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VCE Mathematical Methods ½
Polynomials Exam Skills [0.6]
Workshop

Error Logbook:



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Section A: Recap

Degree of Polynomial Functions



Degree = Highest Power of the Polynomial

Roots of Polynomial Functions



Roots = x-intercept

Polynomial Long Division



➤ Division of polynomials:

$$\begin{array}{r}
 \text{Quotient} \\
 \hline
 \text{Divisor} \overline{) \text{Dividend}} \\
 \vdots \\
 \text{Remainder}
 \end{array}$$

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

Space for Personal Notes



Remainder Theorem

➤ Definition:

- 🔗 Finds the remainder of long division without the need of long division,

when $P(x)$ is divided by $(x - \alpha)$, the remainder is $P(\alpha)$.

➤ Steps

1. Find x -values which make the divisor equal to 0.
2. Substitute it into the dividend function.



Factor Theorem

- For every x -intercept, there is a factor:

If $P(\alpha) = 0$ then, $(x - \alpha)$ is a factor of $P(x)$.



Factorising Polynomials

- The steps are:

- 🔗 Find a single root by trial and error.
 - (Factor Theorem: Substitute into the function and see if we get zero.)
- 🔗 Use long division to find the quadratic factor.
- 🔗 Factorise the quadratic.

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Rational Root Theorem

- Rational Root Theorem **narrows down** the possible roots.

$$\text{Potential root} = \pm \frac{\text{Factors of constant term } a_0}{\text{Factors of leading coefficient } a_n}$$

- If the roots are rational numbers, the roots can only be $\pm \frac{\text{Factors of constant term } a_0}{\text{Factors of leading coefficient } a_n}$.



Sum and Difference of Cubes

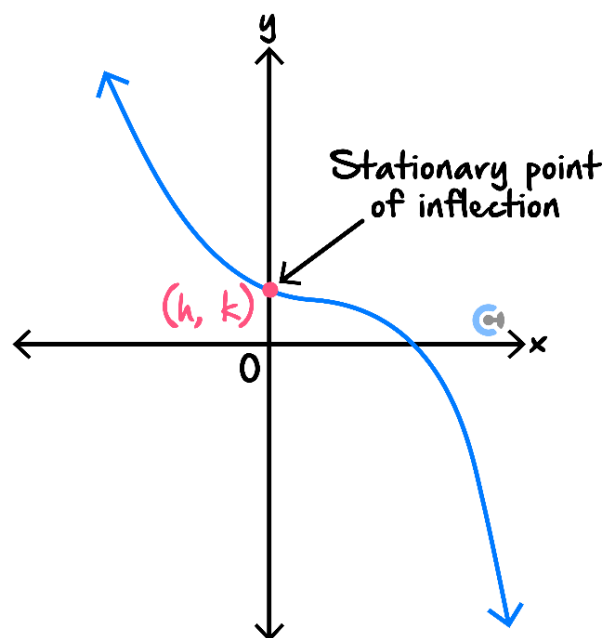
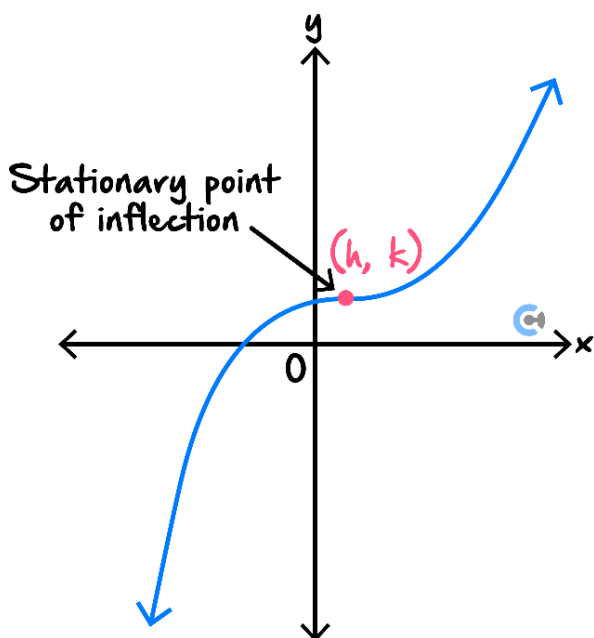
$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$



Graphs of $a(x - h)^n + k$, where n is an Odd Positive Integer

- All graphs look like a "cubic".

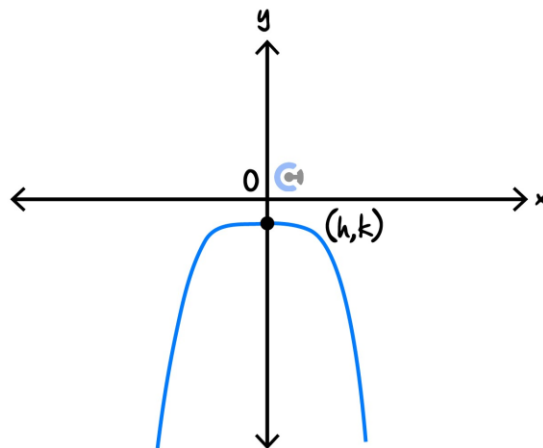
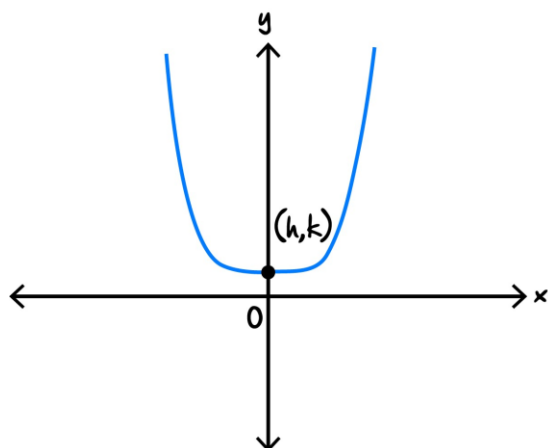


- The point (h, k) gives us the stationary point of inflection.
- n cannot be 1 for this shape to occur!



Graphs of $a(x - h)^n + k$, where n is an Even Positive Integer

- All graphs look like a "quadratic".

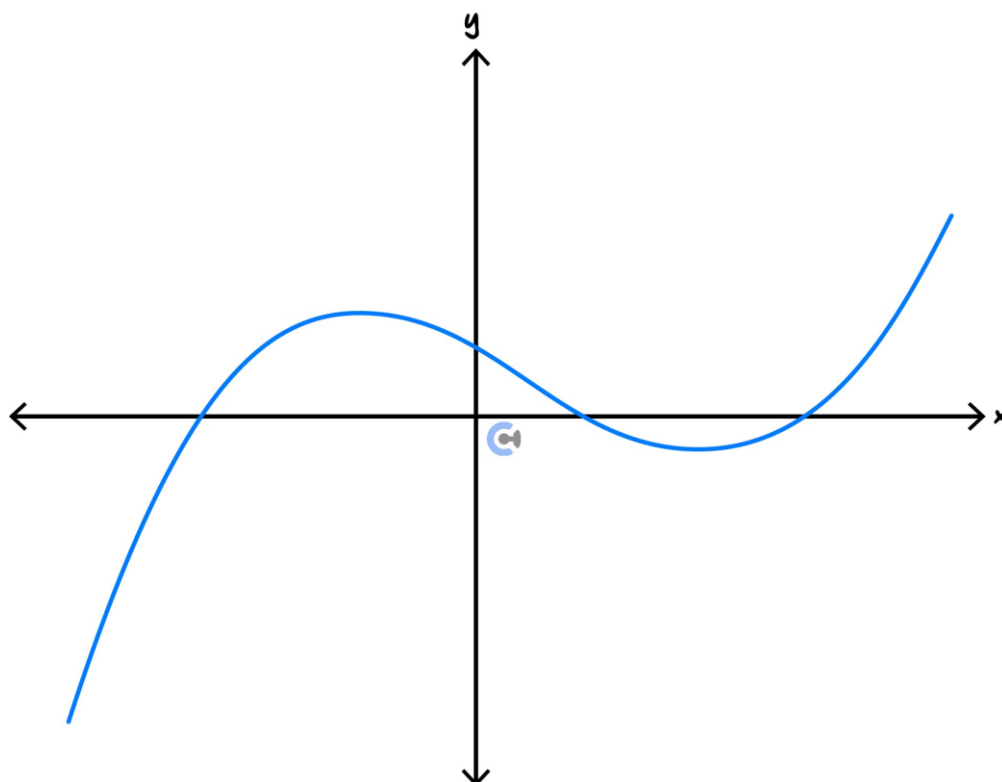


- The point (h, k) gives us the turning point.



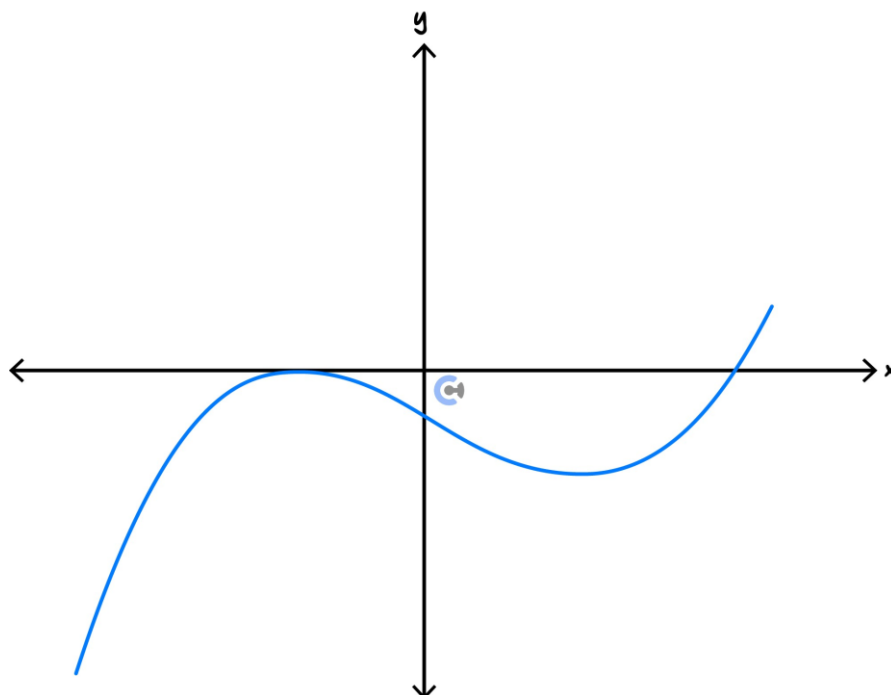
Graphs of Factorised Polynomials

- All non-repeated linear factors correspond to x -intercepts of the graph.

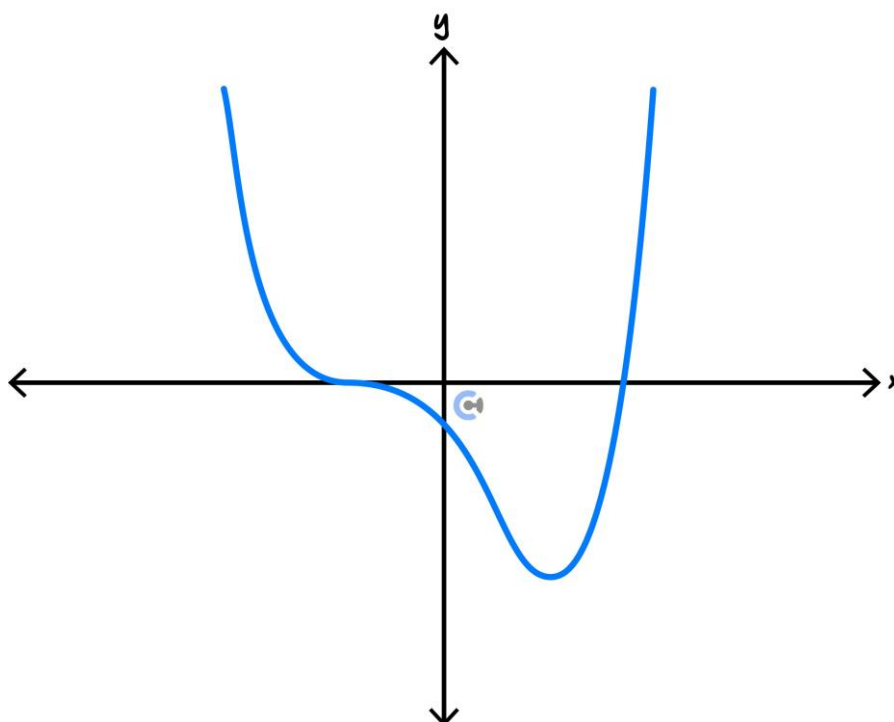


- E.g., $f(x) = (x - a)(x - b)(x - c)$ results in x -intercepts at $(a, 0)$, $(b, 0)$, and $(c, 0)$.

- All squared linear factors correspond to x -intercepts and T.P. of the graph.



- E.g., $f(x) = (x - a)^2(x - b)$ will have an x -intercept $(a, 0)$ which is also a local minimum/maximum.
- All cubed linear factors correspond to x -intercepts and SPI of the graph.



- E.g., $f(x) = (x - a)^3(x - b)$ has an x -intercept $(a, 0)$ which is also a stationary point of inflection.



Steps to Graphing Factorised Polynomials

➤ Steps:

1. Plot x -intercepts.
2. Determine whether the polynomial is positive or negative.
3. Use the repeated factors to deduce the shape.
 - Non - Repeated: Only x -intercept.
 - Even Repeated: x -intercept and a turning point.
 - Odd Repeated: x -intercept and a stationary point of inflection.



Solving the Polynomial Inequality $f(x) > 0$

➤ Steps:

1. Find the x -intercepts.
2. Sketch the polynomial.
3. Shade the places where the y -values are positive.



When does a cubic have n solutions?

➤ Steps:

1. Factorise out the x term.
2. Since the x term gives 1 solution, use discriminant to find when the quadratic has $n - 1$ solutions.

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Bisection Method

➤ Step 1: Pick a random interval $[a, b]$ where $f(a) \times f(b) = \text{Negative}$.

➤ Step 2: Find a midpoint to estimate the root.

$$\text{where } m = \frac{a+b}{2}$$

➤ Step 3: Create a new interval $[a, b]$ by making m either new a or new b .

$$\text{If } f(a) \times f(m) < 0$$

$$\text{New Interval: } [a, m]$$

$$\text{If } f(b) \times f(m) < 0$$

$$\text{New Interval: } [m, b]$$

➤ Step 4: Repeat until the interval becomes short enough for good accuracy.

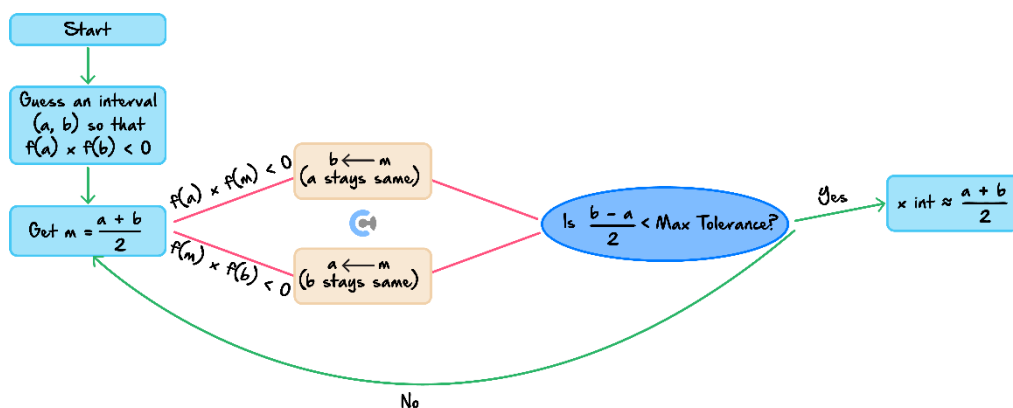
🔄 The smaller the interval $[a, b]$, more accurate our estimation gets.

$$\text{If } \frac{b-a}{2} < \text{Max Tolerance,}$$

We stop.

🔄 Maximum error is half of the width of the interval.

$$\text{Max Error} = \frac{b-a}{2}$$



Section B: Warmup**Question 1**

- a. Solve the inequality $x^3 + x + 6 > 4x^2$.

- b. Find the values of k such that $x^3 + 2kx^2 + 3x = 0$ has only one real solution.

- c. Apply the bisection method with initial interval $[1,2]$ and tolerance 0.1 to find an approximate solution to the equation $x^2 - 2 = 0$.

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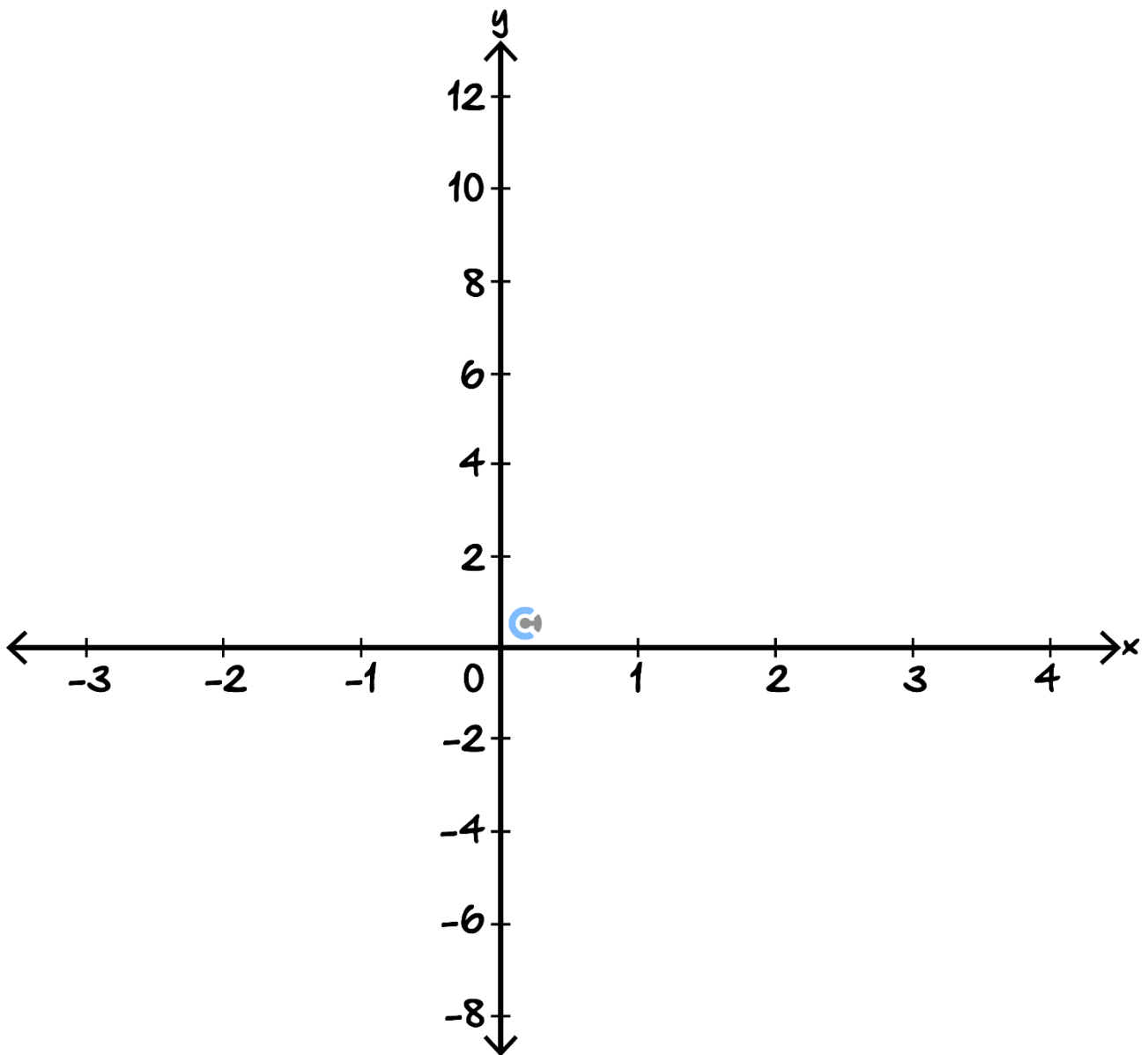
Section C: Exam 1 (23 Marks)**Question 2 (9 marks)**

Let $f(x) = ax^3 - 5x^2 + bx + 9$. When $f(x)$ is divided by $x - 2$ the remainder is -7 and when $f(x)$ is divided by $x + 1$ the remainder is 8 .

- a. Show that $a = 2$ and $b = -6$. (2 marks)

- b. Express $f(x)$ as the product of three linear factors. (3 marks)

- c. Sketch the graph of $y = f(x)$ on the axes below. Label all axes intercepts. Note that f has turning points at approximately $(-0.5, 10.5)$ and $(2.1, -7.1)$. (2 marks)



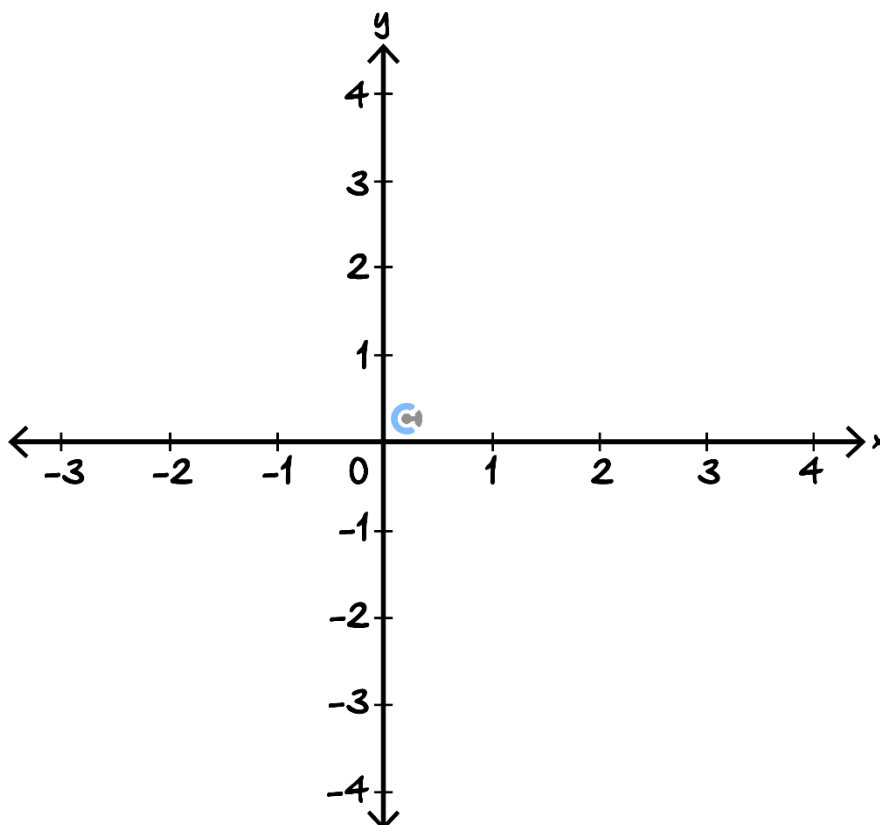
- d. Hence, solve the inequality $2x^3 - 5x^2 - 6x > -9$. (2 marks)

Question 3 (5 marks)

Consider the function $f(x) = \frac{x^3}{4} - \frac{3x^2}{4} + \frac{3x}{4} + \frac{7}{4}$.

- a.** Write $f(x)$ in the form $a(x + b)^3 + c$ for real values a , b , and c . (2 marks)

- b.** Sketch the graph of $y = f(x)$ on the axes below. Label any axes intercepts and stationary points of inflection with coordinates. (3 marks)



Question 4 (4 marks)

The bisection method may be used to approximate $\sqrt{3}$ by finding a root to $x^2 - 3 = 0$.

- a.** Use the bisection method with initial interval $[1,2]$ and tolerance 0.1 to find an approximate solution to $x^2 - 3 = 0$. Leave your answer in the form $\frac{a}{b}$, for positive integers a and b . (3 marks)

- b.** Determine whether $\frac{7}{4}$ is more than or less than $\sqrt{3}$. (1 mark)

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Question 5 (5 marks)

Consider $f(x) = x^3 - 2kx^2 + 4kx + 4x$, where k is a real constant.

Find the values of k such that $f(x) = 0$ has:

a. One solution. (3 marks)

b. Two solutions. (1 mark)

c. Three solutions. (1 mark)

Section D: Tech Active Exam Skills



Calculator Commands: Turning Point

- ALWAYS sketch the graph to find approximate bounds for where the turning point you want is located.
- To find a local maximum we maximise the function over a specific domain.
- To find a local minimum we minimise the function over a specific domain.
- **TI and Casio:** Use $fmin(expression, variable, lower (optional), upper (optional))$ or $fmax(expression, variable, lower (optional), upper (optional))$.
- **TI:** Menu → 4 → $\frac{7}{8}$.

Define $f(x) = x^3 - 4 \cdot x$

Done

$fMin(f(x), x, 0, 2)$

$$x = \frac{2 \cdot \sqrt{3}}{3}$$

$f\left(\frac{2 \cdot \sqrt{3}}{3}\right)$

$$\frac{-16 \cdot \sqrt{3}}{9}$$

- **Casio:** Action → Calculation → $fmin/fmax$

$fmin(x^3 - 4x, x, 0, 2)$

$$\left\{ \text{MinValue} = \frac{-16 \cdot \sqrt{3}}{9}, x = \frac{2 \cdot \sqrt{3}}{3} \right\}$$

NOTE: TI only gives the x -value for the min/max so we then need to sub it back into our function. Casio gives us both!





Calculator Commands

- **Mathematica:** Minimise[] and Maximise[] commands.
- Minimise[$f[x], x$] will minimise $f[x]$ over its whole domain.
- To restrict the domain, we must use Minimise[{ $f[x], a \leq x \leq b$ }, x].

In[34]:= Minimise[{ $x^3 - 4x$, $0 < x < 2$ }, x]

Out[34]= $\left\{-\frac{16}{3\sqrt{3}}, \left\{x \rightarrow \frac{2}{\sqrt{3}}\right\}\right\}$

TI UDF: Bisection Method

➤ Overview:

- 🔗 Apply the bisection method to a function to approximate x -intercepts.

➤ Input:

- 🔗 bisection(< function >, < variable >, < lower bound >, < upper bound >)

➤ Other Notes:

- 🔗 The program will ask for the threshold type to terminate the algorithm.
- 🔗 Select None [0] to provide a specific number of iterations.
- 🔗 Select x [1] to provide a threshold for $b - a$, after which the program will stop if $b - a$ becomes smaller than the threshold.
- 🔗 Select y [2] to provide a threshold for $|f(b) - f(a)|$, after which the program will stop if $|f(b) - f(a)|$ becomes smaller than the threshold.

bisection($x^2 - 2, x, 0, 1$)

Number of Iterations: 5

"n"	"a"	"m"	"b"	"f(a)"	"f(m)"	"f(b)"	"b-a"	" f(b)-f(a) "
0.	0.	0.5	1.	-2.	-1.75	-1.	1.	1.
1.	0.5	0.75	1.	-1.75	-1.4375	-1.	0.5	0.75
2.	0.75	0.875	1.	-1.4375	-1.23438	-1.	0.25	0.4375
3.	0.875	0.9375	1.	-1.23438	-1.12109	-1.	0.125	0.234375
4.	0.9375	0.96875	1.	-1.12109	-1.06152	-1.	0.0625	0.121094
5.	0.96875	0.984375	1.	-1.06152	-1.03101	-1.	0.03125	0.061523



Section E: Exam 2 (25 Marks)

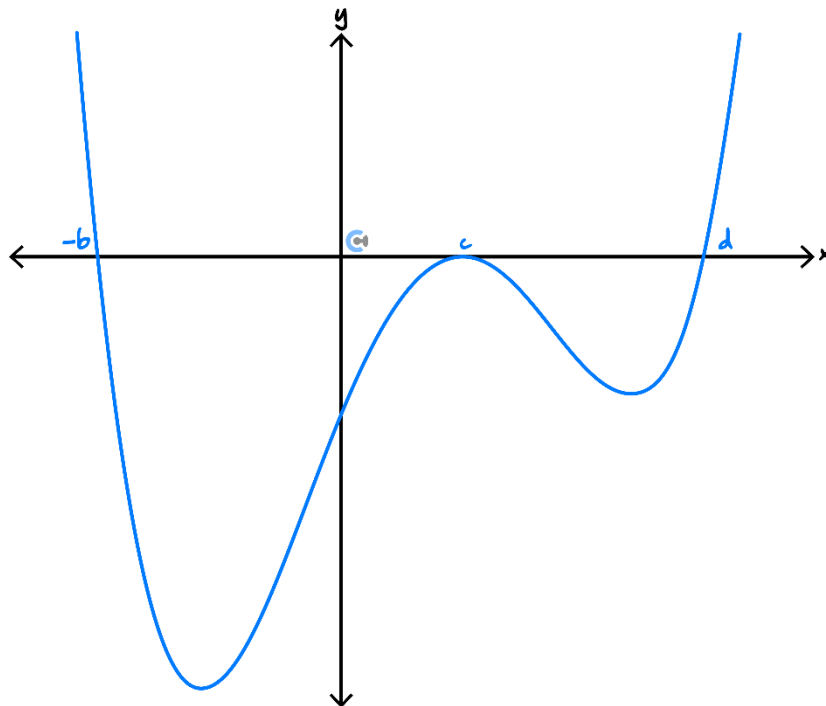
Question 6 (1 mark)

The equation $2x^3 - 3x - 4 = 0$ has one real solution, which lies in the interval $[1, 2]$. Approximate the solution using the bisection method with a maximum error of 0.1. What is the approximate solution?

- A. $x \approx 1.655$
- B. $x \approx 1.6250$
- C. $x \approx 1.6875$
- D. $x \approx 1.6225$

Question 7 (1 mark)

The rule for a function with the graph below, where $b, c, d > 0$, could be:



- A. $y = -2(x + b)(x - c)^2(x - d)$
- B. $y = 3(x + b)(x - c)^2(x - d)$
- C. $y = -2(x - b)(x - c)^2(x - d)$
- D. $y = 2(x - b)(x - c)^2(x - d)$

Question 8 (1 mark)

The polynomial $x^3 + (a + 2)x^2 + bx + 8$ is perfectly divisible by $x + 2$ and has remainder of 2 when divided by $x - 3$. The values (a, b) are:

- A. $(-5, -6)$
- B. $\left(-\frac{21}{5}, -\frac{22}{5}\right)$
- C. $\left(-\frac{3}{5}, -\frac{9}{5}\right)$
- D. $\left(-\frac{7}{5}, \frac{3}{5}\right)$

Question 9 (1 mark)

All real values of x that satisfy the inequality $9x^2 - 2x^3 > 54 - 27x$ are:

- A. $x < -3$ or $\frac{3}{2} < x < 6$.
- B. $x < -6$ or $\frac{3}{2} < x < 3$.
- C. $x < -3$ or $x > -\frac{3}{2}$.
- D. $-3 < x < \frac{3}{2}$ or $x > 6$.

Question 10 (1 mark)

The equation $x^3 - 3kx^2 + 5x = 0$ has exactly one solution when:

- A. $k = \pm \frac{2\sqrt{5}}{3}$
- B. $-\frac{2\sqrt{5}}{3} < k < \frac{2\sqrt{5}}{3}$
- C. $k > \frac{2\sqrt{5}}{3}$
- D. $k < -\frac{2\sqrt{5}}{3}$

Question 11 (9 marks)

A car is travelling along a straight road from A to C .
The car will travel along a section of road ABC .

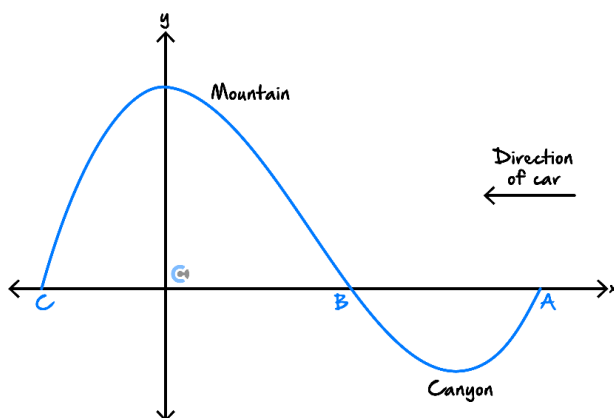
- Section AB passes along a bridge over a canyon.
- Section BC passes through a tunnel in a mountain.

From A to C , the curve of the canyon and then mountain, directly below and above the road, is modelled by the graph of:

$$y = \frac{1}{250}(px^3 + qx^2 + r)$$

Where p , q , and r are real constants.

All measurements are in kilometres and a diagram of this situation is shown below.



- a. The curve defined from A to C passes through the points $(1, 0.652)$, $(2, 0.48)$, and $(5, -0.18)$.
- i. Use this information to write down three simultaneous equations in terms of p , q , and r .
Write these equations with integer coefficients. (3 marks)

ii. Hence, verify that $p = 2$, $q = -19$, and $r = 180$. (1 mark)

b. Find the exact height of the mountain, above the road, in metres. (1 mark)

c. Find the length of the tunnel and the length of the bridge. Give your answers correct to the nearest metre. (3 marks)

d. Find the maximum depth of the canyon below the road. Give your answer to the nearest metre. (1 mark)

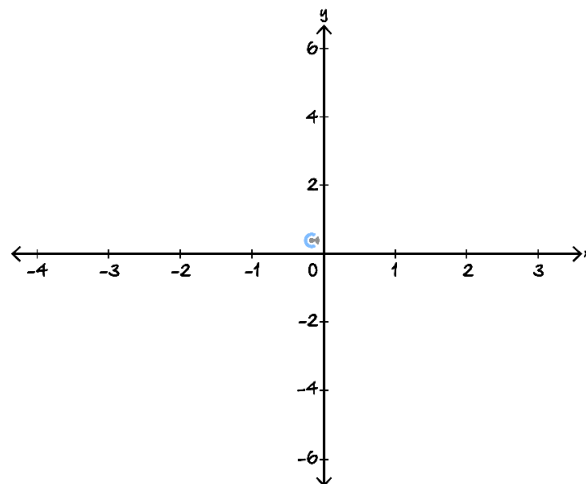
Question 12 (11 marks)

Consider the cubic polynomial $f(x) = x^3 + x^2 - 5x - 2$.

- a. Explain why $f(x)$ must have a root between $x = 1$ and $x = 3$. (1 mark)

- b. Write $f(x)$ in the form $f(x) = (x - a)Q(x)$ where $a > 0$ and $Q(x)$ is a quadratic function. (1 mark)

- c. It is known that the graph of $y = f(x)$ has turning points at x -values that are solutions to the equation $3x^2 + 2x - 5 = 0$. Sketch the graph of $y = f(x)$ on the axes below. Label all axes intercepts and turning points with exact coordinates. (4 marks)



d. Find the values of b such that $f(x) = b$ has:

i. One solution. (2 marks)

ii. Two solutions. (1 mark)

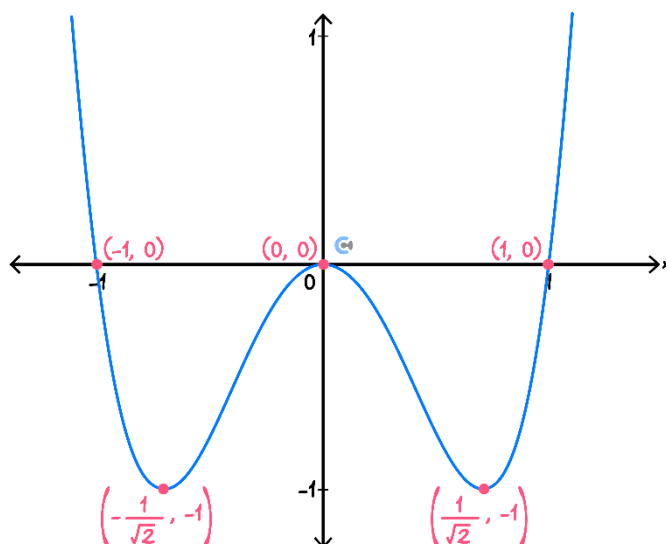
e. Find the values of k for which the equation $x^3 + (k - 2)x^2 + (1 - 2k)x - 2 = 0$ has three solutions. (2 marks)

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Section F: Extension Exam 1 (15 Marks)

Question 13 (4 marks)

The function $f(x)$ is a polynomial of degree 4. The graph of f is shown below.



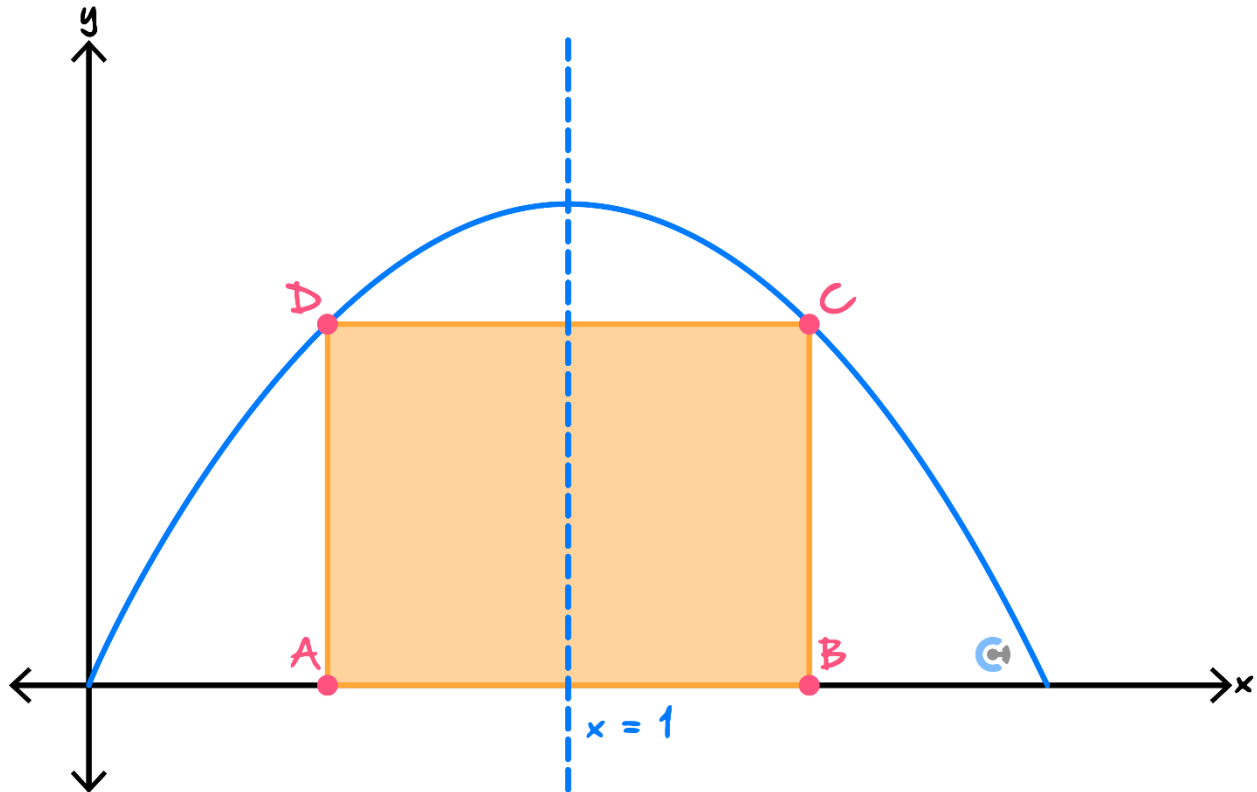
- a. Find the rule of $f(x)$. (2 marks)

- b. Find the values of k such that $f(x) + k = 0$, where k is a real number, has an even number of real solutions. (2 marks)

Question 14 (6 marks)

Consider the parabola $p(x) = x(2 - x)$, where $0 \leq x \leq 2$.

A rectangle $ABCD$ is inscribed between the graph of p and the x -axis. Its vertices are a distance of a units from the axis of symmetry, $x = 1$, as shown below.



- a. Find the value of a when the rectangle is a square. (3 marks)

- b. Find the rational value of a such that the rectangle $ABCD$ has an area of $\frac{3}{4}$ square units. (3 marks)

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Question 15 (5 marks)

Consider the function $g(x) = (x^2 - 4kx + 3)(x^2 - 2x + k)$, where k is a real number.
Find all possible values of k such that $g(x)$ has:

- a.** Four real roots. (3 marks)

- b.** Two real roots. (2 marks)

Section G: Extension Exam 2 (13 Marks)

Question 16 (1 mark)

Let $f(x) = x^3 + 3x^2 - 4x + 8$. The remainder when $f(x)$ is divided by $5x - 4$ is:

- A. 104
- B. 188
- C. $\frac{904}{125}$
- D. $\frac{617}{64}$

Question 17 (1 mark)

Consider the quartic $y = (x - 2)^2(x^2 + 4kx + 6)$. It is known that the quartic has three distinct x -intercepts. The possible values of k are:

- A. $k < -\sqrt{\frac{3}{2}}$ or $k > \sqrt{\frac{3}{2}}$
- B. $-\sqrt{\frac{3}{2}} < k < \sqrt{\frac{3}{2}}$
- C. $k = \pm\sqrt{\frac{3}{2}}$
- D. $k < -\sqrt{3}$ or $k > \sqrt{3}$

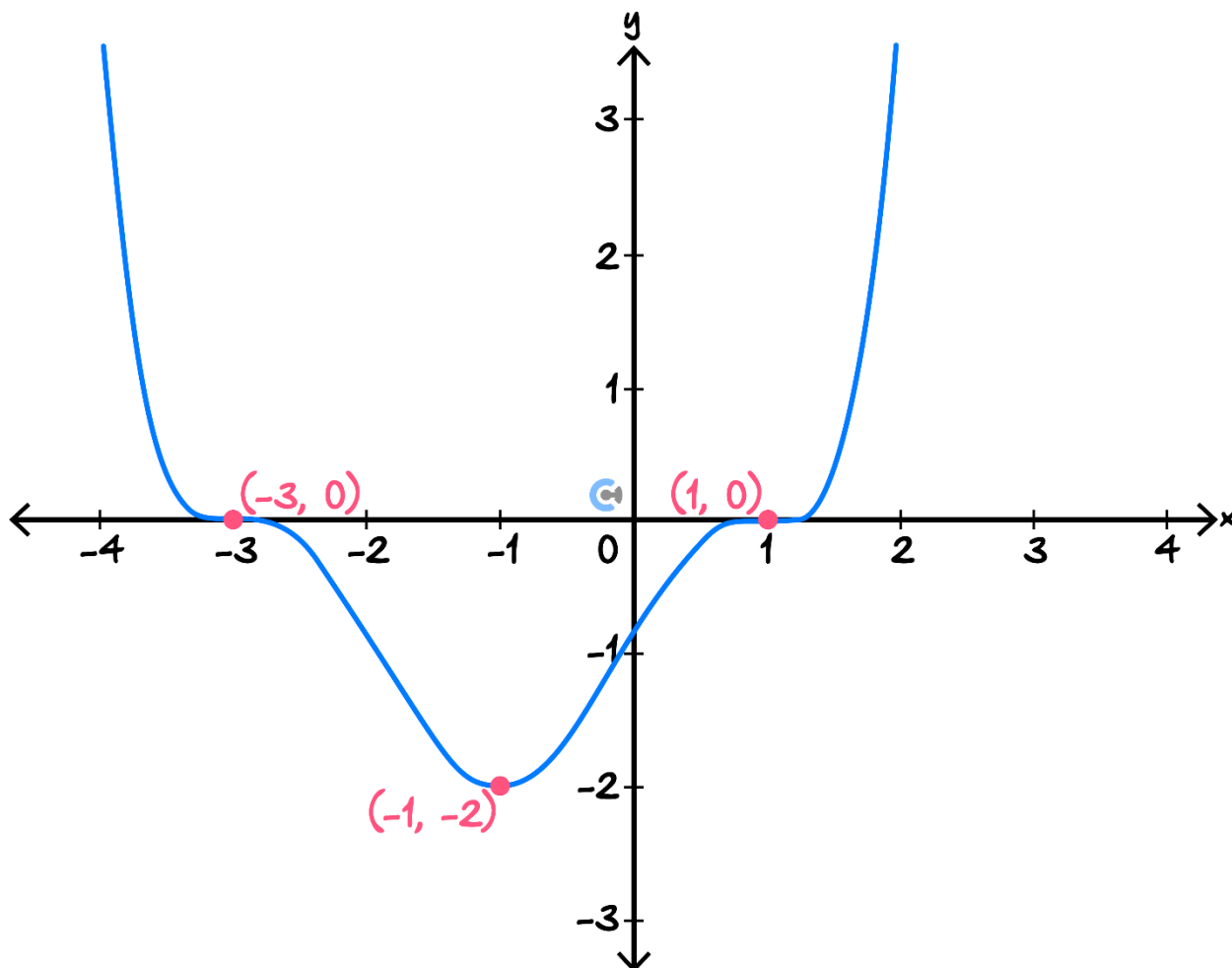
Question 18 (1 mark)

A graph with rule $f(x) = x^3 - 3x^2 + c$, where c is a real number, has one distinct x -intercept. All possible values of c are:

- A. $c > 4$
- B. $0 < c < 4$
- C. $c < 0$ or $c > 4$
- D. $c > 4$

Question 19 (10 marks)

Consider the function of the form $f(x) = a(x - b)^3(x - c)^3$, where $b > c$, depicted on the graph below.



- a. Find the values of a , b , and c . (2 marks)

b. Show that $x = -1$ is an axis of symmetry for the graph of f . (2 marks)

c. Find the value of $d > 0$ such that $f(x) + d = 0$ has one real solution. (1 mark)

d. Consider the function $g(x) = (x + k + 3)^3(x + k - 1)^3$, where $k \in \mathbb{R}$.

i. Find the roots of g in terms of k . (1 mark)

ii. Hence, find the values of k so that $g(x)$ has only positive roots. (2 marks)

iii. A function h is said to be even if $h(x) = h(-x)$ for all x .
Find the value of k such that $g(x)$ is an even function. (2 marks)

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