Website: contoureducation.com.au | Phone: 1800 888 300
Email: hello@contoureducation.com.au

VCE Mathematical Methods ½
Functions & Relations II [0.8]
Workshop

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Section A: Recap

Set Operators

- Intersection: "AND".

$$A \cap B = \textit{What values are in set } A \textit{ AND in set } B.$$

- Union: "OR".

$$A \cup B = \textit{What values are in set } A \textit{ OR in set } B.$$

- Set difference: "Except".

$$A \setminus B = \textit{What values are in set } A \textit{ except those also in set } B.$$

Interval Notation

- Parentheses (non-inclusive):

$$x \in (a, b) \Rightarrow a < x < b$$

- Square brackets [inclusive]:

$$x \in [a, b] \Rightarrow a \leq x \leq b$$

Space for Personal Notes

Maximal Domain

- The maximal domain is the biggest possible domain for a rule without committing a mathematical crime.
- In Methods, we need to consider 3 important rules:

$$\sqrt{z}, \qquad z \geq 0$$

$$\log(z), \qquad z > 0$$

$$\frac{1}{z}, \qquad z \neq 0$$

Range

- The range is the possible value for the output of a function.

Functional Notation

$$f: Domain \rightarrow Codomain, f(x) = Rule$$

- Codomain is simply all the values the function works within.
- Codomain is **not** the same as range.

Piecewise (Hybrid) Functions

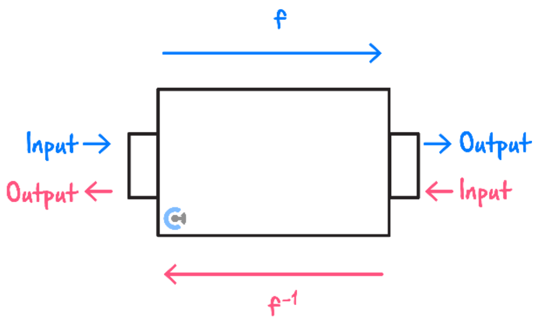
- Series of functions.

$$h(x) = \begin{cases} f(x), & Domain_1 \\ g(x), & Domain_2 \end{cases}$$

- $Domain_1$ and $Domain_2$ represent the x-values for which the two functions are defined.
- The two domains do not have to join!

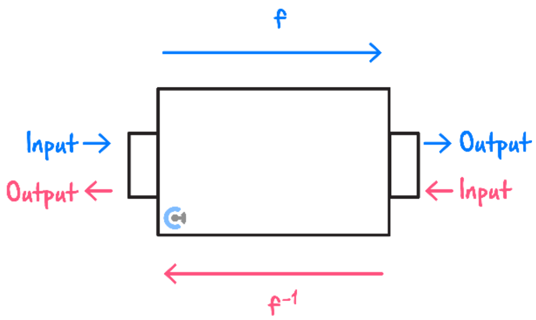
Inverse Relation

- **Definition**: Inverse is a relation that does the opposite.

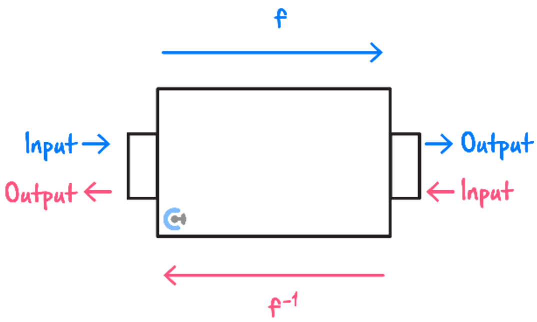


Solving for an Inverse Relation

- Swap x and y.



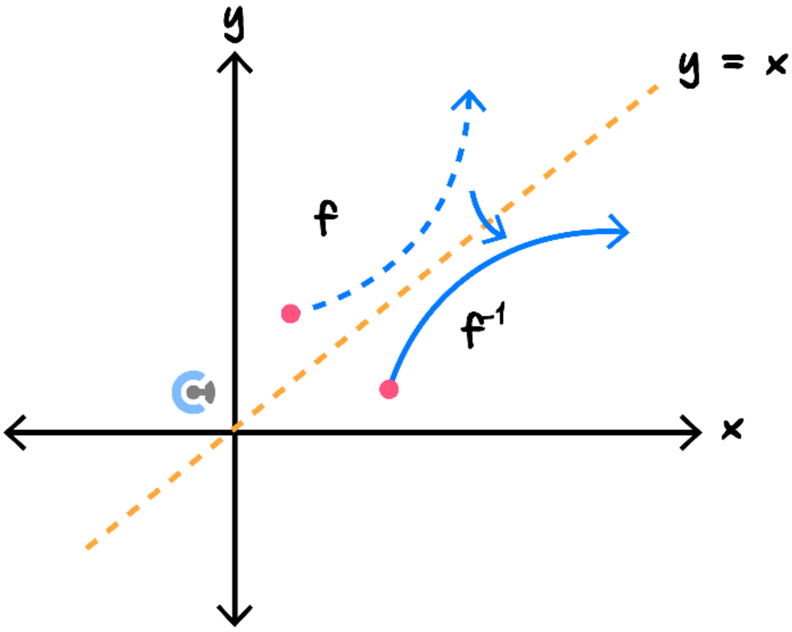
Domain and Range of Inverse Functions



$$\textbf{Dom } f^{-1} = \textbf{Ran } f$$

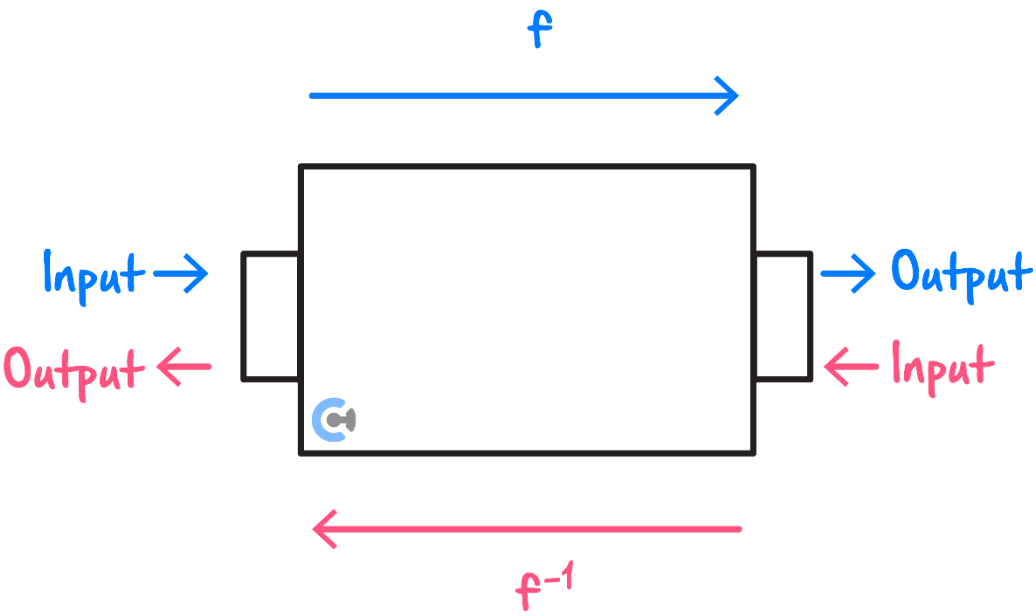
$$\textbf{Ran } f^{-1} = \textbf{Dom } f$$

Symmetry of Inverse Functions



- Inverse functions are always symmetrical around $y = x$.

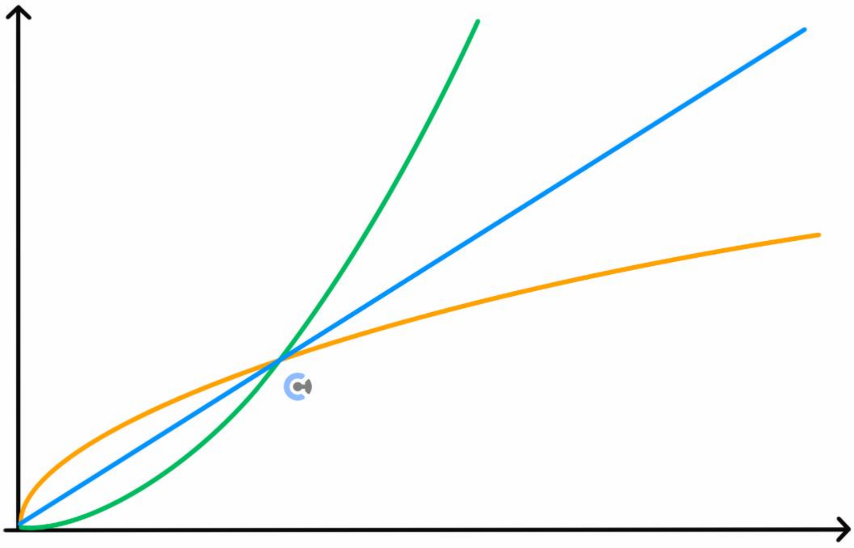
Validity of Inverse Functions



- Requirement for Inverse Function:

$$f \text{ needs to be } 1:1.$$

Intersection between a Function and its Inverse



$$f(x) = x \quad \text{OR} \quad f^{-1}(x) = x$$

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Section B: Warm Up

Question 1

For the sets $A = [-3,5]$ and $B = (-6,2]$, expressing the following in interval notation:

a. $A \cap B$.

$\boxed{[-3, 2]}$

b. $A \cup B$.

$\boxed{(-6, 5]}$

c. $A \backslash B$.

$\boxed{(2, 5]}$

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Question 2

a. Find the maximal domain of $f(x) = \sqrt{(x-3)(x+1)}$ expressing your answer in interval notation.

$\boxed{x \in (-\infty, 1] \cup [3, \infty)}$

b. Find the range of the function $f: [-2, 3) \to \mathbb{R}, f(x) = x^2 - 4$.

$\boxed{[-4, 5)}$

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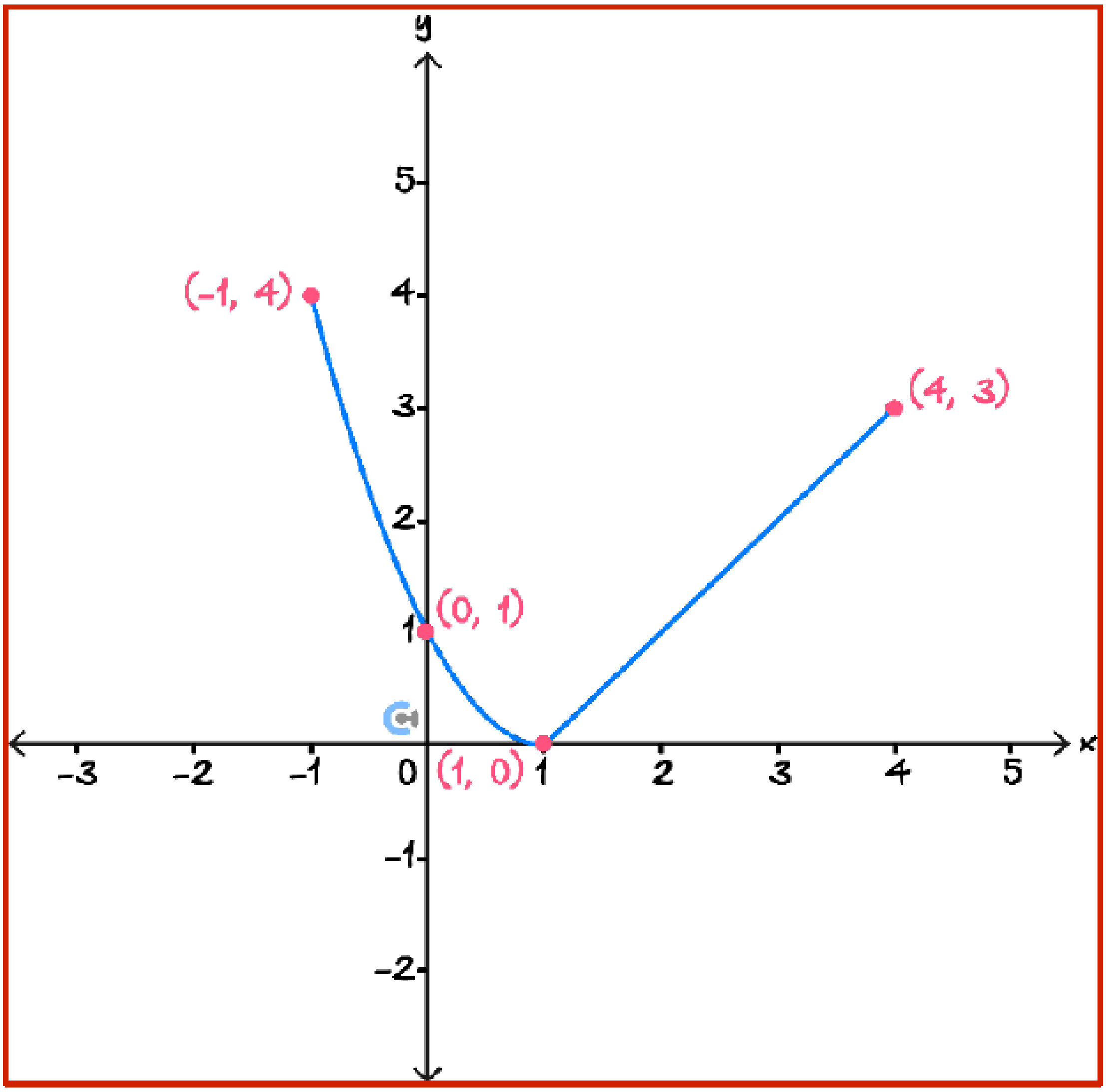
Question 3

Consider the hybrid function f, where:

$$f(x) = \begin{cases} (x-1)^2, & -1 \leq x \leq 1 \\ x-1, & 1 < x < 4 \end{cases}$$

a. Sketch the graph of $y = f(x)$ on the axes below. Label endpoints and axes intercept with coordinates.



b. Hence, state the range of f.

$\boxed{[0, 4)}$

c. State how many solutions $f(x) = 3$ has.

$\boxed{2.}$

Question 4

Consider the function $f: [-2, 4] \to \mathbb{R}, f(x) = 2x - 2$.

a. Find the rule for the inverse function, f^{-1}.

$$f^{-1}(x) = \frac{x+2}{2}$$

b. State the domain and range of f^{-1}.

Solution: dom f^{-1} = ran $f = [-6, 6]$
ran f^{-1} = dom $f = [-2, 4]$

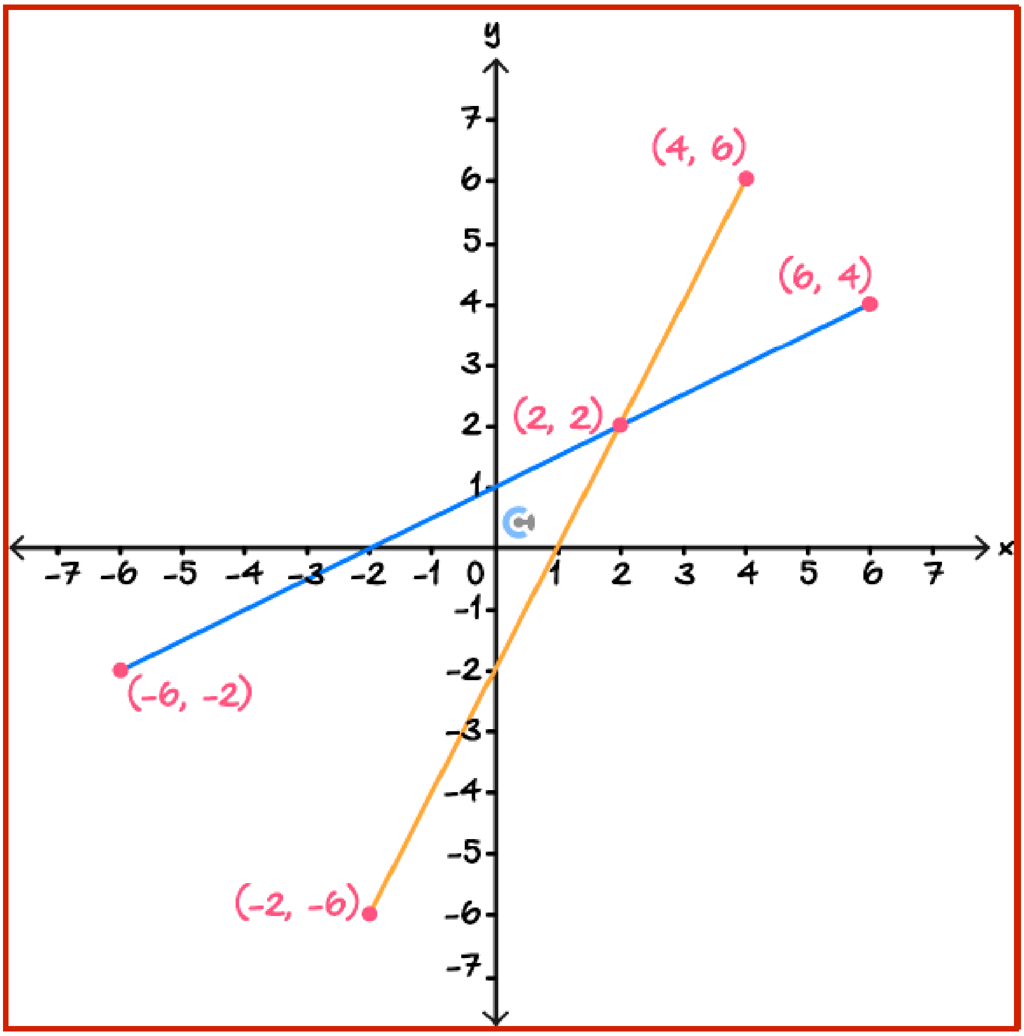
c. Hence, fully define the inverse function using functional notation.

$$f^{-1} : [-6, 6] \to \mathbb{R}, f^{-1}(x) = \frac{1}{2}x + 1$$

d. Find the point of intersection between f and f^{-1}.

Solve $2x - 2 = x \implies x = 2$. Point of intersection is $(2, 2)$

e. Sketch the graphs of f and f^{-1} on the axes below.



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Section C: Exam 1 Questions (21 Marks)

INSTRUCTION: 21 Marks. 27 Minutes Writing.

Question 5 (2 marks)

Consider the function $f: (-\infty, a] \cup [b, \infty) \to R, f(x) = \sqrt{x^2 - 3}$.

≥ 0

Find all possible values of a and b such that $f(x)$ is defined.

$\therefore x^2 - 3 \geq 0$

$x^2 \geq 3 \Rightarrow \therefore x \geq \sqrt{3}$ or $x \leq -\sqrt{3}$

$(-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty)$

$(-\infty, a] \cup [b, \infty)$

$\therefore a \leq -\sqrt{3}$ and $b \geq \sqrt{3}$

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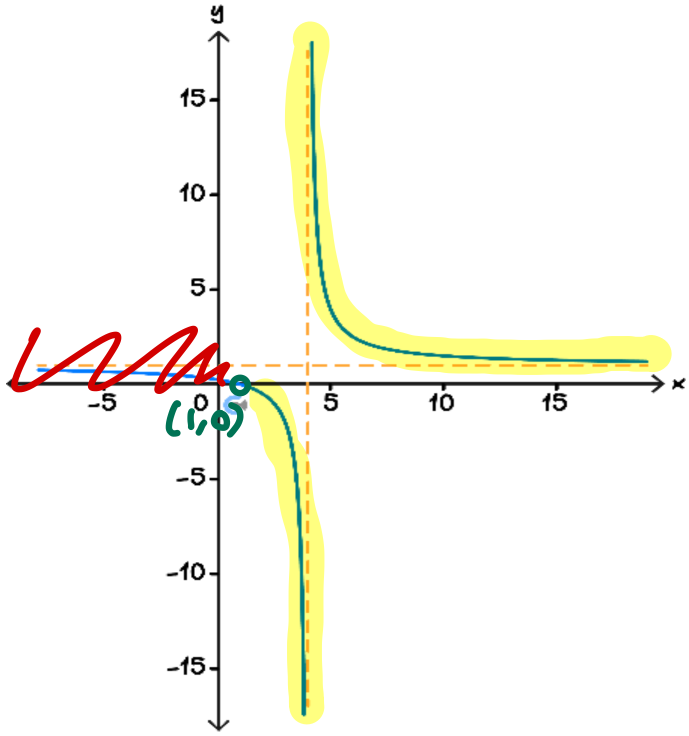
Question 6 (4 marks)

The figure below shows a graph of $y = \frac{3}{x-4} + 1, x \neq 4$.



a. State the equations for the horizontal and vertical asymptotes of the curve, marked as dotted lines in the figure. (1 mark)

$\therefore x=4 \ \& \ y=1$

The function f is defined as $f: (1, \infty)\backslash\{4\} \to \mathbb{R}, f(x) = \frac{3}{x-4} + 1$.

$f(1) = \frac{3}{-3} + 1 = 0$

b. State the range of $f(x)$. (1 mark)

$f(x) \in (-\infty, 0) \cup (1, \infty)$ //

$\in \mathbb{R} \backslash [0,1]$ //

c. Obtain an expression for $f^{-1}(x)$. (1 mark)

$x = \frac{3}{y-4} + 1$

$x - 1 = \frac{3}{y-4}$

$y - 4 = \frac{3}{x-1}$

$y = \frac{3}{x-1} + 4$

$\therefore f^{-1}(x) = \frac{3}{x-1} + 4$

d. State the domain and range of $f^{-1}(x)$. (1 mark)

Dom f^{-1} = Ran f = $\mathbb{R} \setminus [0,1]$ //

Ran f^{-1} = Dom f = $(1, \infty) \setminus \{4\}$ //

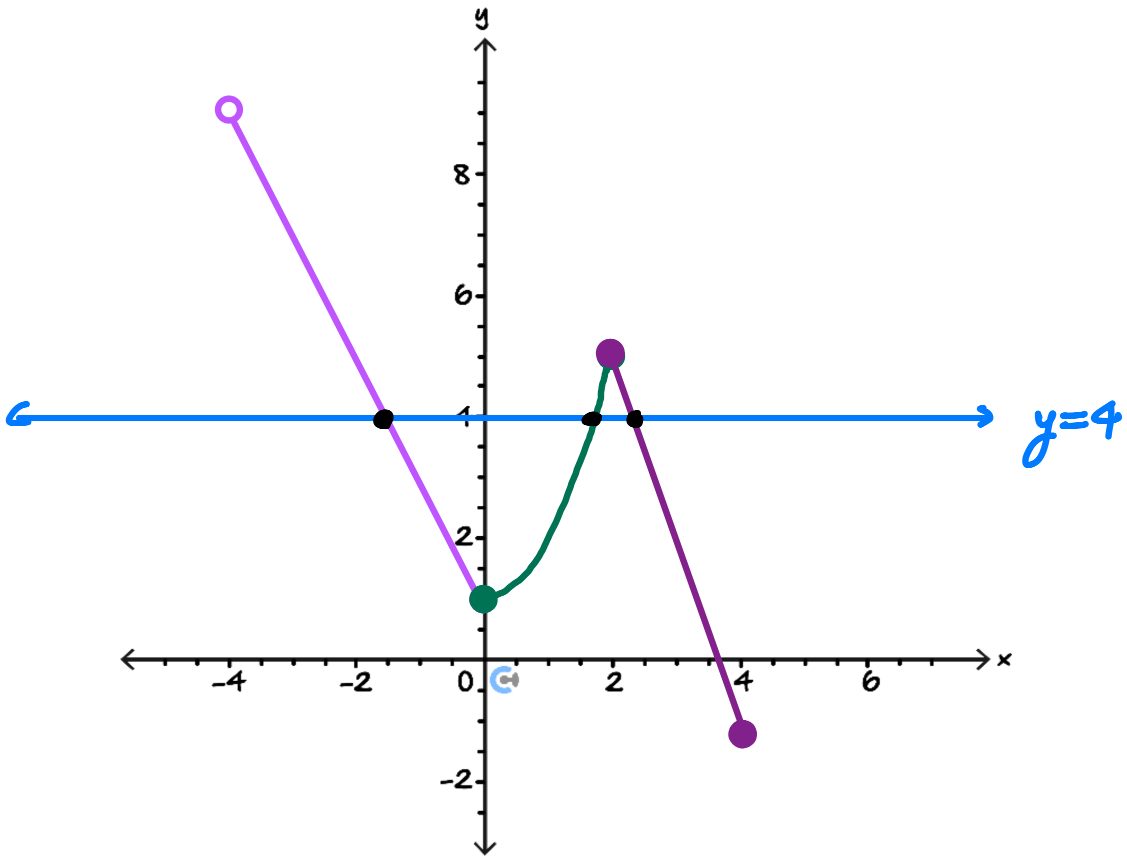
Question 7 (8 marks)

A function f has the definition:

$f(-4) = 9$

$$f(x) = \begin{cases} -2x + 1, & -4 < x < 0 \\ x^2 + 1, & 0 \leq x < 2 \\ 11 - 3x, & 2 \leq x \leq 4 \end{cases}$$

a. Draw the graph of $y = f(x)$ on the axes below. (3 marks)



b. Explain why f does not have an inverse function. (1 mark)

Because f isn't 1:1 ! //

c. How many solutions are there to $f(x) = 0$? (1 mark)

intersections $y=f(x)$ & $y=0$ $\Rightarrow$ $\therefore$ 1 solution //

d. State the range of $f(x)$. (1 mark)

Range $\in [-1, 9)$

e. Solve the equation $f(x) = 4$ for x. (2 marks)

$y=f(x)$ & $y=4$

$-2x+1=4 \Rightarrow 2x=-3 \Rightarrow x=\frac{-3}{2}$

$x^2+1=4 \Rightarrow x^2=3 \Rightarrow x=\pm\sqrt{3} \Rightarrow +\sqrt{3}$ as $x\in(0,2)$

$11-3x=4 \Rightarrow 3x=7 \Rightarrow x=\frac{7}{3}$

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Question 8 (4 marks)

$[0, \infty) + 4 \Rightarrow [4, \infty)$

Consider the function $f(x) = \sqrt{3x-7} + 4$, where f is defined over its maximal domain.

≥ 0

a. Find the domain and the rule for the inverse function f^{-1}. (2 marks)

Let $y = f(x)$:

Swap x & y:

$x = \sqrt{3y-7} + 4$

$x - 4 = \sqrt{3y-7}$

$3y - 7 = (x-4)^2$

$3y = (x-4)^2 + 7$

$y = \frac{1}{3}(x-4)^2 + \frac{7}{3}$

$\therefore f^{-1}(x) = \frac{1}{3}(x-4)^2 + \frac{7}{3}$

$\text{Dom } f^{-1} = \text{Ran } f = [4, \infty)$

b. Find an intersection between $f(x)$ and $f^{-1}(x)$. (2 marks)

Let $f(x) = x$:

$x = \sqrt{3x-7} + 4$

$x - 4 = \sqrt{3x-7}$

$x^2 - 8x + 16 = 3x - 7$

$x^2 - 11x + 23 = 0$

$\therefore x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(1)(23)}}{2(1)}$

$x = \frac{11 \pm \sqrt{121 - 92}}{2}$

$x = \frac{11 \pm \sqrt{29}}{2} = \frac{11 + \sqrt{29}}{2}$

reject $-$ve soln as $x \in [4, \infty)$

$\therefore$ IP: $\left(\frac{11+\sqrt{29}}{2}, \frac{11+\sqrt{29}}{2}\right)$

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$\text{Dom } f = [\frac{7}{3}, \infty)$

$3x - 7 \geq 0 \Rightarrow 3x \geq 7 \Rightarrow x \geq \frac{7}{3}$

$\text{Dom } f^{-1} = [4, \infty)$

$\text{Dom } f \cap \text{Dom } f^{-1} = [4, \infty)$

~~**Question 9** (3 marks)~~

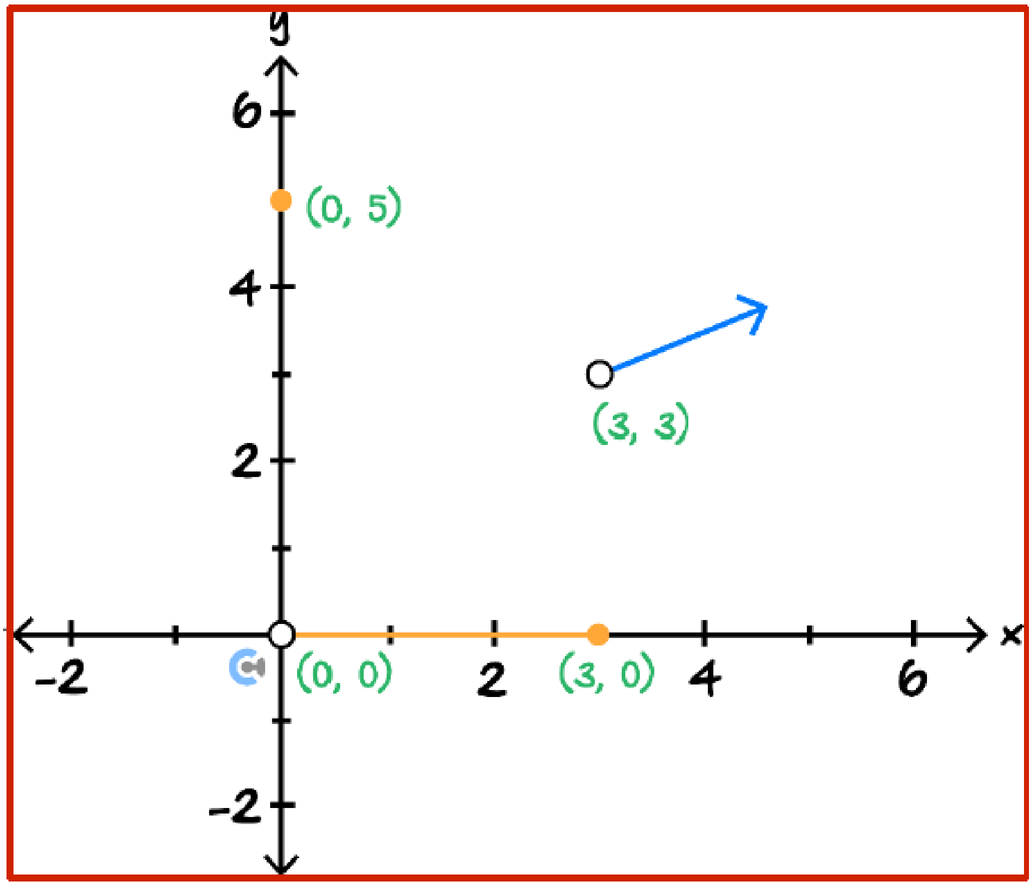
In an effort to reduce the time her children spend in the shower, a mother introduced a penalty scheme with fines to be paid from the children's pocket money according to the following:

If someone spends more than 3 minutes in the shower, the fine in dollars is equal to the shower time in minutes; if someone spends up to and including 3 minutes in the shower, there is no fine. If someone chooses not to shower at all, there is a fine of $5 because that child won't be nice to be near.

Define appropriate symbols, express the penalty scheme as a mathematical rule in hybrid form, and sketch the graph that represents it.

Let the time in the shower be t minutes and the dollar amount be the fine C.

The rule is $C = \begin{cases} 5, & t = 0 \\ 0, & 0 < t \leq 3 \\ t, & t > 3 \end{cases}$



Section D: Tech Active Exam Skills

Calculator Commands: Finding Maximal Domain

Mathematica

```
FunctionDomain[func, x]
```

TI-Nspire

- Type up the domain (or find it under the book button).

domain(func,x)

Casio Classpad

- Sketch the function and analyse.

Calculator Commands: Defining Hybrid Functions on CAS

Mathematica

- Piecewise

Piecewise[{{val_1, $cond_1$}, {val_2, $cond_2$}, …}]

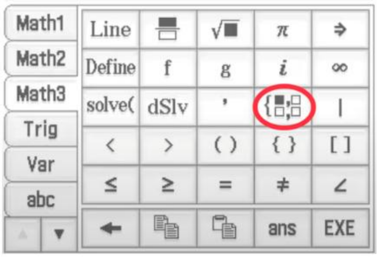
Represents a piecewise function with values val_i in the regions defined by the conditions $cond_i$.

TI-Nspire

- 

$$\begin{cases} func1, & dom1 \\ func2, & dom2 \end{cases}$$

Casio Classpad

- 

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Calculator Commands: Finding the Equation of a Polynomial that Passes Through Points

- Given n points we can find a degree $n-1$ polynomial that passes through all of these points.

- **Example:** Find the equation of the quadratic function that passes through the points $(0,6)$, $(2,2)$, and $(3,3)$.

- **TI:**

Define $f(x)=a\cdot x^2+b\cdot x+c$	*Done*
solve$(f(0)=6$ and $f(2)=2$ and $f(3)=3,a,b,c)$	$a=1$ and $b=^-4$ and $c=6$
$f(x)\|a=1$ and $b=^-4$ and $c=6$	$x^2-4\cdot x+6$

- **Casio:**

```
define f(x) = a*x^2 + b*x + c
                                                done
{f(0)=6
{f(2)=2
{f(3)=3|a,b,c
                                    {a=1,b=-4,c=6}
f(x)|{a=1,b=-4,c=6}
                                         x^2-4·x+6
```

- **Mathematica:**

```
In[9]:= f[x_] := a x^2 + b x + c

In[10]:= Solve[f[0] == 6 && f[2] == 2 && f[3] == 3]
Out[10]= {{a → 1, b → -4, c → 6}}

In[11]:= f[x] /. {a → 1, b → -4, c → 6}
Out[11]= 6 - 4 x + x^2
```

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Section E: Exam 2 Questions (21 Marks)

INSTRUCTION: 21 Marks. 27 Minutes Writing.

Question 10 (1 mark)

The domain of the inverse of $\{(3, -2), (4, -7), (6, -9), (7, -11)\}$ is D. Which of the following statements is true?

A. D is $\{x: -3 < x < 7\}$

B. D is $\{x: 3 < x < 7\}$

C. D is $\{-11, -9, -7, -2\}$

D. D is $\{3, 4, 6, 7\}$

$\text{Dom } f^{-1} = \text{Ran } f$

Question 11 (1 mark)

Which of the following does not have an inverse function?

not 1:1

A. $f: R \to R, f(x) = 4x - 1$

B. $f: [2, \infty) \to R, f(x) = 2(x-2)^2$

$(2,0)$

C. $g: [-4, 4] \to R, g(x) = \sqrt{16 - x^2}$

Not 1:1

D. $f: R \to R, f(x) = \frac{4}{x-3} + 1$

Question 12 (1 mark)

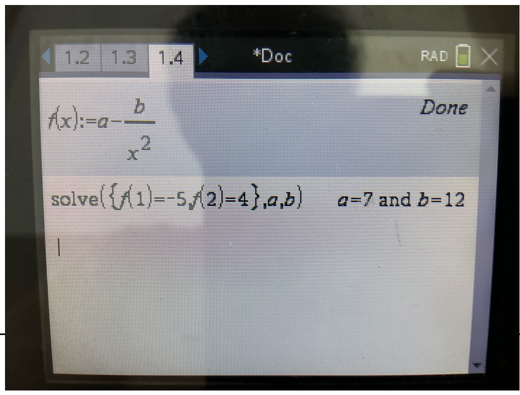
The data $(1, -5)$, $(2, 4)$, $(4, 6.25)$ can be modelled by the equation $y = a - \frac{b}{x^2}$. The values of a and b respectively are:

A. 7 and 12.

B. 12 and 7.

C. -7 and -12.

D. 7 and -12.



f $\quad f^{-1}$

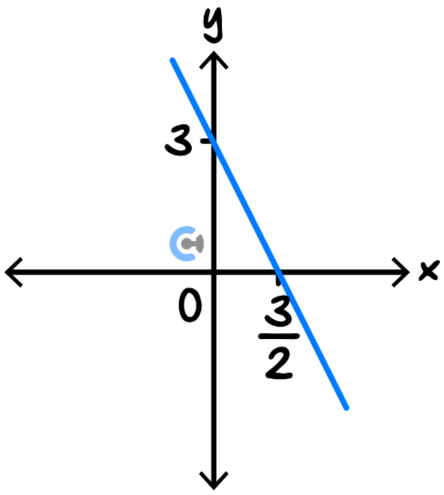
Question 13 (1 mark)

A sketch of the inverse of $f(x) = 2x - 3$ is:

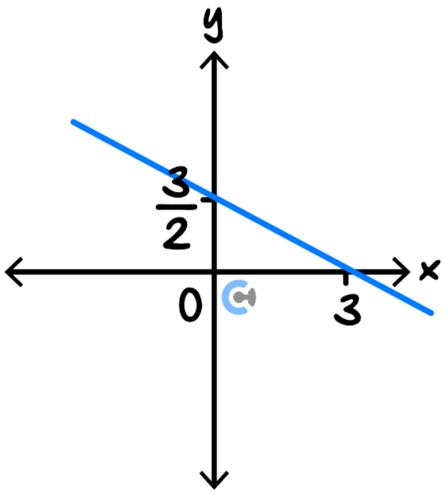
$y\text{-int}: (0,-3) \Rightarrow x\text{-int}: (0,-3)$

$x\text{-int}: (\frac{3}{2},0) \Rightarrow y\text{-int}: (\frac{3}{2},0)$

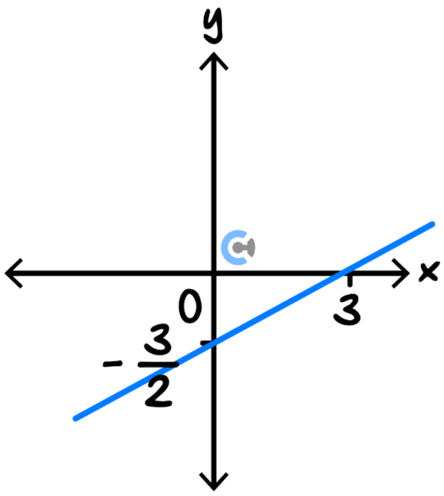
A.



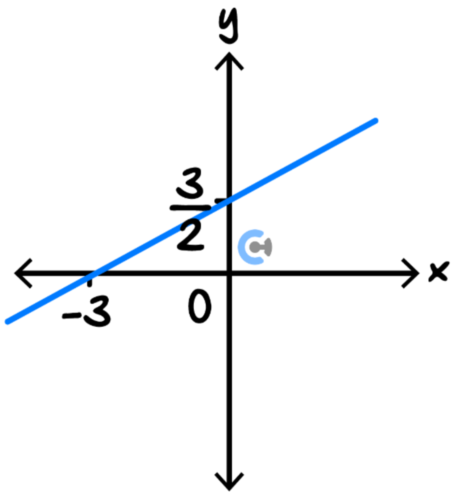
B.



C.



D.



Question 14 (1 mark)

The domain and the range for the graph with the equation $5 - y = -\frac{5}{(x-5)^2}$ respectively are:

A. $\{x: x \in \mathbb{R}\backslash\{5\}\}$ and $\{y: y < 5\}$.

B. $\{x: x \in \mathbb{R}\backslash\{5\}\}$ and $\{y: y > 5\}$.

C. $\{x: x \in \mathbb{R}\backslash\{5\}\}$ and $\{y: y < -5\}$.

D. $\{x: x \in \mathbb{R}\backslash\{5\}\}$ and $\{y: y > -5\}$.

$y = \frac{5}{(x-5)^2} + 5$ $\quad R: y > 5$

$\hookrightarrow$ Dom: $x \in \mathbb{R}\backslash\{5\}$

Question 15 (1 mark)

Which of the following has an inverse which is a function?

must be 1:1

A. $x^2 + y^2 = 4$

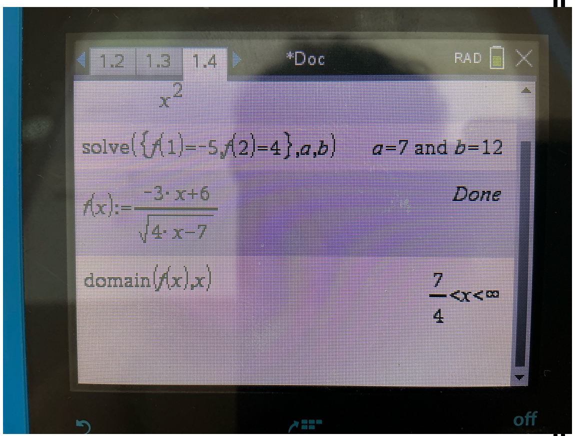
B. $y = \frac{12}{2x-1} - 3$

C. $y = \sqrt{6 - x^2}$

D. $y = 1$

Question 16 (1 mark)

The maximal domain of $y = \frac{-3x+6}{\sqrt{4x-7}}$ is:



A. $\mathbb{R}\backslash\left\{\frac{7}{4}\right\}$.

$4x - 7 > 0$

B. $\mathbb{R}\backslash\left\{\frac{7}{4}, 2\right\}$.

$4x > 7$

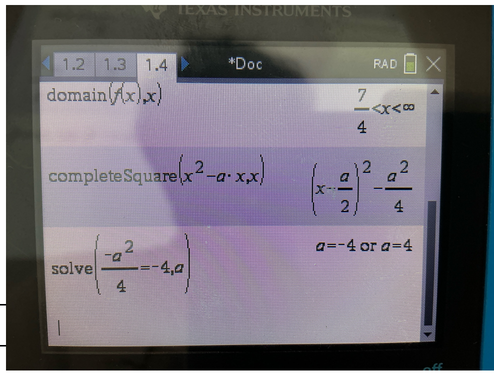
C. $\left[\frac{7}{4}, \infty\right)$.

$x > \frac{7}{4}$

D. $\left(\frac{7}{4}, \infty\right)$.

Question 17 (1 mark)

The graph of $y = x^2 - ax$ has a range of $[-4, \infty)$, where a is a positive constant. The value of a is:



A. 1.

B. 2.

$TP: y = -\frac{a^2}{4} = 4$

C. 8.

$\therefore a = \pm 4$

D. 4.

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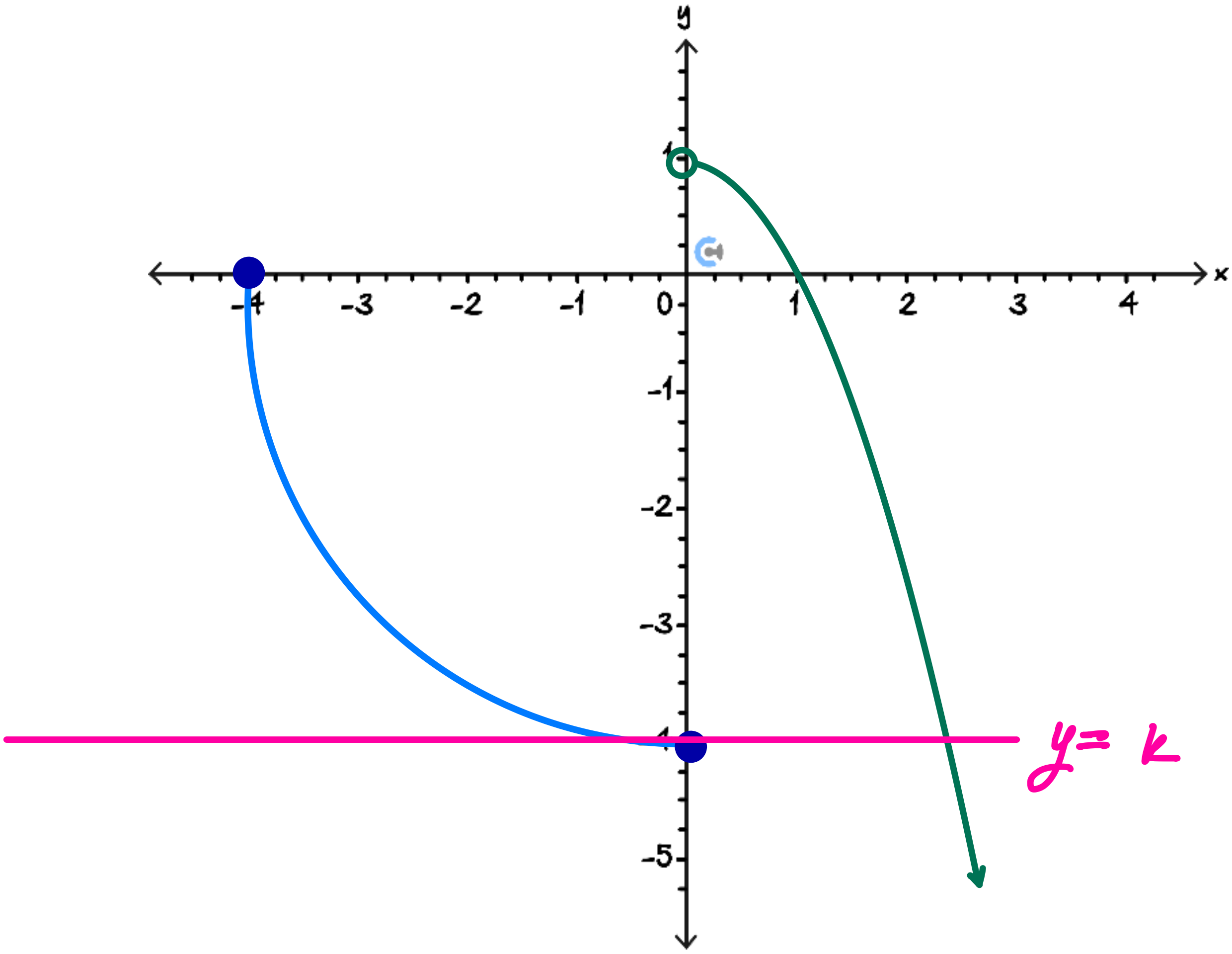
Question 18 (8 marks)

a. Sketch the graph of $y = g(x)$. (2 marks)

$$g(x) = \begin{cases} -\sqrt{16 - x^2}, & -4 \le x \le 0 \\ -x^2 + 1, & x > 0 \end{cases}$$



b. State the range of g. (1 mark)

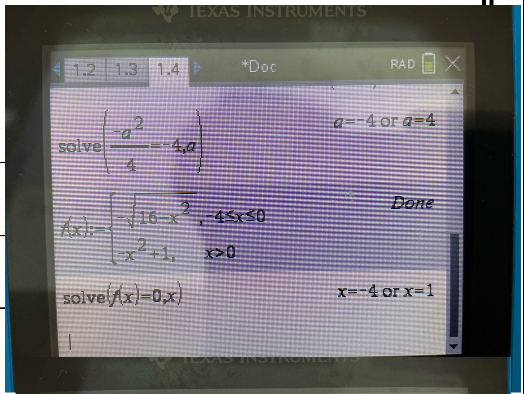
Range of $g \in (-\infty, 1)$

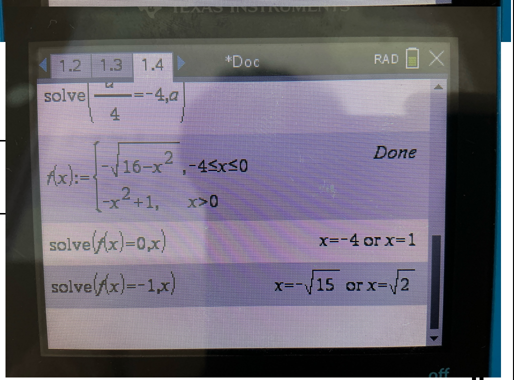
c. Solve for x:

i. $g(x) = 0$. (1 mark)

$y = g(x)$ $y = 0$

$\therefore x = -4, 1$





ii. $g(x) = -1$. (1 mark)

$\therefore x = -\sqrt{15}, 2$ //

$y = g(x)$ & $y = k$

d. Find k if $g(x) = k$ has:

i. 0 solutions. (1 mark)

intersections

$\therefore k \geq 1$

ii. 1 solution. (1 mark)

$\therefore k \in (-\infty, -4) \cup (0, 1)$

OR

$0 < k < 1$ or $k < -4$

iii. 2 solutions. (1 mark)

$\therefore k \in [-4, 0]$ OR $-4 \leq k \leq 0$ //

Question 19 (5 marks)

If a rock falls from a height of 80 metres towards the surface of the Earth, the height, H (in metres) after t seconds is approximately $H(t) = 80 - \frac{7}{12}t^2$.

a. In general, quadratic functions are not one-to-one. However, the function H is one-to-one under its implied domain. Why? (1 mark)

$\therefore$ As $t \geq 0$ (as time is non-negative) the function is 1:1.

b. Find the inverse of H, stating its domain and range given the scenario of the question. (3 marks)

Let $y = H(t)$:

Swap t & y:

$t = 80 - \frac{7}{12}y^2$

$\frac{7}{12}y^2 = 80 - t$

$y^2 = \frac{12(80-t)}{7}$

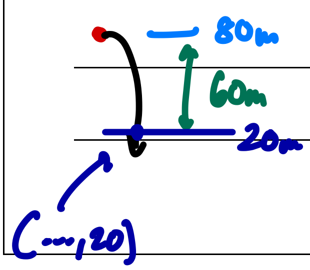
$y = \pm\sqrt{\frac{960-12t}{7}}$ reject −ve as $t \in [0,\infty)$ for $H(t)$

$\therefore H^{-1}(t) = \sqrt{\frac{960-12t}{7}}$

$\therefore$ Dom H^{-1} = Ran $H \in [0,80]$

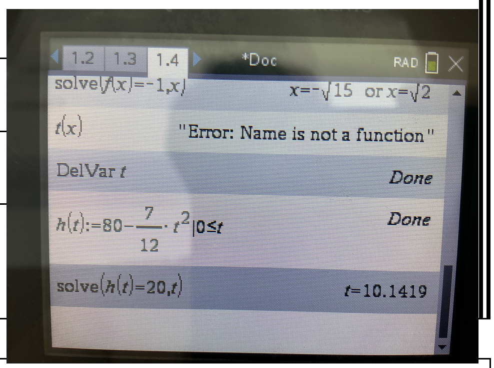
Ran H^{-1} = Dom $H \in [0,\infty)$

c. Find how long it will take for the rock to fall 60 metres to 2 decimal places. (1 mark)



$\therefore H(t) = 20\text{m}$

$\therefore t = 10.14\text{s}$



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Section F: Extension Exam 1 (11 Marks)

INSTRUCTION: 11 Marks. 11 Minutes Writing.

Question 20 (11 marks)

Consider the function $f(x) = \sqrt{6x - x^2} + 1$.

a. Write $f(x)$ in the form $\sqrt{r^2 - (x - h)^2} + k$, and state the values of **positive** integers, r, h, and k. (1 mark)

$f(x) = \sqrt{-(x^2-6x)} + 1$

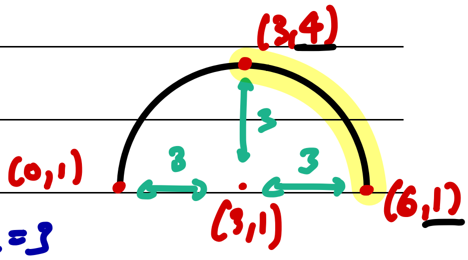
$= \sqrt{-\left((x-3)^2-9\right)} + 1$

$= \sqrt{9-(x-3)^2} + 1$

$r^2 \Rightarrow r = 3$

$\therefore h = 3$

$\therefore k = 1$



The function f has its domain restricted to $[a, 6]$ so that the inverse function f^{-1} exists.

f must be 1:1

b. State the smallest possible value of a. (1 mark)

$\therefore a = 3$

c. Hence, define the inverse function f^{-1}. (3 marks)

Let $y = f(x)$:

Swap x & y:

$x = \sqrt{9-(y-3)^2} + 1$

$(x-1)^2 = 9-(y-3)^2$

$(y-3)^2 = 9-(x-1)^2$

$\therefore y - 3 = \pm\sqrt{9-(x-1)^2}$

$y = 3 \pm \sqrt{9-(x-1)^2} \Rightarrow$ as $3 \le x \le 6$

$\therefore f^{-1}(x) = 3 + \sqrt{9-(x-1)^2}$

$f^{-1}: [1,4] \to \mathbb{R}, \ f^{-1}(x) = 3 + \sqrt{9-(x-1)^2}$

Dom f^{-1} = Ran f = $[1,4]$

d. Find the point of intersection between f and f^{-1}. (2 marks)

Let $f(x) = x$:

$x = \sqrt{9-(x-3)^2} + 1$

$x - 1 = \sqrt{9-(x-3)^2}$

$(x-1)^2 = 9-(x-3)^2$

$x^2 - 2x + 1 = 9 - (x^2 - 6x + 9)$

$x^2 - 2x + 1 = 9 - x^2 + 6x - 9$

$x^2 - 2x + 1 = 6x - x^2$

$2x^2 - 8x + 1 = 0$

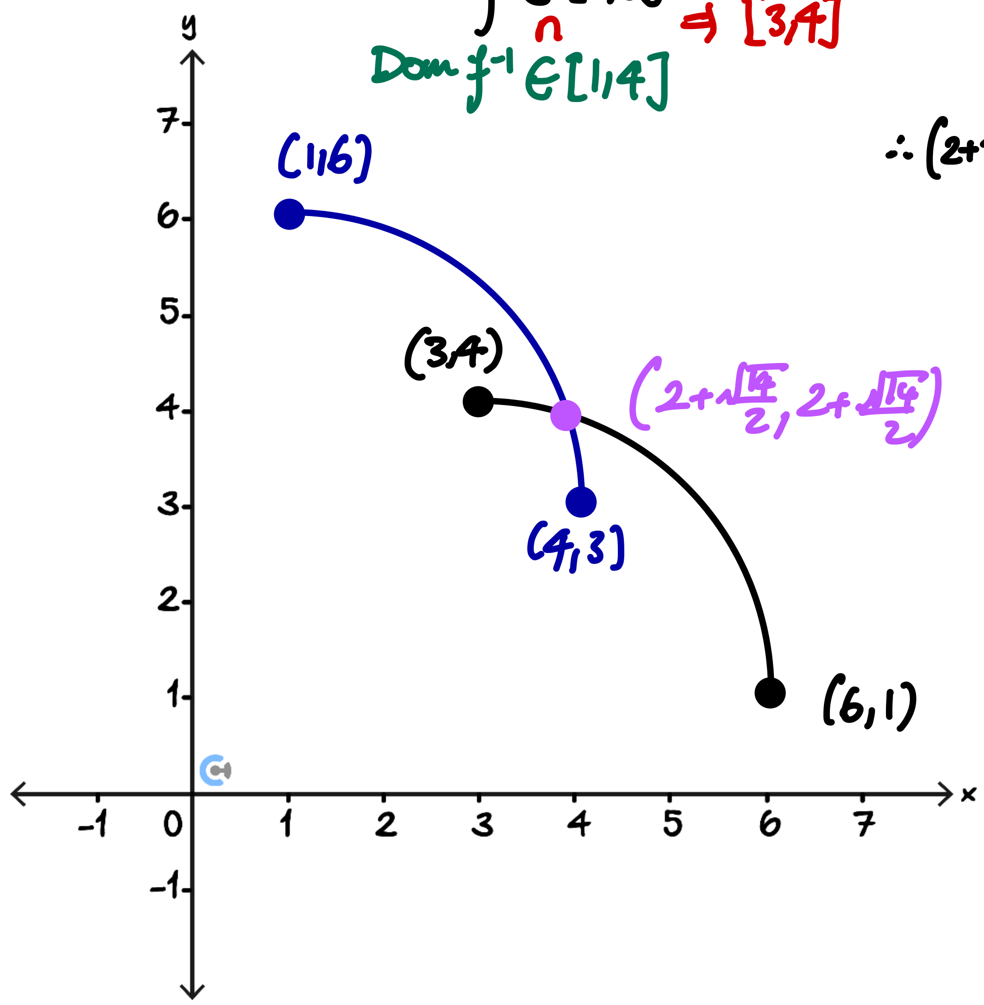
$\therefore x = \dfrac{8 \pm \sqrt{64 - 4(2)(1)}}{2(2)}$

$= \dfrac{8 \pm \sqrt{56}}{4} = \dfrac{8 \pm 2\sqrt{14}}{4} = \dfrac{4 \pm \sqrt{14}}{2} = \dfrac{4 + \sqrt{14}}{2}$ as $x \in [3,4]$

$\therefore \left(2 + \dfrac{\sqrt{14}}{2},\ 2 + \dfrac{\sqrt{14}}{2}\right)$ //

e. Sketch the graphs of f and f^{-1} on the axes below. Label all endpoints and points of intersection with coordinates. (2 marks)

Dom $f \in [3,6]$ $\cap$ Dom $f^{-1} \in [1,4]$ $\Rightarrow [3,4]$

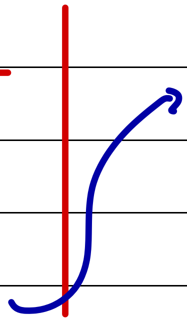


f. Consider all functions of the form $g: [0, r] \to \mathbb{R}, g(x) = \sqrt{r^2 - x^2}$ where $r > 0$. State the x-values for all points of intersection of g and g^{-1}. (2 marks)

let $g(x) = x$:

$x = \sqrt{r^2 - x^2}$

$x^2 = r^2 - x^2$



$2x^2 = r^2$

$x^2 = \frac{r^2}{2}$

$\therefore x = \pm\sqrt{\frac{r^2}{2}}$

$= +\sqrt{\frac{r^2}{2}} = \frac{\sqrt{2}r}{2}$

as $x \in [0, r]$

where $r > 0$

Dom $g \in [0, r]$

Dom g^{-1} = Ran g = $[0, r]$

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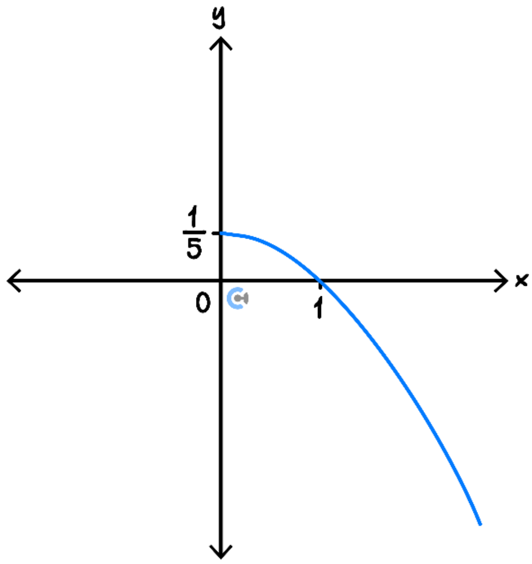
Section G: Extension Exam 2 (13 Marks)

INSTRUCTION: 13 Marks. 13 Minutes Writing.

Question 21 (1 mark)



The graph above represents the inverse of:

A. $f(x) = \sqrt{5 - x}$.

B. $f(x) = \frac{1}{5}\sqrt{1 - x}$.

C. $f(x) = \sqrt{1 - 5x}$.

D. $f(x) = \sqrt{5x - 1}$.

f:
x-int: $(1,0)$
y-int: $(0,\frac{1}{5})$

Swap $\Rightarrow$ $x \Leftrightarrow y$

f^{-1}:
y-int: $(0,1)$
x-int: $(\frac{1}{5},0)$

Question 22 (1 mark)

Which set of ordered pairs represents a function?

A. $\{(1,7), (2,6), (4,3), (4,4), (12,6)\}$

B. $\{(2,4), (2,5), (4,6), (4,7), (4,8)\}$

C. $\{(0,4), (1,4), (2,4), (3,4), (4,4)\}$

D. $\{(0,2), (0,3), (2,4), (3,5), (4,6)\}$

Question 23 (1 mark)

The maximal domain of

$$y = \frac{4x+3}{\sqrt{x^2-2x-8}}$$

is:

A. $x \in [-2,4]$.

B. $x \in (-\infty,-2] \cup [4,\infty)$.

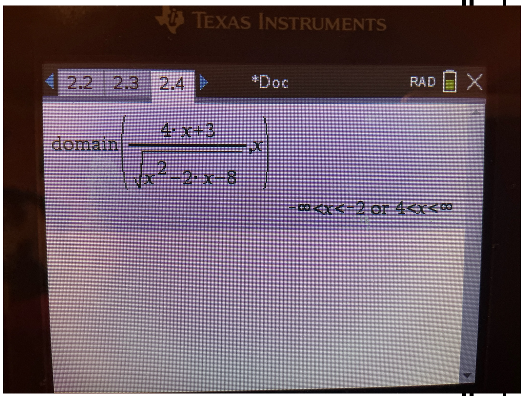
C. $x \in R\backslash[-2,4]$.

D. $x \in R\backslash(-2,4)$.

$x^2-2x-8>0$

$(x-4)(x+2)>0$

$\therefore x<-2$ or $x>4$



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Question 24 (10 marks)

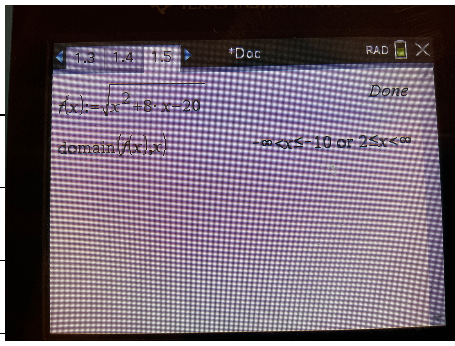
Let $f(x) = \sqrt{x^2 + 8x - 20}$.

a. Determine the maximal domain of f. (1 mark)

$x^2+8x-20 \geq 0$

$(x+10)(x-2) \geq 0$

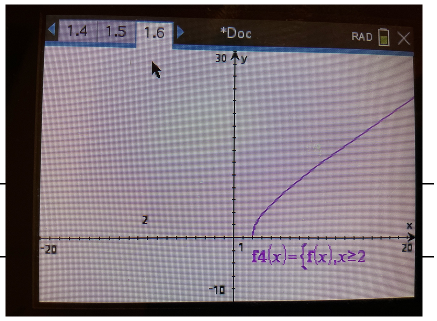
$\therefore x \leq -10$ or $x \geq 2$



Let $g: [2, \infty) \to \mathbb{R}, g(x) = f(x)$.

b. What type of function is g? (1 mark)

1 : 1



c. Define g^{-1}, the inverse function of g. (2 marks)

Let $y = g(x)$:

Swap x & y:

$x = \sqrt{y^2+8y-20}$

$x^2 = y^2+8y-20$

$x^2 = (y+4)^2-36$

$x^2+36 = (y+4)^2$

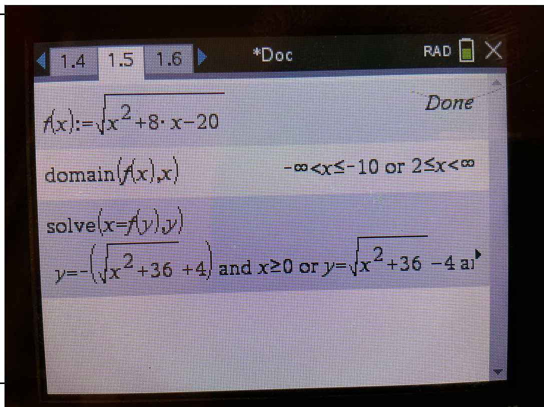
$y+4 = \pm\sqrt{x^2+36}$

$y = -4 \pm \sqrt{x^2+36}$

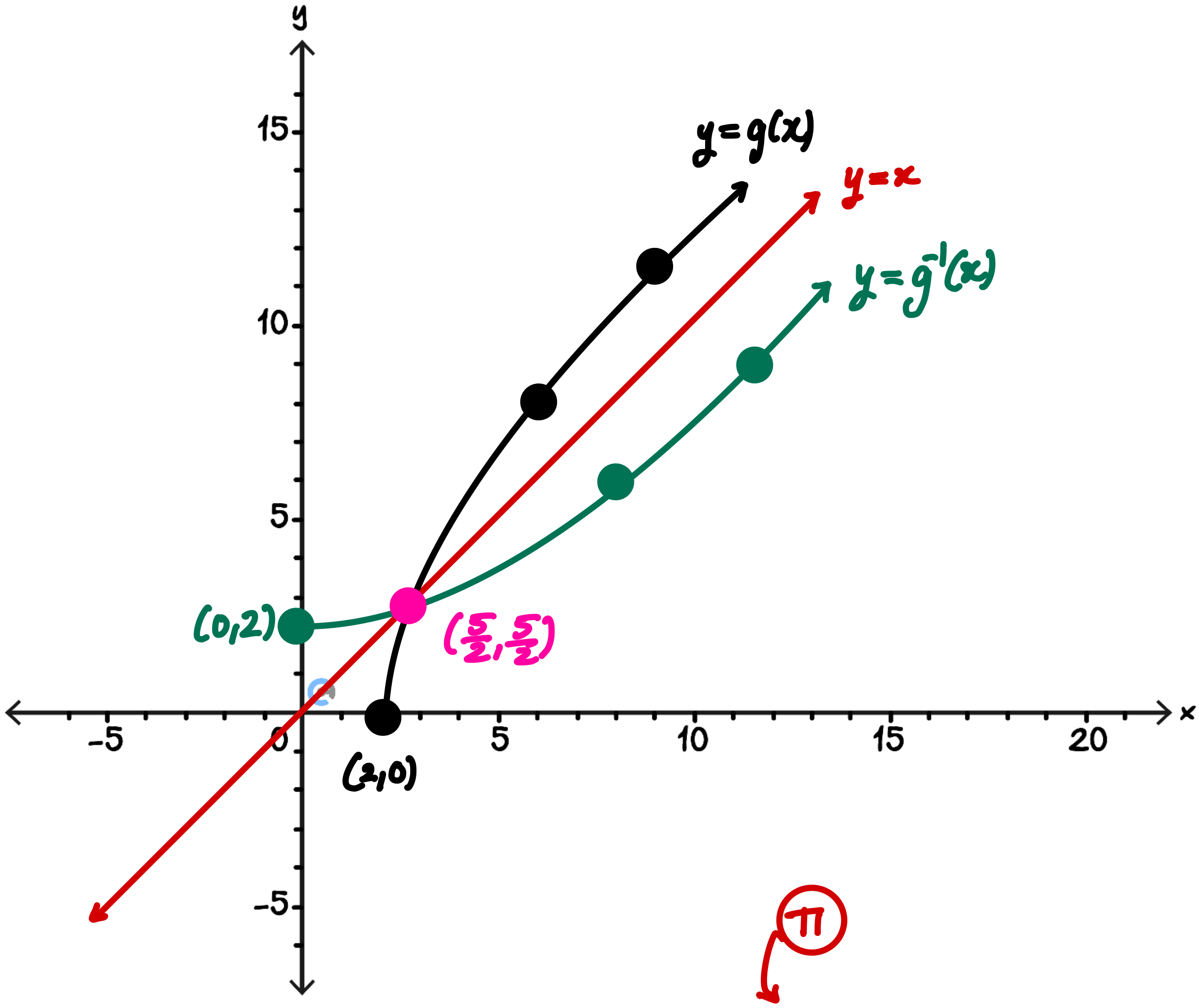
Ran g^{-1} = Dom g $= [2, \infty)$

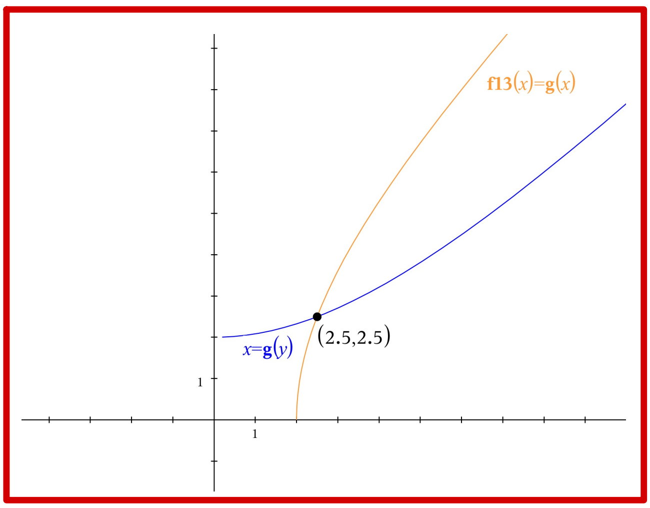
$\therefore g^{-1}(x) = -4+\sqrt{x^2+36}$

$\therefore g^{-1}: [0, \infty) \to \mathbb{R},\ g^{-1}(x) = \sqrt{x^2+36} - 4$



d. Sketch the graph of g and g^{-1} on the axes below. Label all axes intercepts and points of intersection with coordinates. (3 marks)





e. Now, let $h: [2, \infty) \to \mathbb{R}: \cancel{g}\,h(x) = f(x) + k$, where k is a real number. Determine the values of k for which h and h^{-1} have a point of intersection. (3 marks)

let $h(x) = x$:

$\therefore \sqrt{x^2+8x-20} + k = x$

$\sqrt{x^2+8x-20} = x-k$

$x^2+8x-20 = (x-k)^2$

$\cancel{x^2}+8x-20 = \cancel{x^2}-2kx+k^2$

$8x+2kx = k^2+20$

$x(2k+8) = k^2+20$

$$\boxed{\therefore x = \frac{k^2+20}{2k+8}}$$

Dom $h = [2, \infty)$

Dom h^{-1} = Ran h

$= [k, \infty)$

= Ran $f + k$

$= [0, \infty) + k$

$= [k, \infty)$

$\therefore$ intersection occurs on Dom $h \cap$ Dom h^{-1}

$= [2, \infty) \cap [k, \infty)$

Case 1: $k < 2$

$[2, \infty) \cap [k, \infty) = [2, \infty)$

$\therefore \frac{k^2+20}{2k+8} \geq 2 \Rightarrow \therefore k > -4$

Case 2: $k \geq 2$

$[2, \infty) \cap [k, \infty) = [k, \infty)$

$\therefore \frac{k^2+20}{2k+8} \geq k \Rightarrow \therefore -4 < k \leq 2$

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$h(x):=\sqrt{x^2+8\cdot x-20}+k$	*Done*
solve$(h(x)=x,x)$	$x=\frac{k^2+20}{2\cdot(k+4)}$
solve$\left(\frac{k^2+20}{2\cdot k+8}\geq 2,k\right)$	$k>-4$
solve$\left(\frac{k^2+20}{2\cdot k+8}\geq k,k\right)$	$-4<k\leq 2$ or $k\leq -10$

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