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VCE Mathematical Methods ½
Quadratics [0.3]
Extension Workshop

Section A: Recap

Sub-Section: Factorising Quadratics

Factorising Quadratics

$$y = (x - a)(x - b)$$

- **Steps:**

1. Divide by the coefficient of the leading term. (If applicable)
2. Consider the factors of the constant term.
3. (If Positive Constant Term): See which pair of factors can add up to the coefficient of the x term.

   (If Negative Constant Term): See which pair of factors can subtract from the coefficient of the x term.
4. Construct the linear factors.

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Sub-Section: Perfect Squares

Perfect Squares

$$(a+b)^2 = \underline{\hspace{8cm}}$$

$$(a-b)^2 = \underline{\hspace{8cm}}$$

- Perfect squares are special quadratic expressions that are made up of two **identical** linear factors.
- In other words, when a linear factor is squared, it becomes a perfect square.

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Sub-Section: Difference of Squares

Difference of Squares

$$a^2 - b^2 = \underline{\hspace{6cm}}$$

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Sub-Section: Completing the Square

Completing the Square

- When we complete the square of a quadratic $x^2 + bx + c$, we write it in the form:

$$\boldsymbol{x^2 + bx + c = (\_\_\_\_\_\_\_\_\_\_\_\_)^2 - \left(\frac{b}{2}\right)^2 + c}$$

- **Steps:**

1. We halve the coefficient of x.

2. Subtract the half of the coefficient of x squared outside the square bracket.

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Sub-Section: Solving by Factorisation

Solving by Factorisation

$$(x-a)(x-b)=0$$

$$x=a \text{ or } b$$

- Steps:

1. Factorise the quadratic.
2. Equate each factor to 0 and solve for x.

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Sub-Section: Quadratic Formula

The Quadratic Formula

for $ax^2 + bx + c = 0$

$x =$ ______________________________

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Sub-Section: Discriminant

The Discriminant

Definition:

- The discriminant, often denoted by **Δ** (Delta), is the part **inside** the square root of the quadratic formula.

$$Discriminant = \Delta = b^2 - 4ac$$

$\text{if } \Delta > 0$, there are ____.

$\text{if } \Delta = 0$, there is ____.

$\text{if } \Delta < 0$, there are ____.

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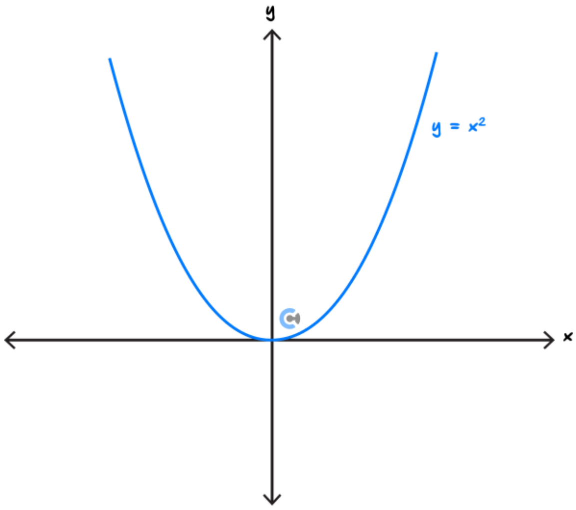
Sub-Section: Parabola and Symmetry

Definition

Parabola

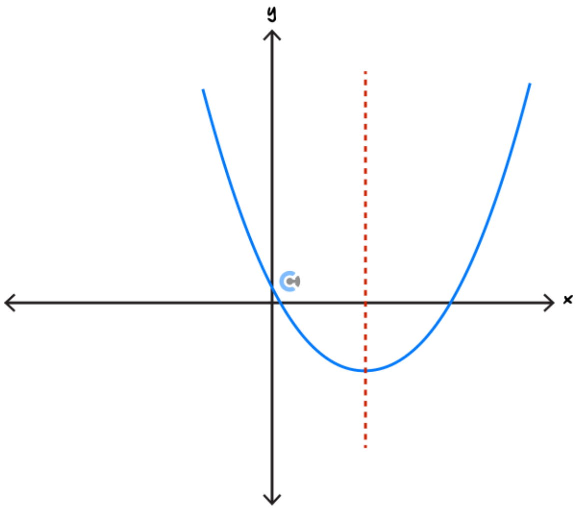
Definition:

- The shape of the graph of a quadratic is known as a ____________________________.



Definition

Axis of Symmetry



$$\textit{Axis of symmetry}: x = -\frac{b}{2a}$$

Sub-Section: Graphing Quadratics

Turning Point Form

- The turning point form of a quadratic is given by:

$$y = a(x-h)^2 + k$$

$$\textit{Turning point} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

- The turning point form is obtained by **completing the square.**

Intercept Form

- The x-intercept form of a quadratic is given by:

$$y = a(x-b)(x-c)$$

$$x\textit{-intercepts}: (b, 0) \textit{ and } (c, 0)$$

- The axis of symmetry is located exactly in the middle of the two x-intercepts.

NOTE: When a is negative, the x-intercepts stay the same, but the **shape** of the parabola becomes a **negative** parabola instead.

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Sub-Section: Finding a Rule of a Quadratic from a Graph

Finding the Equation of a Quadratic

- **Form 1: Turning Point Form**

$$y = a(x - h)^2 + k$$

- Recommended when a turning point is easy to identify.

- **Form 2: x-intercept Form**

$$y = a(x - b)(x - c)$$

- Recommended when both x-intercepts are easy to identify.

NOTE: Never forget the a coefficient!

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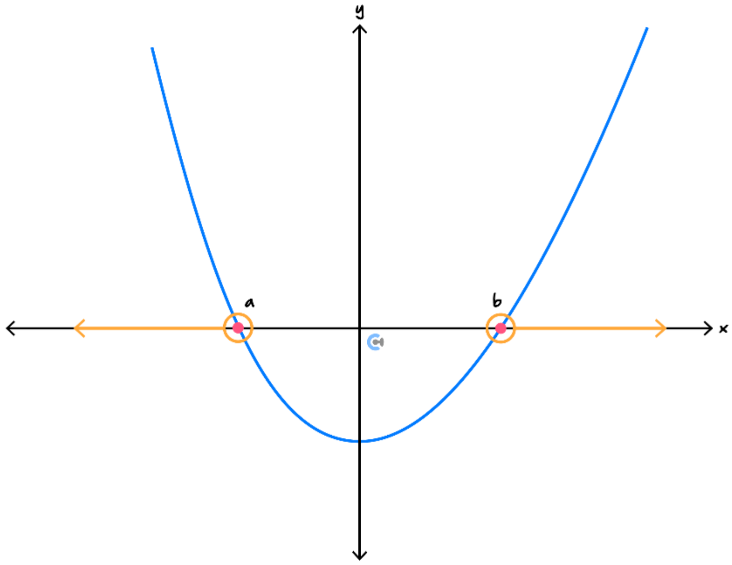
Sub-Section: Quadratic Inequalities

Quadratic Inequalities



- For quadratic inequalities, we always _______________ the function.
- **Steps:**

1. Sketch the function.
2. See where the y-value is within the inequality.
3. Find the corresponding x-values.

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Sub-Section: Hidden Quadratics

Hidden Quadratics

- Instead of:

$$af(x)^2 + bf(x) + c = 0$$

- We can let $f(x) = X$ to have:

$$aX^2 + bX + c = 0$$

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Section B: Warmup

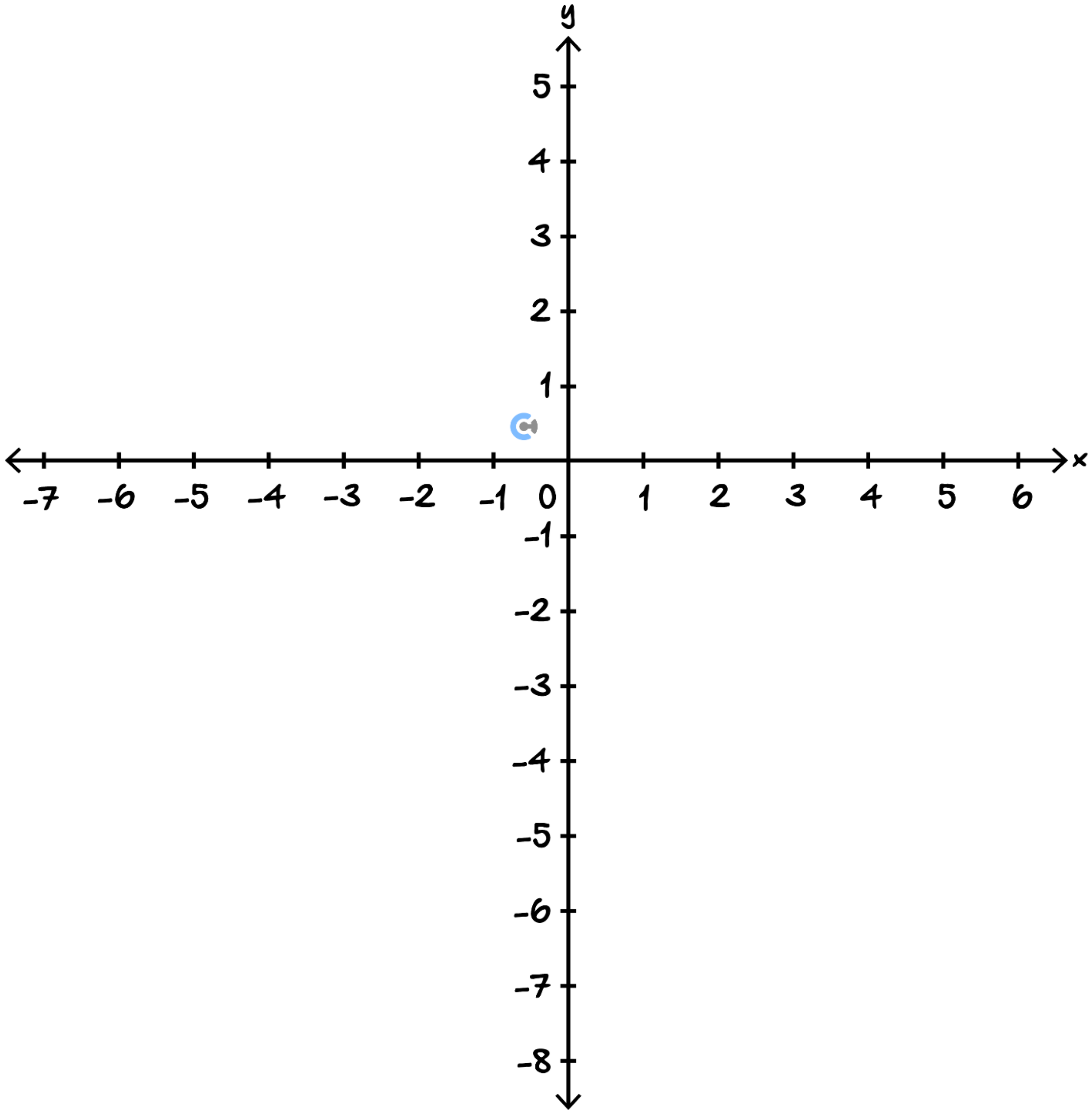
Question 1

Consider the quadratic function $f(x) = x^2 + x - 6$.

a. Find the x-intercepts of the graph $y = f(x)$.

b. Write $f(x)$ in turning point form.

c. Sketch the graph of $y = f(x)$ on the axis below. Label all axes intercepts and the turning point.



d. Find the number of intersections the graph of $y = f(x)$ has with the graph $y = x - 6$.

Section C: Exam 1 (20 Marks)

Question 2 (2 marks)

Determine how many solutions the following equation has:

Δ

$$x^2 + 3x + 1 = 7x - 1$$

$x^2 - 4x + 2 = 0$

$\Delta = (-4)^2 - 4(1)(2)$

$= 16 - 8 = 8$

$\therefore$ As $\Delta > 0 \Rightarrow$ 2 real sols for x!

Question 3 (6 marks)

Solve the following equations for x.

a. $x^2 + 4x - 8 = 0$. (3 marks)

$x = \dfrac{-4 \pm \sqrt{(4)^2 - 4(1)(-8)}}{2(1)}$

$= \dfrac{-4 \pm \sqrt{48}}{2}$

$x = \dfrac{-4 \pm 4\sqrt{3}}{2}$

$\therefore x = -2 + 2\sqrt{3}$ or

$x = -2 - 2\sqrt{3}$ //

b. $x^4 - 13x^2 + 36 = 0$. (3 marks)

Let $a = x^2$;

$a^2 - 13a + 36 = 0$

$\therefore (a-9)(a-4) = 0$

$\therefore a = 9$ or $a = 4$

$\therefore x^2 = 9$ or $x^2 = 4$

$x = \pm 3$ or $x = \pm 2$

Question 4 (5 marks)

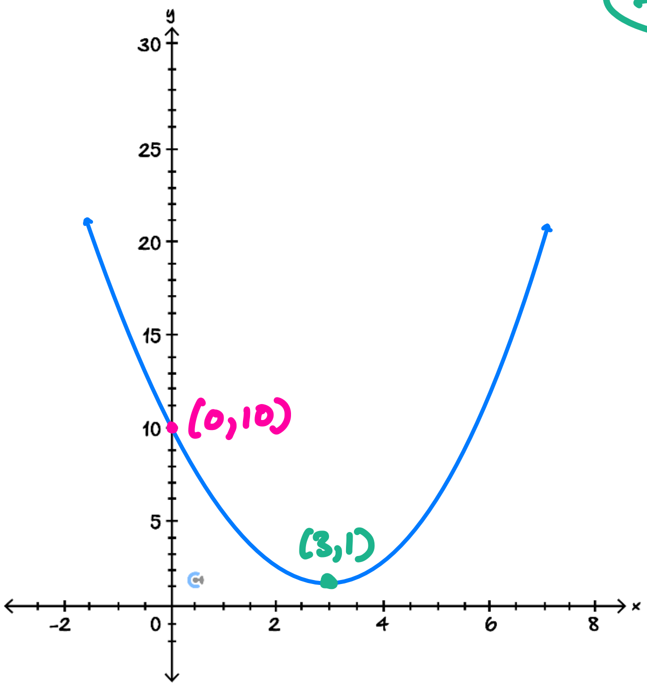
a. Write $x^2 - 6x + 10$ in turning point form. (2 marks)

$= x^2 - 6x + 9 + 1$

$= (x-3)^2 + 1$

b. Hence, graph the curve given by $y = x^2 - 6x + 10$. Label the turning point and all axis intercepts. (3 marks)

TP: $(3,1)$



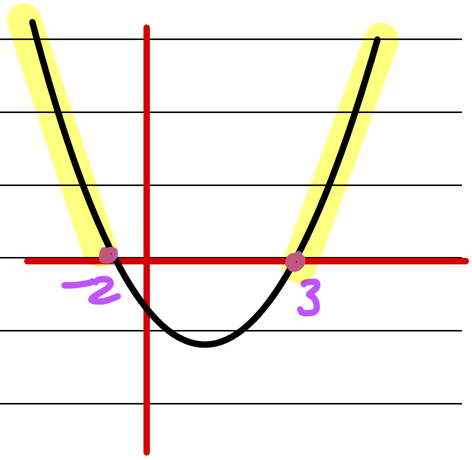
Question 5 (3 marks)

Solve $x^2 - x > 6$ for x.

$x^2 - x - 6 > 0$

$(x-3)(x+2) > 0$

$\therefore x < -2$ or $x > 3$



$x^2 < 4$

$-2 < x < 2$

$-\sqrt{\ } < \quad < +\sqrt{\ }$

$x^2 > 4$

$x < -2 \quad x > 2$

$< -\sqrt{\ } \quad > +\sqrt{\ }$

Question 6 (4 marks)

Find the value(s) of m for which the graphs of $y = mx + 2$ and $y = x^2 + 3x + 2$ have:

a. One point of intersection. (3 marks)

$x^2 + 3x + 2 = mx + 2$

$x^2 + (3-m)x = 0$

$\Delta = 0$:

$(3-m)^2 - 4(1)(0) = 0$

$(3-m)^2 = 0$

$\therefore m = 3$

$x(x + 3 - m) = 0$

$x = 0$ or $x = m - 3$

$m - 3 = 0$

$\therefore m = 3$

b. Two points of intersection. (1 mark)

$\Delta > 0$:

$(3-m)^2 > 0$

$3 - m \neq 0$

$m \neq 3$

$m - 3 \neq 0$

$m \neq 3$

$m \in \mathbb{R} \setminus \{3\}$

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Section D: Tech Active Exam Skills

Calculator Commands: Solving equations

Mathematica

- Solve[].

```
In[122]:= Solve[x^2 - 4 x - 9 == 0, x]
Out[122]= {{x → 2 - √13}, {x → 2 + √13}}
```

TI-Nspire

- Menu→ 3 → 1.

$$\text{solve}\left(x^2-4\cdot x-9=0,x\right) \quad x=-\left(\sqrt{13}-2\right) \text{ or } x=\sqrt{13}+2$$

Casio Classpad

- Action→Advanced→Solve.

$$\text{solve}(x\text{^}2-4x-9=0,x)$$
$$\{x=-\sqrt{13}+2,\ x=\sqrt{13}+2\}$$

Calculator Commands: Completing the Square

TI-Nspire

- Menu→ 3 → 5 completeSquare (func, var).

$$\text{completeSquare}\left(x^2-6\cdot x+8,x\right) \quad (x-3)^2-1$$

Mathematica

- no inbuilt function need udf.

Compsq

Compsq[a x^2 + b x + c, x]
Converts a standard form quadratic to turning point form.

Compsq[a, b, c]
Uses the coefficients of a quadratic to return the turning point form.

```
Compsq[1, 2, 3]
2 + (1 + x)^2
Compsq[2 x^2 + 3 x - 5, x]
Compsq[-5 + 3 x + 2 x^2, x]
```

Casio Classpad

- No function

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Section E: Exam 2 (31 Marks)

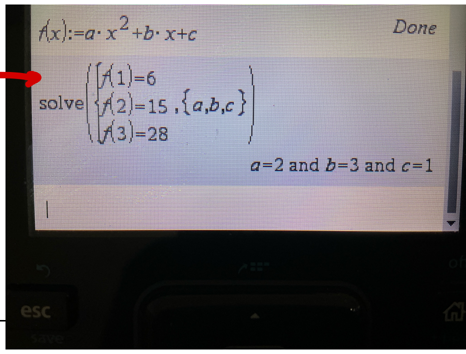
Question 7 (1 mark)

Which of the following is the equation of a parabola which passes through the points (1,6), (2,15) and (3,28)?

A. $3x^2 + 5x - 1$

B. $x^2 + 2x + 1$

Menu → 3 → 7



C. $-x^2 + 3x - 2$

D. $2x^2 + 3x + 1$

Question 8 (1 mark)

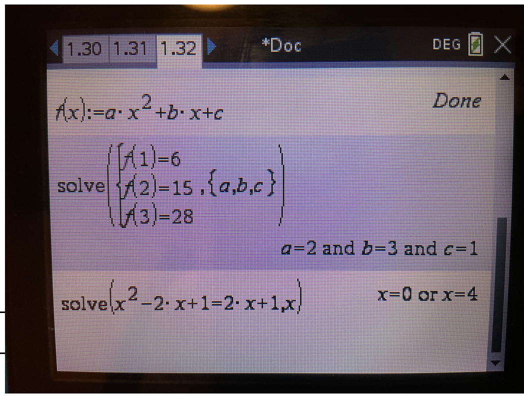
Determine how many intersections the parabola given by $y = x^2 - 2x + 1$ has with the line given by $y = 2x + 1$.

A. 0

B. 1

C. 2

D. 3



Question 9 (1 mark)

Which of the following parabolas is always positive for all real values of x?

A. $(x-3)^2 - 1$ (3,-1) TP

B. $-(x-3)^2 - 1$ (3,-1) TP

C. $(x+1)^2 + 3$ (-1,3) TP

D. $-(x+1)^2 + 3$ (-1,3) TP

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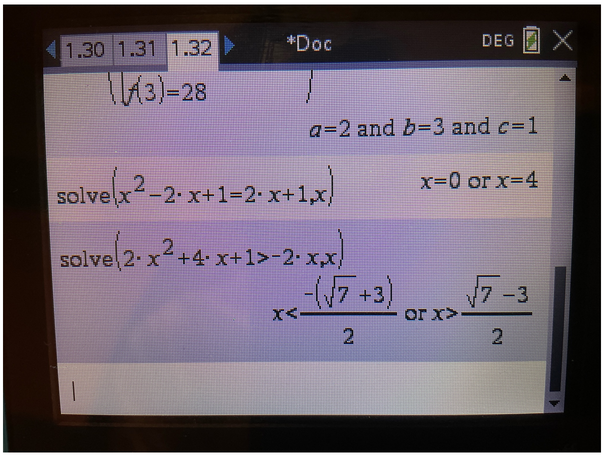
Question 10 (1 mark)

Find the interval(s) of x that satisfies $2x^2 + 4x + 1 > -2x$.



A. $x < \frac{1}{2}(-3-\sqrt{7})$ or $x > \frac{1}{2}(-3+\sqrt{7})$

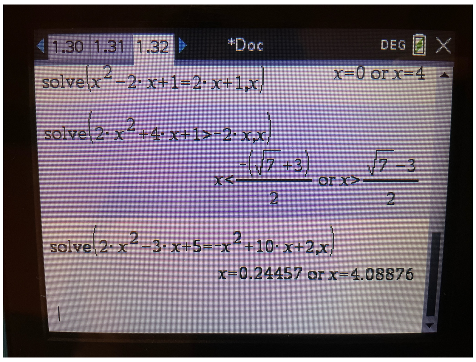
B. $x < 2$ or $x > 4$

C. $x < -\frac{1}{2}(1+\sqrt{3})$ or $x > -\frac{1}{2}(1-\sqrt{3})$

D. $x > -\frac{1}{3}(1+2\sqrt{3})$

Question 11 (1 mark)

Determine the x-coordinates of the intersection between the parabola given by $y = 2x^2 - 3x + 5$ and the parabola given by $y = -x^2 + 10x + 2$ correct to 3 decimal places.



A. $0.245, 4.089$

B. $-0.234, 4.089$

C. $0.345, 1.324$

D. $-0.345, 1.324$

Question 12 (1 mark)

The parabola given by $y = 3x^2 + 4x + 1$ is symmetrical about which of the following?

Axis of symmetry

$= \frac{-b}{2a}$

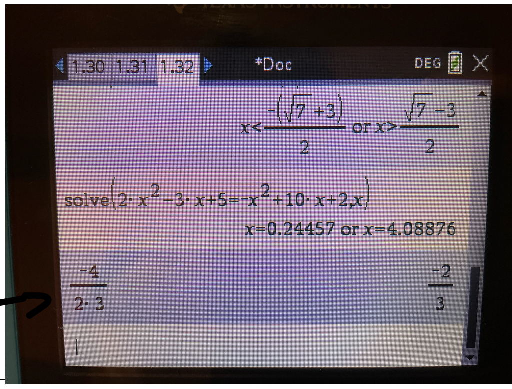
$x = \frac{-4}{2(3)} = \frac{-2}{3}$

A. $x = \frac{1}{3}$

B. $y = \frac{1}{3}$

C. $x = -\frac{2}{3}$

D. $x = \frac{2}{3}$



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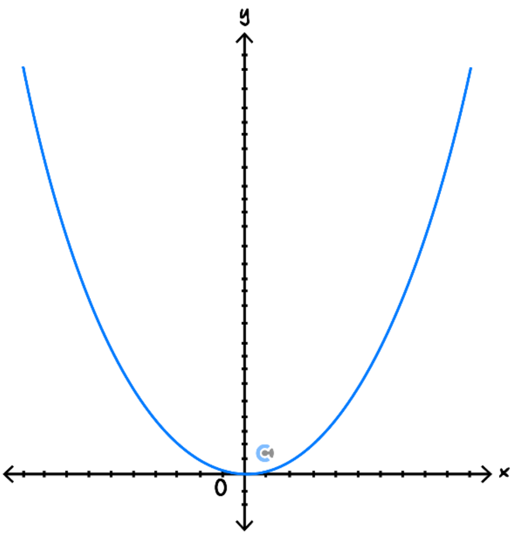
Question 13 (1 mark)

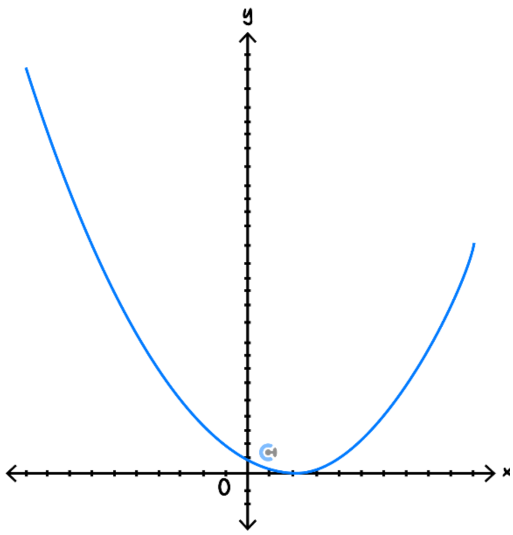
$\rightarrow (a, 0)$

Which of the following could be the graph of $y = (x - a)^2$ where $a < 0$?

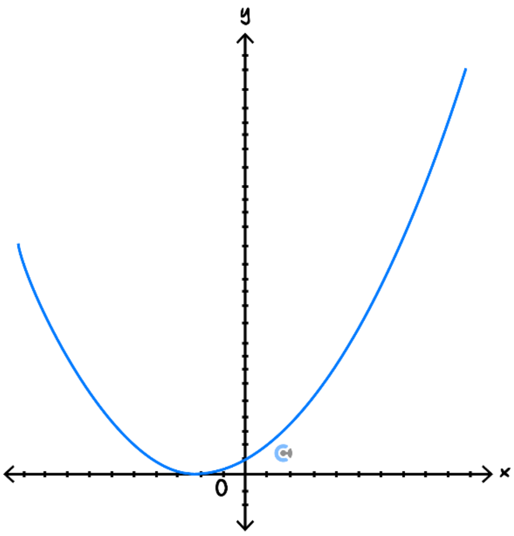
~~A.~~



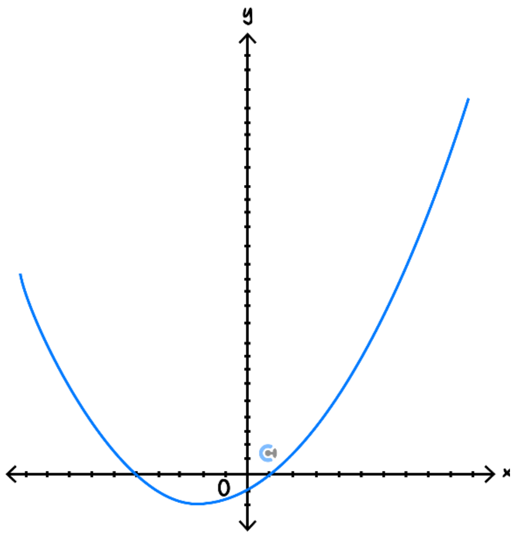
~~B.~~



C.



~~D.~~



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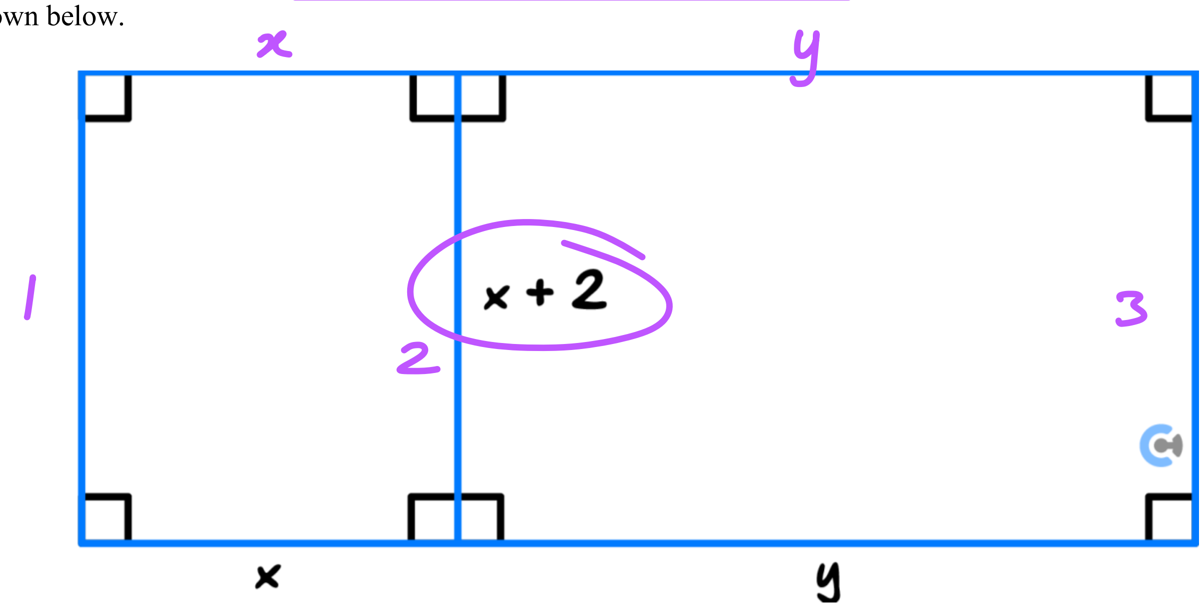
Question 14 (10 marks)

Aluminium window frames are made using 40 metre lengths of aluminium. The dimensions of the frames are shown below.



a. Show that $y = \frac{34-5x}{2}$. (1 mark)

$40 = 3(x+2) + 2x + 2y$

$2y = 40 - 5x - 6$

$= 34 - 5x$

$y = \frac{34-5x}{2}$ Q.E.D

b. Hence, show that the area, A, in square metres, enclosed by one of these frames is given by: (2 marks)

$$A(x) = -\frac{3x^2}{2} + 14x + 34$$

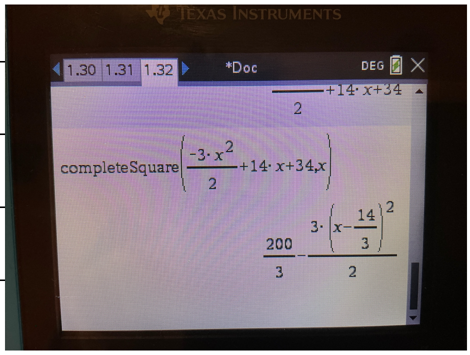
$A(x) = (x+y)(x+2)$

$= \left(x + \frac{34-5x}{2}\right)(x+2)$

$= -\frac{3x^2}{2} + 14x + 34$

c. Express the area function, $A(x)$, given in **part b.** in turning point form. (1 mark)

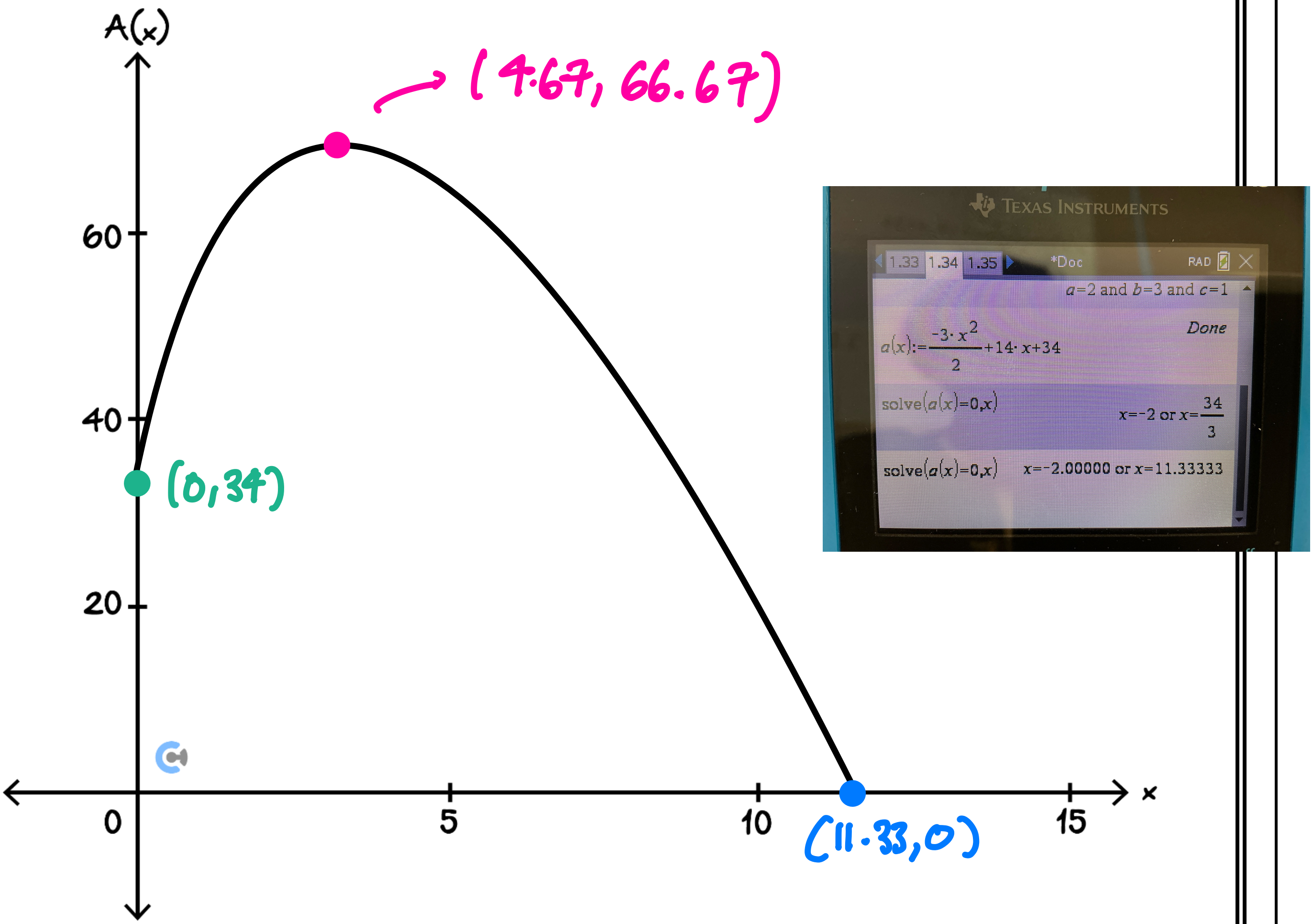
$A(x) = -\frac{3}{2}\left(x - \frac{14}{3}\right)^2 + \frac{200}{3}$



d. Given that this function represents area, sketch the graph of the function:

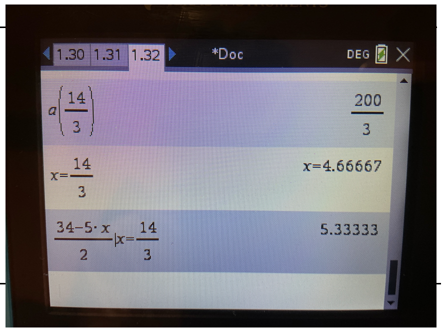
$$A(x) = -\frac{3x^2}{2} + 14x + 34$$

On the set of axes below. Clearly mark on your graph the coordinates of intercepts, endpoints as well as the turning point. Express these coordinates correctly to two decimal places. (3 marks)



e. What is the maximum area enclosed by the frame? Give your answer correct to two decimal places. (1 mark)

$\therefore$ Max Area = $66.67 m^2$ //



f. When the area enclosed by the frame is a maximum, what is:

i. The value of x? Correct to two decimal places. (1 mark)

$\therefore$ $x = 4.67m$

ii. The value of y? Correct to two decimal places. (1 mark)

$\therefore$ $y \approx 5.33m$ $\qquad y = \frac{34-5x}{2}$

Question 15 (14 marks)

Two Contour students are playing catch outside near the reception. The path of the ball is given by the equation $y = -x^2 + x + 1$ where x is the horizontal distance relative to one of the students (given in metres) and y is the height of the ball above the ground (given in metres).

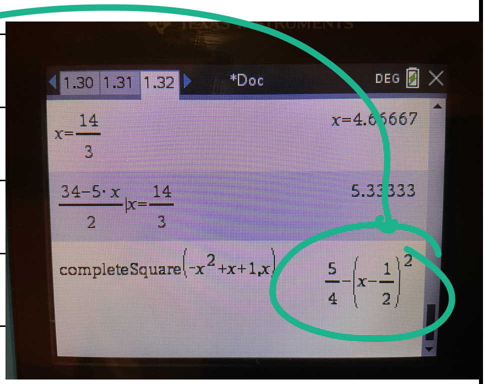
a. Determine the height at which the ball leaves the first student's hands. (1 mark)

$y(0) = 1m$

b. The receptionists see the game of catch occurring and worry about whether the ball will hit the ceiling. Given that the ceiling is 2.5 m tall determine whether the ball will hit the ceiling. (2 marks)

TP: $\left(\frac{1}{2}, \frac{5}{4}\right)$

≈ 1.25m



∴ As TP y-value < 2.5m,

∴ The ball will not hit the ceiling!

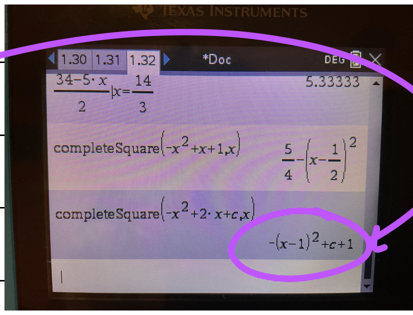
The students decide to propose a new rule for the path that the ball will undertake given by,

$$y = -x^2 + 2x + c$$

where c is a real constant.

c. Determine the coordinates of the ball when it reaches its maximum height in terms of c. (2 marks)

TP: $(1, c+1)$



d. Using this new rule, they wish to determine the value of c such that the ball will just touch the ceiling but not penetrate it. Determine the value of c for this to occur. (1 mark)

$c+1$ 2.5m

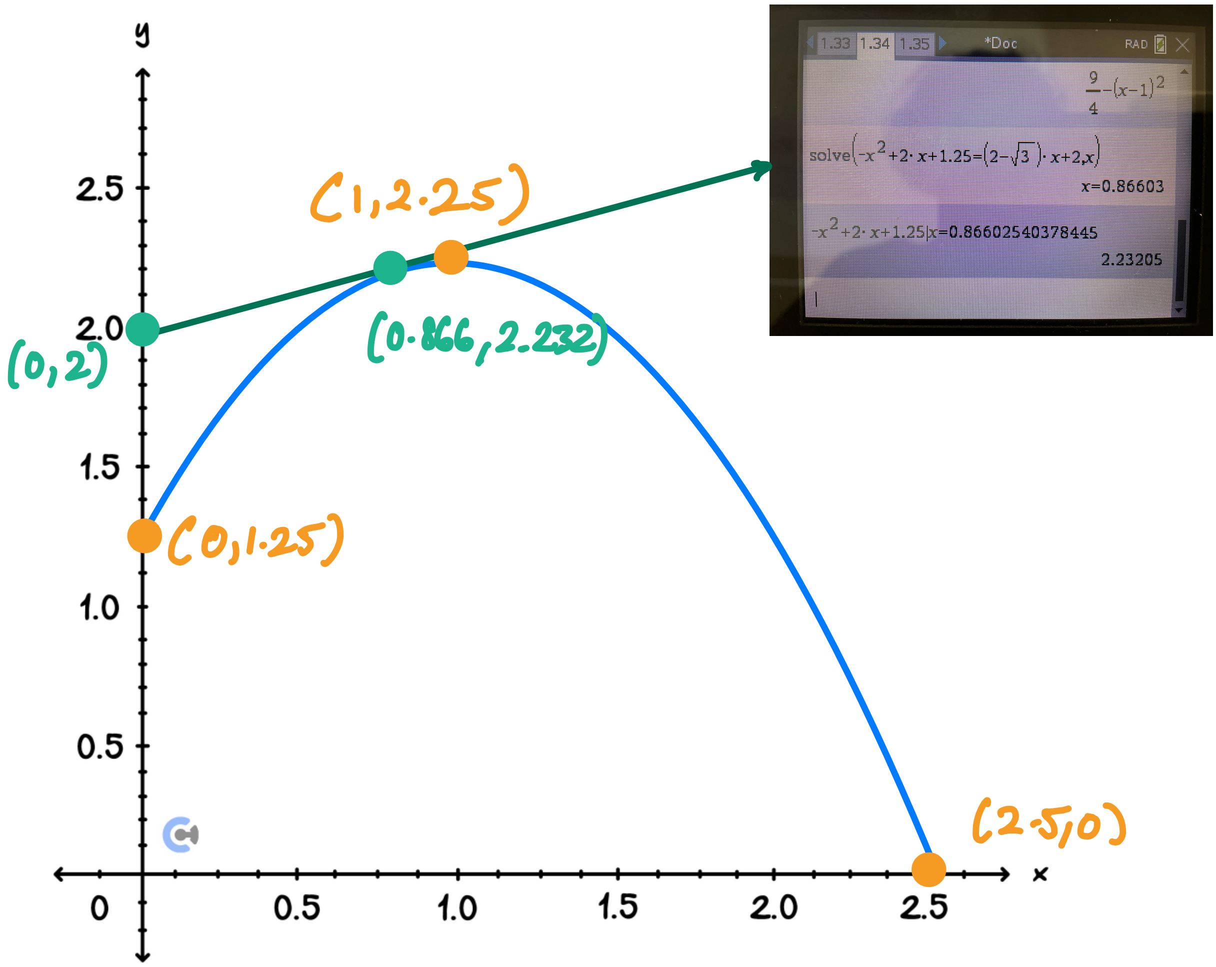
$c+1 = 2.5$

∴ $c = 1.5$m

Deciding to play it safe the Contour students let $c = 1.25$ giving the following equation,

$$y = -x^2 + 2x + 1.25$$

e. Plot the path of this ball on the axes below for $x \in [0,2.5]$ labelling the turning point, y-intercept, and endpoints. (3 marks)



f. Another Contour student is flying a paper plane that travels in a straight line given by the equation $y = mx + 2$ where m is a real constant. Determine the values of m such that the plane's path does not cross the path of the ball. (3 marks)

$-x^2+2x+\frac{5}{4} = mx+2 \Rightarrow$ 0 intersections $\Rightarrow$ 0 solns

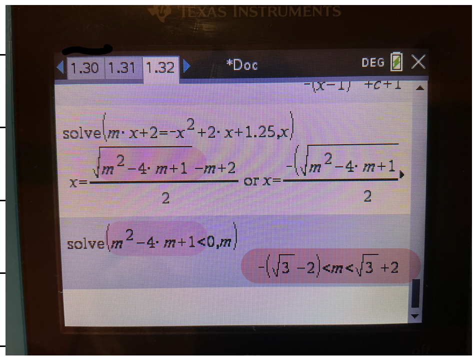
$x^2+(m-2)x+\frac{3}{4}=0$

$\Delta<0$

$(m-2)^2-4(1)\left(\frac{3}{4}\right)<0$

$(m-2)^2<3$

$\hookrightarrow 2-\sqrt{3}<m<2+\sqrt{3}$ //



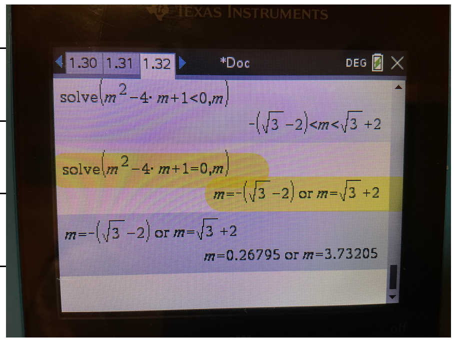
g. The same Contour student who is flying his paper plane realises that he does wish for his plane to cross the path of the ball where $m < 2$. On the axes used to answer **part e.** plot the path of the ball such that the paths cross only once and label this point correct to 3 decimal places. (2 marks)

$\Delta = 0$

$\therefore \Delta = m^2 - 4m + 1 = 0$

$m = 2 - \sqrt{3}$ or $m = 2 + \sqrt{3}$

reject as $m < 2$



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Section F: Extension Exam 1 (14 Marks)

Question 16 (7 marks)

Consider the function $f(x) = kx^2 - 3kx - 2$, where $k > 0$.

a. Show that the graph $y = f(x)$ always has two x-intercepts. (1 mark)

$\Delta > 0$

$(-3k)^2 - 4(k)(-2) > 0$

$9k^2 + 8k > 0$

$k(9k+8) > 0$

↳ $k = 0$ ↳ $k = -\frac{8}{9}$

$k > 0$ or $k < -\frac{8}{9}$

↳ ∴ This is always true as $k > 0$.

b. Find the values of k such that the distance between the two x-intercepts of the graph $y = f(x)$ is less than 5. (3 marks)

$\frac{\sqrt{\Delta}}{a} < 5$

$\frac{\sqrt{9k^2+8k}}{k} < 5$

$\sqrt{9k^2+8k} < 5k$

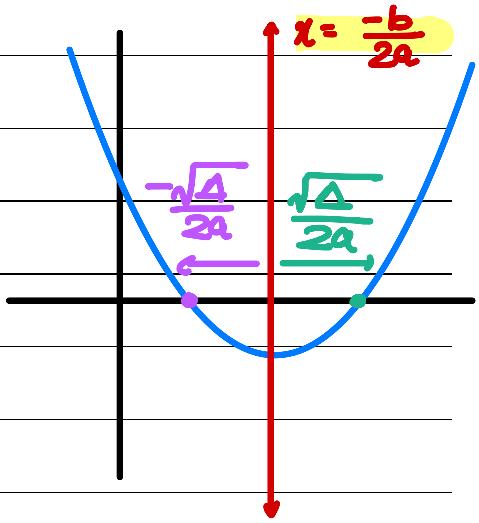
$9k^2 + 8k < 25k^2$

$16k^2 - 8k > 0$

$8k(2k-1) > 0$

↳ $k = 0$ ↳ $k = \frac{1}{2}$

$\therefore k > \frac{1}{2}$ or $k < 0$ ↳ reject as $k > 0$



$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

c. Find the values of k for which the graph of $y = f(x)$ and the graph $y = x - 6k$, where $k > 0$, intersect twice. (3 marks)

$\Delta > 0$

$kx^2 - 3kx - 2 = x - 6k$

$kx^2 + (-3k-1)x + 6k - 2 = 0$

$\Delta = (-3k-1)^2 - 4(k)(6k-2) > 0$

$9k^2 + 6k + 1 - 24k^2 + 8k > 0$

$-15k^2 + 14k + 1 > 0$

$15k^2 - 14k - 1 < 0$

$(15k+1)(k-1) < 0$

$\hookrightarrow k = \frac{-1}{15}$ $\hookrightarrow k = 1$ $\Rightarrow$ $\therefore \frac{-1}{15} < k < 1$

$\hookrightarrow$ $\therefore 0 < k < 1$ or

$k \in (0, 1)$

as $k > 0$

Question 17 (7 marks)

Let $f(x) = x^4 - 2kx^2 + k$, where k is a real constant.

a. Find the values of k for which the graph of $y = f(x)$ has no x-axis intercepts. (3 marks)

$\Delta < 0$

Let $a = x^2$:

$a^2 - 2ka + k = 0$

$\Delta = (-2k)^2 - 4(1)(k) < 0$

$4k^2 - 4k < 0$

$\therefore 0 < k < 1$

$4k(k-1) < 0$

$\hookrightarrow k = 0$ $\hookrightarrow k = 1$

$k(k-1) > 0$

reject $k<0$ as it doesn't fulfill the other condition → $k<0$ or $k>1$

b. Find the values of k for which the graph of $y = f(x)$ has four x-axis intercepts. (2 marks)

$a^2 - 2ka + k = 0$

$(a-k)^2 - k^2 + k = 0$

$(a-k)^2 = k^2 - k$

$a - k = \pm\sqrt{k^2-k}$

$a = k \pm \sqrt{k^2-k}$

$x^2 = k \pm \sqrt{k^2-k}$

$x = \pm\sqrt{\underbrace{k}_{>0} \pm \underbrace{\sqrt{k^2-k}}_{>0}}$

$k^2 - k > 0$

&

$k \pm \sqrt{k^2-k} > 0$

$k + \sqrt{k^2-k} > 0$

$k - \sqrt{k^2-k} > 0$

$\therefore k > 1$

c. Find the values of k for which the graph of $y = f(x)$ has two x-axis intercepts. (1 mark)

$k \pm \sqrt{k^2-k} > 0$

$x = \pm\sqrt{\underbrace{k}_{>0} \pm \underbrace{\sqrt{k^2-k}}_{=0}}$

$k > 0$

$k^2 - k = 0 \Rightarrow k(k-1) = 0$

$k = 0$ or $k = 1$

$\therefore k = 1$ or $k < 0$

$\pm\sqrt{k + \sqrt{k^2-k}}$

$k - \sqrt{k^2-k}$ doesn't work for $k<0$, $\therefore$ 2 solns.

d. Find the values of k for which the graph of $y = f(x)$ has one x-axis intercept. (1 mark)

$x = \cancel{\pm}\sqrt{\underbrace{k}_{=0} \cancel{\pm} \underbrace{\sqrt{k^2-k}}_{=0}}$

$k^2 - k = 0$

$k \pm \sqrt{k^2-k} = 0$

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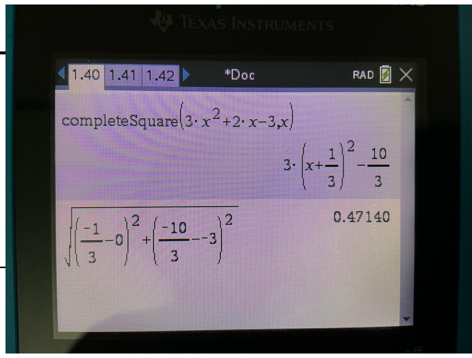
$k(k-1) = 0$

$k = 0$ or $k = 1$ & $k = 0$

$\therefore k = 0$

Section G: Extension Exam 2 (16 Marks)



Question 18 (1 mark)

Find the distance between the turning point and y-intercept of the following function $f(x) = 3x^2 + 2x - 3$ correct to 4 decimal places.

~~A.~~ 0.7455

B. 0.4714

C. 0.4715

~~D.~~ 0.7454

Y-int: $(0,-3)$

TP: $x = \frac{-2}{2(3)} = \frac{-1}{3} \Rightarrow f(\frac{-1}{3}) = \frac{1}{3} - \frac{2}{3} - 3 = -\frac{10}{3}$

TP: $(\frac{-1}{3}, \frac{-10}{3})$

$$\text{distance} = \sqrt{(\tfrac{-1}{3}-0)^2 + (\tfrac{-10}{3} - (-3))^2}$$

$$= \sqrt{\tfrac{1}{9} + \tfrac{1}{9}} = \frac{\sqrt{2}}{3} \approx 0.4714$$

Question 19 (1 mark)

Determine the value(s) of a such that $y = 2x^2 + ax + 2a$ has two real roots.

$\Delta > 0$

A. $a < 0$ or $a > 16$

B. $0 < a < 16$

C. $a < -1$ or $a > 16$

D. $a < 16$

$\therefore (a)^2 - 4(2)(2a) > 0$

$a^2 - 16a > 0$

$a(a-16) > 0$

$\therefore a < 0$ or $a > 16$

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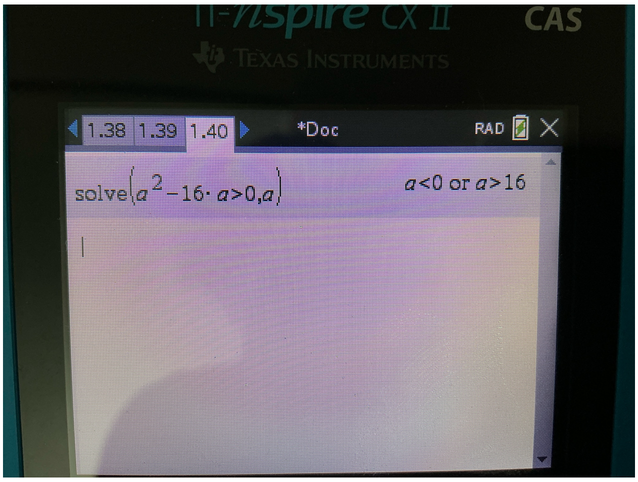
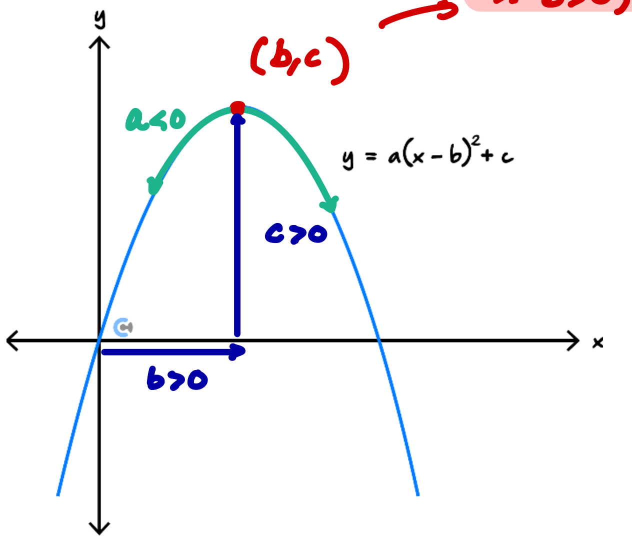


Question 20 (1 mark)

Consider the graph of $y = a(x-b)^2 + c$ shown below.

$\therefore b>0,\ c>0,\ a<0$



For this function, a, b, and c are real constants. It must be that:

~~A.~~ $a < 0,\ b > 0$ and $c < 0$

~~B.~~ $a > 0,\ b < 0$ and $c < 0$

C. $a < 0,\ b > 0$ and $c > 0$

~~D.~~ $a > 0,\ b < 0$ and $c > 0$

Question 21 (1 mark)

The function $g(x) = x^2 - 3px + 5$, where p is a real constant, has a root at $x = 5$, and its other root is when:

$g(5)=0$

A. $x = -1$

B. $x = 0$

$25 - 15p + 5 = 0$

C. $x = 1$

$\therefore p = 2 \longrightarrow g(x) = x^2 - 6x + 5 = (x-5)(x-1)$

D. $x = 2$

$\therefore x = 1$

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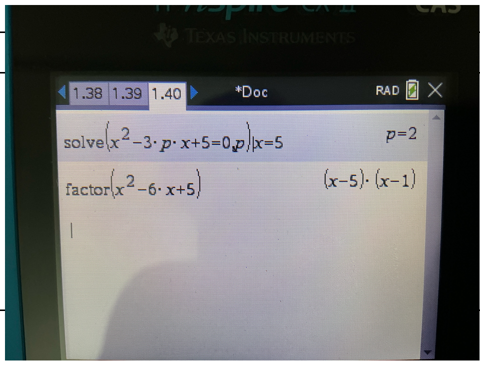
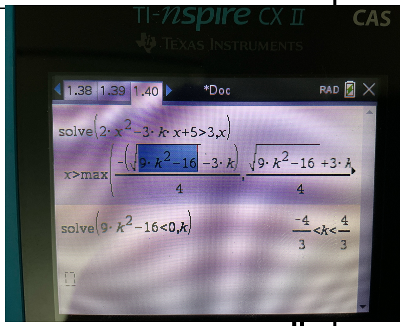


Question 22 (1 mark)

The values of k such that the function $f(x) = 2x^2 - 3kx + 5$ is always greater than 3 are:



A. $-\frac{2}{3} < k < \frac{2}{3}$

B. $-\frac{4}{3} < k < \frac{4}{3}$

C. $-\frac{5}{3} < k < \frac{5}{3}$

D. $k > \frac{2}{3}$

$2x^2 - 3kx + 5 > 3$

$2x^2 - 3kx + 2 > 0$

$\therefore \Delta < 0 \Rightarrow (-3k)^2 - 4(2)(2) < 0$

$9k^2 - 16 < 0$

$k^2 < \frac{16}{9}$

$\therefore -\frac{4}{3} < k < \frac{4}{3}$

Question 23 (11 marks)

Consider the family of functions given by,

$$f(x) = x^2 - 2kx + 3k, \text{ where } k \in \mathbb{R}$$

NOTE: $k \in \mathbb{R}$ means that k is any real number.

a. Write $f(x)$ in turning point form. (1 mark)

$f(x) = (x-k)^2 - k^2 + 3k$ //

b. Find the value of k that maximises the y-value of the turning point of the graph $y = f(x)$. (2 marks)

TP: $(k, 3k - k^2) \Rightarrow$ Max $3k - k^2$.

TP: $k = \frac{-b}{2a} = \frac{-3}{2(-1)} = \frac{3}{2}$

$\therefore k = \frac{3}{2}$

c. Hence, give the coordinates of the turning point of the graph $y = f(x)$ that has the largest y-value for any $k \in \mathbb{R}$. (1 mark)

$3k - k^2$ at $k = \frac{3}{2}$:

$3\left(\frac{3}{2}\right) - \left(\frac{3}{2}\right)^2 = \frac{9}{2} - \frac{9}{4} = \frac{9}{4}$

$\therefore$ TP: $\left(\frac{3}{2}, \frac{9}{4}\right)$ //

d. Find all of k for which the graph of $y = f(x)$ has no x-intercepts. (2 marks)

$\Delta < 0$

$\therefore (-2k)^2 - 4(1)(3k) < 0$

$4k^2 - 12k < 0$

$4k(k-3) < 0$

$\Rightarrow \therefore k \in (0, 3)$ //

OR $0 < k < 3$

$x = \dfrac{-(-2k) \pm \sqrt{4k^2 - 12k}}{2}$

e. Find all values of k for which the graph of $y = f(x)$ has two positive x-intercepts. (1 mark)

$\Delta > 0$ $k > 0$ $x = k \pm \sqrt{k^2 - 3k}$

If $k < 0$, $k - \sqrt{k^2 - 3k} < 0$

$\Delta > 0$ & $k > 0$

$4k(k-3) > 0$ & $k > 0$

$k < 0$ or $k > 3$ & $k > 0$ $\Rightarrow \therefore k > 3$

f. Find all values of k for which the graph of $y = f(x)$ has one positive and one negative x-intercept. (1 mark)

$\Delta > 0$ $k < 0$

If $k > 0$, $k + \sqrt{k^2 - 3k} > 0$ AND $k - \sqrt{k^2 - 3k} > 0$

$\Delta > 0$ & $k < 0$

$4k(k-3) > 0$ & $k < 0$

$k < 0$ or $k > 3$ & $k < 0$ $\Rightarrow \therefore k < 0$

Consider the function $g(x) = kx^2 - 4kx + 2$, where $k \in \mathbb{R}$.

g. Find all values of k such that the graph of $y = f(x)$ has two x-intercepts that have a distance less than 2 between them. (3 marks)

① 2 x-intercepts:

$\Delta > 0$

$(-4k)^2 - 4(k)(2) > 0$

$16k^2 - 8k > 0$

$8k(2k-1) > 0$

$\therefore k < 0$ or $k > \frac{1}{2}$

② Distance b/w x-intercepts is less than 2:

$\frac{\sqrt{\Delta}}{a} < 2$

$\frac{\sqrt{16k^2 - 8k}}{k} < 2$

$\sqrt{16k^2 - 8k} < 2k$

$16k^2 - 8k < 4k^2$

$12k^2 - 8k < 0$

$4k(3k-2) < 0$

$\therefore k \in (0, \frac{2}{3})$

$k > \frac{1}{2}$ & $k \in (0, \frac{2}{3})$

$\therefore k \in (\frac{1}{2}, \frac{2}{3})$

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