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VCE Mathematical Methods ½
Quadratics Exam Skills [0.4]
Workshop Extension

Section A: Recap

Sub-Section: Factorising Quadratics

Factorising Quadratics

$$y = (x - a)(x - b)$$

- **Steps:**

1. Divide by the coefficient of the leading term. (If applicable)
2. Consider the factors of the constant term.
3. (If Positive Constant Term): See which pair of factors can add up to the coefficient of the x term.

   (If Negative Constant Term): See which pair of factors can subtract from the coefficient of the x term.
4. Construct the linear factors.

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<u>Sub-Section</u>: Perfect Squares

Perfect Squares

$$(a+b)^2 = \underline{\hspace{8cm}}$$

$$(a-b)^2 = \underline{\hspace{8cm}}$$

- Perfect squares are special quadratic expressions that are made up of two **identical** linear factors.
- In other words, when a linear factor is squared, it becomes a perfect square.

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Sub-Section: Difference of Squares

Difference of Squares

$$a^2 - b^2 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

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Sub-Section: Completing the Square

Completing the Square

- When we complete the square of a quadratic $x^2 + bx + c$, we write it in the form:

$$x^2 + bx + c = (\_\_\_\_\_\_\_\_\_\_)^2 - \left(\frac{b}{2}\right)^2 + c$$

- **Steps:**

1. We halve the coefficient of x.
2. Subtract the half of the coefficient of x squared outside the square bracket.

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Sub-Section: Solving by Factorisation

Solving by Factorisation

$$(x-a)(x-b)=0$$

$$x = a \text{ or } b$$

- **Steps:**

1. Factorise the quadratic.
2. Equate each factor to 0 and solve for x.

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Sub-Section: Quadratic Formula

The Quadratic Formula

for $ax^2 + bx + c = 0$

$x =$ ______________________

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Sub-Section: Discriminant

The Discriminant

- **Definition:**

  - The discriminant, often denoted by **Δ** (Delta), is the part **inside** the square root of the quadratic formula.

$$\boldsymbol{Discriminant = \Delta = b^2 - 4ac}$$

if $\Delta > 0$, there are ____.

if $\Delta = 0$, there is ____.

if $\Delta < 0$, there are ____.

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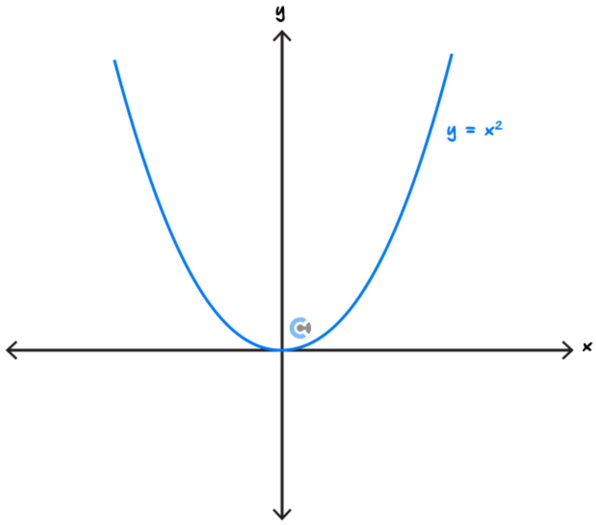
Sub-Section: Parabola and Symmetry

Definition

Parabola

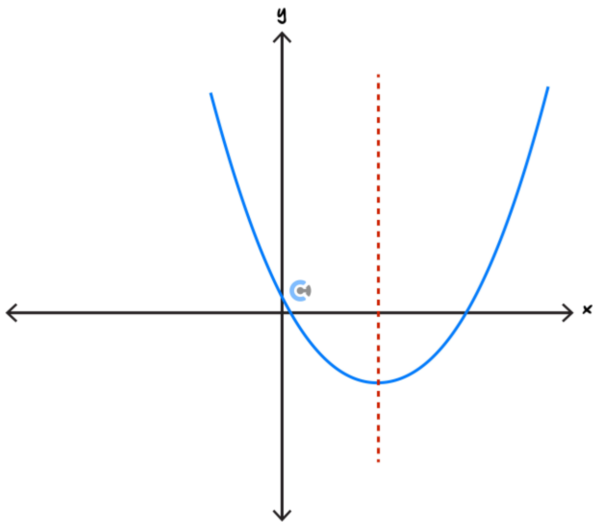
Definition:

- The shape of the graph of a quadratic is known as a __________________________.



Definition

Axis of Symmetry



$$\textit{Axis of symmetry}: x = -\frac{b}{2a}$$

Sub-Section: Graphing Quadratics

Turning Point Form

- The turning point form of a quadratic is given by:

$$y = a(x-h)^2 + k$$

$$\textit{Turning point} = \underline{\hspace{4cm}}$$

- The turning point form is obtained by **completing the square.**

Intercept Form

- The x-intercept form of a quadratic is given by:

$$y = a(x-b)(x-c)$$

$$x\textit{-intercepts}: (b, 0) \textit{ and } (c, 0)$$

- The axis of symmetry is located exactly in the middle of the two x-intercepts.

NOTE: When a is negative, the x-intercepts stay the same, but the **shape** of the parabola becomes a **negative** parabola instead.

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Sub-Section: Finding a Rule of a Quadratic from a Graph

Finding the Equation of a Quadratic

- **Form 1: Turning Point Form**

$$y = a(x-h)^2 + k$$

  - Recommended when a turning point is easy to identify.

- **Form 2: x-intercept Form**

$$y = a(x-b)(x-c)$$

  - Recommended when both x-intercepts are easy to identify.

NOTE: Never forget the a coefficient!

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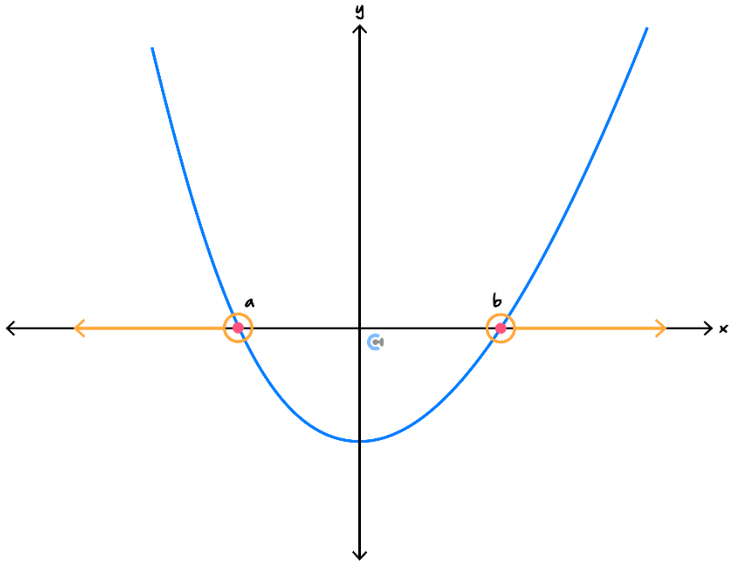
Sub-Section: Quadratic Inequalities

Quadratic Inequalities



- For quadratic inequalities, we always ______________ the function.
- **Steps:**
  1. Sketch the function.
  2. See where the y-value is within the inequality.
  3. Find the corresponding x-values.

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Sub-Section: Hidden Quadratics

Hidden Quadratics

- Instead of:

$$af(x)^2 + bf(x) + c = 0$$

- We can let $f(x) = X$ to have:

$$aX^2 + bX + c = 0$$

Completing the square quickly.

$$y = a(x-h)^2 + k$$

- **Steps**

1. Find the turning point using $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.
2. Use the leading coefficient as a.

Modelling with Quadratics

- Focus on key points such as turning points, x-intercepts and y-intercepts.

Family of Functions

- **Definition:** Functions with unknowns.
- **Question Type:** Find the unknown value to satisfy a certain condition.

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Section B: Warmup

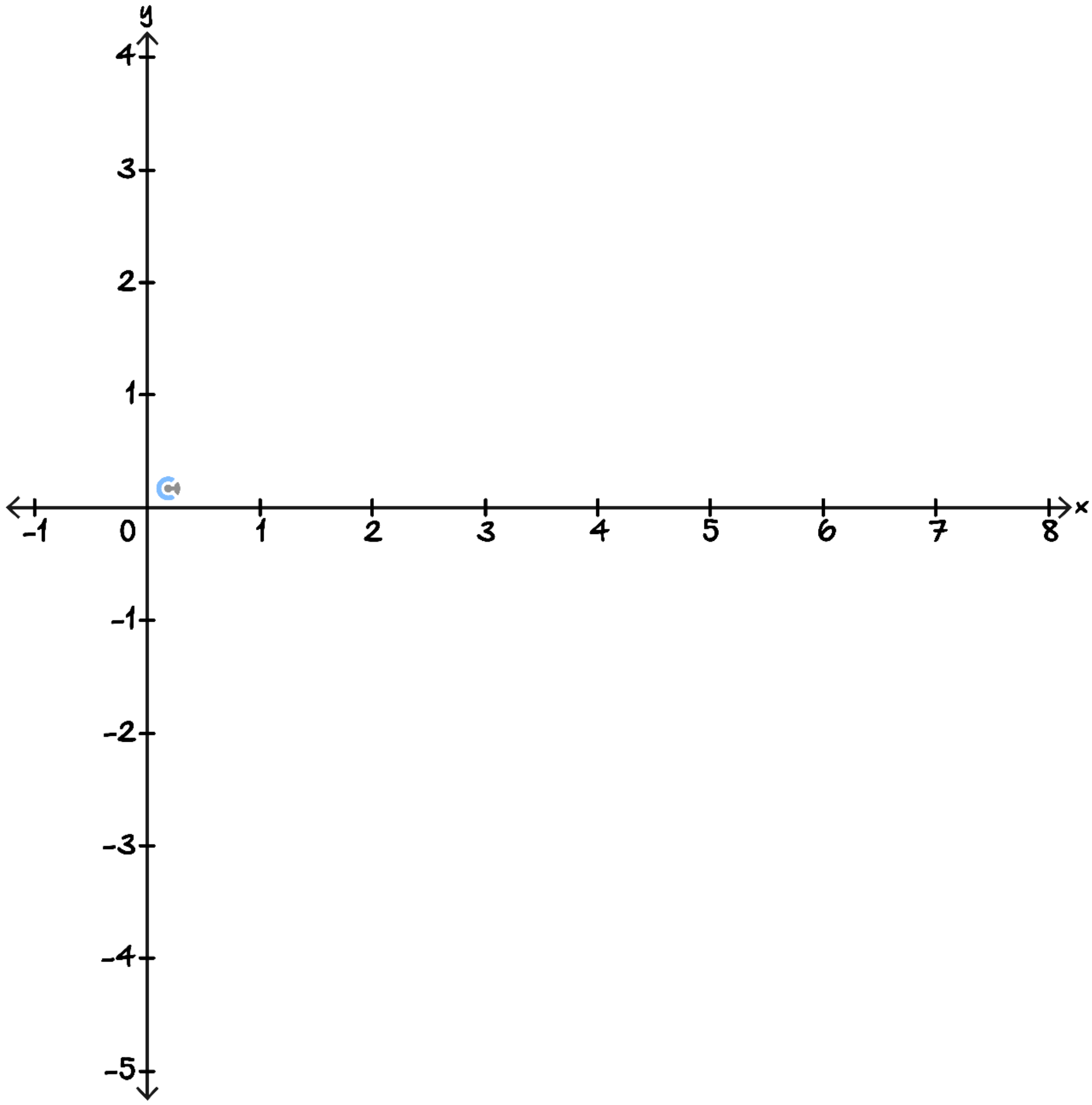
Question 1

Let $f(x) = -\frac{x^2}{2} + 3x - \frac{5}{2}$.

a. Write $f(x)$ in turning point form.

b. Solve the equation $f(x) = 0$.

c. Sketch the graph of $y = f(x)$ on the axes below, Label all axes intercepts and the turning point.



d. Find the value of m such that the line $y = mx - 2$ intersects the graph of $y = f(x)$ exactly once.

Section C: Exam 1 (22 Marks)

Question 2 (3 marks)

The sum of the ages of a man and his son is 30, and the product of their ages is 125.

a. Write down a quadratic equation in the form $ax^2 + bx + c = 0$ that can be solved to find the ages of the man and his son, where x is the age of the son. (1 mark)

$x^2 + 30x + 125 = 0$

b. Find the ages of the man and his son. (2 marks)

$(x+5)(x+25) = 0$

$\therefore$ Ages = 5, 25 //

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Question 3 (6 marks)

Consider the function $f(x) = 2x^2 - 4x - 6$.

a. Solve the equation $f(x) = 0$. (1 mark)

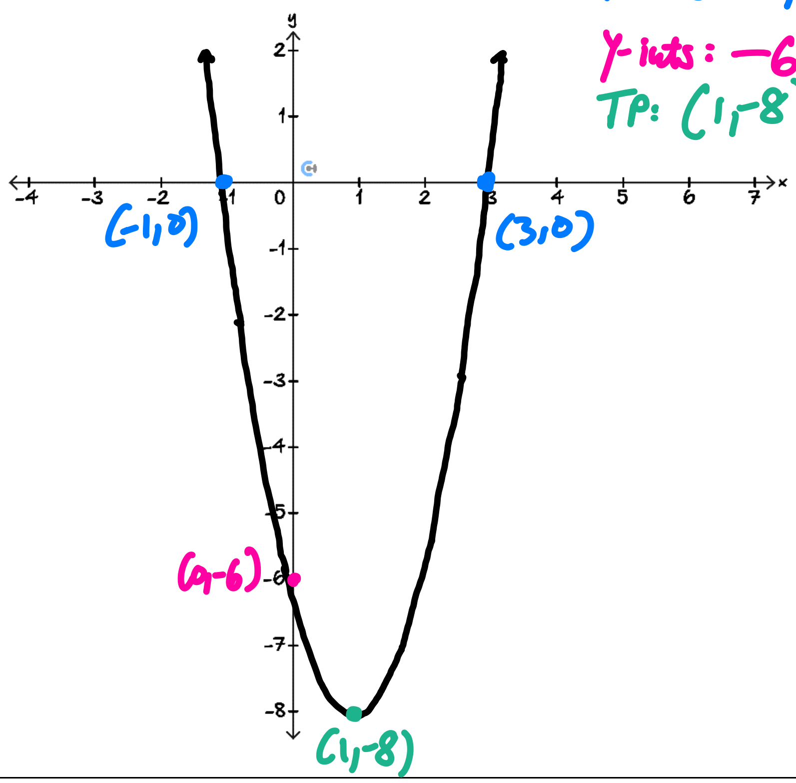
$2(x^2-2x-3)=0$
$(x^2-2x-3)=0$
$(x-3)(x+1)=0$
$\therefore x=-1, 3$ //

b. Write $f(x)$ in turning point form. (1 mark)

$f(x) = 2(x^2-2x)-6$
$= 2((x-1)^2-1)-6$
$= 2(x-1)^2-8$ //

c. Sketch the graph of $y = f(x)$ on the axes below. Label the turning point and all axes intercepts with coordinates. (2 marks)

X-ints: $-1, 3$
Y-ints: -6
TP: $(1,-8)$



d. Find the value(s) of x such that $f(x) + 4 < 0$. (2 marks)

$2x^2 - 4x - 2 < 0$

$2(x^2 - 2x - 1) < 0$

$x^2 - 2x - 1 < 0$

$(x-1)^2 - 2 < 0$

$(x-1)^2 < 2$

$-\sqrt{2} < (x-1) < \sqrt{2}$

$1 - \sqrt{2} < x < 1 + \sqrt{2}$

Question 4 (2 marks)

Solve the inequality $-x^2 + 3x + 18 \geq 0$.

$x^2 - 3x - 18 \leq 0$

$(x-6)(x+3) \leq 0$

$\therefore -3 \leq x \leq 6$

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Question 5 (3 marks)

Solve the equation $2x^4 - 20x^2 + 18 = 0$, for real values of x.

$\Rightarrow (2x^2 - 18)(x^2 - 1) = 0$

$2(x^2 - 9)(x^2 - 1) = 0$

$x^2 - 9 = 0$ or $x^2 - 1 = 0$

$x^2 = 9$ or $x^2 = 1$

$\therefore x = \pm 3$ or $x = \pm 1$ //

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Question 6 (4 marks)

Consider the function $f(x) = x^2 - 3kx + 6$, where k is a real number.

a. Find the turning point of $f(x)$ in terms of k. (2 marks)

$f(x) = \left(x - \frac{3k}{2}\right)^2 - \left(\frac{3k}{2}\right)^2 + 6$

$= \left(x - \frac{3k}{2}\right)^2 - \frac{9k^2}{4} + 6$ //

$\therefore$ TP: $\left(\frac{3k}{2}, 6 - \frac{9k^2}{4}\right)$ //

b. Find all possible values of k if $f(x)$ is always greater than 2. (2 marks)

$f(x) > 2$:

$x^2 - 3kx + 6 > 2$

$\underline{x^2 - 3kx + 4 > 0}$

$\therefore \Delta < 0$

$(-3k)^2 - 4(1)(4) < 0$

$9k^2 - 16 < 0$

$k^2 < \frac{16}{9}$

$\therefore -\frac{4}{3} < k < \frac{4}{3}$

$x^2 - 4 < 0$

$x^2 < 4$

$x^2 - 4 > 0$

$x^2 > 4$

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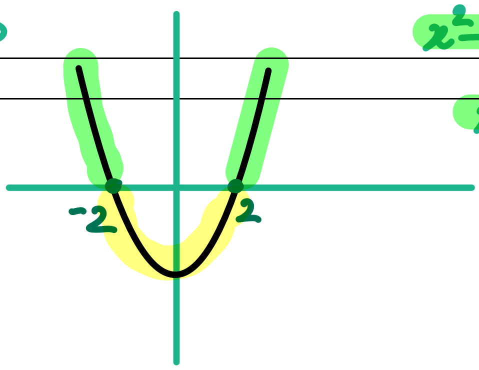


Question 7 (4 marks)

Consider the function $f(x) = x^2 + 2kx - 4$, where k is a real number.

a. Show that the graph $y = f(x)$ always has two x-intercepts. (1 mark)

$\Delta > 0$

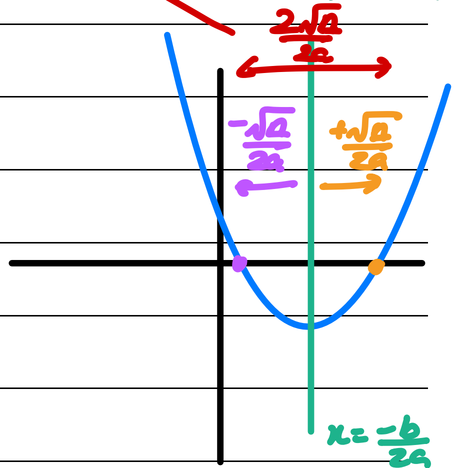
$\Delta = (2k)^2 - 4(1)(-4) = 4k^2 + 16 > 0$

$\therefore k^2 \geq 0$ for all $k \in \mathbb{R}$

$\therefore 4k^2 + 16 > 0$ for all $k \in \mathbb{R}$ and thus $f(x)$ will always have 2 x-ints.

b. Find the values of k such that the distance between the two x-intercepts is less than 6. (3 marks)

$\frac{\sqrt{\Delta}}{a}$ < 6



$\frac{\sqrt{\Delta}}{a} < 6$

$\frac{\sqrt{4k^2+16}}{1} < 6$

$4k^2 + 16 < 36$

$4k^2 < 20$

$k^2 < 5$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

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$-\sqrt{5} < k < \sqrt{5}$

Section D: Tech Active Exam Skills

Calculator Commands: Solving equations

➤ **Mathematica**

- Solve[].

```
In[122]:= Solve[x^2 - 4 x - 9 == 0, x]
Out[122]= {{x → 2 - √13}, {x → 2 + √13}}
```

➤ **TI-Nspire**

- Menu→ 3 → 1.

$\text{solve}(x^2-4\cdot x-9=0,x)$ $\quad x=-(\sqrt{13}-2)$ or $x=\sqrt{13}+2$

➤ **Casio Classpad**

- Action→Advanced→Solve.

```
solve(x^2-4x-9=0, x)
        {x=-√13+2, x=√13+2}
```

Calculator Commands: Completing the Square

➤ **TI-Nspire**

- Menu→ 3 → 5 completeSquare (func, var).

$\text{completeSquare}(x^2-6\cdot x+8,x)$ $\quad (x-3)^2-1$

➤ **Mathematica**

- No inbuilt function need udf.

```
Compsq
Compsq[a x^2 + b x + c, x]
    Converts a standard form quadratic to turning point form.

Compsq[a, b, c]
    Uses the coefficients of a quadratic to return the turning point form.

Compsq[1, 2, 3]
2 + (1 + x)^2

Compsq[2 x^2 + 3 x - 5, x]
Compsq[-5 + 3 x + 2 x^2, x]
```

➤ **Casio Classpad**

- No function

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Section E: Exam 2 (27 Marks)

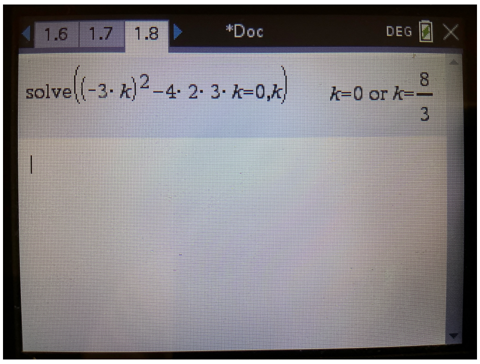
Question 8 (1 mark)

Find the value(s) of k for which the quadratic equation below has exactly one unique real solution.

$$2x^2 - 3kx + 3k = 0$$

$\Delta = 0$

$(-3k)^2 - 4(2)(3k) = 0$



A. $k = \frac{8}{3}$

B. $k = 0, \frac{8}{3}$

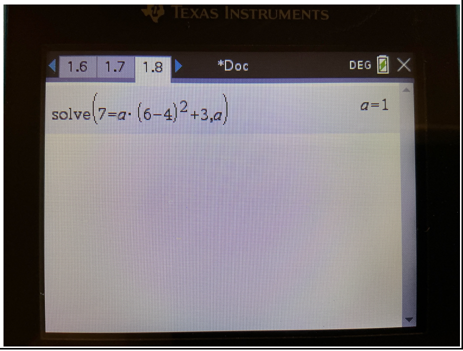
C. $k > \frac{8}{3}$

D. $k = 0, 3$

Question 9 (1 mark)

A quadratic function has a turning point at $(4, 3)$ and goes through the point $(6, 7)$. What is the equation of the function?

$\cancel{y}\,7 = a(\cancel{x}\,6 - 4)^2 + 3$



A. $2(x-4)^2 + 3$

B. $-(x-4)^2 + 3$

C. $(x-3)^2 + 4$

D. $(x-4)^2 + 3$

Question 10 (1 mark)

The function $f(x) = x^2 + mx + 2$ is always greater than -1. The possible values of m are:

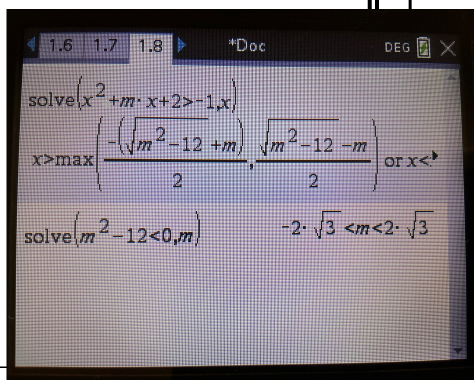
$x^2 + mx + 2 > -1$

$x^2 + mx + 3 > 0$

$\Delta < 0$

$m^2 - 4(1)(3) < 0$

$\Rightarrow m^2 - 12 < 0$



A. $-\sqrt{3} < m < \sqrt{3}$

B. $-2\sqrt{2} < m < 2\sqrt{2}$

C. $-2\sqrt{3} < m < 2\sqrt{3}$

D. $-1 < m < 1$

Question 11 (1 mark)

If one root of the quadratic equation $2x^2 + px - 35 = 0$ is -7 the value of p is:

A. -9

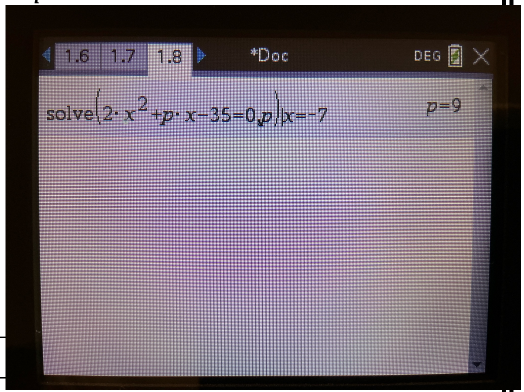
B. 9

C. -4

D. 4

$2(-7)^2 + p(-7) - 35 = 0$

Solve for p



Question 12 (1 mark)

The equation $ax^2 + 6x + c = 0$ has only one real solution if:

$\Delta = 0$

A. $ac > -9$

B. $2ac = 9$

C. $ac = -9$

D. $ac = 9$

$(6)^2 - 4(a)(c) = 0$

$36 = 4ac$

$ac = 9$

Space for Personal Notes

Question 13 (13 marks)

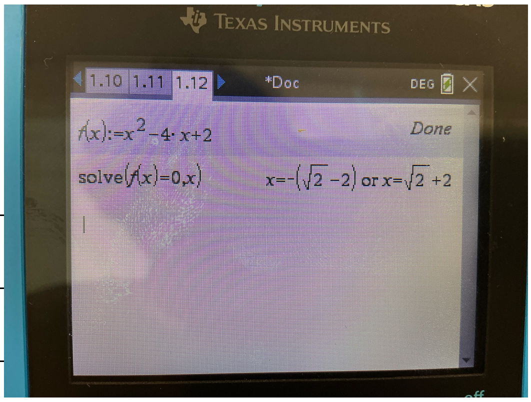
Consider the quadratic function $f(x) = x^2 - 4x + 2$.

a.

i. Solve the equation $f(x) = 0$. (1 mark)

$\therefore x = 2-\sqrt{2}$ or

$x = 2+\sqrt{2}$ //



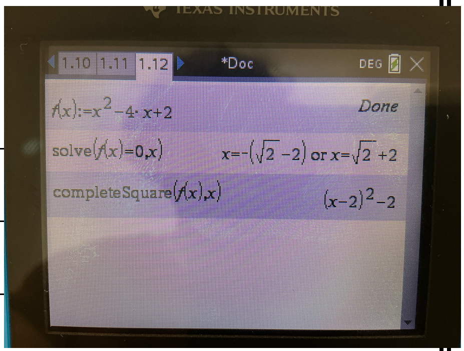
ii. State the distance between the x-axis intercepts. (1 mark)

$2\sqrt{2}$ units /

iii. Find the turning point of the graph of $y = f(x)$. (1 mark)

$f(x) = (x-2)^2 - 2$

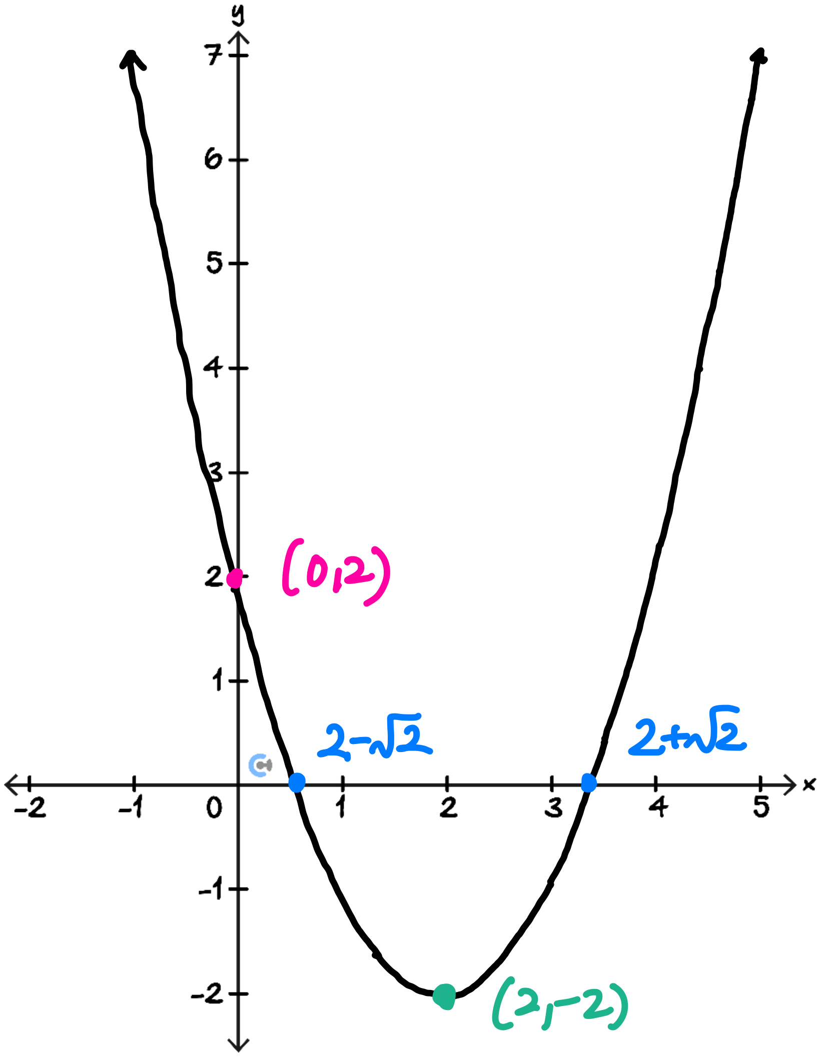
↪ TP: $(2,-2)$



iv. Find the y-intercept of the graph of $y = f(x)$. (1 mark)

$\therefore$ y-int: $(0,2)$ //

b. Sketch the graph of $y = f(x)$ on the axes below. (2 marks)



c. If the graph of $y = f(x)$ is shifted k units to the left, find the values of k for which there is one. Negative x-axis intercept. (2 marks)

Negative X-int

$2-\sqrt{2} - k < 0$

$\therefore k > 2-\sqrt{2}$

Positive X-int

$2+\sqrt{2} - k > 0$

$k < 2+\sqrt{2}$

$\therefore 2-\sqrt{2} < k < 2+\sqrt{2}$

d. The graph of $y = f(x)$ is translated 1 unit to the left and 4 units up and now has the equation:

$$y = a(x-h)^2 + k, \quad a, h, k \in \mathbb{R}$$

Determine the values of a, h, k. (2 marks)

$y = (x-2)^2 - 2$ $(2,-2)$

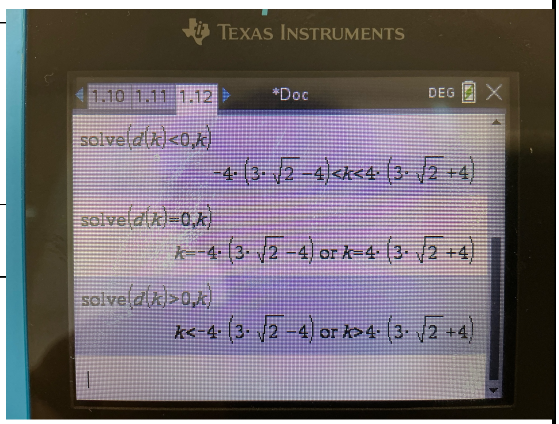
$y = (x-1)^2 + 2$ $(1,2)$

$\therefore a = 1, h = 1, k = 2$

e. Consider the graph of the function $g(x) = 4x^2 + kx + 2(k+1)$. Find the value(s) of k for which the equation $g(x) = 0$ will have:

i. No real root. (1 mark)

$16 - 12\sqrt{2} < k < 16 + 12\sqrt{2}$



ii. One real root. (1 mark)

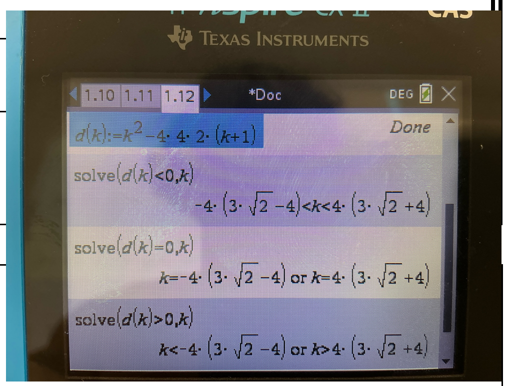
$k = 16 - 12\sqrt{2}$ or

$k = 16 + 12\sqrt{2}$

iii. Two unique real roots. (1 mark)

$\therefore k < 16 - 12\sqrt{2}$ or

$k > 16 + 12\sqrt{2}$



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Question 14 (9 marks)

A cricket player hits a ball, and the ball's trajectory is modelled by the quadratic equation:

$$h(x) = ax^2 + bx + c,$$

where $h(x)$ is the height of the ball (in metres) above the ground, and x is the horizontal distance (in metres) from where the ball was hit.

The following conditions are given:

- The ball is hit from a height of 2 metres, i.e., $h(0) = 2$.
- The ball reaches a height of 15 metres when it has travelled 8 metres horizontally.
- The ball reaches a height of 25 metres when it has travelled 16 metres horizontally.

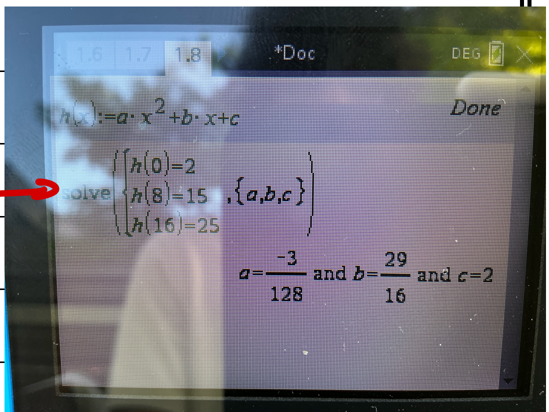
a. Using the, given conditions, set up and solve a system of equations to determine the values of a, b, and c. (3 marks)

$h(0) = 2 \dots (1)$

$h(8) = 15 \dots (2)$

$h(16) = 25 \dots (3)$

Menu 3 7 1

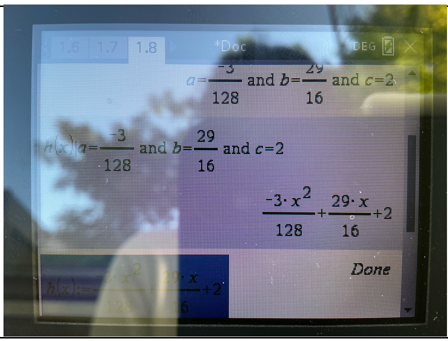


$\therefore a = \frac{-3}{128},\ b = \frac{29}{16},\ c = 2$ //

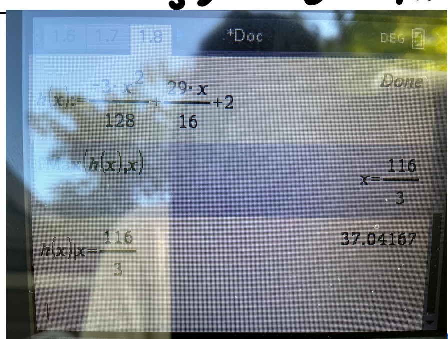
b. Determine the maximum height that the ball reaches. Give your answer correct to 2 decimal places. (1 mark)

$\therefore$ Max value occurs at $x = \frac{116}{3}$ m:

$\therefore h\left(\frac{116}{3}\right) \approx 37.04$ m



Menu 4 → 8



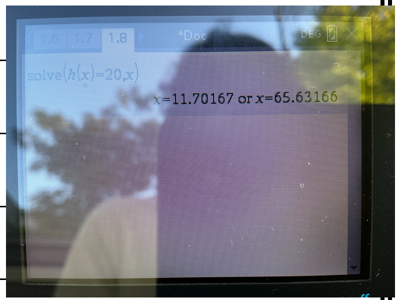
x? $y=20$

c. Determine the horizontal distance the ball has travelled when its height is 20 metres. Provide both possible values of x correct to two decimal places. (2 marks)

$\therefore h(x) = 20$

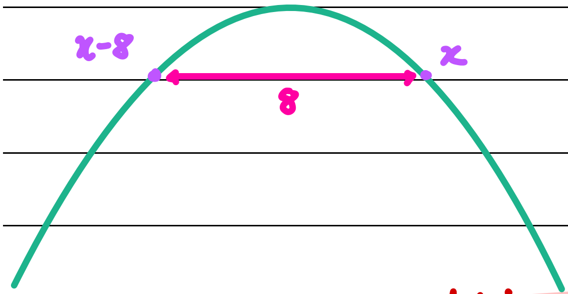
$\therefore x \approx 11.70\text{m}$ or

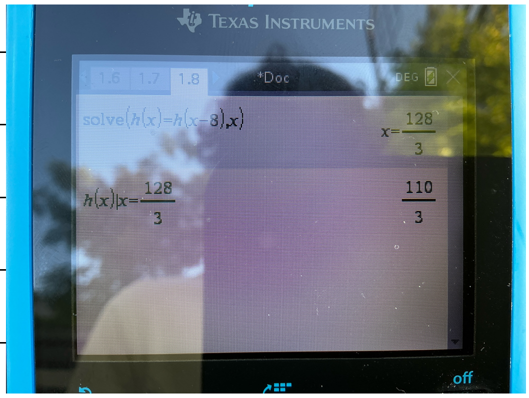
$x \approx 65.63\text{m}$



Find the height where its x-values are 8m apart.

d. ~~Find the exact height where the ball has traveled 8 metres horizontally between the two times that it reaches this height~~. (3 marks)

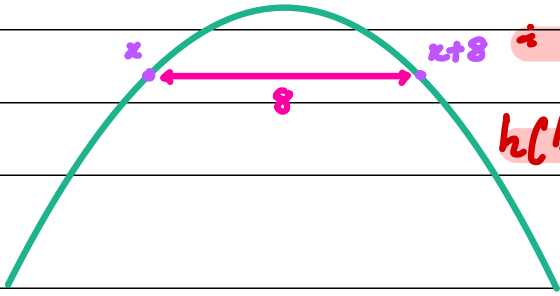


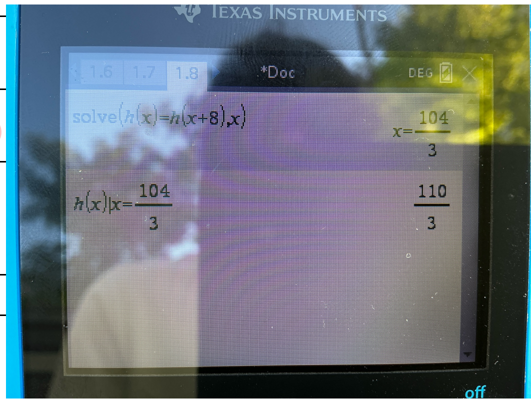


$h(x) = h(x-8)$

$\therefore x = \frac{128}{3}\text{m}$

$h\left(\frac{128}{3}\right) = \frac{110}{3}\text{m}$





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$\therefore \frac{110}{3}\text{m}$

Section F: Extension Exam 1 (16 Marks)

Question 15 (4 marks)

The parabola $y = ax^2 + bx + c$ passes through the points $\left(1, -\frac{1}{2}\right)$, $(4, -5)$, and $(6, -3)$.

Determine the values of real numbers a, b, and c.

$y(1) = -\frac{1}{2}$ ... ① $\Rightarrow a+b+c = -\frac{1}{2}$ ... ①

$y(4) = -5$ ... ② $\Rightarrow 16a+4b+c = -5$ ... ②

$y(6) = -3$ ... ③ $\Rightarrow 36a+6b+c = -3$ ... ③

$$\left.\begin{array}{r} 16a+4b+c = -5 \\ -(a+b+c = -\frac{1}{2}) \end{array}\right\} ② - ①$$

$15a + 3b = \frac{-9}{2}$

$30a + 6b = -9$

$10a + 2b = -3$ ④

$$\left.\begin{array}{r} 36a+6b+c = -3 \\ -(16a+4b+c = -5) \end{array}\right\} ③ - ②$$

$20a + 2b = 2$

$10a + b = 1$ ⑤

$$\left.\begin{array}{r} 10a+b = 1 \\ -(10a+2b = -3) \end{array}\right\} ⑤ - ④$$

$-b = 4$

$\therefore b = -4$

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Sub in ⑤:

$10a - 4 = 1$

$10a = 5 \Rightarrow a = \frac{1}{2}$

Sub in ①:

$\frac{1}{2} - 4 + c = -\frac{1}{2}$

$\therefore c = 3$

Question 16 (4 marks)

Let $f(x) = 2x^2 - 4x + 5$.

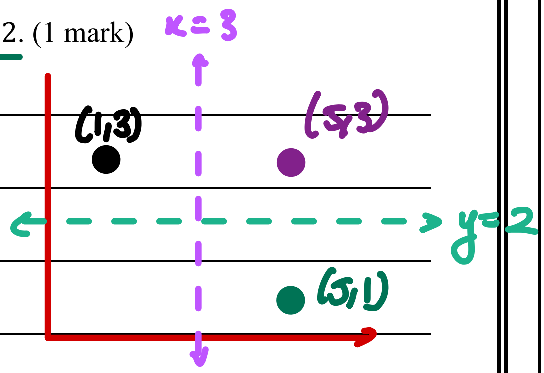
a. Find the turning point of the parabola $y = f(x)$. (1 mark)

$$f(x) = 2(x^2 - 2x) + 5$$
$$= 2\left((x-1)^2 - 1\right) + 5$$
$$= 2(x-1)^2 + 3$$
$$\therefore TP: (1, 3)$$

b. Reflect this turning point in the line $x = 3$ and then in the line $y = 2$. (1 mark)

$(5,3)$ $(5,1)$

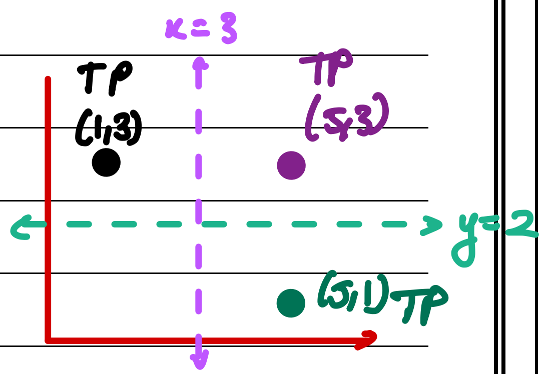
$\therefore (5,1)$



c. The parabola $y = f(x)$ is reflected in the line $x = 3$ and then reflected in the line $y = 2$.
Find the equation of the resulting parabola in the form $y = ax^2 + bx + c$, where $a, b,$ and c are real numbers. (2 marks)

$\therefore y = -2(x-5)^2 + 1$

reflect off $y=2$



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Question 17 (4 marks)

Solve $(x^2+1)^2+4 \geq 8x^2$ for all real x.

Let $a = x^2$:

$(a+1)^2 + 4 \geq 8a$

$a^2 + 2a + 5 \geq 8a$

$a^2 - 6a + 5 \geq 0$

$(a-5)(a-1) \geq 0$

$a \geq 5$ or $a \leq 1$

$x^2 \geq 5$ or $x^2 \leq 1$

$x \geq \sqrt{5}$ or $x \leq -\sqrt{5}$ or $-1 \leq x \leq 1$

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Question 18 (4 marks)

Let $f(x) = x^4 - 4kx^2 + 4 - k^2$, where k is a real constant.

Find the values of k for which the equation $f(x) = 0$ has no real solutions.

Let $f(x) = 0$:

$(x^2 - 2k)^2 - (2k)^2 + 4 - k^2 = 0$

$(x^2 - 2k)^2 = 5k^2 - 4$

$x^2 - 2k = \pm\sqrt{5k^2-4}$ $\quad \therefore -\frac{2\sqrt{5}}{5} < k < \frac{2\sqrt{5}}{5}$

$x^2 = 2k \pm \sqrt{5k^2-4}$ $\quad k^2 < \frac{4}{5}$

$x = \pm\sqrt{\underbrace{2k \pm \sqrt{5k^2-4}}_{<0 \quad <0}}$ $\Rightarrow 5k^2 - 4 < 0$

$2k \pm \sqrt{5k^2-4} < 0$

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$2k + \sqrt{5k^2-4} < 0$

$2k + \sqrt{5k^2-4} = 0$

$2k = -\sqrt{5k^2-4}$

$4k^2 = 5k^2 - 4$

$k^2 = 4$

$k = \pm 2$

$\therefore$ reject $k = 2$

$\therefore k = -2$

$\therefore -2 < k < \frac{2\sqrt{5}}{5}$

$k < -2$? $\quad k > -2$?

$k = -3$ $\quad k = -1$

✗ As > 0 $\quad$ ✓ As < 0

Section G: Extension Exam 2 (16 Marks)

Question 19 (1 mark)

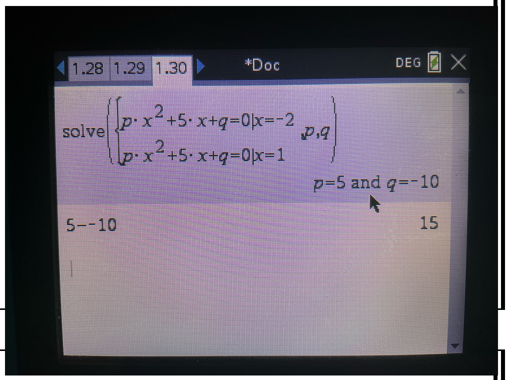
If $px^2 + 5x + q = 0$ has two roots $x = -2$ and $x = 1$, the value of $p - q$ is:

A. -5

B. 5

C. 10

D. 15



Question 20 (1 mark)

The equation of the parabola that passes through the points $(1, 2)$, $(3, 2)$ and $(4, 5)$ is:

A. $y = x^2 - 4x - 5$

B. $y = (x - 2)^2 + 1$

C. $y = x^2 + 4x + 5$

D. $y = (x - 1)^2 + 2$

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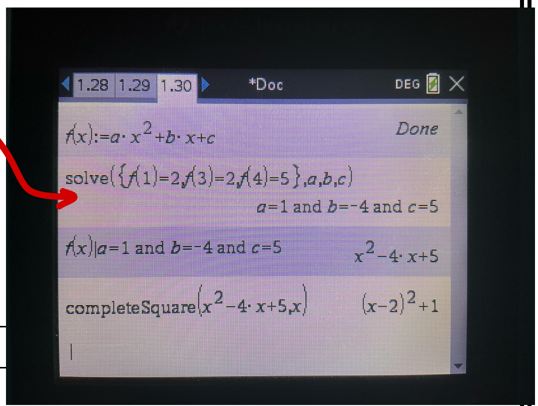


Question 21 (1 mark)

Consider the graph of $y = x^2 - 2kx - 2$ where k is a real constant.

The values of k for which the distance between the two x-intercepts is less than 6 are:

$\frac{\sqrt{\Delta}}{a}$ < 6

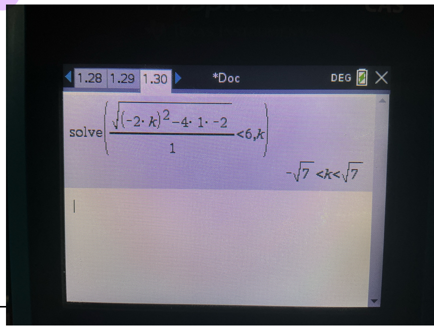
A. $-\sqrt{5} < k < \sqrt{5}$

B. $-\sqrt{6} < k < \sqrt{6}$

C. $-\sqrt{7} < k < \sqrt{7}$

$\frac{\sqrt{\Delta}}{a} < 6$ $\Longrightarrow$

D. $-\sqrt{11} < k < \sqrt{11}$



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Sketch function | w/ domain restriction $(-2<x<3)$

Question 22 (1 mark)

Let $y = 2x^2 - 4x - 2$.

If $-2 < x < 3$, the possible values of y are:

A. $-4 < y \leq 14$

B. $-4 \leq y < 14$

C. $4 < y < 14$

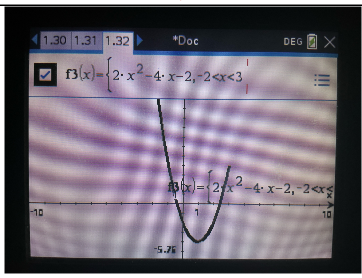
D. $-4 < y < 14$

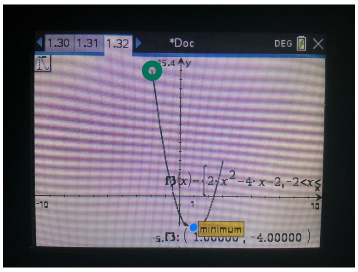
Menu → 5 → 1

Graph Trace

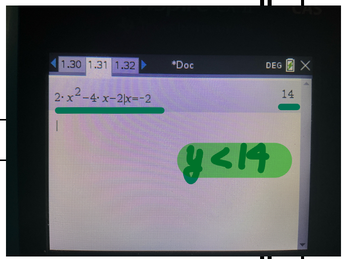
Find minimum

$4 \leq y$





Max y-value occurs at leftmost point. $(x=-2)$



$y<14$

Question 23 (1 mark)

Find all values of k, such that $x^2 + kx + k^2 - 4$ has two real roots for x, where one is positive and one is negative.

A. $k < 2$

B. $k > -2$

C. $-2 < k < 2$

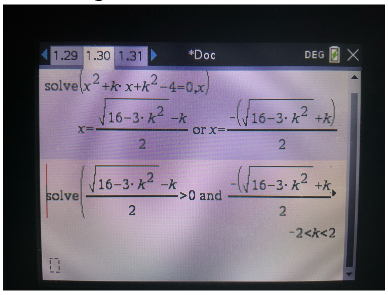
D. $-2 \leq k \leq 2$

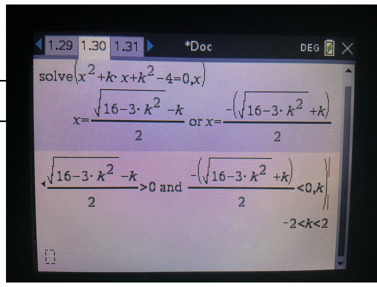
Solve$(x^2+kx+k^2-4=0, x)$

↳ ∴ $x =$ +ve root &

$x =$ −ve root

∴ Solve(+ve root > 0 & −ve root < 0, k)





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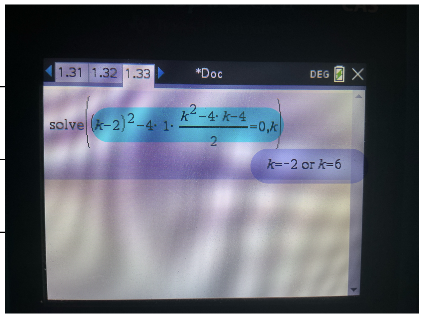
Question 24 (11 marks)

Consider the function $f(x) = x^2 + (k-2)x + \frac{k^2-4k-4}{2}$, where k is a real constant.

a.

i. Find all values of k such that $f(x) = 0$ has one real root.. (1 mark)

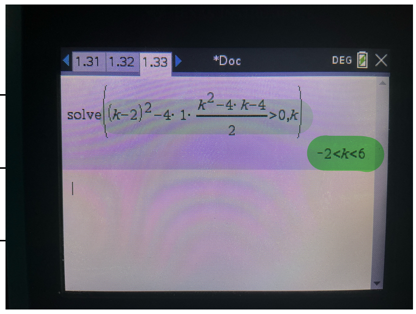
$\Delta = 0$



$\therefore k = -2, 6$

ii. Find all values of k such that $f(x) = 0$ has two real roots. (1 mark)

$\Delta > 0$



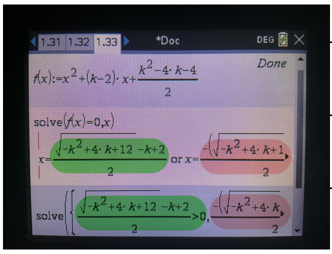
$\therefore -2 < k < 6$

iii. Find all values of k such that $f(x) = 0$ has two real roots, where one is positive and the other is negative. (2 marks)

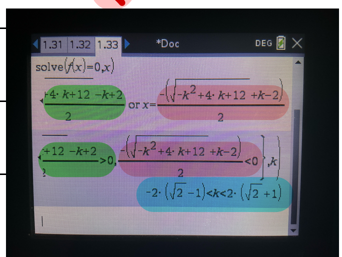
+ve Root > 0   −ve Root < 0

Let $f(x) = 0$:

$x = \frac{\sqrt{-k^2+4k+12} - k + 2}{2}$ OR $\frac{-\sqrt{-k^2+4k+12} - k + 2}{2}$



$\therefore$ +ve Root > 0 & −ve Root < 0



$\therefore -2(\sqrt{2}-1) < k < 2(\sqrt{2}+1)$

OR!

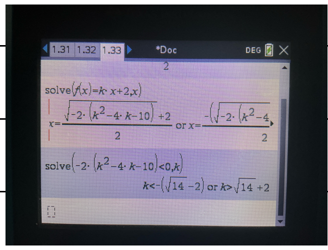
$\therefore 2 - 2\sqrt{2} < k < 2 + 2\sqrt{2}$

b. Find all values of k for which the graph of $y = f(x)$ and the graph $y = kx + 2$ do not intersect. (2 marks)

No solution

$\Delta < 0$

let $f(x) = kx + 2$:



$\therefore x = \frac{2 \pm \sqrt{-2(k^2-4k-10)}}{2}$

$\Delta < 0$:

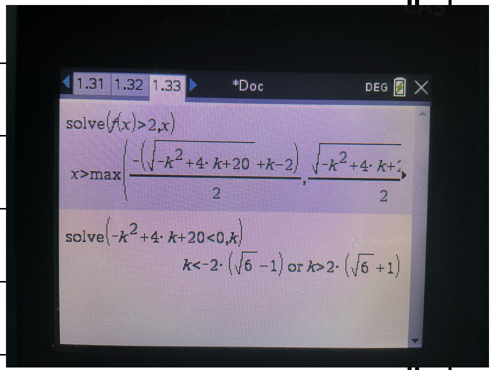
$\therefore -2(k^2-4k-10) < 0 \Rightarrow \therefore k < 2 - \sqrt{14}$ or $k > 2 + \sqrt{14}$

c. Find all values of k such that $f(x) > 2$ for all real x. (2 marks)

$\underbrace{f(x) - 2 > 0}$

$\Delta < 0$ (As no x-ints when y is always >0)

$\therefore k < 2 - 2\sqrt{6}$ or $k > 2 + 2\sqrt{6}$



d. Find all values of k such that the graph of $y = f(x)$ has two x-intercepts that have a distance between them that is less than 2. (3 marks)

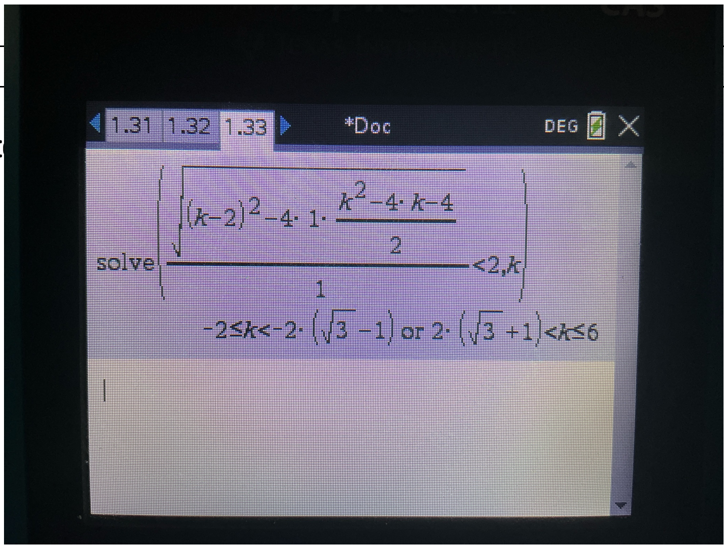
<2

$\frac{\sqrt{\Delta}}{a}$

$\therefore \frac{\sqrt{\Delta}}{a} < 2$

$$\boxed{\therefore \frac{\sqrt{(k-2)^2 - 4(1)\left(\frac{k^2-4k-4}{4}\right)}}{1} < 2}$$

$\therefore -2 \leq k < 2 - 2\sqrt{3}$ or $2 + 2\sqrt{3} < k \leq 6$



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