CONTOUREDUCATION

Website: contoureducation.com.au | Phone: 1800 888 300
Email: hello@contoureducation.com.au

VCE Mathematical Methods ½
Functions & Relations Exam Skills [0.9]
Workshop

Rei – Contacts
- whatsapp/messages 0490 198 272
- email Rei@contoureducation.com.au

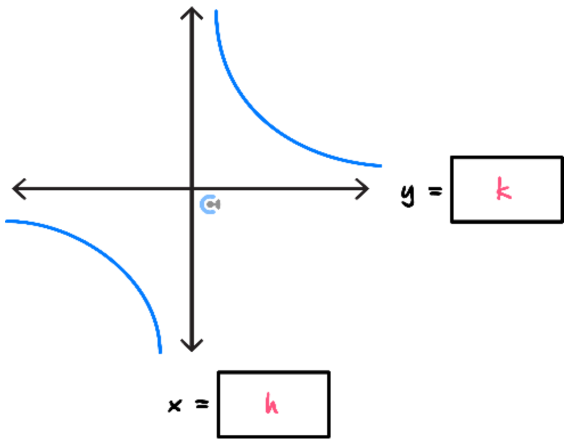
Error Logbook:

Mistake/Misconception #1 Not knowing		Mistake/Misconception #2 Algebra / CAS Mistakes	
Question #:	Page #:	Question #:	Page #:
Notes:		Notes:	
Mistake/Misconception #3 Reading the Question		**Mistake/Misconception #4** Time Management	
Question #:	Page #:	Question #:	Page #:
Notes:		Notes:	

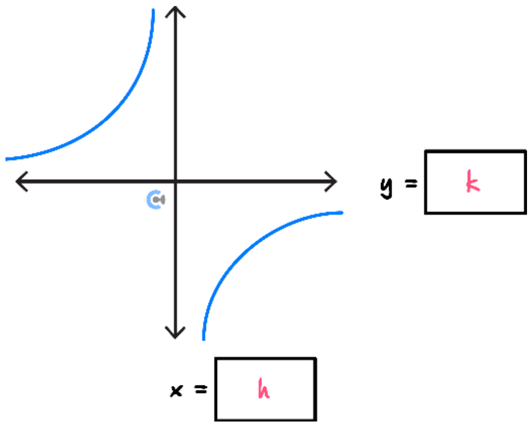
Section A: Recap

Rectangular Hyperbola

$$y = \frac{a}{x-h} + k$$



where $a > 0$



where $a < 0$

- **Steps**

1. Find the horizontal and vertical asymptotes and plot them on the axis.
2. Find the x- and y-intercepts and plot on the axes (if they exist.).
3. Identify the shape of the graph by considering any reflections, and sketch the curve.

Finding the Equation of a Hyperbola from its Graph

- We generally need three facts (h, k, and a) about the hyperbola.

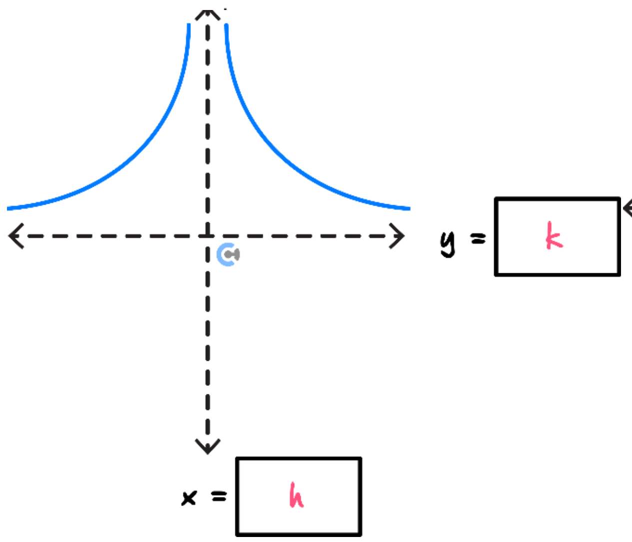
$$y = \frac{a}{x-h} + k$$

- **Steps**
  - Look for the asymptotes.
  - Sub in a point to find the value of a.

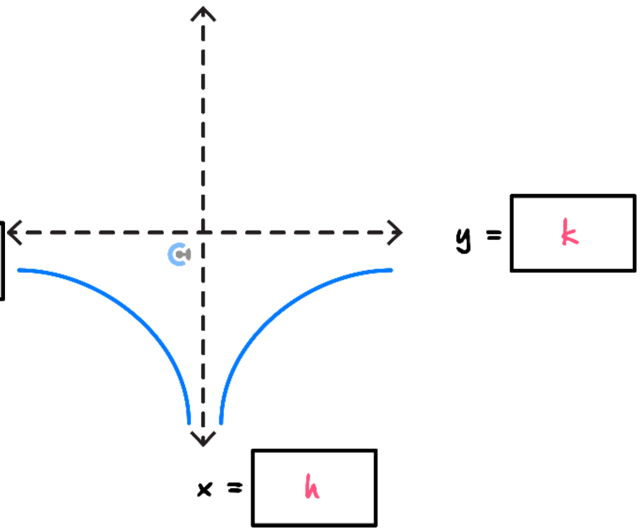
Definition

Truncus

$$y = \frac{a}{(x-h)^2} + k$$



where $a > 0$



where $a < 0$

- Steps

1. Find the horizontal and vertical asymptotes and plot them on the axis.

2. Find the x- and y-intercepts and plot on the axes (if they exist.).

3. Identify the shape of the graph by considering any reflections and sketch the curve.

Finding the Equation of a Truncus from its Graph

- We generally need three facts (h, k, and a) about the truncus.

$$y = \frac{a}{(x-h)^2} + k$$

- Steps

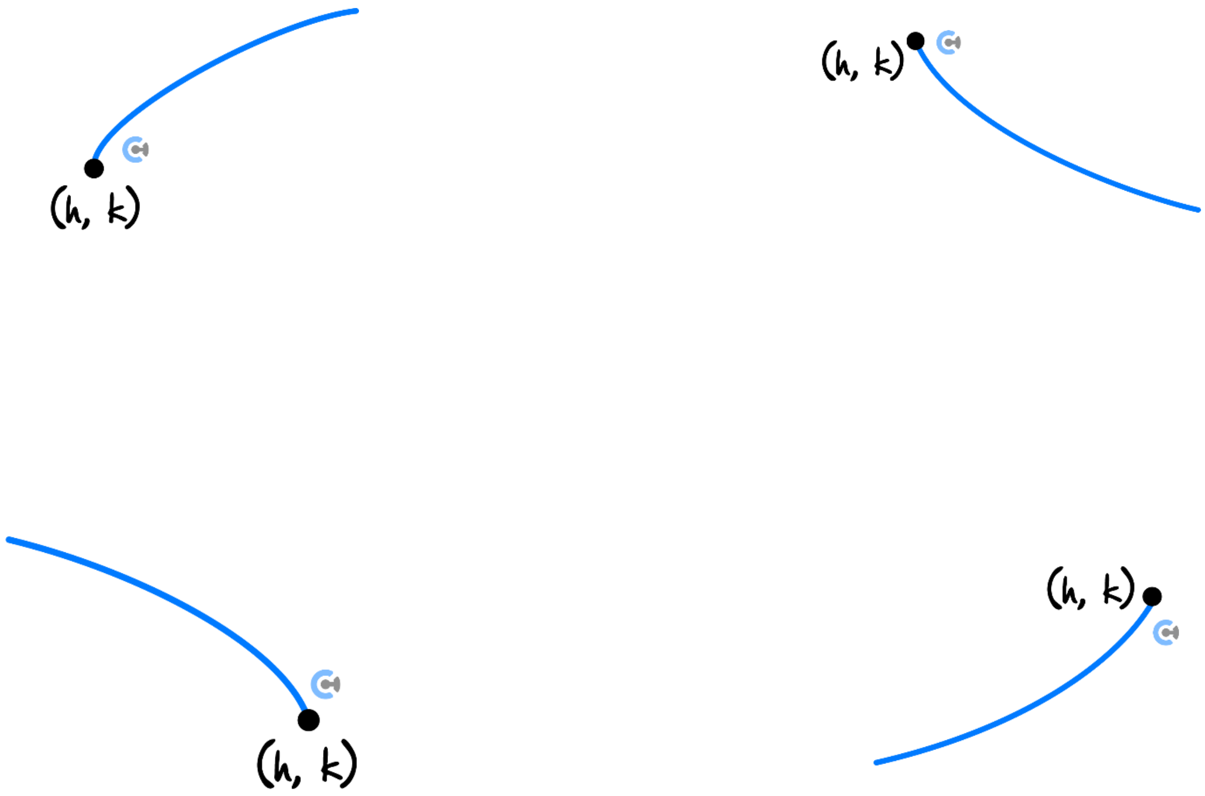
  - Look for the asymptotes.

  - Sub in a point to solve the value of a.

Definition

Square Root Functions

$$y = a\sqrt{b(x-h)} + k$$



- **Steps for sketching roots**

  1. Find the starting point (h, k).
  2. Find the x- and y-intercepts and plot on the axes (if they exist.).
  3. Identify the shape of the graph by considering any reflections and sketch the curve.

Space for Personal Notes

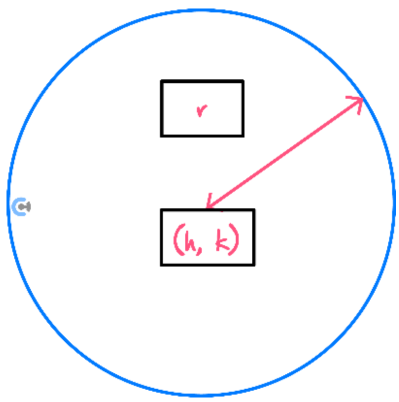
Finding the Equation of a Root Function from its Graph

- We generally need three facts about the root function.

$$y = a\sqrt{\pm(x-h)} + k$$

- **Steps**
  - Look for the starting point (h, k).
  - Sub in a point to solve the value of a.

Circles



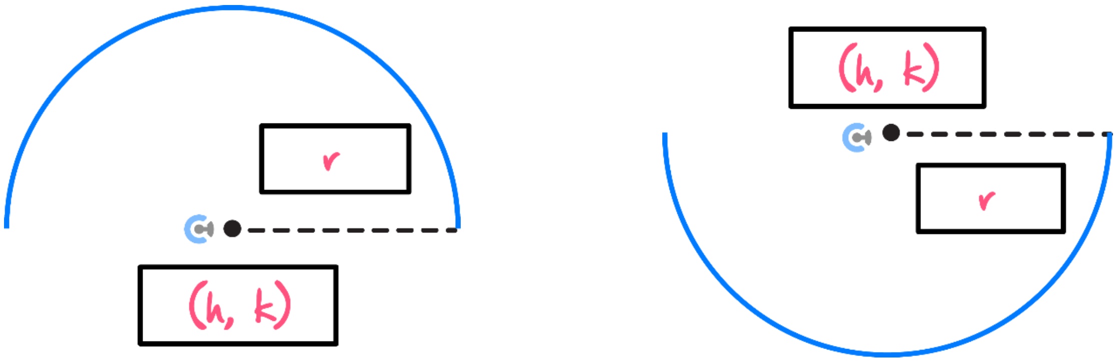
$$(x-h)^2 + (y-k)^2 = r^2$$

$$\text{where } r > 0$$

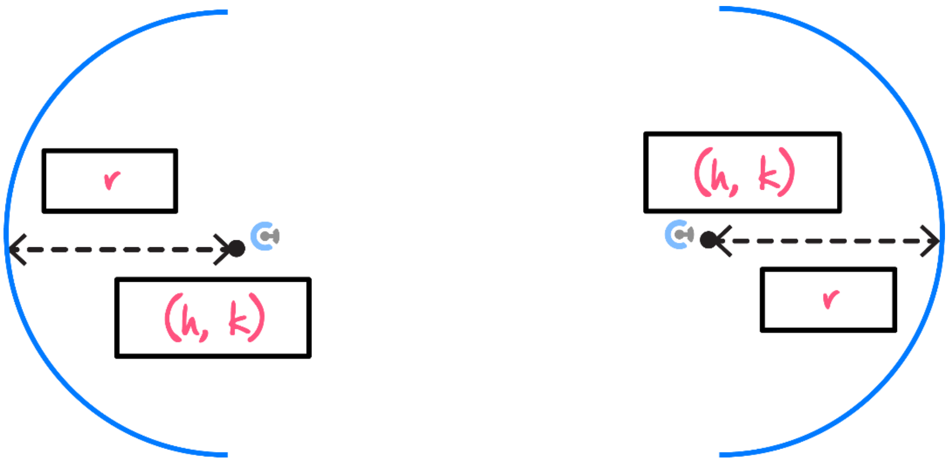
- Centre: (h, k)
- Radius: r
- **Steps**
  1. Find the centre of the circle.
  2. Find the radius of the circle.
  3. Find axes intercepts (if they exist.).
  4. Identify the shape of the graph and sketch the curve.

Definition

Semicircles



$$y = \pm\sqrt{r^2 - (x-h)^2} + k$$



$$x = \pm\sqrt{r^2 - (y-k)^2} + h$$

- **Steps**

1. Find the centre of the semicircle.
2. Find the radius of the circle.
3. Find axes intercepts if they exist.
4. Identify the shape of the graph and sketch the curve.

Space for Personal Notes

Finding the Equation of a Root Function from its Graph

- We need generally three facts about the circles/semicircles.

$$(x-h)^2 + (y-k)^2 = r^2$$

$$y = \pm\sqrt{r^2 - (x-h)^2} + k$$

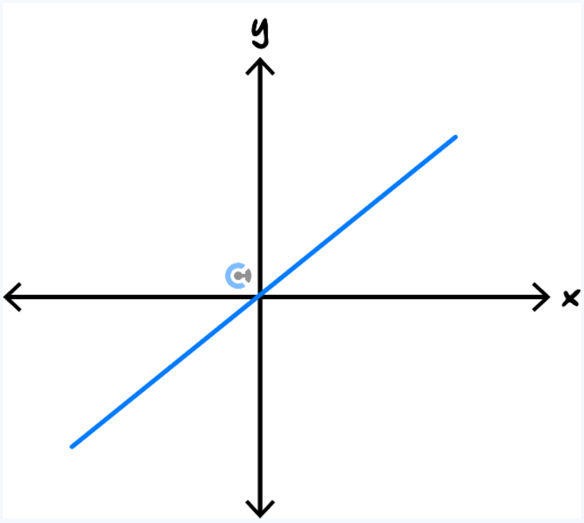
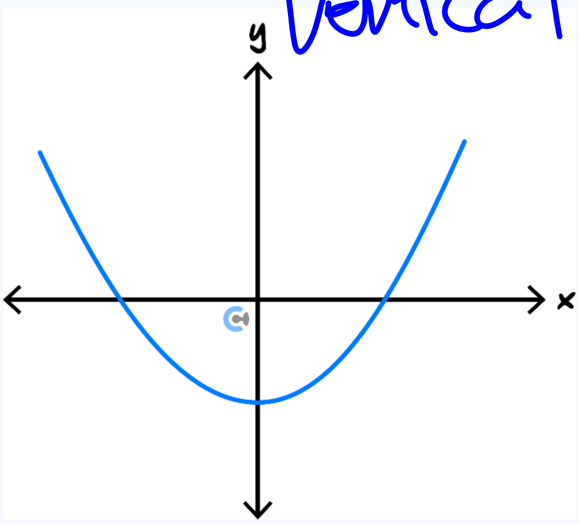
$$x = \pm\sqrt{r^2 - (y-k)^2} + h$$

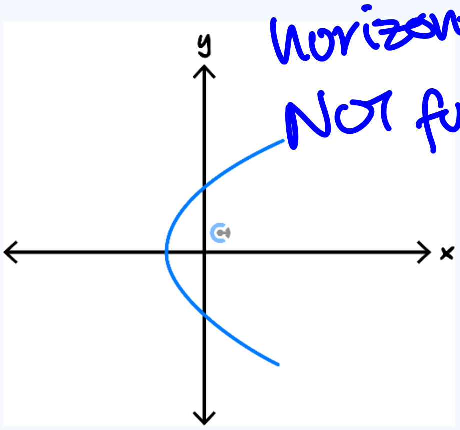
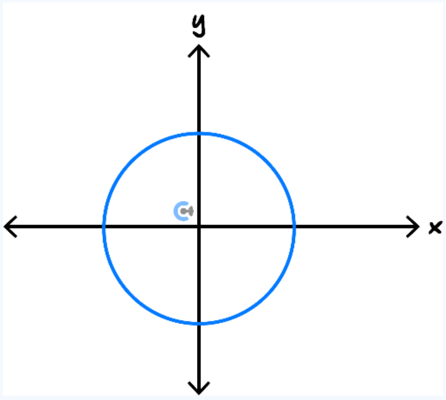
- **Steps**

1. Identify the centre, (h, k).
2. Identify the radius, r.

Types of Relations

- There are four types of relations:

One to One	Many to One
	vertical 
One x to One y.	Many x's to One y.

<u>One to Many</u>	<u>Many to Many</u>
horizontal NOT function 	
One x to Many y's.	Many x's to Many y's.

<u>Functions</u>

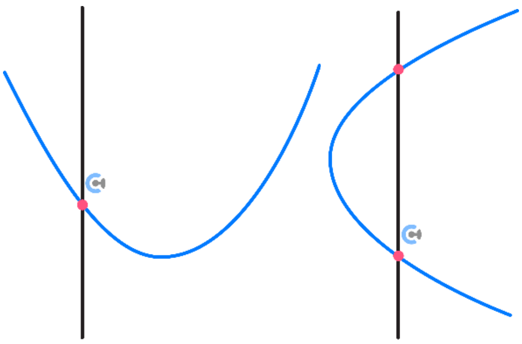
Vertical 1:1

$$y = f(x)$$

- Functions are relations which make one y-value at any given x-value.

<u>Vertical Line Test</u>

- **Definition**: Tells apart between functions and non-function relations.



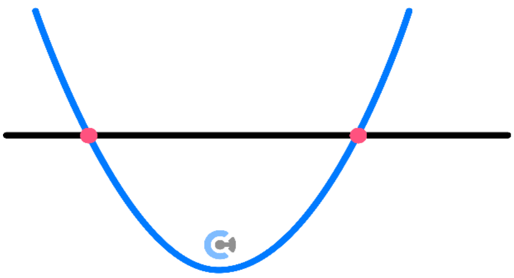
Passes : Function   Fails : Not Function

Every function only intersects a vertical line once.

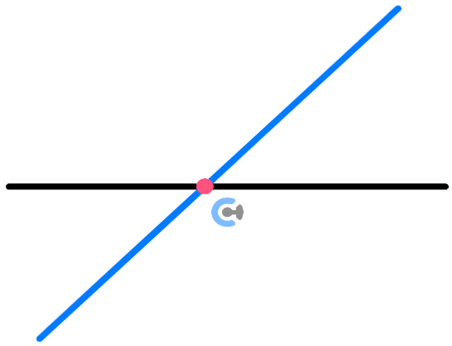
Definition

Horizontal Line Test

- **Definition**: Tells apart between many-to-one and one-to-one functions. (And relations.)



Fails: Many to one



Passes: One to one

Set Operators

- Intersection: "AND".

$$A \cap B = \textit{What values are in set } A \textit{ AND in set } B.$$

- Union: "OR".

$$A \cup B = \textit{What values are in set } A \textit{ OR in set } B.$$

- Set difference: "Except".

$$A \setminus B = \textit{What values are in set } A \textit{ except those also in set } B.$$

B

Interval Notation

$1 \leq x \leq 3$

- Parentheses (non-inclusive):

$x \in [1,3]$

$$x \in (a, b) \Rightarrow a < x < b$$

$x \in (1,3)$

- Square brackets [inclusive]:

$$x \in [a, b] \Rightarrow a \leq x \leq b$$

Maximal Domain

↳ x values

- The maximal domain is the biggest possible domain for a rule without committing a mathematical crime.

- In Methods, we need to consider 3 important rules:

$$\sqrt{z}, \quad z \geq 0$$

$$\log(z), \quad z > 0$$

$$\frac{1}{z}, \quad z \neq 0$$

$z \in R \setminus \{0\}$

Range

- The range is the possible value for the output of a function.

y

Sketch

Functional Notation

name

R

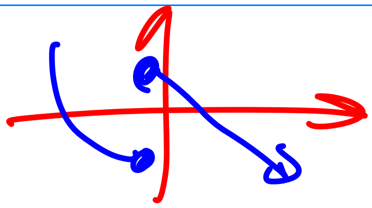
$$f: Domain \to Codomain, f(x) = Rule$$

- Codomain is simply all the values the function works within.

- Codomain is **not** the same as the range.

Piecewise (Hybrid) Functions

- Series of functions.



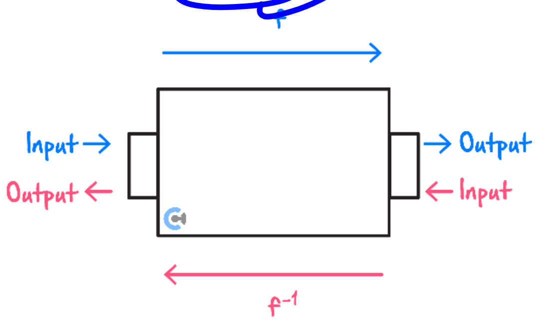
$$h(x) = \begin{cases} f(x), & Domain_1 \\ g(x), & Domain_2 \end{cases}$$

- $Domain_1$ and $Domain_2$ represent the x-values for which the two functions are defined.

- The two domains do not have to join!

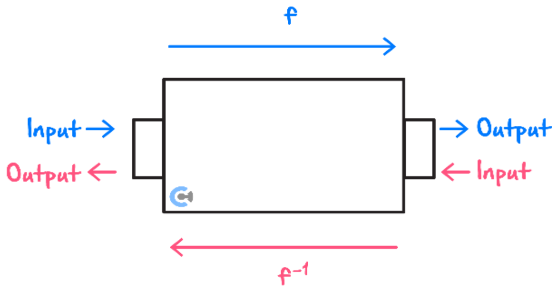
Inverse Relation

- **Definition**: Inverse is a relation that does the opposite.



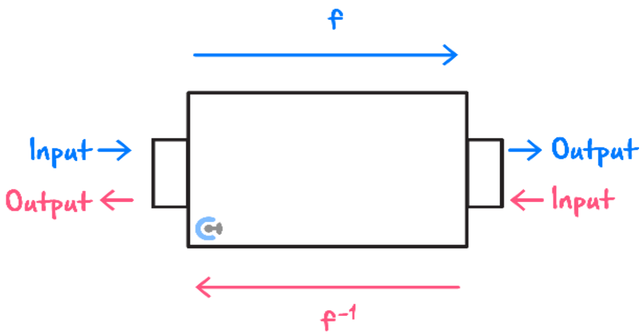
Solving for an Inverse Relation

- Swap x and y.



Domain and Range of Inverse Functions

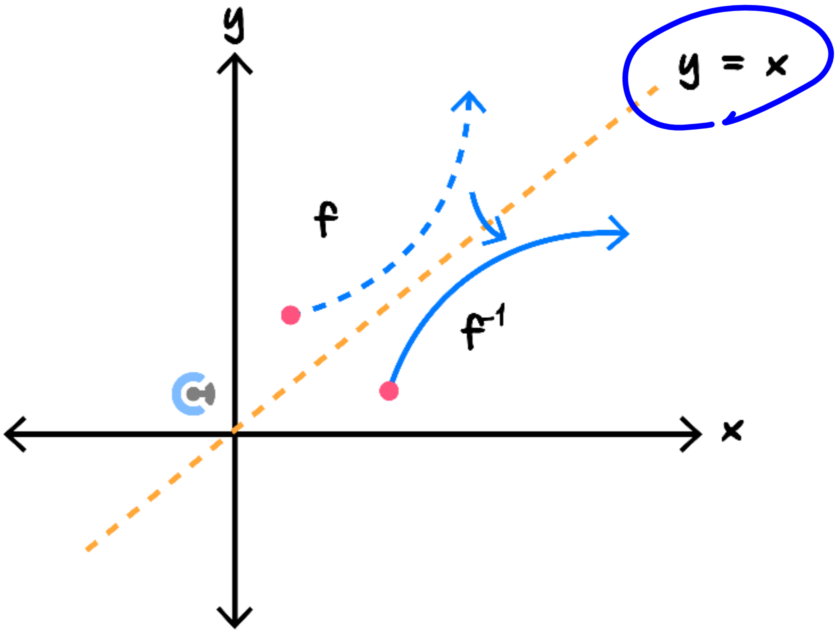
$x \Leftrightarrow y$



$$\textbf{Dom } f^{-1} = \textbf{Ran } f$$

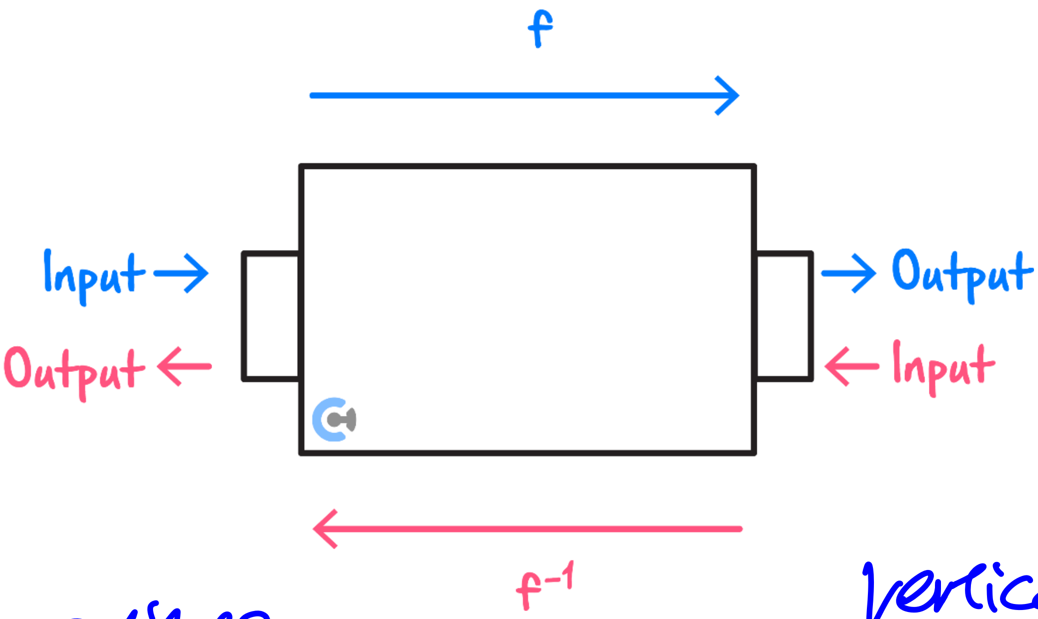
$$\textbf{Ran } f^{-1} = \textbf{Dom } f$$

Symmetry of Inverse Functions



- Inverse functions are always symmetrical around $y = x$.

Validity of Inverse Functions



exists

vertical l:t

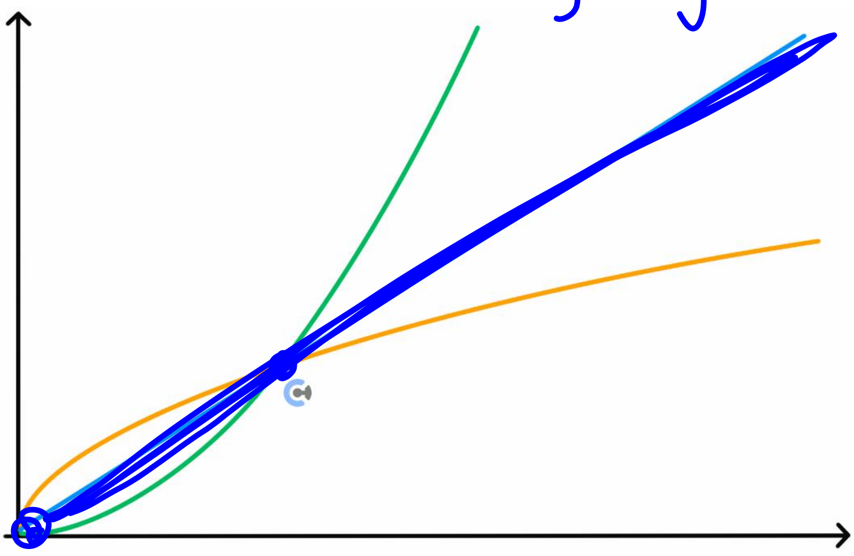
horizontal l:l

- Requirement for Inverse Function:

f needs to be 1:1.

Intersection between a Function and its Inverse

$f = f^{-1}$



$f(x) = x$ OR $f^{-1}(x) = x$

Space for Personal Notes

Section B: Warmup (14 Marks)

INSTRUCTION: 14 Marks. 15 Minutes Writing.

Question 1 (5 marks)

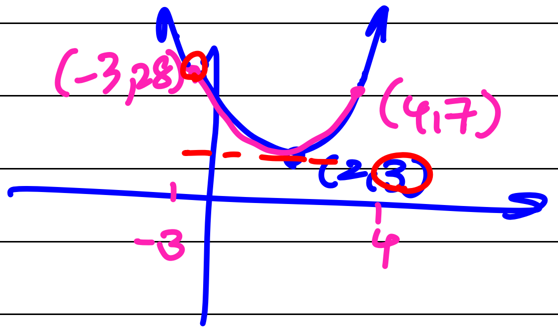
→ sketch

a. Let $h : (-3, 4] \to \mathbb{R}, f(x) = x^2 - 4x + 7$. Find the range of h. (2 marks)

$f(x) = (x-2)^2 + 3$

$f(-3) = 9 + 12 + 7$
$= 28$

$f(4) = 16 - 16 + 7 = 7$



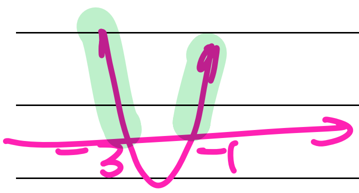
$Ran = [3, 28)$

b. Find the maximal domain of the function $f(x) = \frac{1}{\sqrt{x^2+4x+3}}$. Express your answer in interval notation. (2 marks)

$\sqrt{0}$

$x^2 + 4x + 3 > 0$

$(x+3)(x+1) > 0$



$x \in (-\infty, -3) \cup (-1, \infty)$

domain

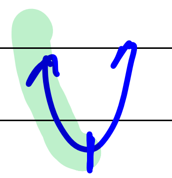
c. Let $g : (-\infty, b] \to \mathbb{R}, f(x) = x^2 + 4x + 1$.

→ one-to-one

Determine the maximum value of b such that g^{-1} exists. (1 mark)

↳ vertical l.t.
horizontal l.t

$(x+2)^2 - 3$



$(-2, -3)$

$(-\infty, -2]$

$\boxed{b = -2}$

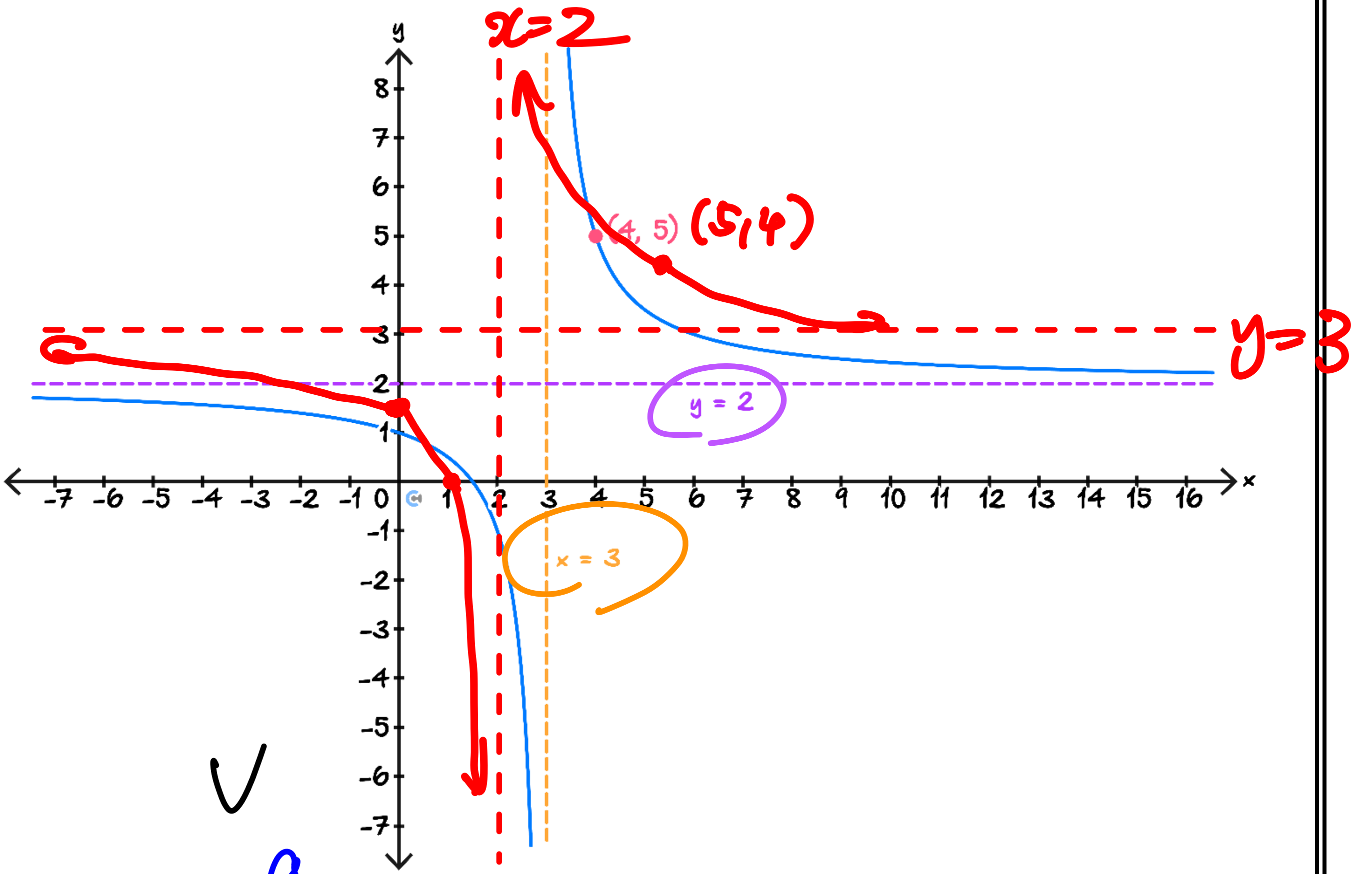
Space for Personal Notes

$x^2 + 4x + 1 = (x+2)^2 - 3$

$x^2 + 4x + 4$

Question 2 (4 marks)

a. The function f is sketched on the axes below. Find the rule for $f(x)$.



$$y = \frac{a}{x-3} + 2$$

$$5 = \frac{a}{4-3} + 2$$

$5 = a + 2$

$a = 3$

$$\boxed{f(x) = \frac{3}{x-3} + 2}$$

b. Sketch the inverse function f^{-1} on the same axes as f. Label the equations of the asymptotes.

Space for Personal Notes

Question 3 (5 marks)

Consider the function $f(x) = \sqrt{4x - 4} + 1$, defined on its maximal domain.

$\downarrow y$

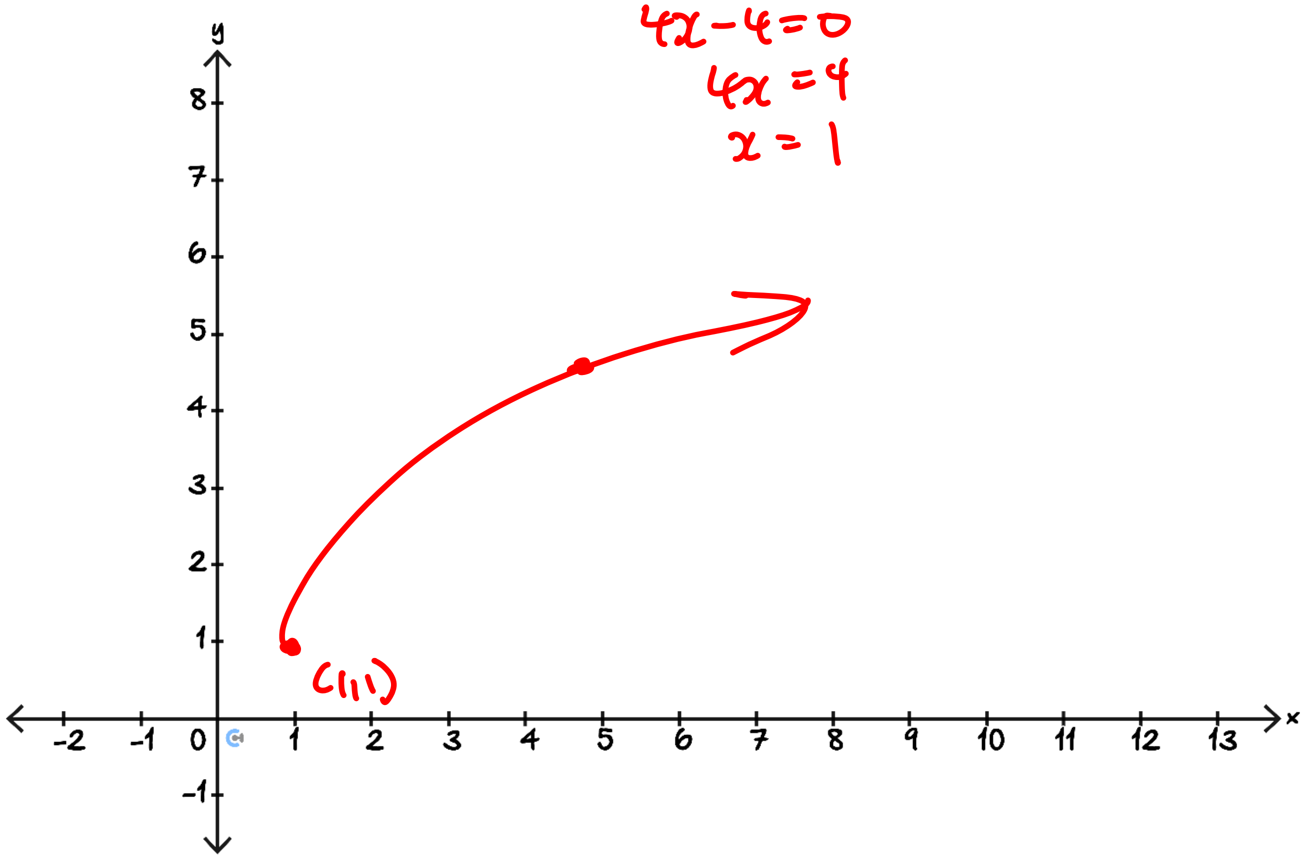
$= 0$

a. Sketch the graph of $y = f(x)$ on the axes below. Label the starting point with coordinates.. (2 marks)

$4x - 4 = 0$

$4x = 4$

$x = 1$



b. Find the rule and domain for the inverse function f^{-1}. (2 marks)

swap x & y

$x = \sqrt{4y - 4} + 1$

$x - 1 = \sqrt{4y - 4}$

$(x-1)^2 = 4y - 4$

$(x-1)^2 + 4 = 4y$

$\div 4$

$y = \frac{1}{4}(x-1)^2 + 1$

$f^{-1}(x) = \frac{1}{4}(x-1)^2 + 1$

$\text{dom} f^{-1} = \text{ran} f$

$= [1, \infty)$

c. State the coordinates of any points of intersection between f and f^{-1}. (1 mark)

$f(x) = x$

$\sqrt{4x-4}+1 = x$

$\sqrt{4x-4} = x-1$

$4x-4 = (x-1)^2$

$4x-4 = x^2-2x+1$

$0 = x^2-6x+5$

Space for Personal Notes

$0 = (x-5)(x-1)$

$x = 1, 5$

$y=x$

$(1,1)$
$(5,5)$

Section C: Exam 1 Questions (21 Marks)

INSTRUCTION: 21 Marks. 30 Minutes Writing.

Question 4 (8 marks)

Consider the function:

$$f : [a, \infty) \to \mathbb{R}, f(x) = x^2 - 3x + 4$$

$4 - \frac{9}{4} = \frac{7}{4}$

Complete the square

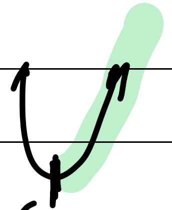
a.

i. Write $f(x)$ in turning point form. (1 mark)

$f(x) = \left(x - \frac{3}{2}\right)^2 + \frac{7}{4}$

→ one-to-one

ii. Hence, find the ~~largest~~ smallest value of a such that the inverse function f^{-1} exists. (1 mark)



$\left(\frac{3}{2}, \frac{7}{4}\right)$

$a = \frac{3}{2}$

Use the value of a found in **part a. ii.** for all subsequent questions.

b. Define f^{-1}, the inverse function of f. (2 marks)

Rule

↳ function notation

Domain

Rule

Swap x & y

$x = \left(y - \frac{3}{2}\right)^2 + \frac{7}{4}$

$x - \frac{7}{4} = \left(y - \frac{3}{2}\right)^2$

$y - \frac{3}{2} = \pm\sqrt{x - \frac{7}{4}}$

$y = \frac{3}{2} + \sqrt{x - \frac{7}{4}}$

+ve

$f^{-1}(x) = \frac{3}{2} + \sqrt{x - \frac{7}{4}}$

dom f^{-1} = ran f = $\left[\frac{7}{4}, \infty\right)$

$f^{-1}: \left[\frac{7}{4}, \infty\right) \to R$, $f^{-1}(x) = \sqrt{x - \frac{7}{4}} + \frac{3}{2}$

c. Write the rule for $f^{-1}(x)$, in the form $f^{-1}(x) = a\sqrt{4x - b} + \frac{3}{2}$, where $a, b \in \mathbb{R}$. (1 mark)

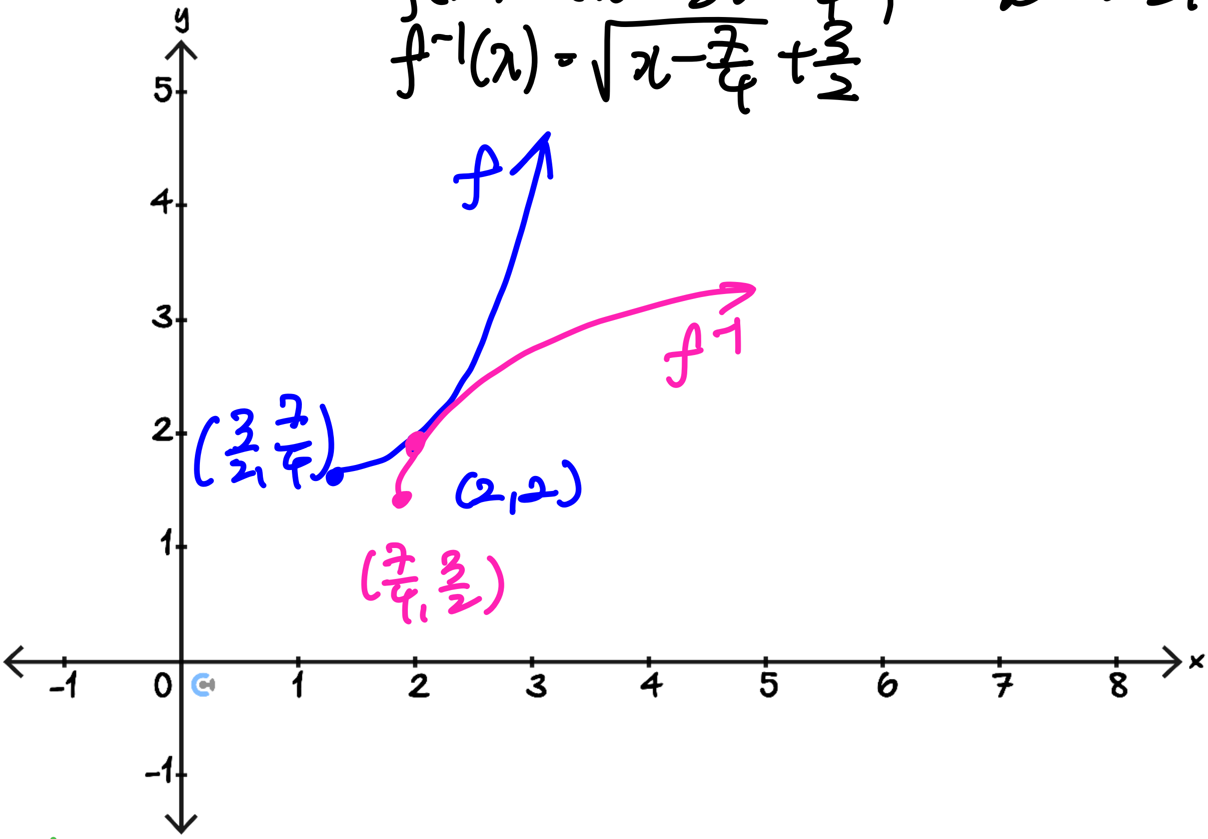
$f^{-1}(x) = \frac{\sqrt{4\left(x - \frac{7}{4}\right)}}{\sqrt{4}} + \frac{3}{2}$

$= \frac{1}{2}\sqrt{4x - 7} + \frac{3}{2}$

d. Sketch the graph of $y = f(x)$ and $y = f^{-1}(x)$ on the axes below. Label all endpoints and points of intersection with coordinates. (3 marks)

$f(x) = \left(x - \frac{3}{2}\right)^2 + \frac{7}{4}, \quad x \in \left[\frac{3}{2}, \infty\right)$

$f^{-1}(x) = \sqrt{x - \frac{7}{4}} + \frac{3}{2}$



$f(x) = x$

$x^2 - 3x + 4 = x$

$x^2 - 4x + 4 = 0$

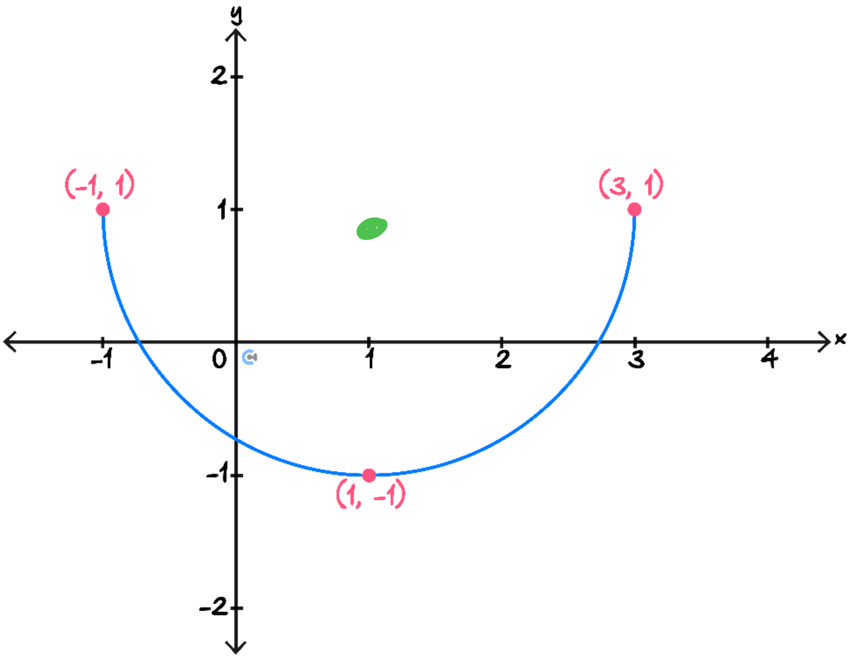
$(x-2)^2 = 0$

$x = 2$

Space for Personal Notes

Question 5 (9 marks)

Consider the function f that describes a semi-circle. The graph of f is shown below.



a. State the domain of f. (1 mark)

$\text{Dom} = [-1, 3]$

b. Find the rule for $f(x)$. (2 marks)

Centre $(1,1)$

$y = \pm\sqrt{r^2 - (x-h)^2} + k$

radius $= 2$

$y = -\sqrt{2^2 - (x-1)^2} + 1$

c. Hence, find all axes intercepts of the graph of $y = f(x)$. (3 marks)

↳ x-int ↳ y-int

$y = -\sqrt{4-(x-1)^2}+1$

y-int

$x=0$

$y=-\sqrt{4-(-1)^2}+1$

$=-\sqrt{3}+1$

$(0, 1-\sqrt{3})$

x-int

$0=-\sqrt{4-(x-1)^2}+1$

$\sqrt{4-(x-1)^2}=1$

$4-(x-1)^2=1$

$4-(x^2-2x+1)=1$

$-x^2+2x+2=0$

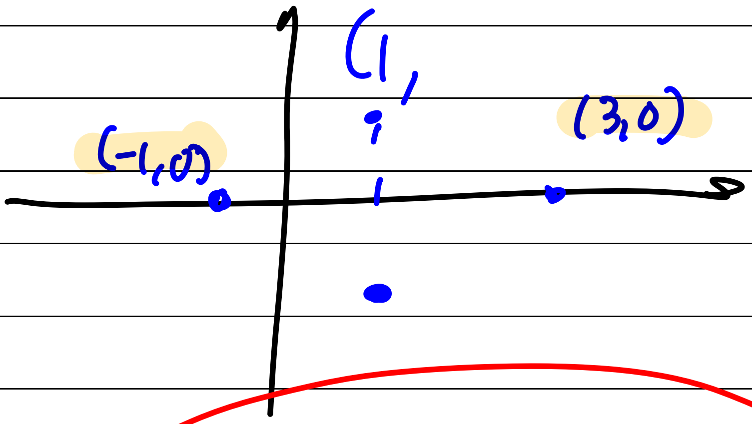
$x^2-2x-2=0$

$x=1\pm\sqrt{3}$

$(1+\sqrt{3},0)$ $(1-\sqrt{3},0)$

The function g describes a semi-circle of the same shape as f, whose minimum occurs when $x = 1$ and has x-axis intercepts at $(-1, 0)$ and $(3, 0)$.

d. Determine all possible rules for g in terms of the radius r of the semi-circle. (2 marks)



$y=-\sqrt{r^2-(x-1)^2}+k$

$0=-\sqrt{r^2-(-2)^2}+k$

$0=-\sqrt{r^2-4}+k$

$k=\sqrt{r^2-4}$ (1)

$0=-\sqrt{r^2-2^2}+k$

$k=\sqrt{r^2-4}$ (2)

$y=-\sqrt{r^2-(x-1)^2}+\sqrt{r^2-4}$

e. Hence find the rule for g that has the smallest possible radius. (1 mark)

$r = 2$

$g(x) = -\sqrt{4-(x-1)^2} + \sqrt{4-4}$

$g(x) = -\sqrt{4-(x-1)^2}$

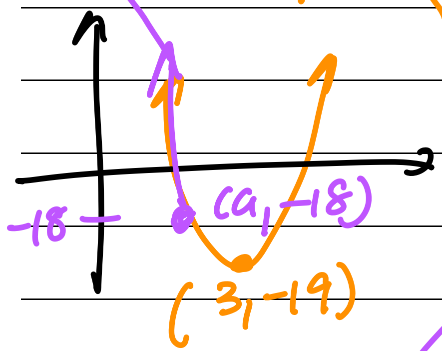
Question 6 (4 marks)

Let $f: [-2a, a] \to \mathbb{R}, f(x) = x^2 - 6x - 10$, where $a > 0$, be a function.

$(-2a, 30)$ -ve +ve y

a. Find the value of a if the range of f is $[-18, 30]$. (2 marks)

$(x-3)^2 - 19$



$f(a) = -18$

$f(-2a) = 30$

$-18 = a^2 - 6a - 10$

$0 = a^2 - 6a + 8$

$0 = (a-4)(a-2)$

$a = 4, 2$

$a = 2$

$(-2a)^2 - 6(-2a) - 10 = 30$

$4a^2 + 12a - 10 = 30$

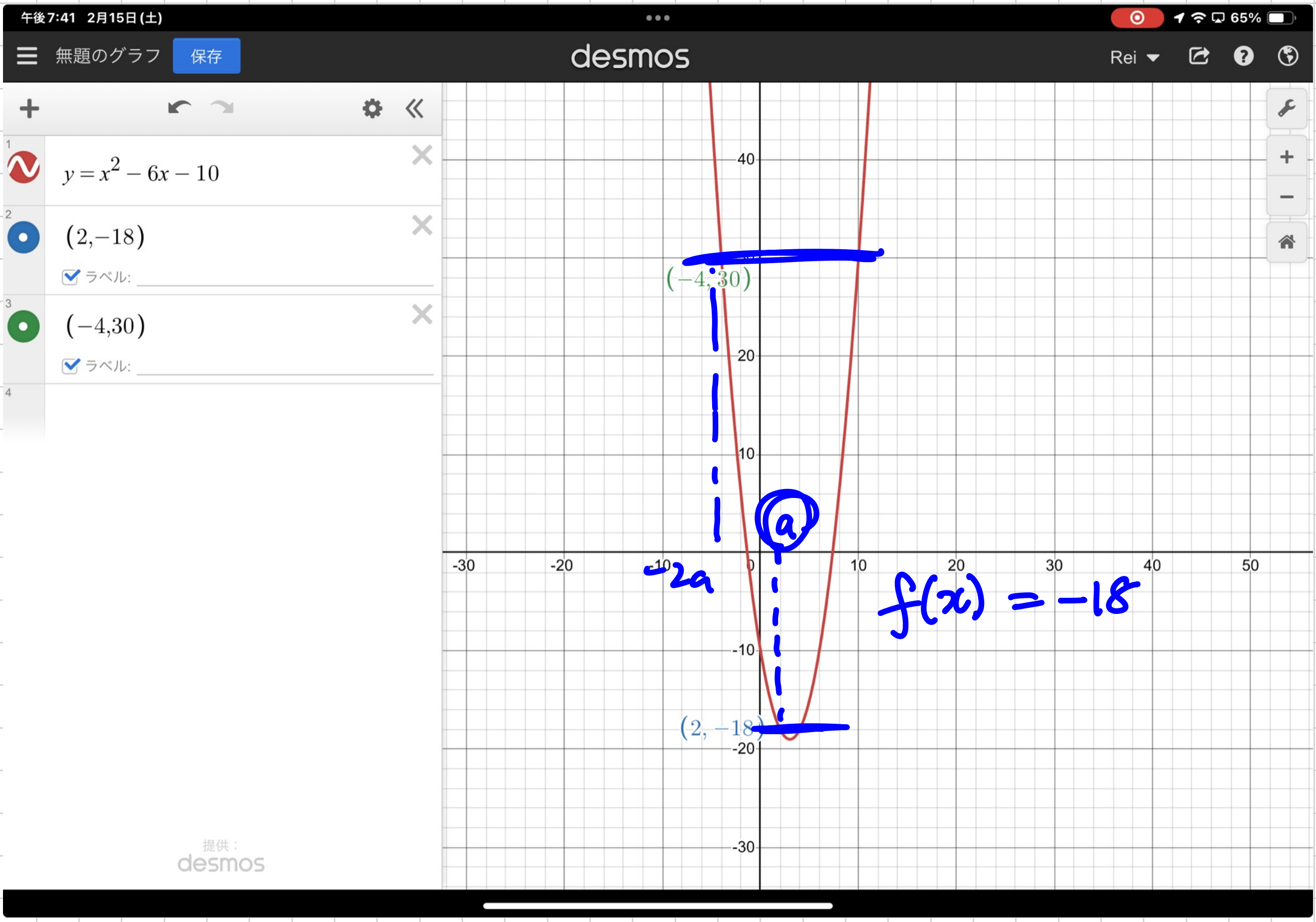
$4a^2 + 12a - 40 = 0$

$a^2 + 3a - 10 = 0$

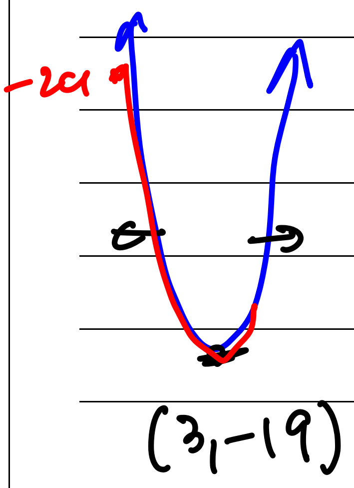
$(a+5)(a-2) = 0$

$a = -5, 2$

$a = 2$


午後7:41 2月15日(土)
65%
無題のグラフ
保存
desmos
Rei
$y = x^2 - 6x - 10$
(2,−18)
ラベル:
(−4,30)
ラベル:
提供：
desmos
(−4, 30)
(2, −18)
a
−2a
f(x) = −18

b. Determine the range of f in terms of a, for any value of a. (2 marks)



$f(-2a) = 4a^2+12a-10$

$f(a) = a^2-6a-10$

$a>3$ $\quad [-19, 4a^2+12a-10]$

$a<3$ $\quad [a^2-6a-10, 4a^2+12a-10]$

Space for Personal Notes

Section D: Tech Active Exam Skills

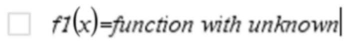
Calculator Commands: Using Sliders/Manipulate on CAS

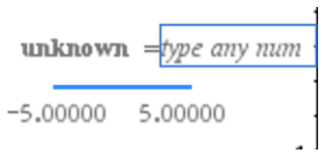
Mathematica

```
Manipulate[Plot[function, {x, xmin, xmax}],
    {unknown, lowerbound, upperbound}]
```

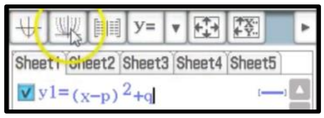
NOTE: The function **must** be typed out instead of using its saved name.

TI-Nspire





Casio Classpad



Calculator Commands: Finding the maximal domain

Mathematica

```
FunctionDomain[func, x]
```

TI-Nspire

Type up the domain (or find it under the book button.).

$$\text{domain}(\text{func},x)$$

Casio Classpad

Sketch the function and analyse.

Space for Personal Notes

Defining Hybrid Functions on CAS

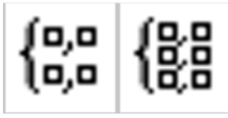
Mathematica

- Piecewise

Piecewise[{{val_1, $cond_1$}, {val_2, $cond_2$}, …}]

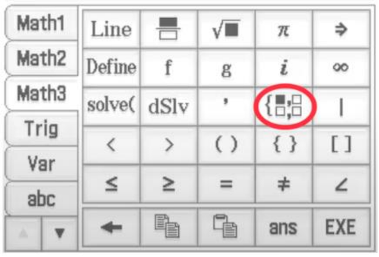
- Represents a piecewise function with values val_i in the regions defined by the conditions $cond_i$.

TI-Nspire



$$\begin{cases} func1, dom1 \\ func2, dom2 \end{cases}$$

Casio Classpad



Space for Personal Notes

Calculator Commands: Finding the equation of a polynomial that passes through points.

- Given n points we can find a degree $n-1$ polynomial that passes through all of these points.

- **Example:** Find the equation of the quadratic function that passes through the points $(0,6)$, $(2,2)$ and $(3,3)$.

- **TI:**

Define $f(x)=a\cdot x^2+b\cdot x+c$	*Done*
solve$(f(0)=6$ and $f(2)=2$ and $f(3)=3,a,b,c)$	$a=1$ and $b=-4$ and $c=6$
$f(x)\|a=1$ and $b=-4$ and $c=6$	$x^2-4\cdot x+6$

- **Casio:**

```
define f(x) = a*x^2 + b*x + c
                                        done
{f(0)=6
{f(2)=2
{f(3)=3 |a, b, c
                                        {a=1, b=-4, c=6}
f(x)|{a=1, b=-4, c=6}
                                        x^2-4·x+6
```

- **Mathematica:**

```
In[9]:=  f[x_] := a x^2 + b x + c
In[10]:= Solve[f[0] == 6 && f[2] == 2 && f[3] == 3]
Out[10]= {{a → 1, b → -4, c → 6}}
In[11]:= f[x] /. {a → 1, b → -4, c → 6}
Out[11]= 6 - 4 x + x^2
```

Space for Personal Notes

Calculator Commands: Turning Point

- ALWAYS sketch the graph to find approximate bounds for where the turning point you want is located.
- To find a local maximum, we maximise the function over a specific domain.
- To find a local minimum, we minimise the function over a specific domain.
- **TI and Casio:** Use $fmin(expression, variable, lower\ (optional), upper\ (optional))$ or $fmax(expression, variable, lower\ (optional), upper\ (optional))$.
- **TI:** Menu→ $4 \rightarrow \frac{7}{8}$.

Define $f(x)=x^3-4\cdot x$	*Done*
$\text{fMin}(f(x),x,0,2)$	$x=\dfrac{2\cdot\sqrt{3}}{3}$
$f\left(\dfrac{2\cdot\sqrt{3}}{3}\right)$	$\dfrac{-16\cdot\sqrt{3}}{9}$

- **Casio:** Action→Calculation→ $fmin/fmax$

$$\text{fmin}(x\hat{\ }3-4x, x, 0, 2)$$

$$\left\{\text{MinValue}=\frac{-16\cdot\sqrt{3}}{9}, x=\frac{2\cdot\sqrt{3}}{3}\right\}$$

.

- **Mathematica:** Minimise[ ] and Maximise[ ] commands.
- Minimise[$f[x], x$] will minimise $f[x]$ over its whole domain.
- To restrict the domain, we must use Minimise[$\{f[x], a \leq x \leq b\}, x$].

```
In[34]:= Minimize[{x^3 - 4 x, 0 < x < 2}, x]
```

Out[34]= $\left\{-\dfrac{16}{3\sqrt{3}}, \left\{x \rightarrow \dfrac{2}{\sqrt{3}}\right\}\right\}$

Section E: Exam 2 Questions (33 Marks)

INSTRUCTION: 33 Marks. 48 Minutes Writing.

Q7, 9, 11, 13

Question 7 (1 mark)

one-to-one

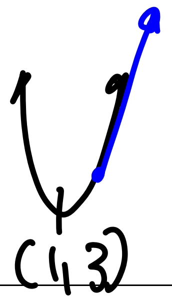
The function f defined by $f : A \to \mathbb{R}, f(x) = (x-1)^2 + 3$ will have an inverse function if its domain A is:

A. ~~$\mathbb{R}$~~

B. ~~$(-\infty, 3)$~~

C. $[3, 10]$

D. $[0, \infty)$



Question 8 (1 mark)

Which one of the following functions does **not** have an inverse function?

A. $f : \mathbb{R} \to \mathbb{R},\ f(x) = 2x - 5$

B. $g : (0, \infty) \to \mathbb{R},\ \ g(x) = x^2$

C. $h : \mathbb{R} \to \mathbb{R},\ h(x) = x^3$

D. $k : [-2,2] \to \mathbb{R},\ \ k(x) = \sqrt{4 - x^2}$

Question 9 (1 mark)

$y = 3 - x$

$(-3, 6)$

$6 = 3 - x$

The linear function $f : D \to \mathbb{R},\ f(x) = 3 - x$ has a range of $[-4, 6)$. The domain of f is:

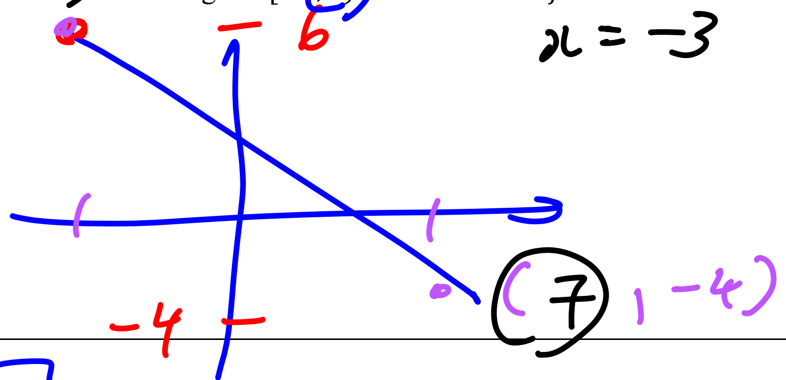
$x = -3$

A. $(-5, 1]$

B. $(-3, 7]$

C. $(-2, 7)$

D. $[-3, 7]$



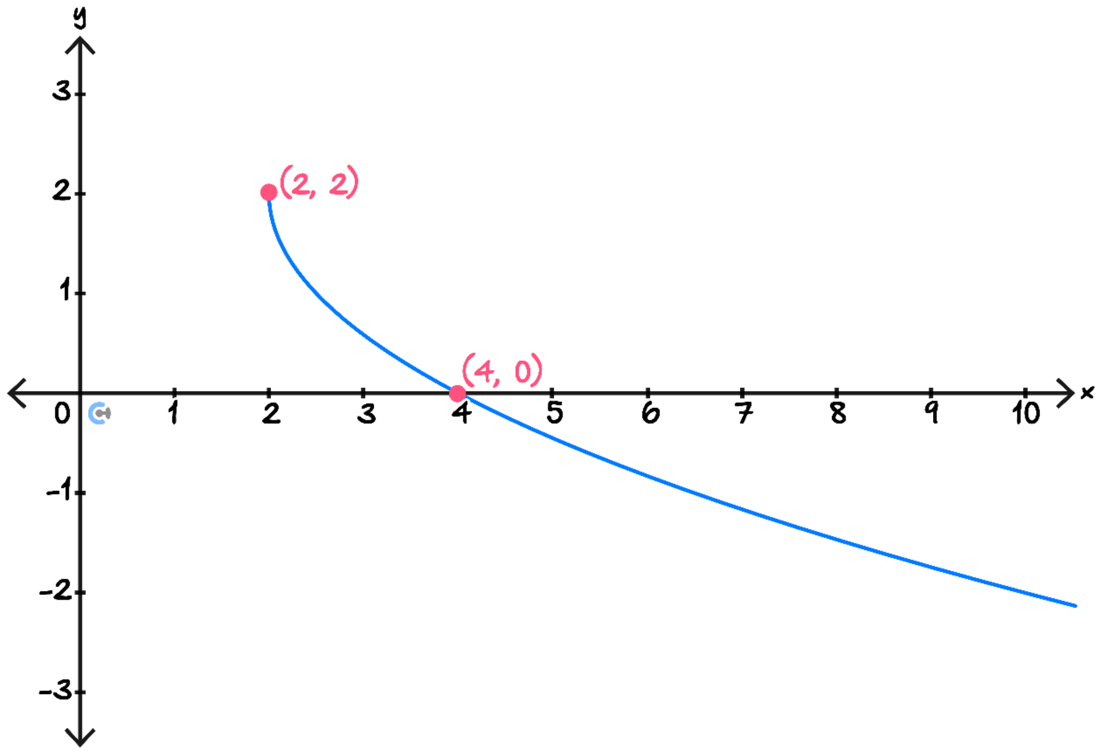
$[-3, 7]$

$-4 = 3 - x$

$x = 7$

Question 10 (1 mark)

The rule for the function shown in the graph below could be:



A. $y = \sqrt{2x - 4} + 2$

B. $y = -\sqrt{2x - 4} + 2$

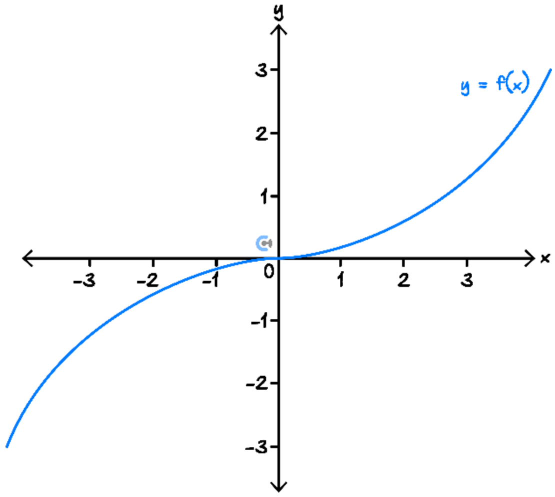
C. $y = \sqrt{x - 2} + 2$

D. $y = -\sqrt{x - 2} + 2$

Space for Personal Notes

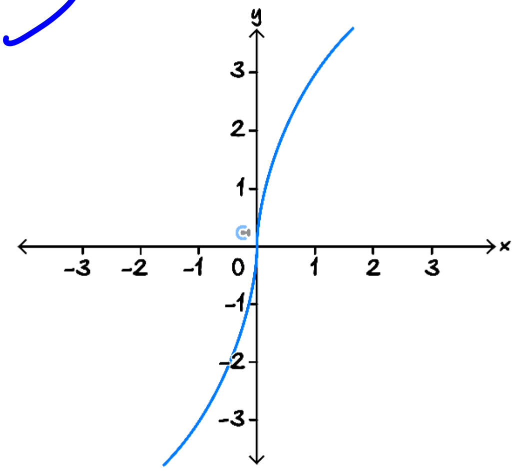
Question 11 (1 mark)

The graph of the function with equation $y = f(x)$ is shown below.

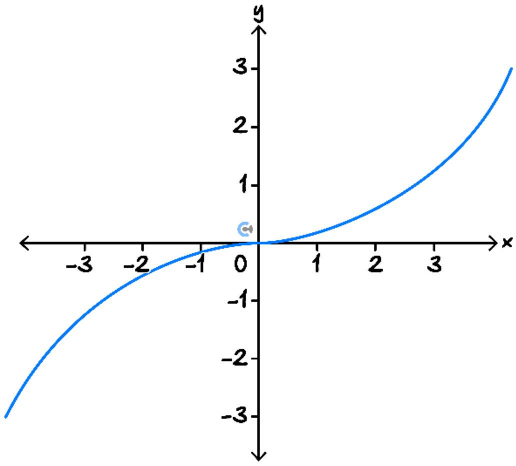


Which one of the following is most likely to be the graph of the inverse function?

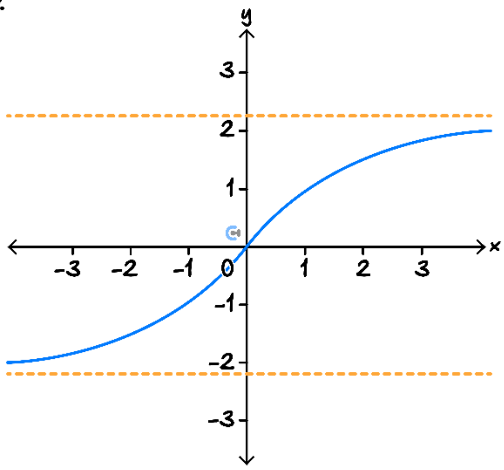
A.



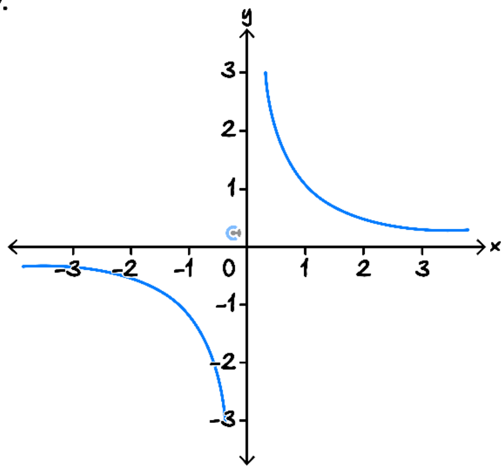
B.



C.



D.



Question 12 (1 mark)

Consider a one-to-one function f which satisfies $f(2) = 4$ and $f(3) = 2$. The equation of a line that intersects f^{-1} at the points $x = 2$ and $x = 4$ is:

A. $y = 6 - 2x$

B. $y = 4 - \frac{1}{2}x$

C. $y = 8 - 2x$

D. $y = 6 - \frac{1}{2}x$

Question 13 (13 marks)

The temperature of a cooling object follows a hyperbolic model given by T.

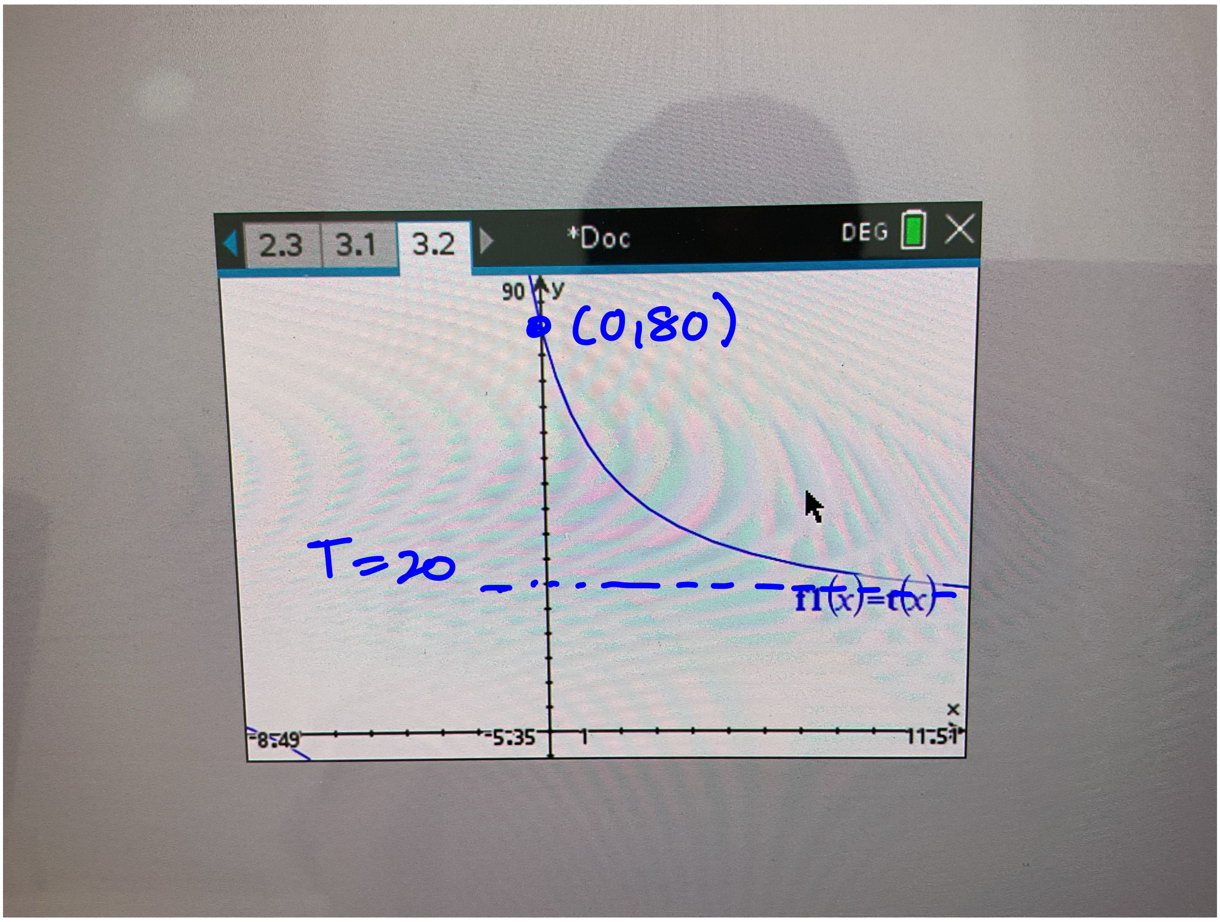
$$T(x) = \frac{120}{x+2} + 20$$

Where $T(x)$ represents the temperature (in degrees Celsius) of the object x minutes after it was removed from an oven.

a. What is the implied domain of the function T (i.e. what values make sense)? (1 mark)

$x \geq 0$

b. Sketch the graph of $T(x)$ over its implied domain, on the axes below. Label any endpoints with coordinates and asymptotes with equations. (2 marks)



c. Find the temperature of the object after $x = 5$ minutes. (1 mark)

$T(5) = \frac{260}{7}\ °C$

d. Determine the time x when the temperature of the object is 50°C. (2 marks)

Solve$(50 = T(x))$

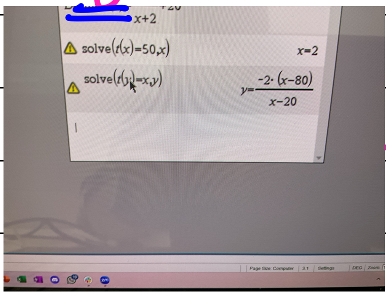
$x = 2$ mins

e. Find the rule and domain of the inverse function $T^{-1}(x)$. (2 marks)

$T(x) = y$

solve$(T(y) = x, y)$

$y =$



$T^{-1}(x) = \dfrac{-2(x-80)}{x-20}$

dom f^{-1} = ran f

$= (20, 80]$

f. Describe the information that $T^{-1}(30)$ gives us in relation to this scenario. (1 mark)

"y"

T

time when temperature is 30°C

The temperature of the oven is changed so that now when the object is taken out of the oven its temperature is given by $T_1(x) = T(x) + a$, where $a > 0$.

up

g. Determine the range of $T_1(x)$ on its implied domain. (2 marks)

$(20 + a, 80 + a]$

h. Determine the value of a if the object's temperature 28 minutes after it is taken out of the oven is 39 degrees Celsius. (2 marks)

$x =$

T

solve $(T(28)+a = 39, a)$

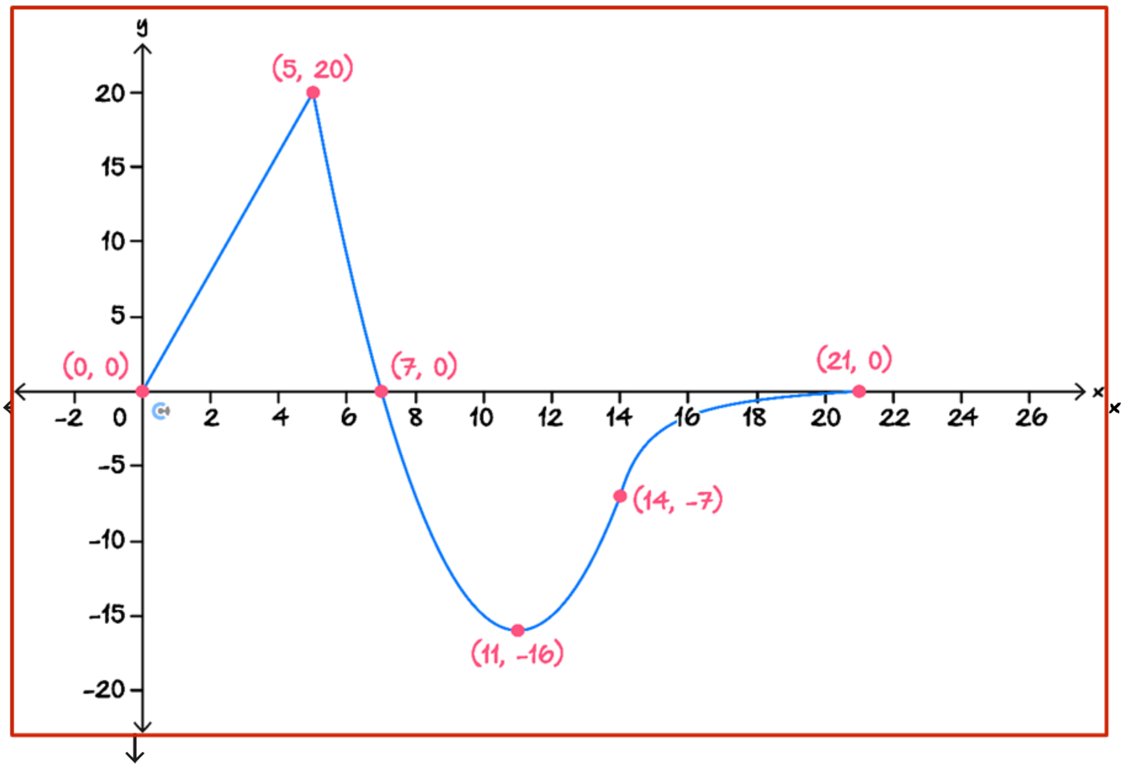
$a = 15$

Question 14 (14 marks)

Contour Park constructs a roller that is made up of three different sections of track. Let h be the function that determines the height of the roller coaster above the ground, according to its horizontal position x. h is modelled by the rule:

$$\begin{cases} 4x & 0 \le x \le 5 \\ x^2 - 22x + 105 & 5 < x \le 14 \\ -\dfrac{8}{x-13} + 1 & 14 < x \le 22 \end{cases}$$

a. Sketch the graph of $h(x)$ on the axes below. Label all endpoints, intercepts and turning points with coordinates. (4 marks)



b. State the maximum height of the roller coaster above the ground. (1 mark)

20 metres.

c.

i. Find the values of x for which the roller coaster is 15 metres below the ground. (2 marks)

Solution: See that this occurs during the quadratic section.
Solve x^2-22x+105=-15 --> x=10,12

ii. Find the horizontal distance(s) when the height is 15 metres **above** the ground. Give your answers correct to two decimal places. (2 marks)

Solution: See that this occurs during a linear section and quadratic section.
Solve 4x = 15--> x = 3.75 and solve x^2- 22x -105=15--> x= 5.43 (only solution in domain).
So horizontal distances are 3.75 and 5.43 for a height of 15 metres above the ground.

d. Find the values of x for which the roller coaster is below the ground. Express your answer using interval notation. (2 marks)

this is when h<0 so xe(7,22)

The roller coaster is a huge success, however a complaint is that the ride is too quick. To rectify this issue it is decided that instead of the roller coaster track ending at $x = 21$, a new track with the exact same shape as $h(x)$ will be constructed from this point.

e. Define the function $h_1(x)$ which describes the linear section of the new track. (2 marks)

Solution: Has a gradient of 4 and starts from the point $(21, 0)$ and goes for 5 units to the right. Therefore

$$h_1(x) = 4(x - 21) = 4x - 84 \text{ for } x \in [21, 26]$$

f. At what x value will the roller coaster be 16 metres below the ground for a second time.. (1 mark)

x=21+11=32

Space for Personal Notes

Section F: Extension Exam 1 (9 Marks)

INSTRUCTION: 9 Marks. 13 Minutes Writing.

Question 15 (9 marks)

Consider the function $f(x) = \frac{1}{x-4}$.

a. Find the values of x for which $f^{-1}(x) > f(x)$. (4 marks)

Solution: $x = \frac{1}{y-4} \implies y = \frac{1}{x} + 4$. Solve,

$$\begin{aligned} \frac{1}{x-4} &= x \\ x^2 - 4x - 1 &= 0 \\ (x-2)^2 &= 5 \\ x &= 2 \pm \sqrt{5} \end{aligned}$$

Consider the shapes of the two graphs to conclude that $f^{-1}(x) > f(x)$ for

$$x \in (-\infty, 2 - \sqrt{5}) \cup (0, 4) \cup (2 + \sqrt{5}, \infty)$$

Now let $g : (-\infty, k) \to \mathbb{R}, g(x) = \frac{1}{k-x}$, where k is a real constant.

b. Find the rule and domain for the inverse function, g^{-1}, in terms of k. (2 marks)

$$g^{-1} : (0, \infty) \to \mathbb{R},\ g^{-1}(x) = k - \frac{1}{x}$$

c. Find the exact value of k so that g and g^{-1} have one point of intersection. (3 marks)

Solution: Must intersect on the line $y = x$. Solve

$$\begin{aligned} \frac{1}{k-x} &= x \\ 1 &= kx - x^2 \\ x^2 - kx + 1 &= 0 \\ x &= \frac{k \pm \sqrt{k^2-4}}{2} \end{aligned}$$

So one solution if $k^2 - 4 = 0 \implies k = \pm 2$.
Then checking domain of g and g^{-1} only $k = 2$ is valid.

Space for Personal Notes

Section G: Extension Exam 2 (15 Marks)

INSTRUCTION: 15 Marks. 22 Minutes Writing.

Question 16 (1 mark)

The range of the function given by $f : (0, 4] \to \mathbb{R}, f(x) = x^2 - 2x + b$ is:

A. $(b - 1, b + 8)$

B. $[b - 1, b + 8]$

C. (,8]

D. $(b - 1, b + 8]$

Question 17 (1 mark)

The functions $f(x) = \log_2(a - x)$ and $g(x) = -\sqrt{x + a}$ are defined on their maximal domains and $a \in \mathbb{R}^+$. The domain of $f(x) \times g(x)$ is:

A. $[-a, a)$

B. $[-a, a]$

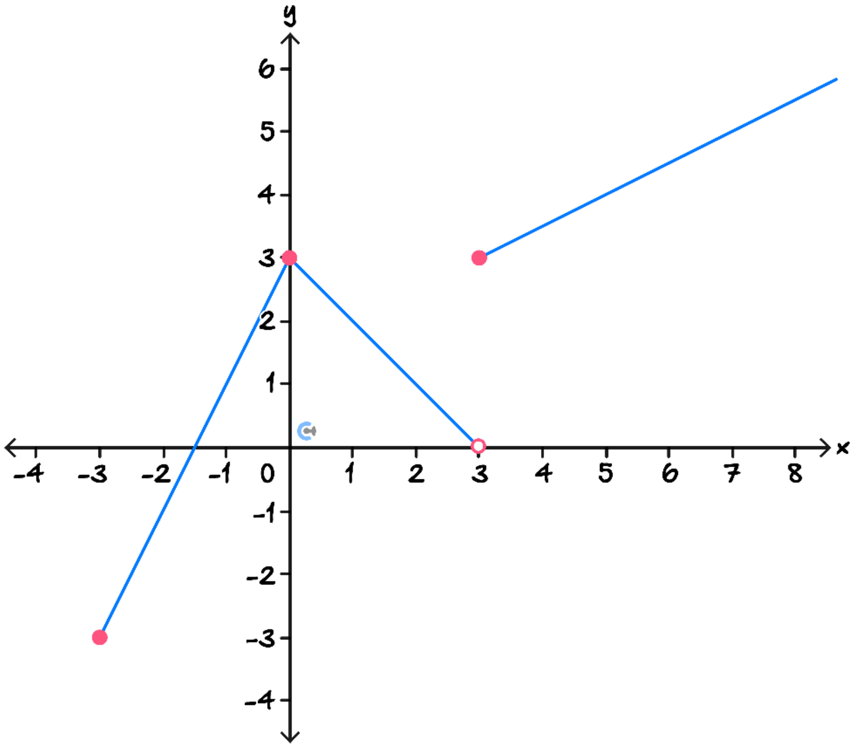
C. $(-a, a)$

D. $\mathbb{R}\backslash\{a\}$

Space for Personal Notes

Question 18 (1 mark)

The graph of the function f is shown below.



In order for the inverse f^{-1} to exist, a possible restricted domain of f is:

A. $x \in [-3,0] \cup [3,0]$

B. $x \in [-1,2)$

C. $x \in [0,3]$

D. $x \in [-3,0) \cup [3,0]$

Question 19 (1 mark)

The equation $12x^5 + 15x^4 - 60x^3 - 30x^2 + 120x = k$ has one real solution for:

A. $k \in (-87,57)$

B. $k \in (-\infty, -87) \cup (-24, \infty)$

C. $k \in (-87, -24)$

D. $k \in (-\infty, 57)$

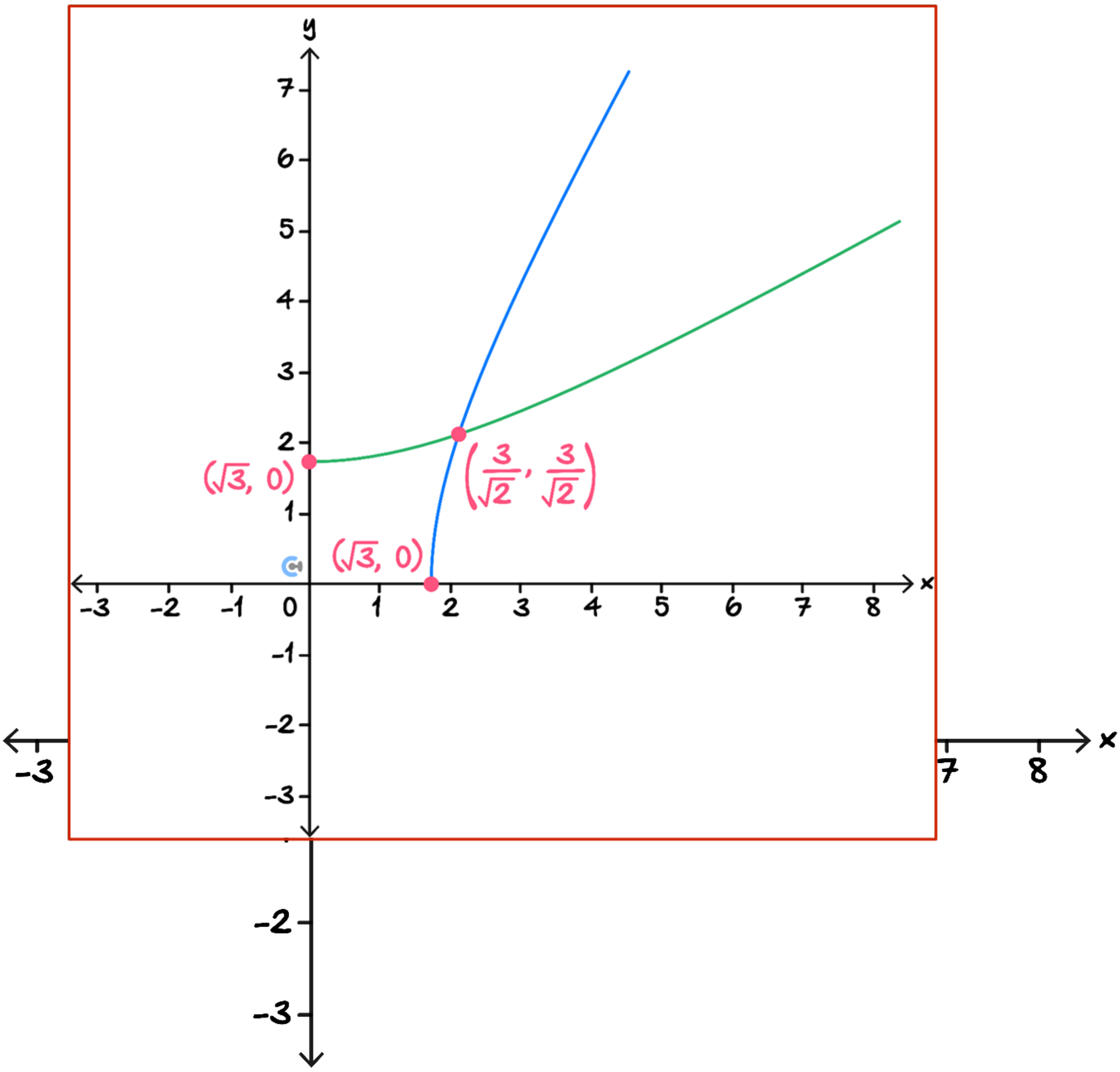
Question 20 (11 marks)

Consider the function $f\colon \left[\sqrt{3}, \infty\right) \to \mathbb{R}, f(x) = \sqrt{3x^2 - 9}$.

a. Define f^{-1}, the inverse function of f. (2 marks)

$f^{-1} : [0, \infty) \to \mathbb{R},\ f^{-1}(x) = \sqrt{\dfrac{x^2+9}{3}}$

b. Sketch the graphs of $y = f(x), y = f^{-1}(x)$ on the axes below. Label all axes intercepts and points of intersection with coordinates. (3 marks)



Now consider the one-to-one function, defined on its maximal domain, $g : [a, \infty) \to \mathbb{R}$, where $g(x) = \sqrt{kx^2 - 9}$ and $a, k \in \mathbb{R}^+$.

c.

i. Find the value of a in terms of k. (1 mark)

Solution: $kx^2 - 9 \geq 0 \implies x \geq \frac{3}{\sqrt{k}}$.

So $a = \frac{3}{\sqrt{k}}$

ii. Find the value of k such that g and g^{-1} intersect at $(2, 2)$. (2 marks)

Solution: We require $g(2) = 2 \implies \sqrt{4k - 9} = 2 \implies k = \frac{13}{4}$

iii. Find the value(s) of k for which g and g^{-1} do not intersect each other. (2 marks)

Solution: Intersection must occur on the line $y = x$. Solve $g(x) = x \implies x = \frac{3}{\sqrt{k-1}}$.

Then this solution is only valid for $k > 1$.

Therefore intersection for $0 < k < 1$.

d. As x gets larger and larger (i.e. as $x \to \infty$)), the function $g(x)$ approaches, but never touches a linear function of the form $y = mx$.

State the value of m in terms of k. (1 mark)

Solution: $g(x) \to \sqrt{kx^2} = \sqrt{k}x$ as $x \to \infty$.
$m = \sqrt{k}$

Space for Personal Notes

CONTOUREDUCATION

Website: contoureducation.com.au | **Phone:** 1800 888 300 | **Email:** hello@contoureducation.com.au

VCE Mathematical Methods ½

Free 1-on-1 Consults

What Are 1-on-1 Consults?

- **Who Runs Them?** Experienced Contour tutors (45 + raw scores and 99 + ATARs).
- **Who Can Join?** Fully enrolled Contour students.
- **When Are They?** 30-minute 1-on-1 help sessions, after-school weekdays, and all-day weekends.
- **What To Do?** Join on time, ask questions, re-learn concepts, or extend yourself!
- **Price?** Completely free!
- **One Active Booking Per Subject:** Must attend your current consultation before scheduling the next. :)

SAVE THE LINK, AND MAKE THE MOST OF THIS (FREE) SERVICE!

Booking Link

bit.ly/contour-methods-consult-2025

Website: contoureducation.com.au | **Phone:** 1800 888 300 | **Email:** hello@contoureducation.com.au

VCE Mathematical Methods ½

Free 1-on-1 Consults

What Are 1-on-1 Consults?

- **Who Runs Them?** Experienced Contour tutors ($45+$ raw scores and $99+$ ATARs).
- **Who Can Join?** Fully enrolled Contour students.
- **When Are They?** 30-minute 1-on-1 help sessions, after-school weekdays, and all-day weekends.
- **What To Do?** Join on time, ask questions, re-learn concepts, or extend yourself!
- **Price?** Completely free!
- **One Active Booking Per Subject:** Must attend your current consultation before scheduling the next. :)

SAVE THE LINK, AND MAKE THE MOST OF THIS (FREE) SERVICE!

Booking Link

bit.ly/contour-methods-consult-2025

CONTOUREDUCATION

Website: contoureducation.com.au | **Phone:** 1800 888 300 | **Email:** hello@contoureducation.com.au

VCE Mathematical Methods ½

Free 1-on-1 Consults

What Are 1-on-1 Consults?

- **Who Runs Them?** Experienced Contour tutors ($45 +$ raw scores and $99 +$ ATARs).
- **Who Can Join?** Fully enrolled Contour students.
- **When Are They?** 30-minute 1-on-1 help sessions, after-school weekdays, and all-day weekends.
- **What To Do?** Join on time, ask questions, re-learn concepts, or extend yourself!
- **Price?** Completely free!
- **One Active Booking Per Subject:** Must attend your current consultation before scheduling the next. :)

SAVE THE LINK, AND MAKE THE MOST OF THIS (FREE) SERVICE!

Booking Link

bit.ly/contour-methods-consult-2025