Website: contoureducation.com.au | Phone: 1800 888 300
Email: hello@contoureducation.com.au

VCE Mathematical Methods ½
Polynomials Exam Skills [0.6]
Workshop Solutions

Error Logbook:

Mistake/Misconception #1		Mistake/Misconception #2	
Question #:	Page #:	Question #:	Page #:
Notes:		Notes:	

Mistake/Misconception #3		Mistake/Misconception #4	
Question #:	Page #:	Question #:	Page #:
Notes:		Notes:	

Section A: Recap

Degree of Polynomial Functions

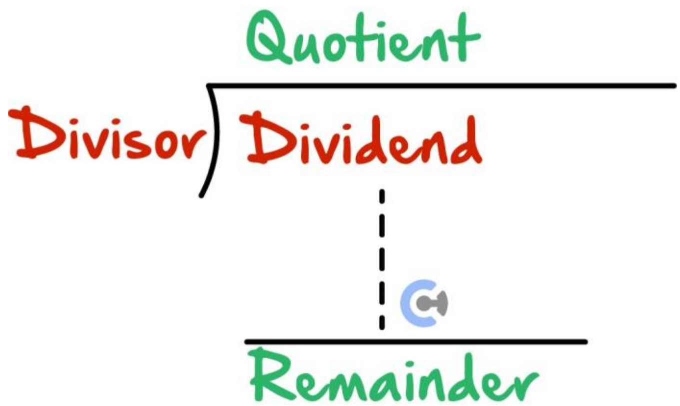
$$\textit{Degree} = \textit{Highest Power of the Polynomial}$$

Roots of Polynomial Functions

$$\textit{Roots} = x\textit{-intercept}$$

Polynomial Long Division

- Division of polynomials:



$$\frac{\textit{Dividend}}{\textit{Divisor}} = \textit{Quotient} + \frac{\textit{Remainder}}{\textit{Divisor}}$$

Space for Personal Notes

Remainder Theorem

- **Definition**:
  - Finds the remainder of long division without the need of long division,

$$\textbf{when } P(x) \textbf{ is divided by } (x-\alpha), \textbf{the remainder is } P(\alpha).$$

- **Steps**
  1. Find x-values which make the divisor equal to 0.
  2. Substitute it into the dividend function.

Factor Theorem

- For every x-intercept, there is a factor:

$$\textbf{If } P(\alpha) = 0 \textbf{ then}, (x-\alpha) \textbf{ is a factor of } P(x).$$

Factorising Polynomials

- The steps are:
  - Find a single root by trial and error.
    - (Factor Theorem: Substitute into the function and see if we get zero.)
  - Use long division to find the quadratic factor.
  - Factorise the quadratic.

Space for Personal Notes

Definition

Rational Root Theorem

- Rational Root Theorem **narrows down** the possible roots.

$$Potential\ root = \pm\frac{Factors\ of\ constant\ term\ a_0}{Factors\ of\ leading\ coefficient\ a_n}$$

- If the roots are rational numbers, the roots can only be $\pm\frac{\text{Factors of constant term } a_0}{\text{Factors of leading coefficient } a_n}$.

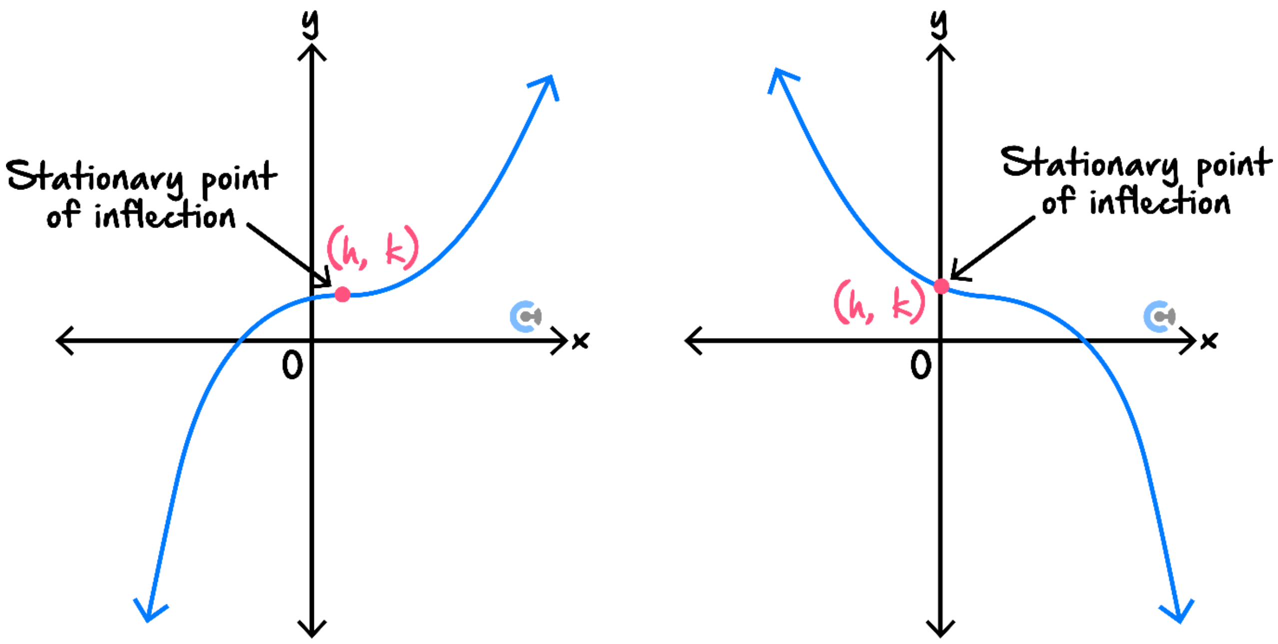
Sum and Difference of Cubes

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Graphs of $a(x-h)^n + k$, where n is an Odd Positive Integer

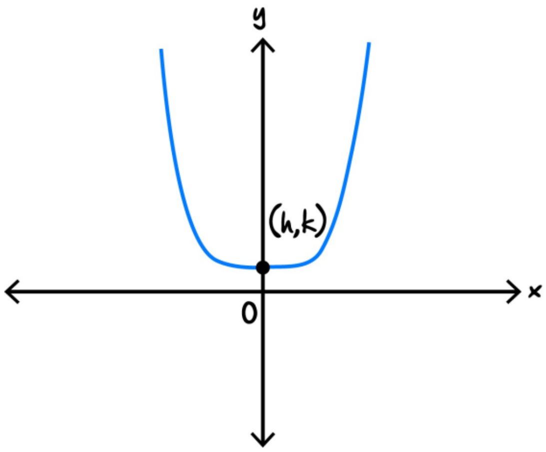
- All graphs look like a "cubic".

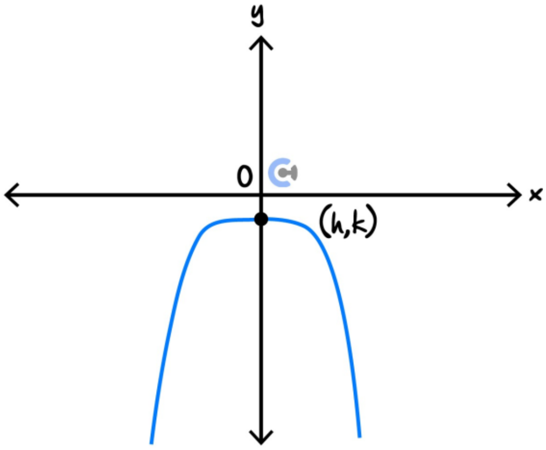


- The point (h, k) gives us the stationary point of inflection.
- n cannot be 1 for this shape to occur!

Graphs of $a(x-h)^n + k$, where n is an Even Positive Integer

- All graphs look like a "quadratic".

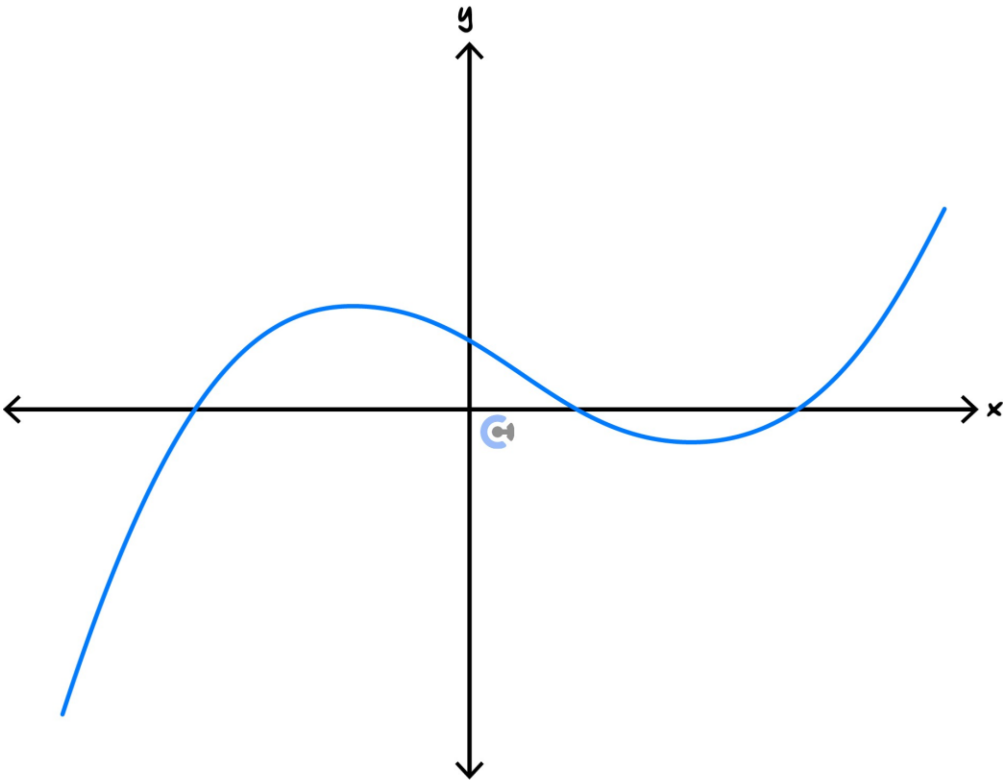




- The point (h, k) gives us the turning point.

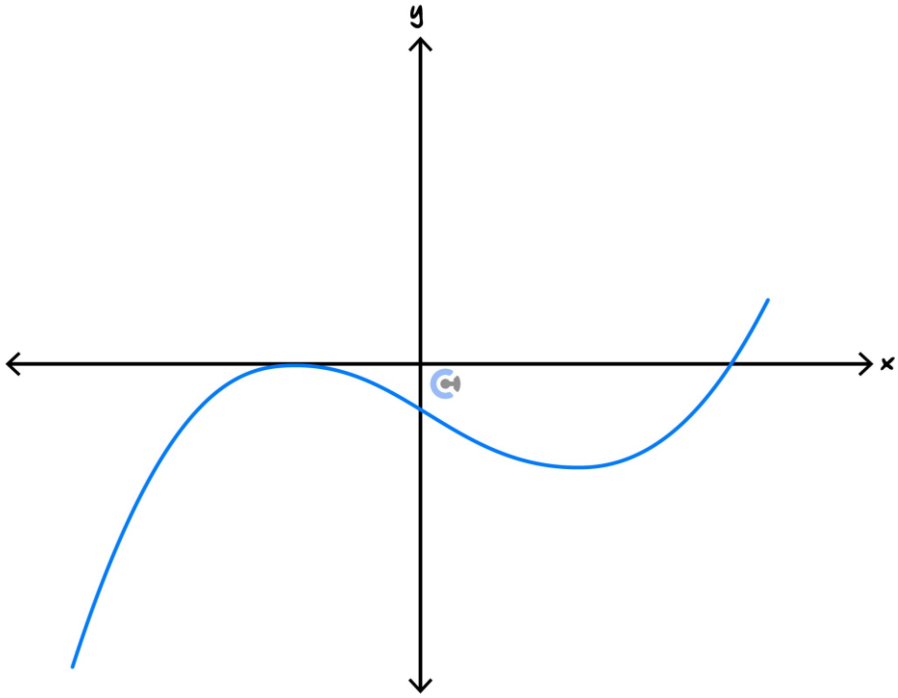
Graphs of Factorised Polynomials

- **All non-repeated linear factors correspond to x-intercepts of the graph.**



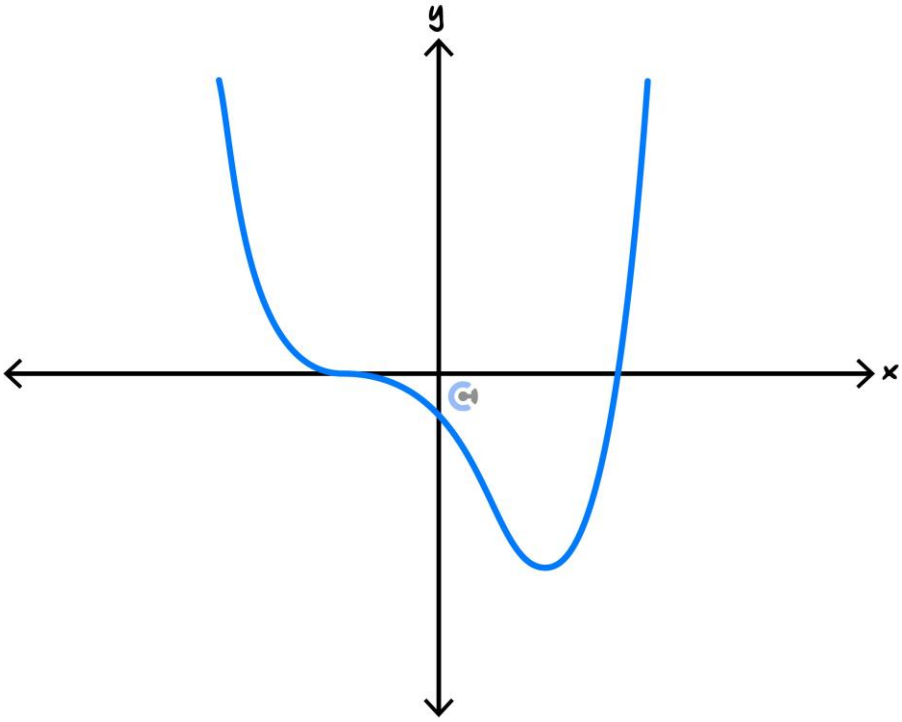
- E.g., $f(x) = (x-a)(x-b)(x-c)$ results in x-intercepts at $(a, 0)$, $(b, 0)$, and $(c, 0)$.

- All squared linear factors correspond to x-intercepts and T.P. of the graph.



- E.g., $f(x) = (x-a)^2(x-b)$ will have an x-intercept $(a, 0)$ which is also a local minimum/maximum.

- All cubed linear factors correspond to x-intercepts and SPI of the graph.



- E.g., $f(x) = (x-a)^3(x-b)$ has an x-intercept $(a, 0)$ which is also a stationary point of inflection.

Definition

Steps to Graphing Factorised Polynomials

- **Steps:**

1. Plot x-intercepts.
2. Determine whether the polynomial is positive or negative.
3. Use the repeated factors to deduce the shape.
   - Non - Repeated: Only x-intercept.
   - Even Repeated: x-intercept and a turning point.
   - Odd Repeated: x-intercept and a stationary point of inflection.

Definition

Solving the Polynomial Inequality $f(x) > 0$

- Steps:

1. Find the x-intercepts.
2. Sketch the polynomial.
3. Shade the places where the y-values are positive.

Definition

When does a cubic have n solutions?

- Steps:

1. Factorise out the x term.
2. Since the x term gives 1 solution, use discriminant to find when the quadratic has $n - 1$ solutions.

Space for Personal Notes

Definition

Bisection Method

- Step 1: Pick a random interval $[a, b]$ where $f(a) \times f(b) =$ Negative.
- Step 2: Find a midpoint to estimate the root.

$$\text{where } m = \frac{a+b}{2}$$

- Step 3: Create a new interval $[a, b]$ by making m either new a or new b.

$$\text{If } f(a) \times f(m) < 0$$

$$\text{New Interval: } [a, m]$$

$$\text{If } f(b) \times f(m) < 0$$

$$\text{New Interval: } [m, b]$$

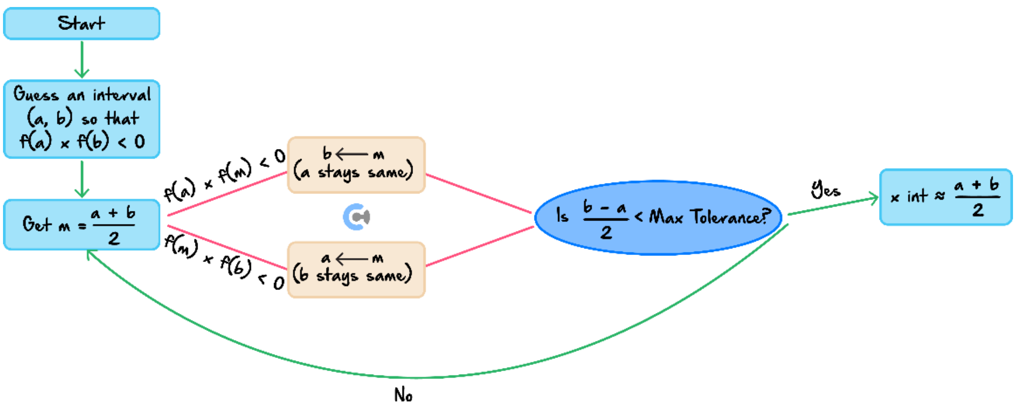
- Step 4: Repeat until the interval becomes short enough for good accuracy.
  - The smaller the interval $[a, b]$, more accurate our estimation gets.

$$\text{If } \frac{b-a}{2} < \text{Max Tolerance,}$$

$$\text{We stop.}$$

  - Maximum error is half of the width of the interval.

$$\text{Max Error} = \frac{b-a}{2}$$



Section B: Warmup

Question 1

a. Solve the inequality $x^3 + x + 6 > 4x^2$.

Solution: Write this as $x^3 - 4x^2 + x + 6 > 0$. This can be factored as

$$(x+1)(x-2)(x-3) > 0$$

and by considering the shape of the cubic the inequality holds for $-1 < x < 2$ or $x > 3$.

b. Find the values of k such that $x^3 + 2kx^2 + 3x = 0$ has only one real solution.

Solution: Factor as $x(x^2 + 2kx + 3)$. Only one solution $x = 0$ if quadratic has no solutions. So consider the discriminant.

$$4k^2 - 12 < 0$$
$$k^2 < 3$$

so only one solution for $-\sqrt{3} < k < \sqrt{3}$.

c. Apply the bisection method with initial interval [1,2] and tolerance 0.1 to find an approximate solution to the equation $x^2 - 2 = 0$.

Solution: Our intervals are: $[1,2] \to \left[1, \frac{3}{2}\right] \to \left[\frac{5}{4}, \frac{3}{2}\right] \to \left[\frac{11}{8}, \frac{3}{2}\right]$.

This last interval has width $\frac{1}{8} < 2 \times 0.1$. So our estimate is

$$\frac{11/8 + 12/8}{2} = \frac{23}{16}$$

Space for Personal Notes

Section C: Exam 1 (23 Marks)

Question 2 (9 marks)

Let $f(x) = ax^3 - 5x^2 + bx + 9$. When $f(x)$ is divided by $x - 2$ the remainder is -7 and when $f(x)$ is divided by $x + 1$ the remainder is 8.

a. Show that $a = 2$ and $b = -6$. (2 marks)

Solution: We have that $f(2) = -7$ and $f(-1) = 8$. This gives us the equations:

$$8a + 2b - 11 = -7 \quad (1)$$
$$-a - b + 4 = 8 \quad (2)$$

adding two times second equation to first gives

$$6a - 3 = 9 \implies a = 2$$

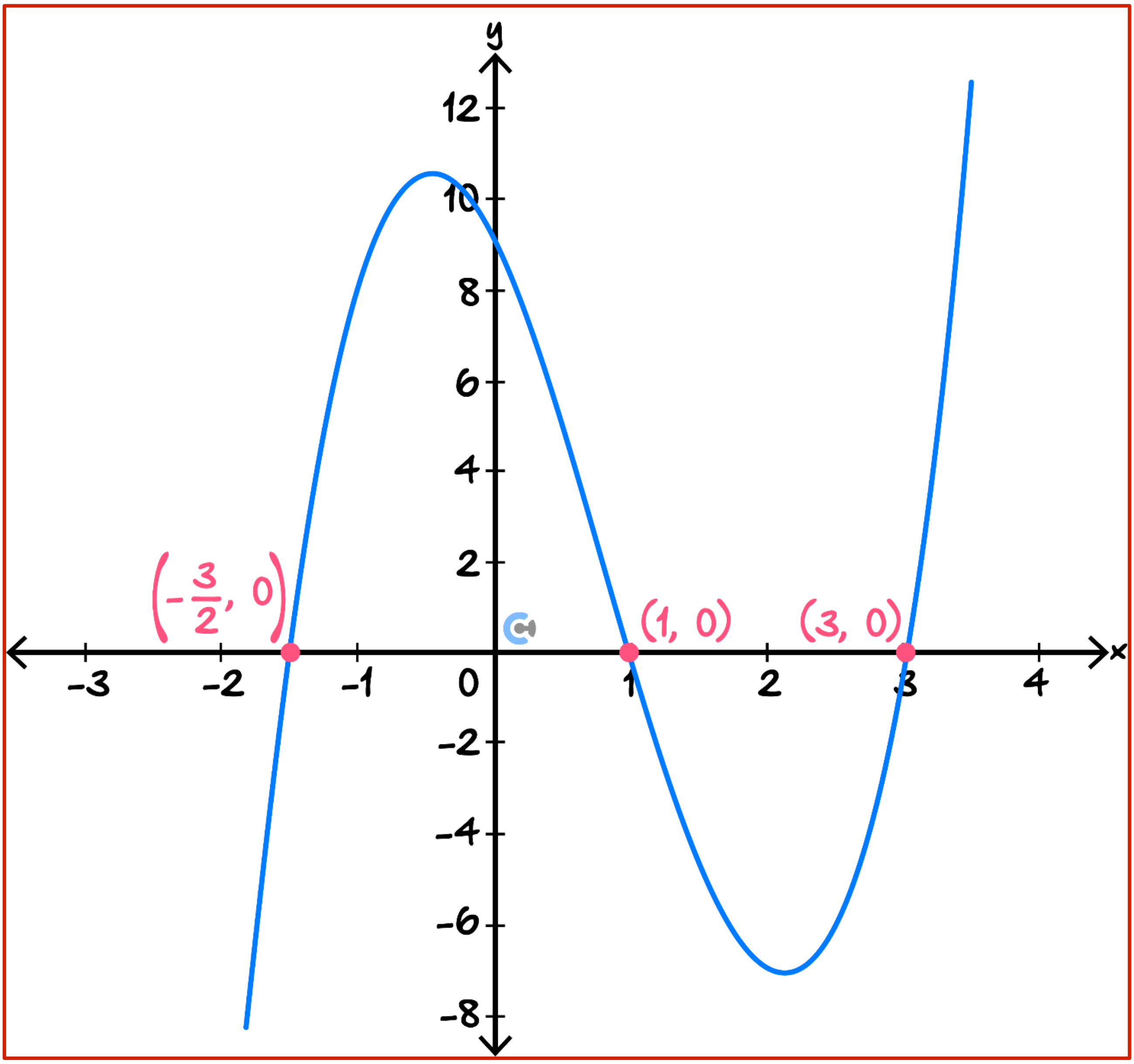
then $2 - b = 8 \implies b = -6$

b. Express $f(x)$ as the product of three linear factors. (3 marks)

Solution: Note that $f(x) = 2x^3 - 5x^2 - 6x + 9$ and $f(1) = 0$ so $x - 1$ is a factor. Then

$$\begin{aligned} f(x) &= (x-1)(2x^2 - 3x - 9) \\ &= (x-1)(2x+3)(x-3) \end{aligned}$$

c. Sketch the graph of $y = f(x)$ on the axes below. Label all axes intercepts. Note that f has turning points at approximately $(-0.5, 10.5)$ and $(2.1, -7.1)$. (2 marks)



d. Hence, solve the inequality $2x^3 - 5x^2 - 6x > -9$. (2 marks)

Solution: $-\frac{3}{2} < x < 1$ or $x > 3$

Question 3 (5 marks)

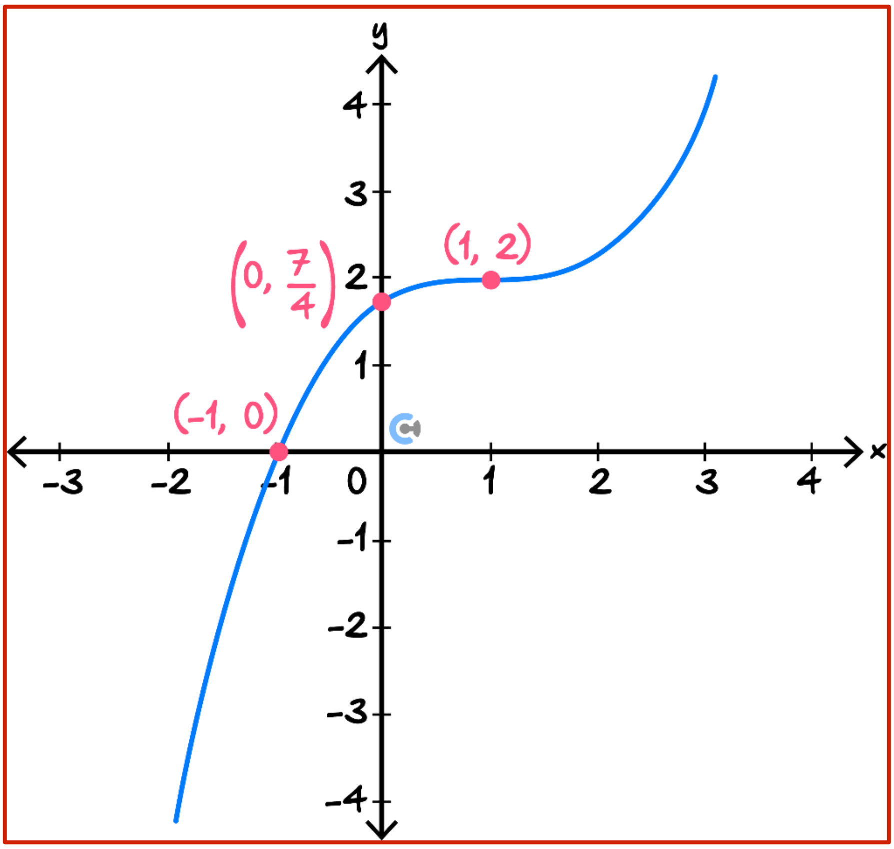
Consider the function $f(x) = \frac{x^3}{4} - \frac{3x^2}{4} + \frac{3x}{4} + \frac{7}{4}$.

a. Write $f(x)$ in the form $a(x+b)^3 + c$ for real values a, b, and c. (2 marks)

Solution:

$$\begin{aligned} f(x) &= \frac{1}{4}(x^3 - 3x^2 + 3x + 7) \\ &= \frac{1}{4}((x-1)^3 + 8) \\ &= \frac{1}{4}(x-1)^3 + 2 \end{aligned}$$

b. Sketch the graph of $y = f(x)$ on the axes below. Label any axes intercepts and stationary points of inflection with coordinates. (3 marks)



Question 4 (4 marks)

The bisection method may be used to approximate $\sqrt{3}$ by finding a root to $x^2 - 3 = 0$.

a. Use the bisection method with initial interval $[1,2]$ and tolerance 0.1 to find an approximate solution to $x^2 - 3 = 0$. Leave your answer in the form $\frac{a}{b}$, for positive integers a and b. (3 marks)

Solution: Let $f(x) = x^2 - 3$. $f(1) < 0$ and $f(2) > 0$.

$m = \frac{3}{2}$ and $f(m) = -\frac{3}{4}$. New interval $\left[\frac{3}{2}, 2\right]$

$m = \frac{7}{4}$ and $f(m) = \frac{49}{16} - \frac{48}{16} > 0$. New interval $\left[\frac{3}{2}, \frac{7}{4}\right]$

$m = \frac{13}{8}$ and $f(m) = \frac{169}{64} - \frac{192}{64} < 0$. New interval $\left[\frac{13}{8}, \frac{7}{4}\right]$. Interval has width $\frac{1}{8} < 2 \times 0.1$.

So our final estimate is $m = \frac{13/8 + 14/8}{2} = \frac{27}{16}$

b. Determine whether $\frac{7}{4}$ is more than or less than $\sqrt{3}$. (1 mark)

Solution: In our bisection method $\frac{7}{4}$ was a right end point of the interval. Therefore $\frac{7}{4} > \sqrt{3}$

Space for Personal Notes

Question 5 (5 marks)

Consider $f(x) = x^3 - 2kx^2 + 4kx + 4x$, where k is a real constant.

Find the values of k such that $f(x) = 0$ has:

a. One solution. (3 marks)

Solution: Factorise as $x(x^2 - 2kx + 4(k+1))$
We find that the only solution will be $x = 0$ if the discriminant of the quadratic factor is less than 0.

$$4k^2 - 16(k+1) < 0$$
$$k^2 - 4k - 4 < 0$$
$$(k-2)^2 < 8$$

There will be only one solution if

$$2 - 2\sqrt{2} < k < 2 + 2\sqrt{2}$$

b. Two solutions. (1 mark)

Solution: $k = 2 - 2\sqrt{2}, 2 + 2\sqrt{2}$ or $k = -1$

c. Three solutions. (1 mark)

Solution: $k < 2 - 2\sqrt{2}$ and $k \neq -1$ or $k > 2 + 2\sqrt{2}$

<u>Section D:</u> Tech Active Exam Skills

<u>**Calculator Commands:**</u> **Turning Point**

- ALWAYS sketch the graph to find approximate bounds for where the turning point you want is located.
- To find a local maximum we maximise the function over a specific domain.
- To find a local minimum we minimise the function over a specific domain.
- **TI and Casio:** Use $fmin(expression, variable, lower\ (optional), upper\ (optional))$ or $fmax(expression, variable, lower\ (optional), upper\ (optional))$.
- **TI:** Menu→ $4 \rightarrow \frac{7}{8}$.

$$\text{Define } f(x)=x^3-4\cdot x \qquad \textit{Done}$$

$$\text{fMin}(f(x),x,0,2) \qquad x=\frac{2\cdot\sqrt{3}}{3}$$

$$f\left(\frac{2\cdot\sqrt{3}}{3}\right) \qquad \frac{-16\cdot\sqrt{3}}{9}$$

- **Casio:** Action→Calculation→ $fmin/fmax$

$$\text{fmin}(x\hat{\ }3-4x,x,0,2)$$

$$\left\{\text{MinValue}=\frac{-16\cdot\sqrt{3}}{9},\ x=\frac{2\cdot\sqrt{3}}{3}\right\}$$

.

NOTE: TI only gives the x-value for the min/max so we then need to sub it back into our function. Casio gives us both!

Calculator Commands

- **Mathematica:** Minimise[ ] and Maximise[ ] commands.
- Minimise[$f[x], x$] will minimise $f[x]$ over its whole domain.
- To restrict the domain, we must use Minimise[$\{f[x], a \leq x \leq b\}, x$].

```
In[34]:= Minimize[{x^3 - 4 x, 0 < x < 2}, x]
```

Out[34]= $\left\{-\frac{16}{3\sqrt{3}}, \left\{x \to \frac{2}{\sqrt{3}}\right\}\right\}$

TI UDF: Bisection Method

- **Overview:**
  - Apply the bisection method to a function to approximate x-intercepts.
- **Input:**
  - $\text{bisection}(<\text{function}>, <\text{variable}>, <\text{lower bound}>, <\text{upper bound}>)$
- **Other Notes:**
  - The program will ask for the threshold type to terminate the algorithm.
  - Select None $[0]$ to provide a specific number of iterations.
  - Select x $[1]$ to provide a threshold for $b - a$, after which the program will stop if $b - a$ becomes smaller than the threshold.
  - Select y $[2]$ to provide a threshold for $|f(b) - f(a)|$, after which the program will stop if $|f(b) - f(a)|$ becomes smaller than the threshold.

$bisection(x^2-2,x,0,1)$

Number of Iterations: 5

"n"	"a"	"m"	"b"	"f(a)"	"f(m)"	"f(b)"	"b−a"	"|f(b)−f(a)|"
0.	0.	0.5	1.	-2.	-1.75	-1.	1.	1.
1.	0.5	0.75	1.	-1.75	-1.4375	-1.	0.5	0.75
2.	0.75	0.875	1.	-1.4375	-1.23438	-1.	0.25	0.4375
3.	0.875	0.9375	1.	-1.23438	-1.12109	-1.	0.125	0.234375
4.	0.9375	0.96875	1.	-1.12109	-1.06152	-1.	0.0625	0.121094
5.	0.96875	0.984375	1.	-1.06152	-1.03101	-1.	0.03125	0.061523

Section E: Exam 2 (25 Marks)

Question 6 (1 mark)

The equation $2x^3 - 3x - 4 = 0$ has one real solution, which lies in the interval $[1,2]$. Approximate the solution using the bisection method with a maximum error of 0.1. What is the approximate solution?

A. $x \approx 1.655$

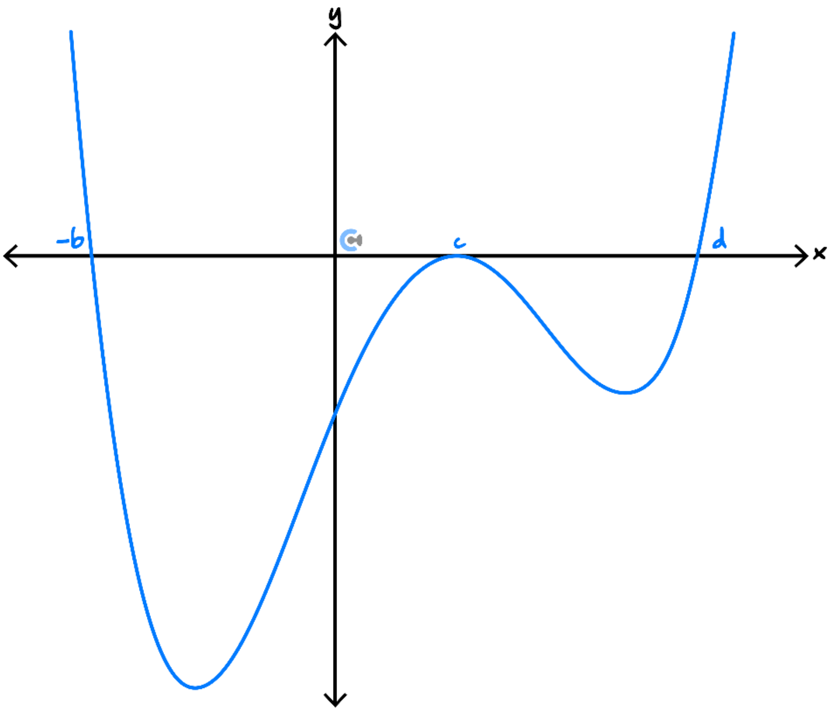
B. $x \approx 1.6250$

C. $x \approx 1.6875$

D. $x \approx 1.6225$

Question 7 (1 mark)

The rule for a function with the graph below, where $b, c, d > 0$, could be:



A. $y = -2(x + b)(x - c)^2(x - d)$

B. $y = 3(x + b)(x - c)^2(x - d)$

C. $y = -2(x - b)(x - c)^2(x - d)$

D. $y = 2(x - b)(x - c)^2(x - d)$

Question 8 (1 mark)

The polynomial $x^3 + (a + 2)x^2 + bx + 8$ is perfectly divisible by $x + 2$ and has remainder of 2 when divided by $x - 3$. The values (a, b) are:

A. $(-5, -6)$

B. $\left(-\frac{21}{5}, -\frac{22}{5}\right)$

C. $\left(-\frac{3}{5}, -\frac{9}{5}\right)$

D. $\left(-\frac{7}{5}, \frac{3}{5}\right)$

Question 9 (1 mark)

All real values of x that satisfy the inequality $9x^2 - 2x^3 > 54 - 27x$ are:

A. $x < -3$ or $\frac{3}{2} < x < 6$.

B. $x < -6$ or $\frac{3}{2} < x < 3$.

C. $x < -3$ or $x > -\frac{3}{2}$.

D. $-3 < x < \frac{3}{2}$ or $x > 6$.

Question 10 (1 mark)

The equation $x^3 - 3kx^2 + 5x = 0$ has exactly one solution when:

A. $k = \pm\frac{2\sqrt{5}}{3}$

B. $-\frac{2\sqrt{5}}{3} < k < \frac{2\sqrt{5}}{3}$

C. $k > \frac{2\sqrt{5}}{3}$

D. $k < -\frac{2\sqrt{5}}{3}$

Question 11 (9 marks)

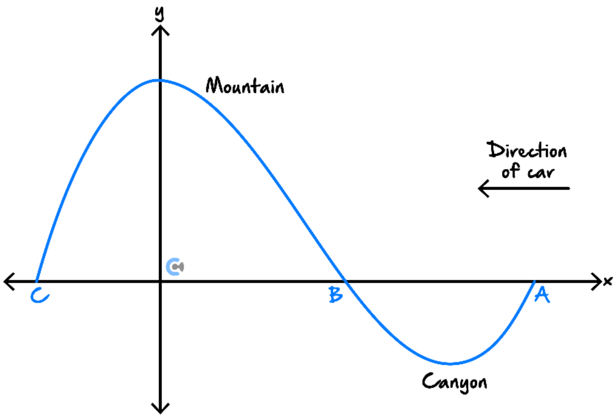
A car is travelling along a straight road from A to C.
The car will travel along a section of road ABC.

- Section AB passes along a bridge over a canyon.
- Section BC passes through a tunnel in a mountain.

From A to C, the curve of the canyon and then mountain, directly below and above the road, is modelled by the graph of:

$$y = \frac{1}{250}(px^3 + qx^2 + r)$$

Where p, q, and r are real constants.
All measurements are in kilometres and a diagram of this situation is shown below.



a. The curve defined from A to C passes through the points $(1, 0.652)$, $(2, 0.48)$, and $(5, -0.18)$.

i. Use this information to write down three simultaneous equations in terms of p, q, and r. Write these equations with integer coefficients. (3 marks)

Solution: We use the equations $f(1) = 0.652$, $f(2) = 0.48$ and $f(5) = -0.18$ to obtain

$$p + q + r = 163$$
$$8p + 4q + r = 120$$
$$125p + 25q + r = -45$$

ii. Hence, verify that $p = 2, q = -19$, and $r = 180$. (1 mark)

Solution: These values are obtained by solving the system of equations from the previous part and they can be verified by subbing them into the equations.

$$2 - 19 + 180 = 163$$
$$8(2) + 4(-19) + 180 = -60 + 180 = 12$$
$$125(2) + 25(-19) + 180 = -225 + 180 = -45$$

b. Find the exact height of the mountain, above the road, in metres. (1 mark)

Solution: $\frac{18}{25} = 0.72$ km $= 720$ metres.

c. Find the length of the tunnel and the length of the bridge. Give your answers correct to the nearest metre. (3 marks)

Solution: Let $f(x) = \frac{1}{250}\left(2x^3 - 19x^2 + 180\right)$. We solve $f(x) = 0$, which yields

$$x = -2.71446 \text{ or } x = 4.0719 \text{ or } x = 8.14256$$

Therefore bridge length $= 4.07066$ km $= 4071$ metres.
Tunnel length $= 6.78637$ km $= 6786$ metres.

d. Find the maximum depth of the canyon below the road. Give your answer to the nearest metre. (1 mark)

Solution: 296 metres.

Question 12 (11 marks)

Consider the cubic polynomial $f(x) = x^3 + x^2 - 5x - 2$.

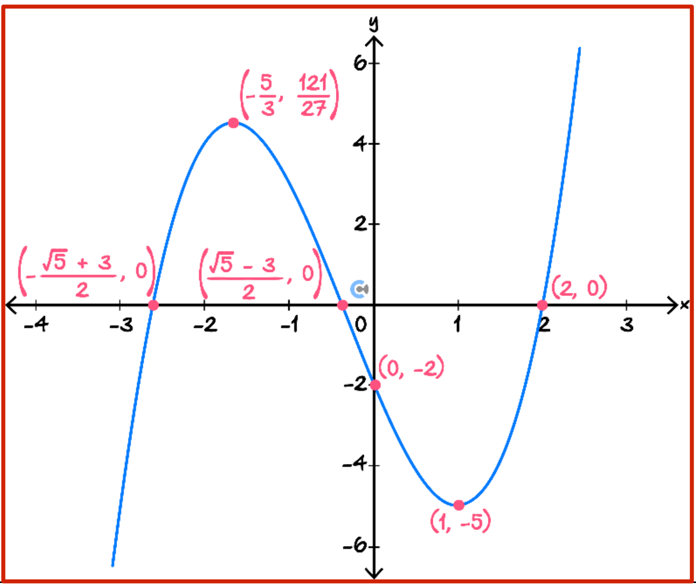
a. Explain why $f(x)$ must have a root between $x = 1$ and $x = 3$. (1 mark)

Solution: $f(1) = -5$ and $f(3) = 19$
so there must be a root between $x = 1$ and $x = 3$.

b. Write $f(x)$ in the form $f(x) = (x - a)Q(x)$ where $a > 0$ and $Q(x)$ is a quadratic function. (1 mark)

Solution: $f(x) = (x-2)(x^2 + 3x + 1)$

c. It is known that the graph of $y = f(x)$ has turning points at x-values that are solutions to the equation $3x^2 + 2x - 5 = 0$. Sketch the graph of $y = f(x)$ on the axes below. Label all axes intercepts and turning points with exact coordinates. (4 marks)



Solution: x-intercepts: $x = 2, \frac{1}{2}\left(-\sqrt{5} - 3\right), \frac{1}{2}\left(\sqrt{5} - 3\right)$
y-intercept: $(0, -2)$
Turning points: $\left(-\frac{5}{3}, \frac{121}{27}\right), (1, -5)$

d. Find the values of b such that $f(x) = b$ has:

i. One solution. (2 marks)

Solution: $b < -5$ or $b > \frac{121}{27}$

ii. Two solutions. (1 mark)

Solution: $b = -5$ or $b = \frac{121}{27}$

e. Find the values of k for which the equation $x^3 + (k-2)x^2 + (1-2k)x - 2 = 0$ has three solutions. (2 marks)

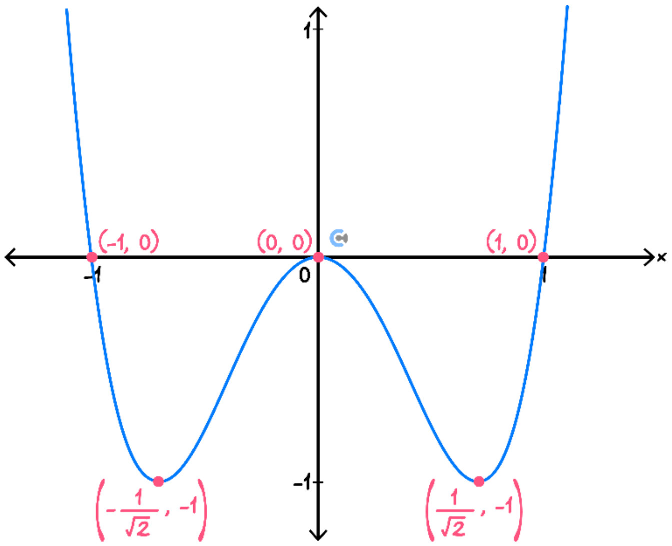
Solution: We can write this as $(x-2)(x^2 + kx + 1) = 0$.
Three solutions if the quadratic has two solutions.
Therefore $k^2 - 4 > 0 \implies k < -2$ or $k > 2$

Space for Personal Notes

Section F: Extension Exam 1 (15 Marks)

Question 13 (4 marks)

The function $f(x)$ is a polynomial of degree 4. The graph of f is shown below.



a. Find the rule of $f(x)$. (2 marks)

Solution: $f(x) = 4x^2(x+1)(x-1)$

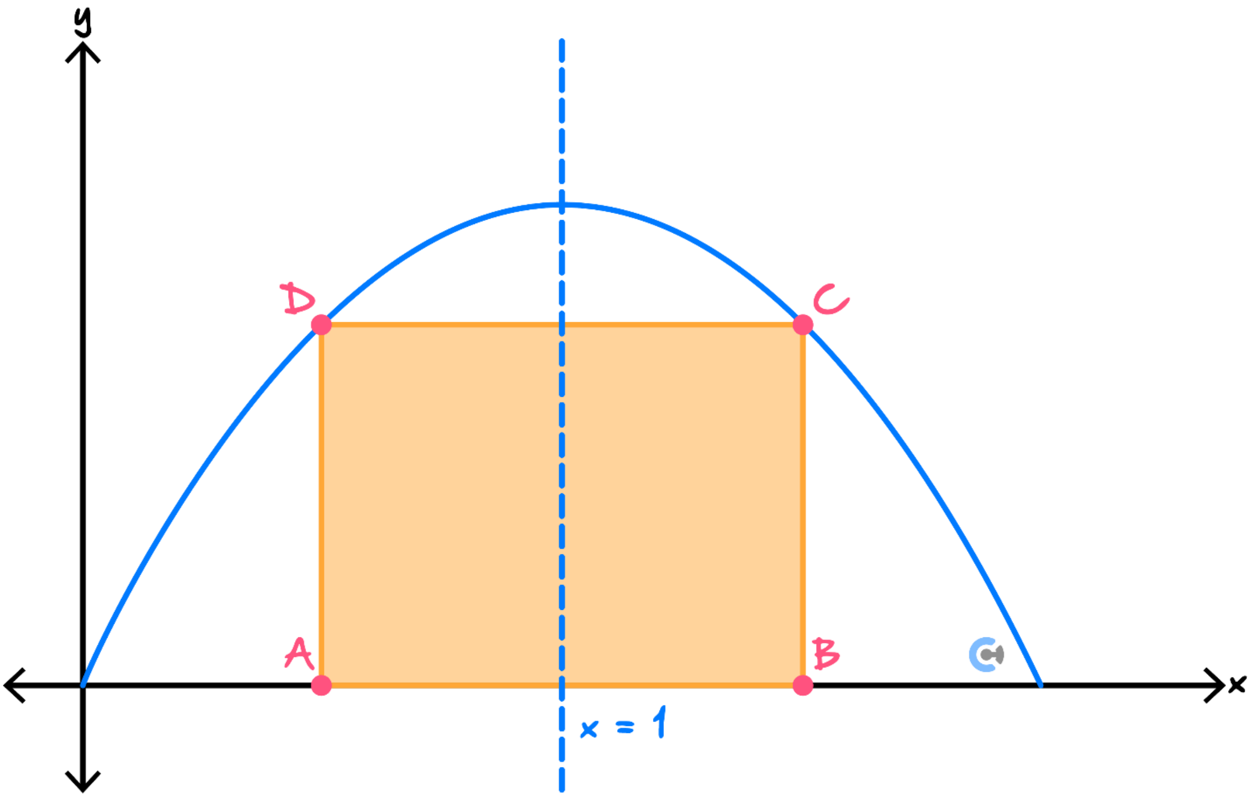
b. Find the values of k such that $f(x) + k = 0$, where k is a real number, has an even number of real solutions. (2 marks)

Solution: Only time there is an odd number of solutions is when $k = 0$. Therefore $k \neq 0$.

Question 14 (6 marks)

Consider the parabola $p(x) = x(2 - x)$, where $0 \leq x \leq 2$.

A rectangle $ABCD$ is inscribed between the graph of p and the x-axis. Its vertices are a distance of a units from the axis of symmetry, $x = 1$, as shown below.



a. Find the value of a when the rectangle is a square. (3 marks)

Solution: We need base equal to height.

$$\begin{aligned} p(a+1) &= 2a \\ (a+1)(1-a) &= 2a \\ 1 - a^2 &= 2a \\ a^2 + 2a &= 1 \\ (a+1)^2 &= 2 \\ a &= -1 \pm \sqrt{2} \end{aligned}$$

Only $a = \sqrt{2} - 1$ is a valid solution.

b. Find the rational value of a such that the rectangle $ABCD$ has an area of $\frac{3}{4}$ square units. (3 marks)

Solution: Area is given by $2a \times p(a+1) = 2a(1+a)(1-a) = 2a - 2a^3$.
We must solve $2a - 2a^3 = \frac{3}{4} \implies 8a - 8a^3 = 3$

$$8a^3 - 8a + 3 = 0$$

Use the rational root theorem to find that $a = \frac{1}{2}$ is a solution.

Space for Personal Notes

Question 15 (5 marks)

Consider the function $g(x) = (x^2 - 4kx + 3)(x^2 - 2x + k)$, where k is a real number.
Find all possible values of k such that $g(x)$ has:

a. Four real roots. (3 marks)

Solution: We require the discriminant for both quadratic functions to be greater than zero.

$$\Delta_1 = 16k^2 - 12 > 0 \implies k < -\frac{\sqrt{3}}{2} \text{ or } k > \frac{\sqrt{3}}{2}$$
$$\Delta_2 = 4 - 4k > 0 \implies k < 1$$

Therefore $g(x)$ has four real roots for $k < -\frac{\sqrt{3}}{2}$ or $\frac{\sqrt{3}}{2} < k < 1$

b. Two real roots. (2 marks)

Solution: First quadratic no roots for $-\frac{\sqrt{3}}{2} < k < \frac{\sqrt{3}}{2}$ and second quadratic two roots for $k < 1$. This gives two roots for $-\frac{\sqrt{3}}{2} < k < \frac{\sqrt{3}}{2}$

Second quadratic no roots for $k > 1$ and first quadratic will give two roots for $k > \frac{\sqrt{3}}{2}$.
However, note that when $k = 1$,

$$g(x) = (x^2 - 4x + 3)(x^2 - 2x + 1) = (x-1)(x-3)(x-1)^2 = (x-1)^3(x-3)$$

so two roots when $k = 1$.
Conclude that there are two real roots when $-\frac{\sqrt{3}}{2} < k < \frac{\sqrt{3}}{2}$ or $k \geq 1$

Section G: Extension Exam 2 (13 Marks)

Question 16 (1 mark)

Let $f(x) = x^3 + 3x^2 - 4x + 8$. The remainder when $f(x)$ is divided by $5x - 4$ is:

A. 104

B. 188

C. $\frac{904}{125}$

D. $\frac{617}{64}$

Question 17 (1 mark)

Consider the quartic $y = (x-2)^2(x^2 + 4kx + 6)$. It is known that the quartic has three distinct x-intercepts. The possible values of k are:

A. $k < -\sqrt{\frac{3}{2}}$ or $k > \sqrt{\frac{3}{2}}$

B. $-\sqrt{\frac{3}{2}} < k < \sqrt{\frac{3}{2}}$

C. $k = \pm\sqrt{\frac{3}{2}}$

D. $k < -\sqrt{3}$ or $k > \sqrt{3}$

Question 18 (1 mark)

A graph with rule $f(x) = x^3 - 3x^2 + c$, where c is a real number, has one distinct x-intercept.
All possible values of c are:

A. $c > 4$

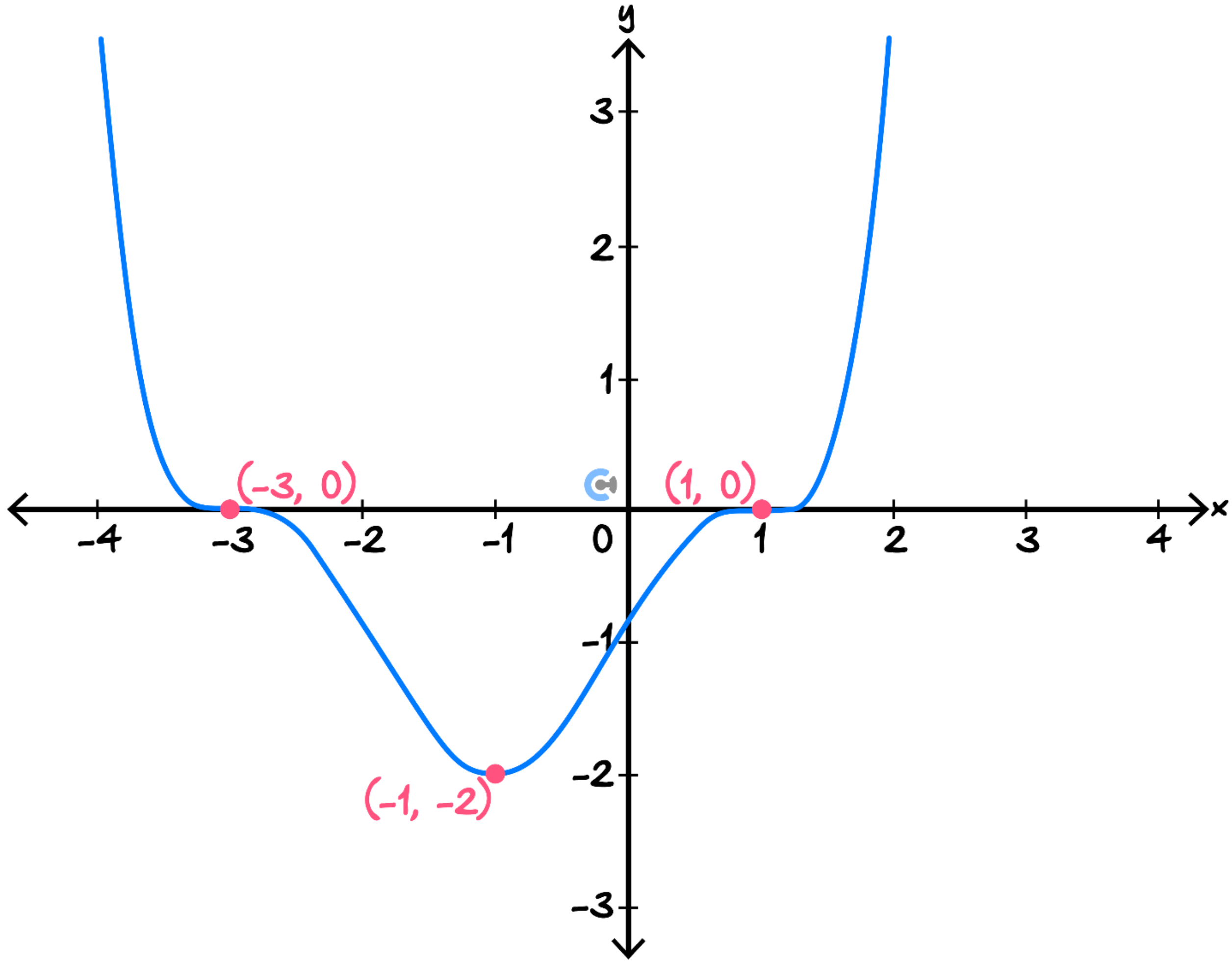
B. $0 < c < 4$

C. $c < 0$ or $c > 4$

D. $c > 4$

Question 19 (10 marks)

Consider the function of the form $f(x) = a(x-b)^3(x-c)^3$, where $b > c$, depicted on the graph below.



a. Find the values of a, b, and c. (2 marks)

Solution: From the inflection points we see that $b = 1$ and $c = -3$. Then using the point $(-1, -2)$ we find that $a = \frac{1}{32}$.

$$f(x) = \frac{1}{32}(x+3)^3(x-1)^3$$

b. Show that $x = -1$ is an axis of symmetry for the graph of f. (2 marks)

Solution:

$$\begin{aligned} f(-1+m) &= \frac{1}{32}(m+2)^3(m-2)^3 \\ &= \frac{1}{32}(-m-2)^3(2-m)^3 \\ &= f(-1-m) \end{aligned}$$

since this holds for any $m \in \mathbb{R}$, $x = -1$ is an axis of symmetry for f.

c. Find the value of $d > 0$ such that $f(x) + d = 0$ has one real solution. (1 mark)

Solution: $d = 2$

d. Consider the function $g(x) = (x + k + 3)^3(x + k - 1)^3$, where $k \in \mathbb{R}$.

i. Find the roots of g in terms of k. (1 mark)

Solution: $x = -k - 3, 1 - k$

ii. Hence, find the values of k so that $g(x)$ has only positive roots. (2 marks)

Solution: We require both $-k - 3 > 0$ and $1 - k > 0$. This yields $k < -3$.

iii. A function h is said to be even if $h(x) = h(-x)$ for all x.
Find the value of k such that $g(x)$ is an even function. (2 marks)

Solution: $k = -1$

Space for Personal Notes

CONTOUREDUCATION

Website: contoureducation.com.au | **Phone:** 1800 888 300 | **Email:** hello@contoureducation.com.au

VCE Mathematical Methods ½

Free 1-on-1 Support

Be Sure to Make The Most of These (Free) Services!

- Experienced Contour tutors (45+ raw scores, 99+ ATARs).
- For fully enrolled Contour students with up-to-date fees.
- After school weekdays and all-day weekends.

1-on-1 Video Consults	Text-Based Support
➤ Book via bit.ly/contour-methods-consult-2025 (or QR code below). ➤ One active booking at a time (must attend before booking the next).	➤ Message +61 440 138 726 with questions. ➤ Save the contact as "Contour Methods".

Booking Link for Consults
bit.ly/contour-methods-consult-2025

Number for Text-Based Support
+61 440 138 726