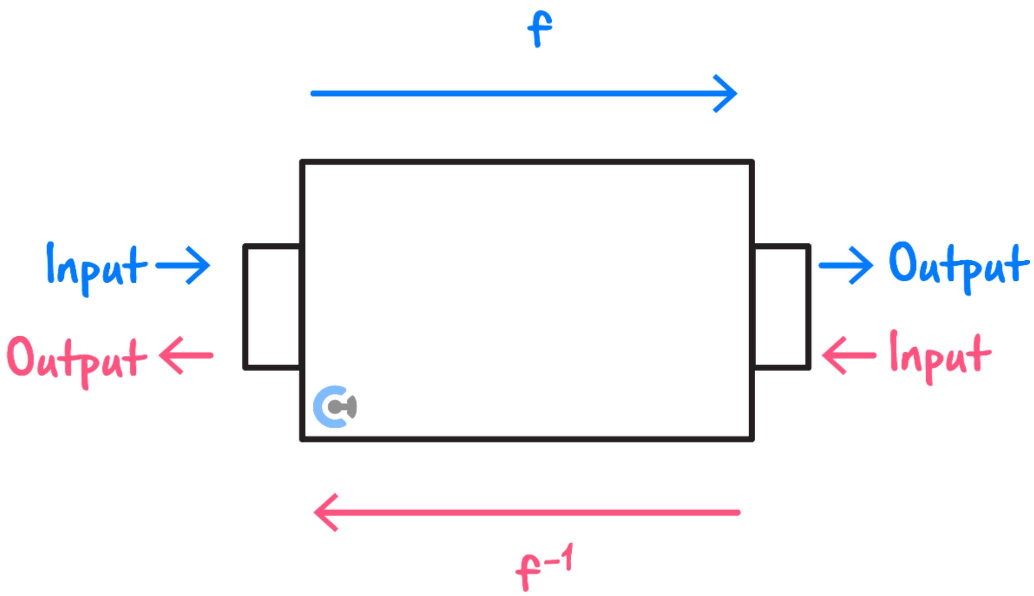
$$\textbf{Ran } f^{-1} = \textbf{Dom } f$$

Symmetry of Inverse Functions



- Inverse functions are always symmetrical around $y = x$.

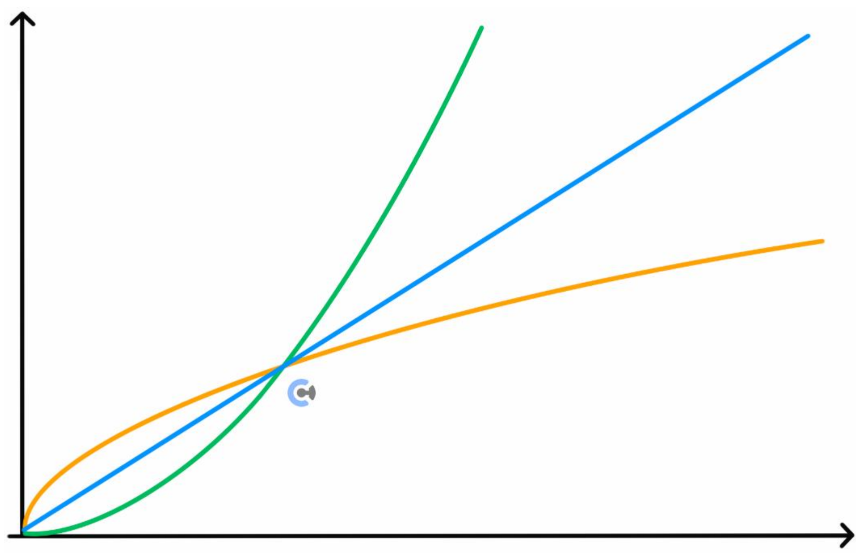
Validity of Inverse Functions



- Requirement for Inverse Function:

f needs to be $1:1$.

Intersection between a Function and its Inverse



$f(x) = x$ OR $f^{-1}(x) = x$

Section B: Warmup (5 Marks)

INSTRUCTION:

- Regular: 5 Marks. 5 Minutes Writing.
- Extension: Skip

Question 1 (5 marks)

a. Let $f : [a, \infty) \to \mathbb{R}, f(x) = (x-3)^2 + 4$.

Determine the minimal value of a such that, f^{-1} exists.

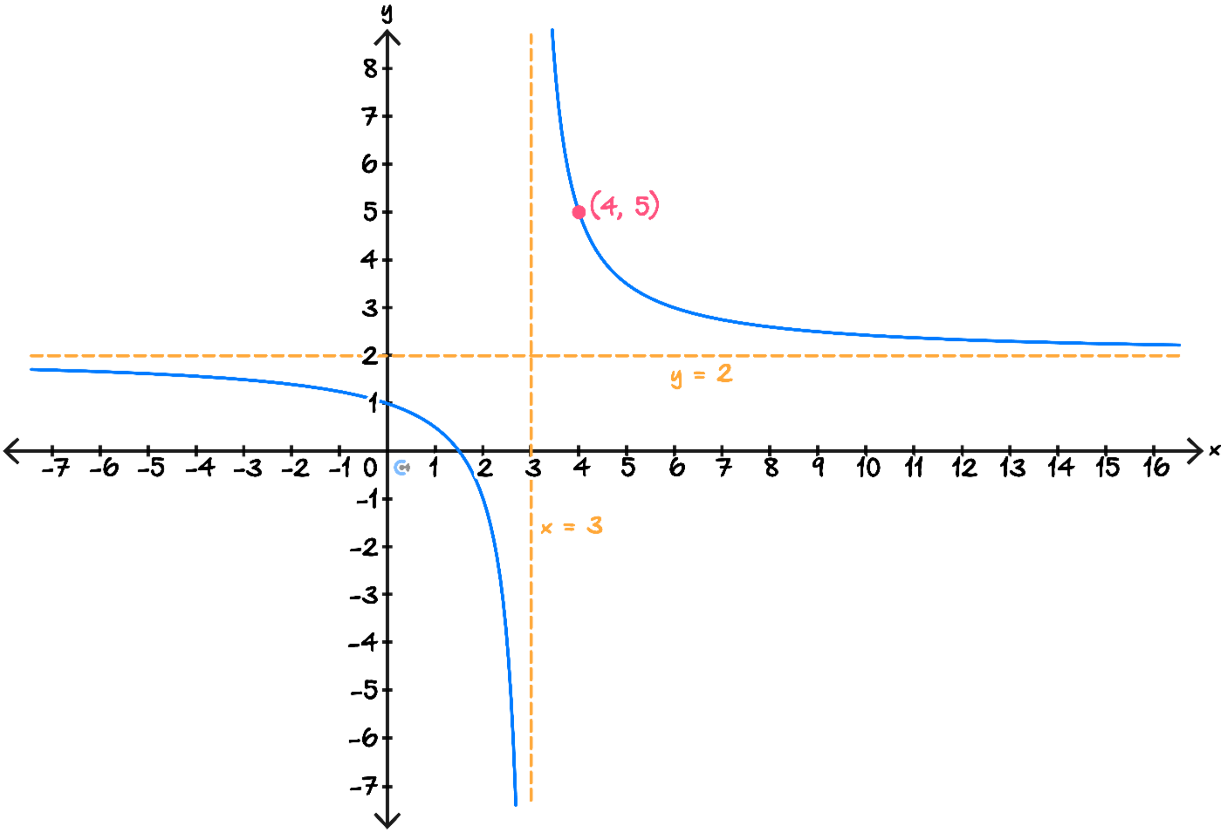
$$\boxed{a = 3}$$

b. Let $g : (-\infty, b] \to \mathbb{R}, f(x) = x^2 + 4x + 1$.

Determine the minimal value of b such that, g^{-1} exists.

$g(x) = (x+2)^2 - 3$.
So, $b = -2$.

c. Find the equation that best represents the graph below.



Solution: Hyperbola of the form $y = \frac{a}{x-h} + k$.
We see the asymptotes so $h = 3$ and $k = 2$. Then using the point $(4, 5)$

$$5 = \frac{a}{4-3} + 2 \implies a + 2 = 5 \implies a = 3.$$

So graph if $y = \frac{3}{x-3} + 2$

Space for Personal Notes

Section C: Exam 1 Questions (19 Marks)

INSTRUCTION:

- Regular: 19 Marks. 28 Minutes Writing.
- Extension: 19 Marks. 19 Minutes Writing.

Question 2 (5 marks)

Consider the function $f(x) = \frac{3}{x-3} + 5$, defined on its maximal domain.

a. Write down the maximal domain of f. (1 mark)

$\boxed{\mathbb{R} \setminus \{3\}}$

b. Find the rule and domain of the inverse function, h^{-1}, of h. (2 marks)

Solution:

$$x = \frac{3}{y-3} + 6$$

$$x - 5 = \frac{3}{y-3}$$

$$y = \frac{3}{x-5} + 3$$

dom h^{-1} = ran $h = \mathbb{R} \setminus \{3\}$. Therefore,

$$h^{-1} : \mathbb{R} \setminus \{3\} \to \mathbb{R},\ h^{-1}(x) = \frac{2}{x-3} + 2$$

c. Find the point(s) of intersection between h and h^{-1}. (2 marks)

Solution: Intersect on the line $y = x$.

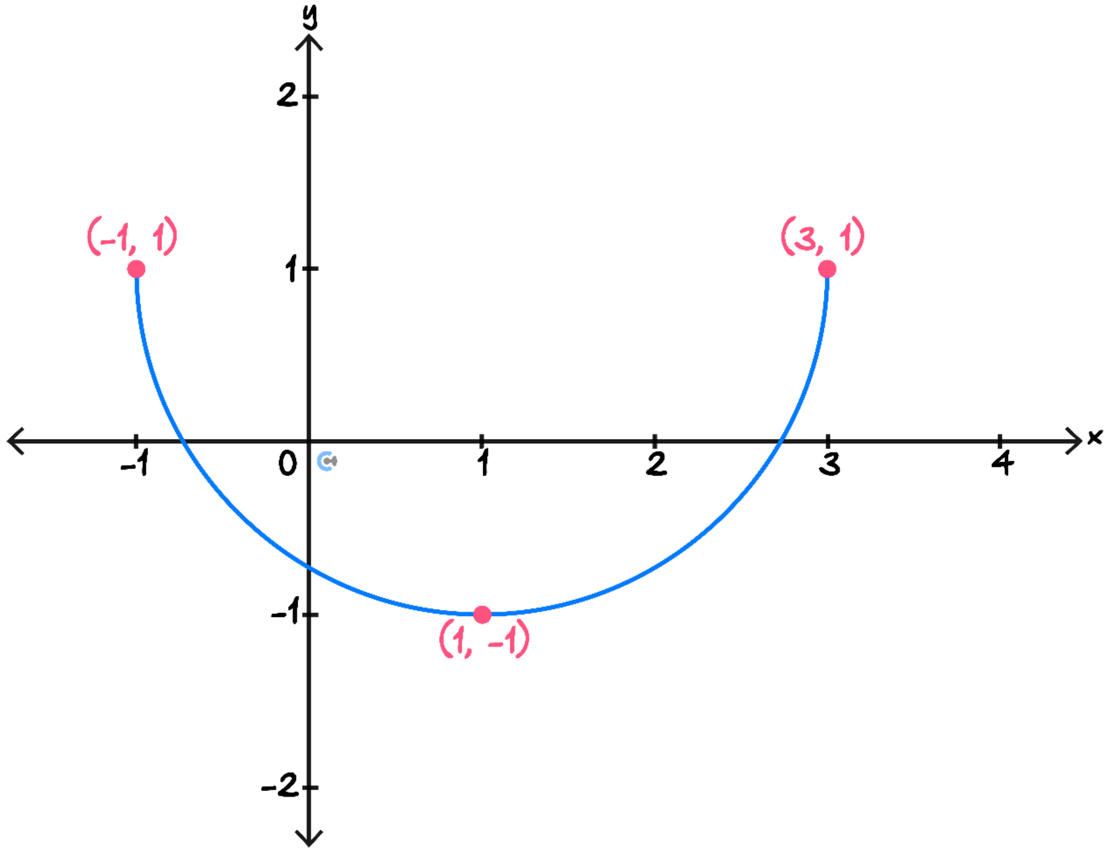
$$\begin{aligned} \frac{3}{x-3} + 5 &= x \\ (x-5)(x-3) &= 3 \\ x^2 - 8x + 15 - 3 &= 0 \\ x^2 - 8x + 12 &= 0 \\ (x-2)(x-6) &= 0 \\ x &= 2, 6 \end{aligned}$$

Therefore points of intersection are $(2, 2)$ and $(6, 6)$

Space for Personal Notes

Question 3 (6 marks)

Consider the function f that describes a semi-circle. The graph of f is shown below.



a. State the domain of f. (1 mark)

$x \in [-1, 3]$

b. Find the rule for $f(x)$. (2 marks)

Solution: We see that the semi-circle comes from a circle with centre $(1, 1)$ and radius 2. It is the bottom half, therefore

$$f(x) = -\sqrt{4 - (x-1)^2} + 1$$

c. Hence, find all axes intercepts of the graph of $y = f(x)$. (3 marks)

Solution: $f(x) = 0 \implies \sqrt{4-(x-1)^2} = 1$. So we solve

$$\begin{aligned} 4-(x-1)^2 &= 1 \\ x^2-2x+1+1 &= 4 \\ x^2-2x-2 &= 0 \\ (x-1)^2 &= 3 \\ x &= 1 \pm \sqrt{3} \end{aligned}$$

and $f(0) = -\sqrt{4-1}+1 = 1-\sqrt{3}$.
Therefore, x-intercepts at $(1-\sqrt{3}, 0)$ and $(1+\sqrt{3}, 0)$ and y-intercept at $(0, 1-\sqrt{3})$.

Space for Personal Notes

Question 4 (8 marks)

Consider the function:

$$f : [a, \infty) \to \mathbb{R}, f(x) = x^2 - 3x + 4$$

a.

i. Write $f(x)$ in turning point form. (1 mark)

$$\boxed{f(x) = \left(x - \frac{3}{2}\right)^2 + \frac{7}{4}}$$

ii. Hence, find the largest value of a such that, the inverse function f^{-1} exists. (1 mark)

$$\boxed{a = \frac{3}{2}}$$

Use the value of a found in **part a. ii.** for all subsequent questions.

b. Define f^{-1}, the inverse function of f. (2 marks)

Solution: dom $f^{-1} =$ ran $f = \left[\frac{7}{4}, \infty\right)$ and ran $f^{-1} =$ dom $f = \left[\frac{3}{2}, \infty\right)$

$$x = \left(y - \frac{3}{2}\right)^2 + \frac{7}{4}$$

$$x - \frac{7}{4} = \left(y - \frac{3}{2}\right)^2$$

$$y = \pm\sqrt{x - \frac{7}{4}} + \frac{3}{2}$$

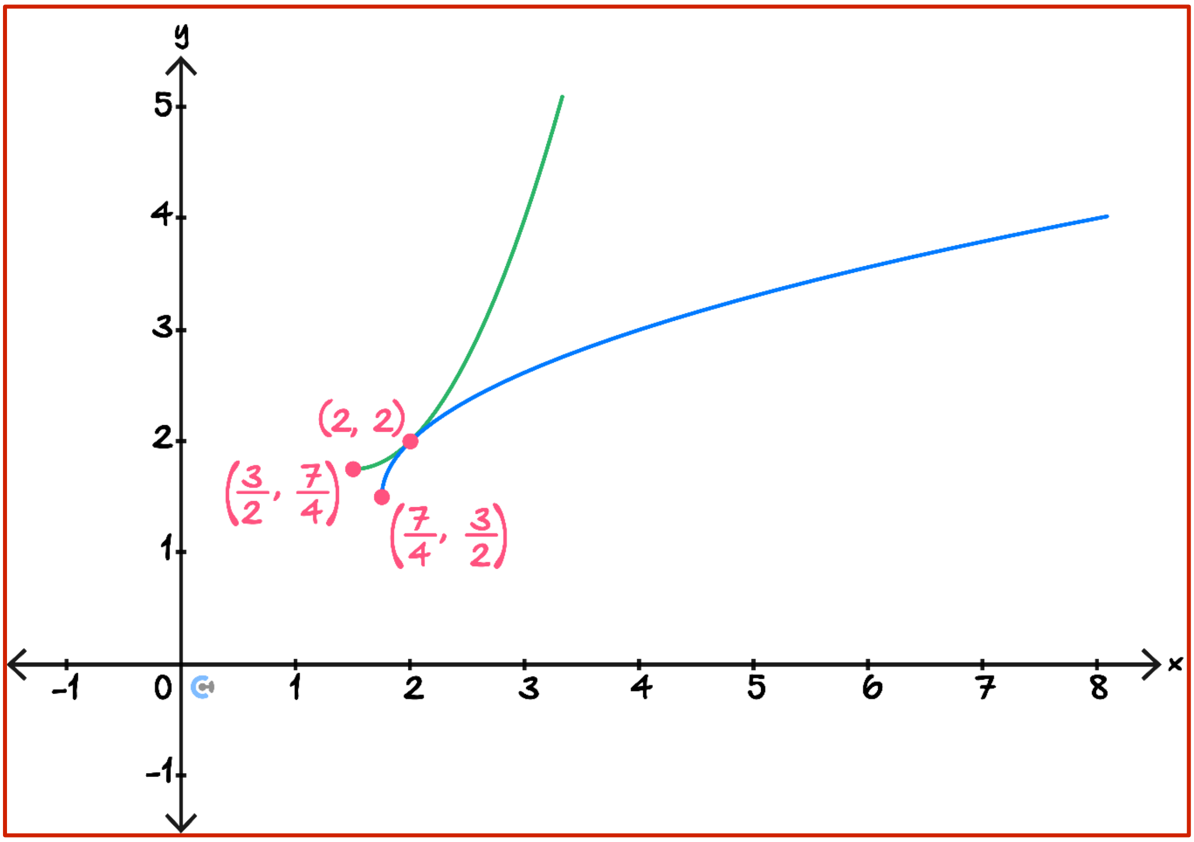
By considering ran f^{-1} conclude that

$$f^{-1} : \left[\frac{7}{4}, \infty\right) \to \mathbb{R},\ f^{-1}(x) = \sqrt{x - \frac{7}{4}} + \frac{3}{2}.$$

c. Write the rule for $f^{-1}(x)$ in the form $f^{-1}(x) = a\sqrt{4x - b} + \frac{3}{2}$, where $a, b \in \mathbb{R}$. (1 mark)

Solution: $f^{-1}(x) = \sqrt{\frac{1}{4}(4x-7)} + \frac{3}{2} = \frac{1}{2}\sqrt{4x-7} + \frac{3}{2}$.
$a = \frac{1}{2}, b = 7$

d. Sketch the graph of $y = f(x)$ and $y = f^{-1}(x)$ on the axes below. Label all endpoints and points of intersection with coordinates. (3 marks)



Solution: Intersect when $x^2 - 3x + 4 = x \implies x^2 - 4x + 4 = 0 \implies (x-2)^2 = 0 \implies x = 2$.

Section D: Tech Active Exam Skills

Calculator Commands: Using Sliders/Manipulate on CAS

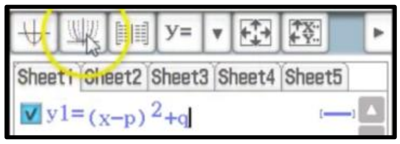
Mathematica

```
Manipulate[Plot[function, {x, xmin, xmax}],
{unknown, lowerbound, upperbound}]
```

NOTE: The function **must** be typed out instead of using its saved name.

TI-Nspire

Casio Classpad



Calculator Commands: Finding Maximal Domain

Mathematica

```
FunctionDomain[func, x]
```

TI-Nspire

Type up domain (or find it under the book button).

$$\text{domain}(\text{func},x)$$

Casio Classpad

Sketch the function and analyse.

Space for Personal Notes

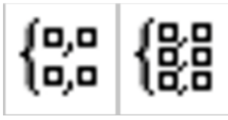
Calculator Commands: Defining Hybrid Functions on CAS

- **Mathematica**
  - Piecewise

    Piecewise[{{val_1, $cond_1$}, {val_2, $cond_2$}, …}]

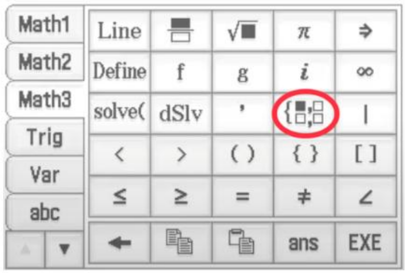
  - Represents a piecewise function with values val_i in the regions defined by the conditions $cond_i$.

- **TI-Nspire**



$$\begin{cases} func1, dom1 \\ func2, dom2 \end{cases}$$

- **Casio Classpad**



Calculator Commands: Finding the Equation of a Polynomial that Passes Through Points

- Given n points, we can find a degree $n-1$ polynomial that passes through all of these points.

- **Example:** Find the equation of the quadratic function that passes through the points $(0, 6)$, $(2, 2)$, and $(3, 3)$.

- **TI:**

Define $f(x)=a\cdot x^2+b\cdot x+c$	*Done*
solve($f(0)=6$ and $f(2)=2$ and $f(3)=3,a,b,c$)	$a=1$ and $b=^-4$ and $c=6$
$f(x)\vert a=1$ and $b=^-4$ and $c=6$	$x^2-4\cdot x+6$

- **Casio:**

```
define f(x) = a*x^2 + b*x + c
                                        done
{f(0)=6
{f(2)=2
{f(3)=3|a,b,c
                           {a=1,b=-4,c=6}
f(x)|{a=1,b=-4,c=6}
                                 x^2-4·x+6
```

- **Mathematica:**

```
In[9]:= f[x_] := a x^2 + b x + c

In[10]:= Solve[f[0] == 6 && f[2] == 2 && f[3] == 3]
Out[10]= {{a → 1, b → -4, c → 6}}

In[11]:= f[x] /. {a → 1, b → -4, c → 6}
Out[11]= 6 - 4 x + x^2
```

<u>Calculator Commands:</u> Turning Point

- ALWAYS sketch the graph to find approximate bounds for where the turning point you want is located.
- To find a local maximum, we maximise the function over a specific domain.
- To find a local minimum, we minimise the function over a specific domain.
- **TI and Casio:** Use $fmin(expression, variable, lower\ (optional), upper\ (optional))$ or $fmax(expression, variable, lower\ (optional), upper\ (optional))$.
- **TI:** Menu $\rightarrow 4 \rightarrow \frac{7}{8}$.

Define $f(x)=x^3-4\cdot x$	*Done*
$\mathrm{fMin}(f(x),x,0,2)$	$x=\frac{2\cdot\sqrt{3}}{3}$
$f\left(\frac{2\cdot\sqrt{3}}{3}\right)$	$\frac{-16\cdot\sqrt{3}}{9}$

- **Casio:** Action $\rightarrow$ Calculation $\rightarrow$ $fmin/fmax$

$$\mathrm{fmin}(x\text{^}3-4x, x, 0, 2)$$

$$\left\{\mathrm{MinValue}=\frac{-16\cdot\sqrt{3}}{9}, x=\frac{2\cdot\sqrt{3}}{3}\right\}$$

.

- **Mathematica:** Minimise[ ] and Maximise[ ] commands.
- Minimise[$f[x], x$] will minimise $f[x]$ over its whole domain.
- To restrict the domain, we must use Minimise[$\{f[x], a \leq x \leq b\}, x$].

```
In[34]:= Minimize[{x^3 - 4 x, 0 < x < 2}, x]
```

Out[34]= $\left\{-\frac{16}{3\sqrt{3}}, \left\{x \to \frac{2}{\sqrt{3}}\right\}\right\}$

Space for Personal Notes

Section E: Exam 2 Questions (30 Marks)

INSTRUCTION:

- Regular: 30 Marks. 45 Minutes Writing.
- Extension: 30 Marks. 30 Minutes Writing.

Question 5 (1 mark)

The function, f defined by $f : A \to \mathbb{R}, f(x) = (x-1)^2 + 3$ will have an inverse function if its domain A is:

A. $\mathbb{R}$

B. $(-\infty, 3]$

C. $[3, 10]$

D. $[0, \infty)$

Question 6 (1 mark)

Which one of the following functions does **not** have an inverse function?

A. $f : \mathbb{R} \to \mathbb{R}, f(x) = 2x - 5$

B. $g : [0, \infty) \to \mathbb{R}, g(x) = x^2$

C. $h : \mathbb{R} \to \mathbb{R}, h(x) = x^3$

D. $k : [-2, 2] \to \mathbb{R}, k(x) = \sqrt{4 - x^2}$

Space for Personal Notes

Question 7 (1 mark)

The linear function, $f : D \to \mathbb{R}, f(x) = 3 - x$ has a range of $[-4, 6)$. The domain of f is:

A. $(-5, 1]$

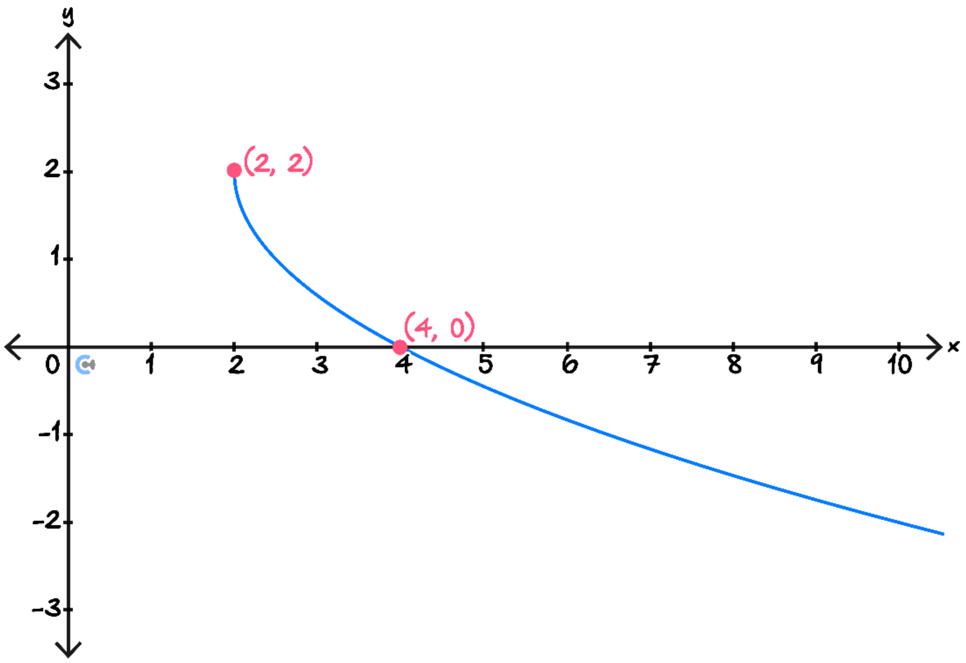
B. $(-3, 7]$

C. $(-2, 7)$

D. $[-3, 7]$

Question 8 (1 mark)

The rule for the function shown in the graph below could be:



A. $y = \sqrt{2x - 4} + 2$

B. $y = -\sqrt{2x - 4} + 2$

C. $y = \sqrt{x - 2} + 2$

D. $y = -\sqrt{x - 2} + 2$

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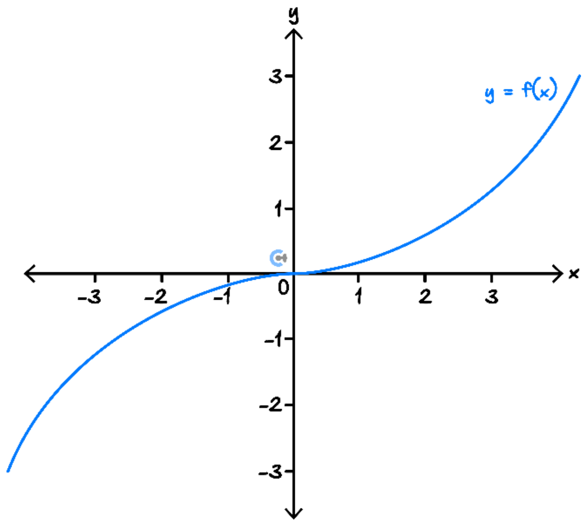
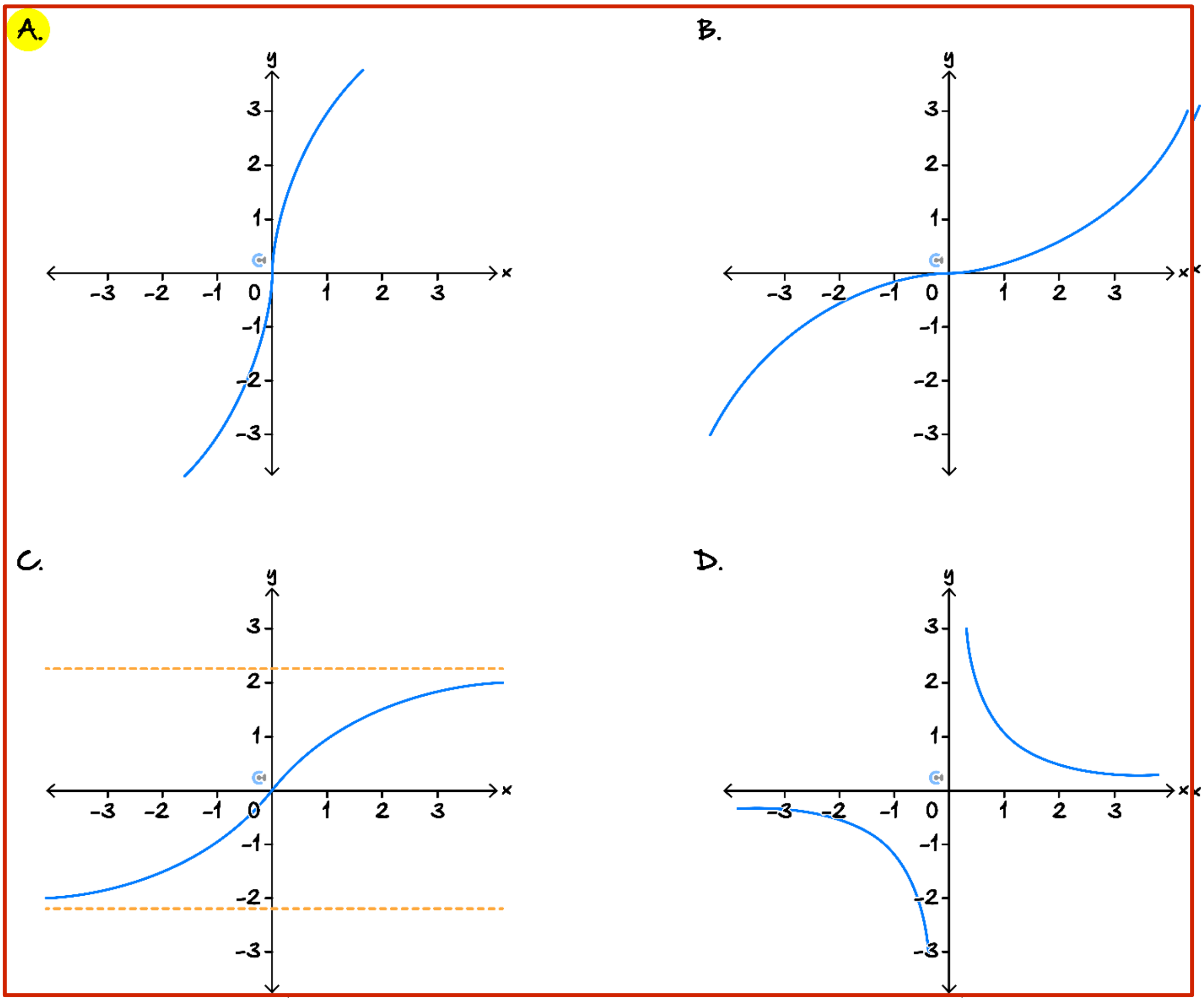
Question 9 (1 mark)

The graph of the function with equation, $y = f(x)$ is shown below.



Which one of the following is most likely to be the graph of the inverse function?



Question 10 (1 mark)

The equation $x^3 - 3x = k$ always has three real solutions for:

A. $k > 2$

B. $k \in [-2, 2]$

C. $k \in (-2, 2)$

D. $k < 2$

Question 11 (13 marks)

The temperature of a cooling object follows a hyperbolic model given by T:

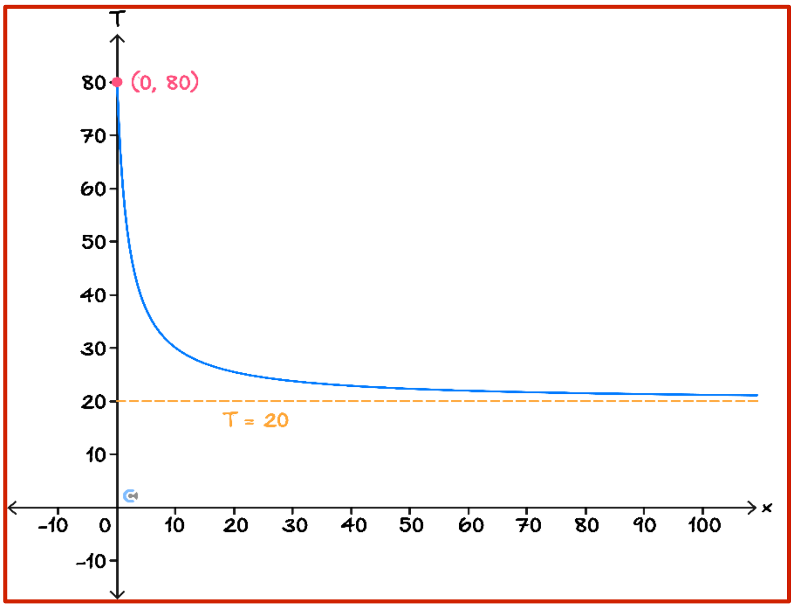
$$T(x) = \frac{120}{x + 2} + 20$$

where, $T(x)$ represents the temperature (in degrees Celsius) of the object, x minutes after it was removed from an oven.

a. What is the implied domain of the function T? (i.e. what values of x make sense?) (1 mark)

$\boxed{x \in [0, \infty)}$

b. Sketch the graph of $T(x)$, over its implied domain, on the axes below. Label any endpoints with coordinates and asymptotes with equations. (2 marks)



c. Find the temperature of the object after $x = 5$ minutes. (1 mark)

$$\boxed{T(5) = \frac{260}{7} \text{ degrees Celsius.}}$$

d. Determine the time x, when the temperature of the object is 50°C. (2 marks)

$$\boxed{\text{Solve } T(x) = 50 \implies x = 2 \text{ minutes.}}$$

e. Find the rule and domain of the inverse function $T^{-1}(x)$. (2 marks)

$$\boxed{\text{Domain: } (20, 80] \text{ and } T^{-1}(x) = \frac{120}{x - 20} - 2.}$$

f. Describe the information that $T^{-1}(30)$ gives us in relation to this scenario. (1 mark)

$$\boxed{T^{-1}(30) \text{ tells us how long it takes for the temperature to reach 30 degrees.}}$$

g. Calculate the average change in temperature in degrees per minute from $x = 1$ to $x = 11$ minutes. Give your answer correct to two decimal places. (2 marks)

Solution: This is the gradient of the line connecting the points on the curve T when $x = 1$ and $x = 11$.

$$\frac{T(11) - T(1)}{11 - 1} = -\frac{40}{13} = -3.07 \text{ degrees per minute}$$

h. The object's temperature is said to be "stabilising" when the average rate of change in temperature from time $x = b$ to $x = 60$ is less than -0.1 degrees per minute. Find the time, correct to the nearest minute, at which the object's temperature first begins stabilising. (2 marks)

Solution: We solve $\dfrac{T(60) - T(b)}{60 - b} = -\dfrac{1}{10} \implies b == \dfrac{538}{31} \approx 17.$
So 17 minutes.

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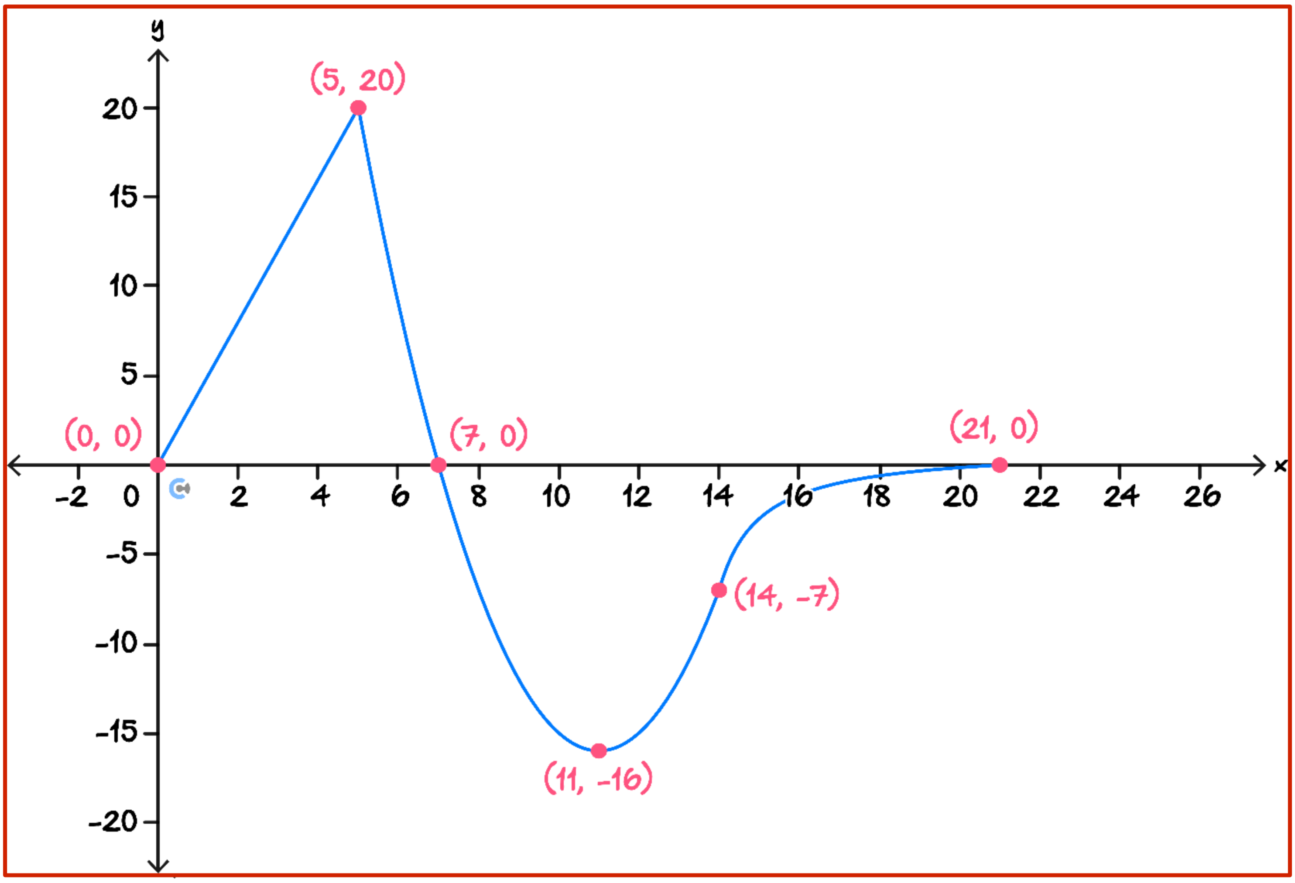
Question 12 (11 marks)

Contour Park constructs a roller that is made up of three different sections of track. Let h be the function that determines the height of the roller coaster above the ground, according to its horizontal position x. h is modelled by the rule:

$$\begin{cases} 4x & 0 \le x \le 5 \\ x^2 - 22x + 105 & 5 < x \le 14 \\ -\dfrac{8}{x-13} + 1 & 14 < x \le 22 \end{cases}$$

a. Sketch the graph of $h(x)$ on the axes below. Label all endpoints, intercepts, and turning points with coordinates. (4 marks)



b. State the maximum height of the roller coaster above the ground. (1 mark)

$\boxed{\text{20 metres.}}$

c. Find the values of x for which, the roller coaster is 15 metres **below** the ground. (2 marks)

Solution: See that this occurs during the quadratic section. Solve $x^2 - 22x + 105 = -15 \implies x = 10, 12$.

d. Find the values of x for which, the roller coaster is below the ground. Express your answer using interval notation. (2 marks)

This is when $h < 0$ so $x \in (7, 22)$.

The roller coaster is a huge success, however a complaint is that the ride is too quick. To rectify this issue, it is decided that instead of the roller coaster track ending at $x = 21$, a new track with the exact same shape as $h(x)$ will be constructed from this point.

e. Define the function $h_1(x)$ which describes the linear section of the new track. (2 marks)

Solution: Has a gradient of 4 and starts from the point $(21, 0)$ and goes for 5 units to the right. Therefore

$$h_1(x) = 4(x - 21) = 4x - 84 \text{ for } x \in [21, 26]$$

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Section F: Extension Exam 1 (9 Marks)

INSTRUCTION:

- Regular: Skip
- Extension: 9 Marks. 13 Minutes Writing.

Question 13 (9 marks)

Consider the function, $f(x) = \frac{1}{x-4}$.

a. Find the values of x for which, $f^{-1}(x) > f(x)$. (4 marks)

Solution: $x = \dfrac{1}{y-4} \implies y = \dfrac{1}{x} + 4$. Solve,

$$\begin{aligned} \frac{1}{x-4} &= x \\ x^2 - 4x - 1 &= 0 \\ (x-2)^2 &= 5 \\ x &= 2 \pm \sqrt{5} \end{aligned}$$

Consider the shapes of the two graphs to conclude that $f^{-1}(x) > f(x)$ for

$$x \in (-\infty, 2-\sqrt{5}) \cup (0,4) \cup (2+\sqrt{5}, \infty)$$

Now, let $g : (-\infty, k) \to \mathbb{R}, g(x) = \frac{1}{k-x}$, where k is a real constant.

b. Find the rule and domain for the inverse function, g^{-1}, in terms of k. (2 marks)

$$\boxed{g^{-1} : (0, \infty) \to \mathbb{R},\ g^{-1}(x) = k - \frac{1}{x}}$$

c. Find the exact value of k so that g and g^{-1} have one point of intersection. (3 marks)

Solution: Must intersect on the line $y = x$. Solve

$$\begin{aligned} \frac{1}{k-x} &= x \\ 1 &= kx - x^2 \\ x^2 - kx + 1 &= 0 \\ x &= \frac{k \pm \sqrt{k^2 - 4}}{2} \end{aligned}$$

So one solution if $k^2 - 4 = 0 \implies k = \pm 2$.
Then checking domain of g and g^{-1} only $k = 2$ is valid.

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Section G: Extension Exam 2 (15 Marks)

INSTRUCTION:

- Regular: Skip
- Extension: 15 Marks. 22 Minutes Writing.

Question 14 (1 mark)

The range of the function given by $f : (0, 4] \to \mathbb{R}, f(x) = x^2 - 2x + b$ is:

A. $(b - 1, b + 8)$

B. $[b - 1, b + 8]$

C. $(b, 8]$

D. $(b - 1, b + 8]$

Question 15 (1 mark)

The functions, $f(x) = \log_2(a - x)$ and $g(x) = -\sqrt{x + a}$ are defined on their maximal domains and $a \in \mathbb{R}^+$.

The domain of $f(x) \times g(x)$ is:

A. $[-a, a)$

B. $[-a, a]$

C. $(-a, a)$

D. $\mathbb{R} \setminus \{a\}$

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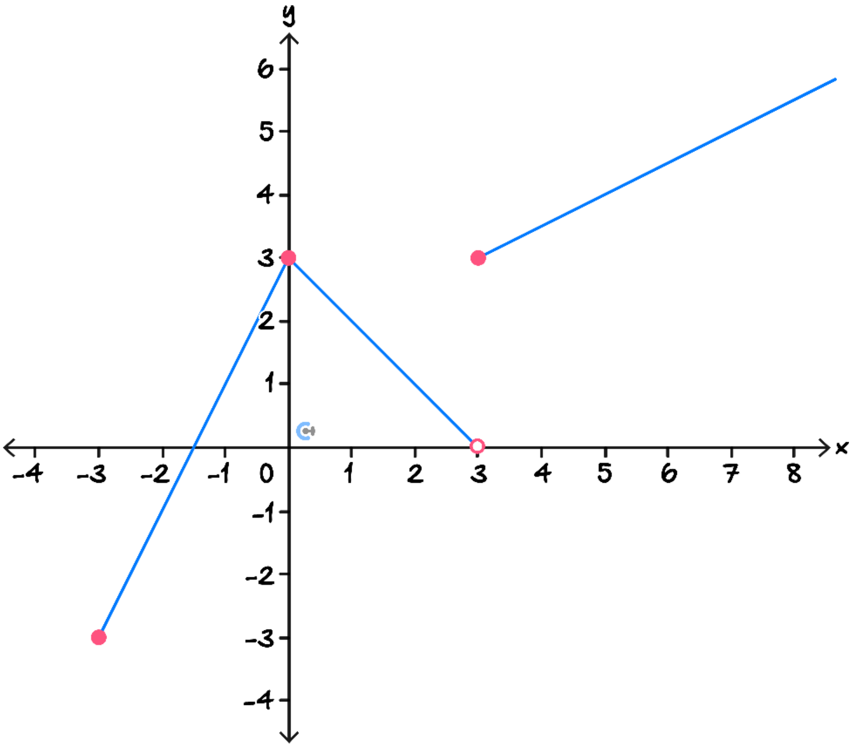
Question 16 (1 mark)

The graph of the function f is shown below.



In order for the inverse f^{-1} to exist, a possible restricted domain of f is:

A. $x \in [-3, 0] \cup [3, 0]$

B. $x \in [-1, 2)$

C. $x \in [0, 3]$

D. $x \in [-3, 0) \cup [3, 0]$

Question 17 (1 mark)

The equation $12x^5 + 15x^4 - 60x^3 - 30x^2 + 120x = k$ has one real solution for:

A. $k \in (-87, 57)$

B. $k \in (-\infty, -87) \cup (-24, \infty)$

C. $k \in (-87, -24)$

D. $k \in (-\infty, 57)$

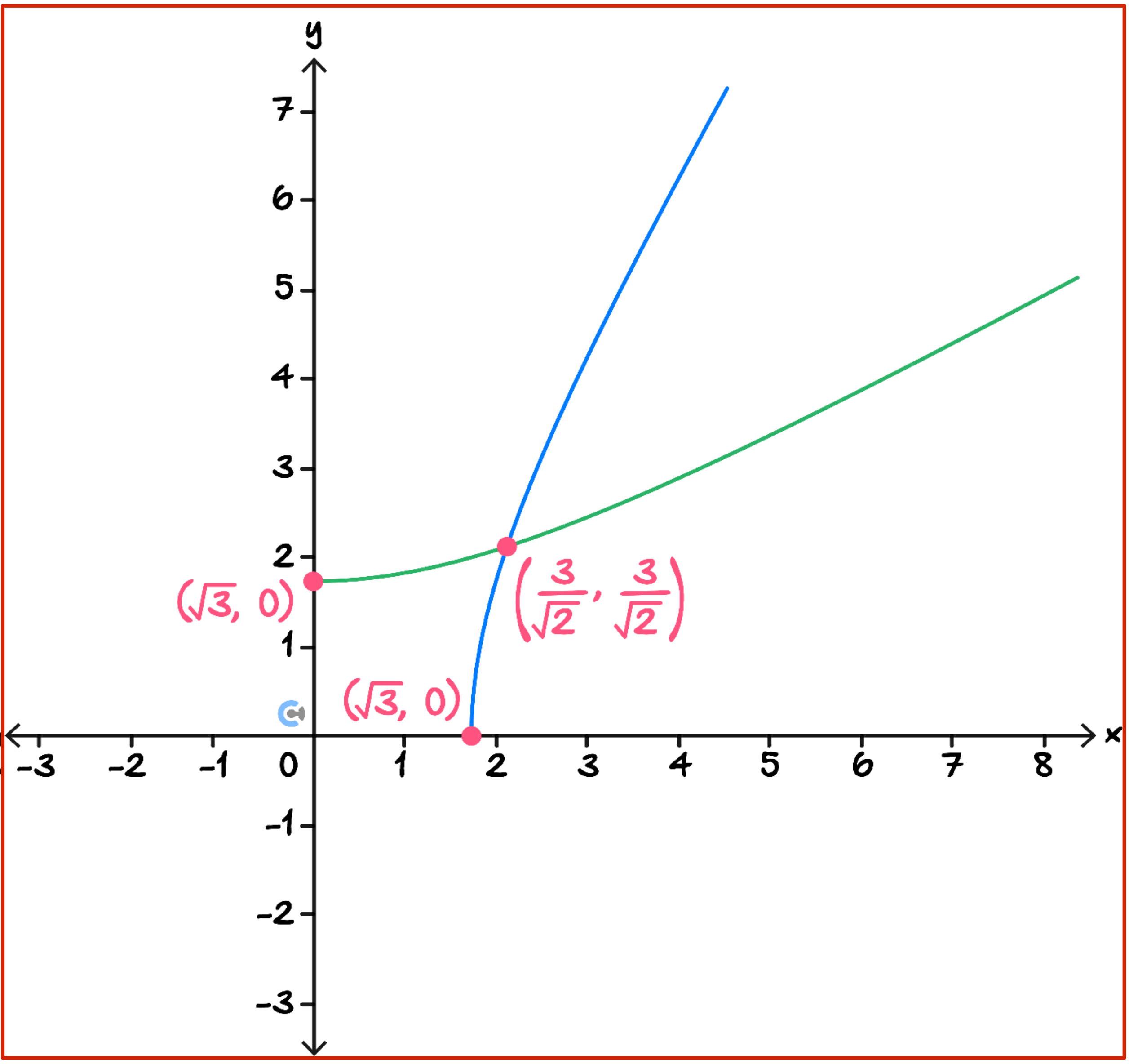
Question 18 (11 marks)

Consider the function, $f:\left[\sqrt{3},\infty\right)\to\mathbb{R}, f(x)=\sqrt{3x^2-9}$.

a. Define f^{-1}, the inverse function of f. (2 marks)

$$\boxed{f^{-1}:[0,\infty)\to\mathbb{R},\ f^{-1}(x)=\sqrt{\frac{x^2+9}{3}}}$$

b. Sketch the graphs of $y=f(x)$, $y=f^{-1}(x)$, on the axes below. Label all axes intercepts and points of intersection with coordinates. (3 marks)



Now, consider the one-to-one function, defined on its maximal domain, $g : [a, \infty) \to \mathbb{R}$, where $g(x) = \sqrt{kx^2 - 9}$ and $a, k \in \mathbb{R}^+$.

c.

i. Find the value of a in terms of k. (1 mark)

Solution: $kx^2 - 9 \geq 0 \implies x \geq \dfrac{3}{\sqrt{k}}$.

So $a = \dfrac{3}{\sqrt{k}}$

ii. Find the value of k such that, g and g^{-1} intersect at $(2, 2)$. (2 marks)

We require $g(2) = 2 \implies \sqrt{4k - 9} = 2 \implies k = \dfrac{13}{4}$

iii. Find the value(s) of k for which, g and g^{-1} do not intersect each other. (2 marks)

Solution: Intersection must occur on the line $y = x$. Solve $g(x) = x \implies x = \dfrac{3}{\sqrt{k-1}}$.

Then this solution is only valid for $k > 1$.

Therefore intersection for $0 < k < 1$.

d. As x gets larger and larger (i.e. as $x \to \infty$), the function $g(x)$ approaches, but never touches, a linear function of the form $y = mx$. State the value of m in terms of k. (1 mark)

Solution: $g(x) \to \sqrt{kx^2} = \sqrt{k}x$ as $x \to \infty$.
$m = \sqrt{k}$

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