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VCE Mathematical Methods ½
Quadratics Exam Skills [0.4]
Workshop Solutions

Section A: Recap

Sub-Section: Factorising Quadratics

Factorising Quadratics

$$y = (x - a)(x - b)$$

- **Steps:**

1. Divide by the coefficient of the leading term. (If applicable)
2. Consider the factors of the constant term.
3. (If Positive Constant Term): See which pair of factors can add up to the coefficient of the x term.

   (If Negative Constant Term): See which pair of factors can subtract from the coefficient of the x term.
4. Construct the linear factors.

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Sub-Section: Perfect Squares

Perfect Squares

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

- Perfect squares are special quadratic expressions that are made up of two **identical** linear factors.
- In other words, when a linear factor is squared, it becomes a perfect square.

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Sub-Section: Difference of Squares

Difference of Squares

$$a^2 - b^2 = (a + b)(a - b)$$

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Sub-Section: Completing the Square

Completing the Square

- When we complete the square of a quadratic $x^2 + bx + c$, we write it in the form:

$$x^2 + bx + c = \left(\boxed{x + \frac{b}{2}}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$

- **Steps:**

1. We halve the coefficient of x.

2. Subtract the half of the coefficient of x squared outside the square bracket.

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Sub-Section: Solving by Factorisation

Solving by Factorisation

$$(x-a)(x-b)=0$$

$$x = a \text{ or } b$$

- **Steps:**

1. Factorise the quadratic.
2. Equate each factor to 0 and solve for x.

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Sub-Section: Quadratic Formula

The Quadratic Formula

for $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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Sub-Section: Discriminant

The Discriminant

- **Definition:**

  - The discriminant, often denoted by **Δ** (Delta), is the part **inside** the square root of the quadratic formula.

$$Discriminant = \Delta = b^2 - 4ac$$

if $\Delta > 0$, there are **two solutions**.

if $\Delta = 0$, there is **one solution**.

if $\Delta < 0$, there are **no solutions**.

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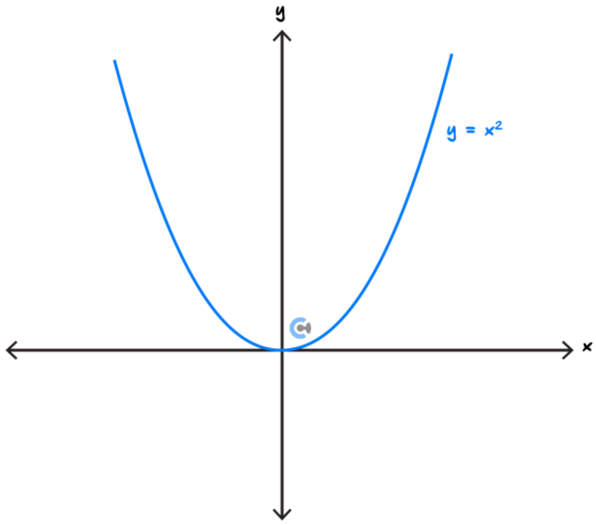
Sub-Section: Parabola and Symmetry

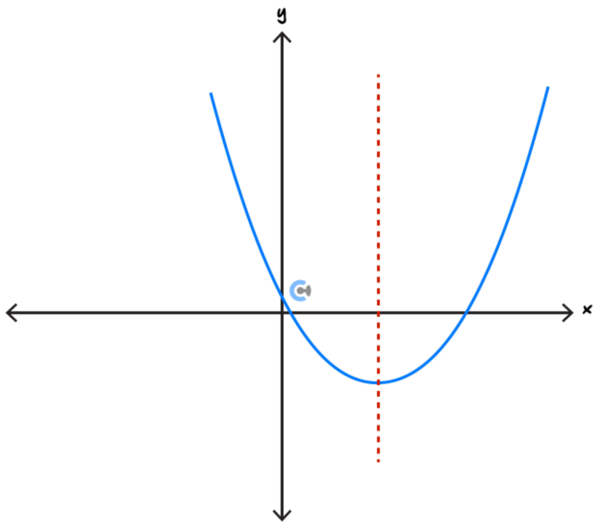
Parabola

- **Definition:**

  - The shape of the graph of a quadratic is known as a ______parabola______.



Axis of Symmetry



$$\textit{Axis of symmetry}: x = -\frac{b}{2a}$$

Sub-Section: Graphing Quadratics

Turning Point Form

- The turning point form of a quadratic is given by:

$$y = a(x-h)^2 + k$$

$$\textit{Turning point} = \underline{\quad (h, k) \quad}$$

- The turning point form is obtained by **completing the square.**

Intercept Form

- The x-intercept form of a quadratic is given by:

$$y = a(x-b)(x-c)$$

$$x\textit{-intercepts}: (b, 0) \textit{ and } (c, 0)$$

- The axis of symmetry is located exactly in the middle of the two x-intercepts.

NOTE: When a is negative, the x-intercepts stay the same, but the **shape** of the parabola becomes a **negative** parabola instead.

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Sub-Section: Finding a Rule of a Quadratic from a Graph

Finding the Equation of a Quadratic

- **Form 1: Turning Point Form**

$$y = a(x - h)^2 + k$$

  - Recommended when a turning point is easy to identify.

- **Form 2: x-intercept Form**

$$y = a(x - b)(x - c)$$

  - Recommended when both x-intercepts are easy to identify.

NOTE: Never forget the a coefficient!

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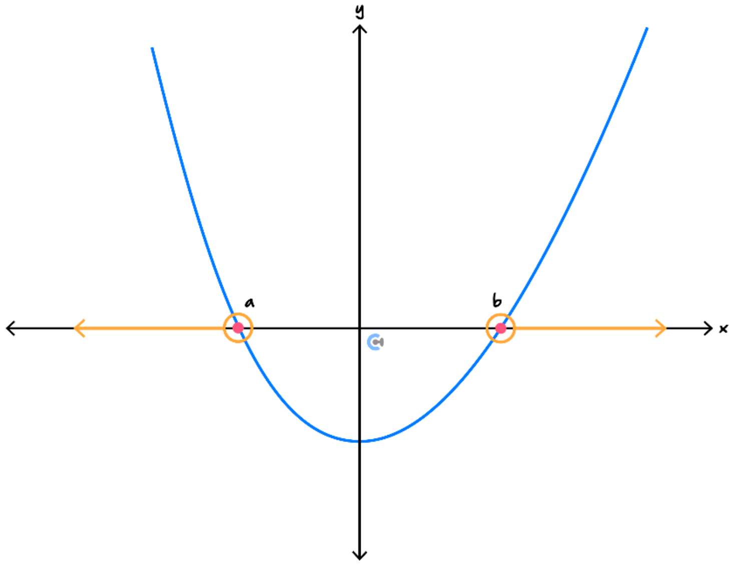
Sub-Section: Quadratic Inequalities

Quadratic Inequalities



- For quadratic inequalities, we always ___ sketch ___ the function.
- **Steps:**

1. Sketch the function.
2. See where the y-value is within the inequality.
3. Find the corresponding x-values.

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Sub-Section: Hidden Quadratics

Hidden Quadratics

- Instead of:

$$af(x)^2 + bf(x) + c = 0$$

- We can let $f(x) = X$ to have:

$$aX^2 + bX + c = 0$$

Completing the square quickly.

$$y = a(x - h)^2 + k$$

- **Steps**

1. Find the turning point using $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.
2. Use the leading coefficient as a.

Modelling with Quadratics

- Focus on key points such as turning points, x-intercepts and y-intercepts.

Family of Functions

- **Definition:** Functions with unknowns.
- **Question Type:** Find the unknown value to satisfy a certain condition.

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Section B: Warmup

Question 1

Let $f(x) = -\frac{x^2}{2} + 3x - \frac{5}{2}$.

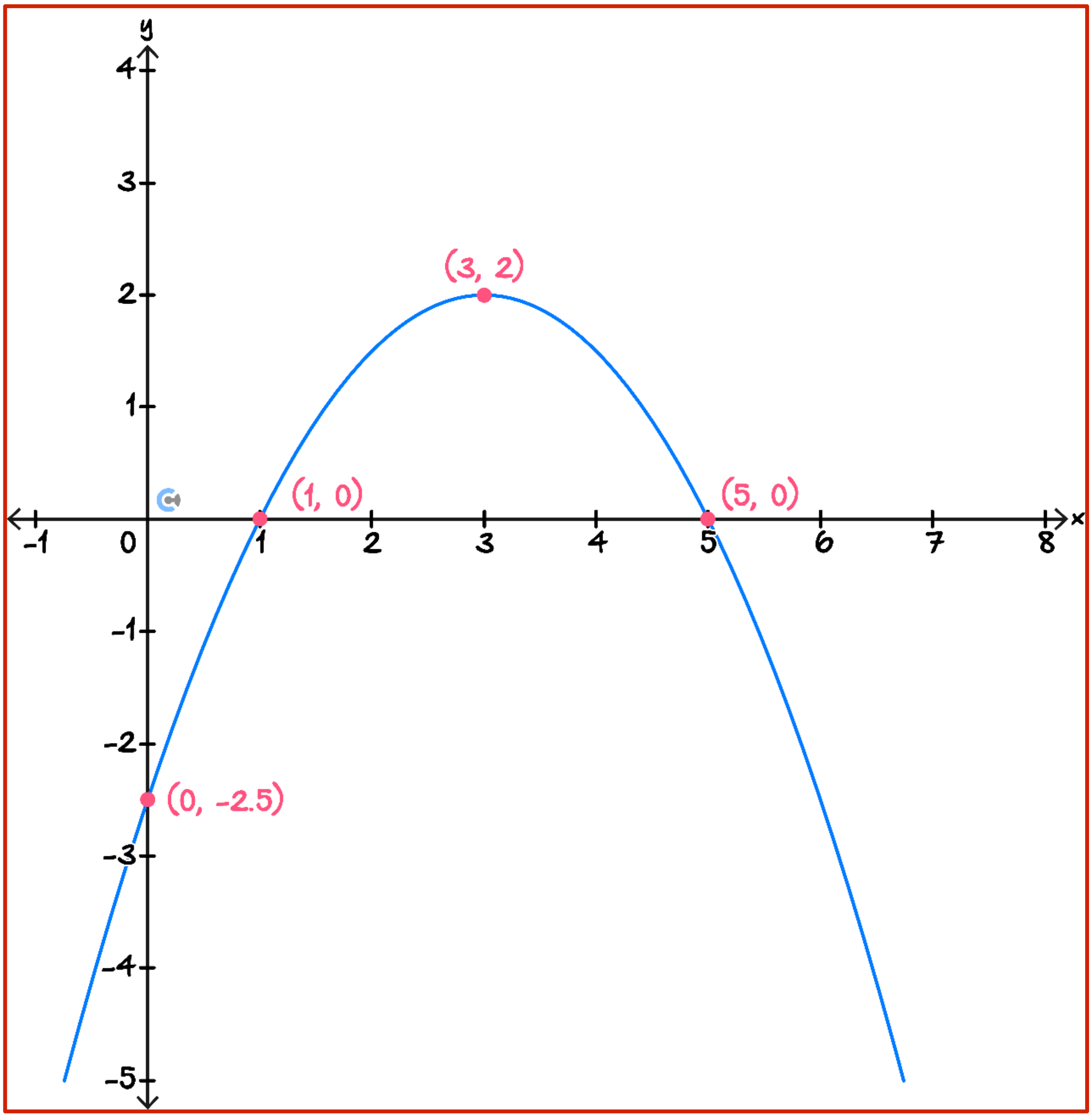
a. Write $f(x)$ in turning point form.

$$f(x) = -\frac{1}{2}(x-3)^2 + 2$$

b. Solve the equation $f(x) = 0$.

$$x = 1, 5$$

c. Sketch the graph of $y = f(x)$ on the axes below, Label all axes intercepts and the turning point.



d. Find the value of m such that the line $y = mx - 2$ intersects the graph of $y = f(x)$ exactly once.

$m = 2,4$

Section C: Exam 1 (22 Marks)

Question 2 (3 marks)

The sum of the ages of a man and his son is 30, and the product of their ages is 125.

a. Write down a quadratic equation in the form $ax^2 + bx + c = 0$ that can be solved to find the ages of the man and his son, where x is the age of the son. (1 mark)

$$x(30 - x) = 125$$
$$x^2 - 30x + 125 = 0$$

b. Find the ages of the man and his son. (2 marks)

$$x^2 - 30x + 125 = 0$$
$$(x - 5)(x - 25) = 0$$
$$x = 5, 25$$

There ages are 25 and 5

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Question 3 (6 marks)

Consider the function $f(x) = 2x^2 - 4x - 6$.

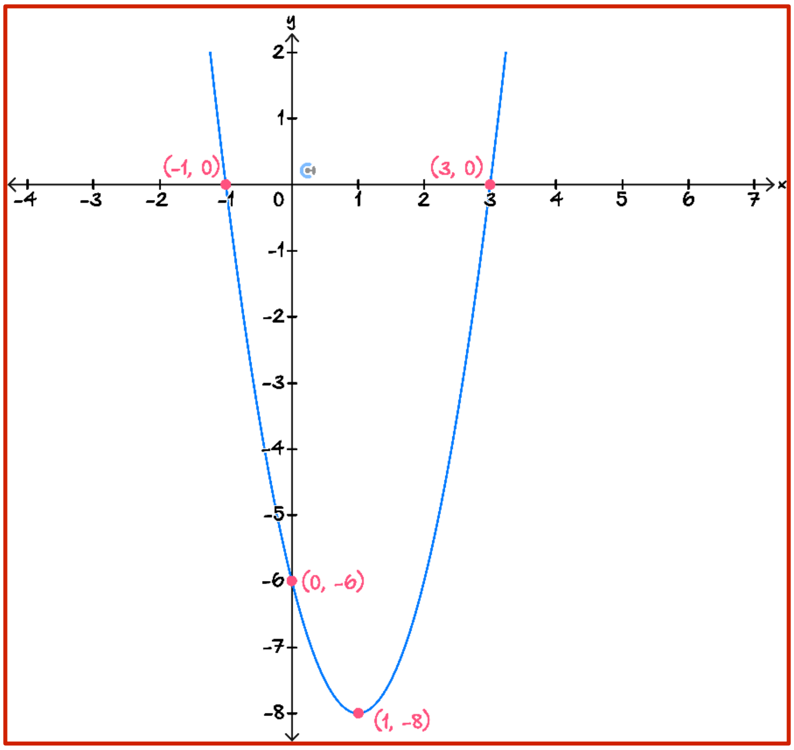
a. Solve the equation $f(x) = 0$. (1 mark)

Divide both sides by 2 to solve $x^2 - 2x - 3 = 0$
$(x-3)(x+1) = 0$
$x = -1, 3$

b. Write $f(x)$ in turning point form. (1 mark)

$f(x) = 2(x-1)^2 - 8$

c. Sketch the graph of $y = f(x)$ on the axes below. Label the turning point and all axes intercepts with coordinates. (2 marks)



d. Find the value(s) of x such that $f(x) + 4 < 0$. (2 marks)

Solve $f(x) = -4$

$$x = 1 \pm \sqrt{2}$$

Therefore, $1 - \sqrt{2} < x < 1 + \sqrt{2}$

Question 4 (2 marks)

Solve the inequality $-x^2 + 3x + 18 \geq 0$.

Now considering the shape of the quadratic we conclude that $-3 \leq x \leq 6$.

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Question 5 (3 marks)

Solve the equation $2x^4 - 20x^2 + 18 = 0$, for real values of x.

$x = -3, -1, 1, 3$

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Question 6 (4 marks)

Consider the function $f(x) = x^2 - 3kx + 6$, where k is a real number.

a. Find the turning point of $f(x)$ in terms of k. (2 marks)

Solution: $f(x) = \left(x - \frac{3k}{2}\right)^2 + 6 - \frac{9k^2}{4}$.
The turning point is at

$$\left(\frac{3k}{2}, 6 - \frac{9k^2}{4}\right)$$

b. Find all possible values of k if $f(x)$ is always greater than 2. (2 marks)

Solution: We want no solutions to the equation $f(x) = 2$. So we want the discriminant of this equation < 0

$$x^2 - 3kx + 4 = 0$$
$$\Delta = 9k^2 - 16 < 0$$
$$k^2 < \frac{16}{9}$$

Therefore, $-\frac{4}{3} < k < \frac{4}{3}$
Alternatively, we use **part a.** and solve

$$6 - \frac{9k^2}{4} > 2$$
$$\implies -\frac{4}{3} < k < \frac{4}{3}$$

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Question 7 (4 marks)

Consider the function$f(x) = x^2 + 2kx - 4$, where k is a real number.

a. Show that the graph $y = f(x)$ always has two x-intercepts. (1 mark)

Consider the discriminant for $x^2 + 2kx - 4 = 0$

$$\Delta = 4k^2 + 16 > 0$$

Therefore, must have two x-intcrcepts.

b. Find the values of k such that the distance between the two x-intercepts is less than 6. (3 marks)

$$-\sqrt{5} < k < \sqrt{5}$$

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Section D: Tech Active Exam Skills

Calculator Commands: Solving equations

Mathematica

- Solve[].

```
In[122]:= Solve[x^2 - 4 x - 9 == 0, x]
Out[122]= {{x → 2 - √13}, {x → 2 + √13}}
```

TI-Nspire

- Menu→ 3 → 1.

$\text{solve}(x^2-4\cdot x-9=0,x)$ $\quad x=-(\sqrt{13}-2)$ or $x=\sqrt{13}+2$

Casio Classpad

- Action→Advanced→Solve.

$\text{solve}(x\verb|^|2-4x-9=0, x)$

$\{x=-\sqrt{13}+2, x=\sqrt{13}+2\}$

Calculator Commands: Completing the Square

TI-Nspire

- Menu→ 3 → 5 completeSquare (func, var).

$\text{completeSquare}(x^2-6\cdot x+8,x) \quad (x-3)^2-1$

Mathematica

- No inbuilt function need udf.

Compsq

Compsq[a x^2 + b x + c, x]
Converts a standard form quadratic to turning point form.

Compsq[a, b, c]
Uses the coefficients of a quadratic to return the turning point form.

```
Compsq[1, 2, 3]
2 + (1 + x)^2

Compsq[2 x^2 + 3 x - 5, x]
Compsq[-5 + 3 x + 2 x^2, x]
```

Casio Classpad

- No function

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Section E: Exam 2 (27 Marks)

Question 8 (1 mark)

Find the value(s) of k for which the quadratic equation below has exactly one unique real solution.

$$2x^2 - 3kx + 3k = 0$$

A. $k = \frac{8}{3}$

B. $k = 0, \frac{8}{3}$

C. $k > \frac{8}{3}$

D. $k = 0, 3$

Question 9 (1 mark)

A quadratic function has a turning point at $(4, 3)$ and goes through the point $(6, 7)$. What is the equation of the function?

A. $2(x-4)^2 + 3$

B. $-(x-4)^2 + 3$

C. $(x-3)^2 + 4$

D. $(x-4)^2 + 3$

Question 10 (1 mark)

The function $f(x) = x^2 + mx + 2$ is always greater than -1. The possible values of m are:

A. $-\sqrt{3} < m < \sqrt{3}$

B. $-2\sqrt{2} < m < 2\sqrt{2}$

C. $-2\sqrt{3} < m < 2\sqrt{3}$

D. $-1 < m < 1$

Question 11 (1 mark)

If one root of the quadratic equation $2x^2 + px - 35 = 0$ is -7 the value of p is:

A. -9

B. 9

C. -4

D. 4

Question 12 (1 mark)

The equation $ax^2 + 6x + c = 0$ has only one real solution if:

A. $ac > -9$

B. $2ac = 9$

C. $ac = -9$

D. $ac = 9$

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Question 13 (14 marks)

Consider the quadratic function $f(x) = x^2 - 4x + 2$.

a.

i. Solve the equation $f(x) = 0$. (1 mark)

$x = 2 \pm \sqrt{2}$

ii. State the distance between the x-axis intercepts. (1 mark)

$2\sqrt{2}$

iii. Find the turning point of the graph of $y = f(x)$. (1 mark)

$(2, -2)$

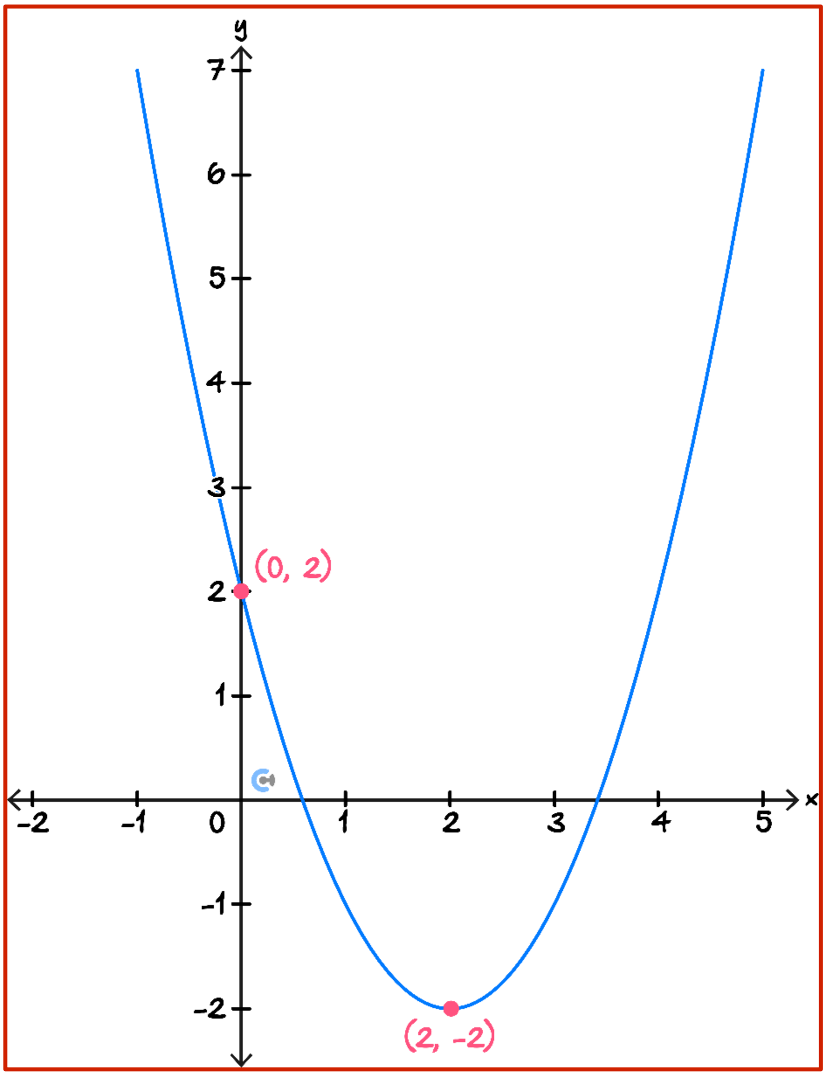
iv. Hence, write $f(x)$ in turning point form. (1 mark)

$f(x) = (x - 2)^{2-2}$

v. Find the y-intercept of the graph of $y = f(x)$. (1 mark)

$(0,2)$

b. Sketch the graph of $y = f(x)$ on the axes below. (2 marks)



c. If the graph of $y = f(x)$ is shifted k units to the left, find the values of k for which there is one, negative x-axis intercept. (2 marks)

Solution: Negative x-axis intercept if we shift greater than $2 - \sqrt{2}$ units left. Another negative x-axis intercept if we shift greater than $2 + \sqrt{2}$ units left. Therefore, $2 - \sqrt{2} < k \le 2 + \sqrt{2}$

d. The graph of $y = f(x)$ is translated 1 unit to the left and 4 units up and now has the equation:

$$y = a(x-h)^2 + k, \quad a, h, k \in \mathbb{R}$$

Determine the values of a, h, k. (2 marks)

$y = (x-1)^2 + 2. a = 1, h = 1$ and $k = 2$

e. Consider the graph of the function $g(x) = 4x^2 + kx + 2(k+1)$. Find the value(s) of k for which $g(x)$ will have:

i. No real root. (1 mark)

$16 - 12\sqrt{2} < k < 16 + 12\sqrt{2}$

ii. One real root. (1 mark)

$k = 16 \pm 12\sqrt{2}$

iii. Two unique real roots. (1 mark)

$k < 16 - 12\sqrt{2}$ or $k > 16 - 12\sqrt{2}$

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Question 14 (9 marks)

A cricket player hits a ball, and the ball's trajectory is modelled by the quadratic equation:

$$h(x) = ax^2 + bx + c,$$

where $h(x)$ is the height of the ball (in metres) above the ground, and x is the horizontal distance (in metres) from where the ball was hit.

The following conditions are given:

- The ball is hit from a height of 2 metres, i.e., $h(0) = 2$.
- The ball reaches a height of 15 metres when it has travelled 8 metres horizontally.
- The ball reaches a height of 25 metres when it has travelled 16 metres horizontally.

a. Using the given conditions, set up and solve a system of equations to determine the values of $a, b,$ and c. (3 marks)

$$a = -\frac{3}{128}, b = \frac{29}{16}, c = 2$$

b. Determine the maximum height that the ball reaches. Give your answer correct to 2 decimal places. (1 mark)

37.04 metres

c. Determine the horizontal distance the ball has travelled when its height is 20 metres. Provide both possible horizontal distances correct to two decimal places. (2 marks)

Solve $h(x) = 12$.
$x = 11.70, 65.63$ metres

d. After reaching a certain height, the ball travels 8 metres horizontally to drop down to that height again. Find this exact height. (3 marks)

$\frac{110}{3}$ metres

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Section F: Extension Exam 1 (16 Marks)

Question 15 (4 marks)

The parabola $y = ax^2 + bx + c$ passes through the points $\left(1, -\frac{1}{2}\right)$, $(4, -5)$, and $(6, -3)$.

Determine the values of real numbers a, b, and c.

Solution: Let $y = f(x)$ then we have that

$$f(1) = a + b + c = -\frac{1}{2}$$
$$f(4) = 16a + 4b + c = -5$$
$$f(6) = 36a + 6b + c = -3$$

Solving the three simultaneous equations yields

$$a = \frac{1}{2} \quad b = -4 \quad c = 3$$

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Question 16 (4 marks)

Let $f(x) = 2x^2 - 4x + 5$.

a. Find the turning point of the parabola $y = f(x)$. (1 mark)

Solution: $y = 2(x-1)^2 + 3$. TP at $(1, 3)$.

b. Reflect this turning point in the line $x = 3$ and then in the line $y = 2$. (1 mark)

Solution: $(1,3) \mapsto (5,3) \mapsto (5,1)$

c. The parabola $y = f(x)$ is reflected in the line $x = 3$ and then reflected in the line $y = 2$. Find the equation of the resulting parabola in the form $y = ax^2 + bx + c$, where $a, b,$ and c are real numbers. (2 marks)

Solution: The new parabola has a turning point at $(5, 1)$
The reflection in the line $y = 2$ inverts the parabola so it has equation

$$y = -2(x-5)^2 + 1 = -2x^2 + 20x - 49$$

$a = -2, b = 20$ and $c = -49$

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Question 17 (4 marks)

Solve $(x^2+1)^2+4 \geq 8x^2$ for all real x.

Solution: Let $a = x^2+1$ then we have the inequality

$$a^2+4 \geq 8(a-1)$$
$$\implies a \leq 2 \text{ or } a \geq 6$$

Now we solve

$$x^2+1 \leq 2 \implies -1 \leq x \leq 1$$

and also

$$x^2+1 \geq 6 \implies x \leq -\sqrt{5} \text{ or } x \geq \sqrt{5}$$

So all solutions to the inequality are

$$x \leq -\sqrt{5} \text{ or } -1 \leq x \leq 1 \text{ or } x \geq \sqrt{5}$$

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Question 18 (4 marks)

Let $f(x) = x^4 - 4kx^2 + 4 - k^2$, where k is a real constant.

Find the values of k for which the equation $f(x) = 0$ has no real solutions.

Solution: Let $a = x^2$ then we solve

$$a^2 - 4ka + 4 - k^2 = 0$$
$$(a - 2k)^2 = 5k^2 - 4$$
$$a = 2k \pm \sqrt{5k^2 - 4}$$

Now note that there will be no solutions if a is not a real number. So no solutions if

$$5k^2 - 4 < 0 \implies -\frac{2}{\sqrt{5}} < k < \frac{2}{\sqrt{5}}$$

also no solution if $a < 0$.
Note that if $2k + \sqrt{5k^2 - 4} < 0$ then it must be that $2k - \sqrt{5k^2 - 4} < 0$ since square root is ≥ 0. Therefore we must now just solve

$$2k + \sqrt{5k^2 - 4} < 0$$

Solve

$$2k + \sqrt{5k^2 - 4} = 0$$
$$4k^2 = 5k^2 - 4$$
$$k = \pm 2.$$

Then by considering the shape/domain of a graph we have that

$$2k + \sqrt{5k^2 - 4} < 0 \implies -2 < k \leq -\frac{2}{\sqrt{5}}$$

Combining our answers we have that there is no solution to $f(x) = 0$ if

$$-2 < k < \frac{2}{\sqrt{5}}$$

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Section G: Extension Exam 2 (16 Marks)

Question 19 (1 mark)

If $px^2 + 5x + q = 0$ has two roots $x = -2$ and $x = 1$, the value of $p - q$ is:

A. -5

B. 5

C. 10

D. 15

Question 20 (1 mark)

The equation of the parabola that passes through the points $(1, 2), (3, 2)$ and $(4, 5)$ is:

A. $y = x^2 - 4x - 5$

B. $y = (x - 2)^2 + 1$

C. $y = x^2 + 4x + 5$

D. $y = (x - 1)^2 + 2$

Question 21 (1 mark)

Consider the graph of $y = x^2 - 2kx - 2$ where k is a real constant.

The values of k for which the distance between the two x-intercepts is less than 6 are:

A. $-\sqrt{5} < k < \sqrt{5}$

B. $-\sqrt{6} < k < \sqrt{6}$

C. $-\sqrt{7} < k < \sqrt{7}$

D. $-\sqrt{11} < k < \sqrt{11}$

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Question 22 (1 mark)

Let $y = 2x^2 - 4x - 2$.

If $-2 < x < 3$, the possible values of y are:

A. $-4 < y \leq 14$

B. $-4 \leq y < 14$

C. $4 < y < 14$

D. $-4 < y < 14$

Question 23 (1 mark)

Find all values of k, such that $x^2 + kx + k^2 - 4$ has two real roots for x, where one is positive and one is negative.

A. $k < 2$

B. $k > -2$

C. $-2 < k < 2$

D. $-2 \leq k \leq 2$

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Question 24 (11 marks)

Consider the function $f(x) = x^2 + (k-2)x + \frac{k^2-4k-4}{2}$, where k is a real constant.

a.

i. Find all values of k such that $f(x) = 0$ has one real root.. (1 mark)

$k = -2, 6$

ii. Find all values of k such that $f(x) = 0$ has two real roots. (1 mark)

$-2 < k < 6$

iii. Find all values of k such that $f(x) = 0$ has two real roots, where one is positive and the other is negative. (2 marks)

Solution: We solve $f(0) = 0 \implies k = 2 \pm 2\sqrt{2}$
Then inspect a graph to see that there is one positive root and one negatie root for

$$2 - 2\sqrt{2} < k < 2 + 2\sqrt{2}$$

b. Find all values of k for which the graph of $y = f(x)$ and the graph $y = kx + 2$ do not intersect. (2 marks)

Solution: Solve $f(x) = kx + 2 \implies x = \frac{1}{2}\left(2 \pm \sqrt{2}\sqrt{-k^2 + 4k + 10}\right)$.
This will have no solution if $-k^2 + 4k + 10 < 0$.
Therefore $k < 2 - \sqrt{14}$ or $k > 2 + \sqrt{14}$

c. Find all values of k such that $f(x) > 2$ for all real x. (2 marks)

Solution: We want $f(x) = 2$ to have no real solutions.
$f(x) = 2 \implies x = \frac{1}{2}\left(2 - k \pm \sqrt{-k^2 + 4k + 20}\right)$
No solution if $-k^2 + 4k + 20 < 0$
Therefore, $k < 2 - 2\sqrt{6}$ or $k > 2 + 2\sqrt{6}$

d. Find all values of k such that the graph of $y = f(x)$ has two x-intercepts that have a distance between them that is less than 2. (3 marks)

Solution: Solve $f(x) = 0 \implies x = \frac{1}{2}\left(2 - k \pm \sqrt{-k^2 + 4k + 12}\right)$
Let x_1 and x_2 be the respective x-intercepts. We then solve

$$|x_1 - x_2| < 2$$

which yields $-2 \leq k < 2 - 2\sqrt{3}$ or $2 + 2\sqrt{3} < k \leq 6$

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