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VCE Mathematical Methods ½
Linear and Coordinate Geometry Exam Skills [0.2]
Workshop

Section A: Recap

Definition

Linear equations

- **Definition:** Equations where the highest power of a variable is 1.
  - **Gradient-intercept form:**

$$y = mx + c$$

Where $m = \text{gradient} = \frac{\text{rise}}{\text{run}} =$

and $c =$

- No singular solution for a linear equation in two variables.
  - All pairs of coordinates (x, y) that satisfy the equation lie on a **line.** (Hence, linear equations.)

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Sub-Section: Inequality

Inequalities rule

$$x > \frac{b}{a}, \text{ where } a < 0$$

- Multiplying both sides by a negative number ____________ the inequality sign.

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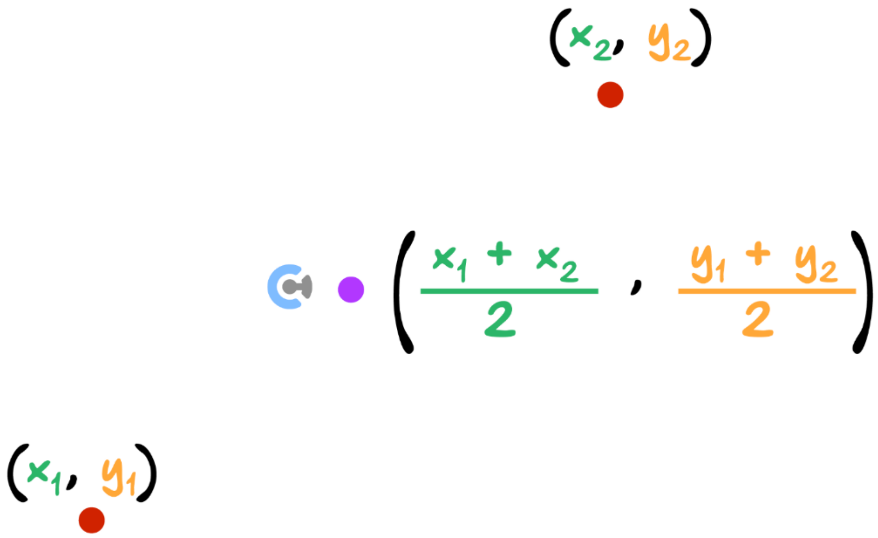
Sub-Section: Midpoint

Midpoint



- **Definition:**
  - The midpoint, M, of two points A and B is the point halfway between A and B.

$$M(x_m, y_m) = \left(\qquad\qquad\qquad\right)$$

- The midpoint can be found by taking the ____________________ of the x-coordinate and y-coordinate of the two points.

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Sub-Section: Distance between Two Points

Distance between two points

- **Definition:**
  - The distance between two points (x_1, x_2) and (y_1, y_2) can be found using Pythagoras' theorem:

$$\textbf{Distance} = \underline{\hspace{8cm}}$$

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Sub-Section: Vertical Distance vs Horizontal Distance

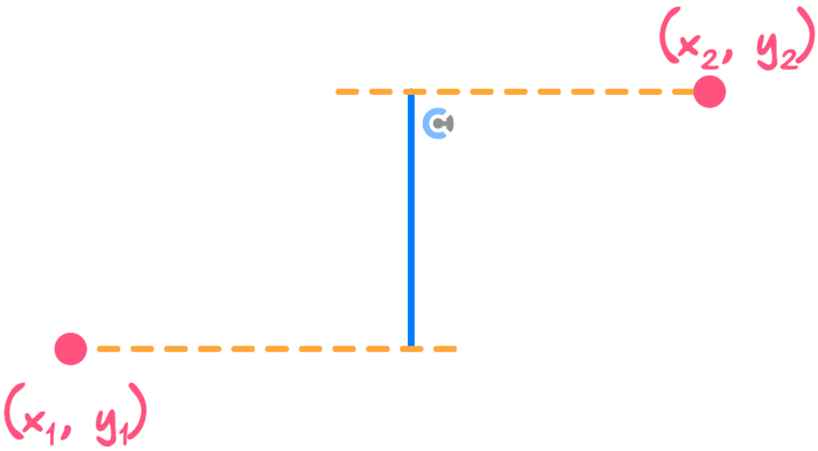
Horizontal distance

Horizontal Distance $= x_2 - x_1$ where, ________________.

- Find the difference between their x-values.

What about vertical distance then?

Vertical distance

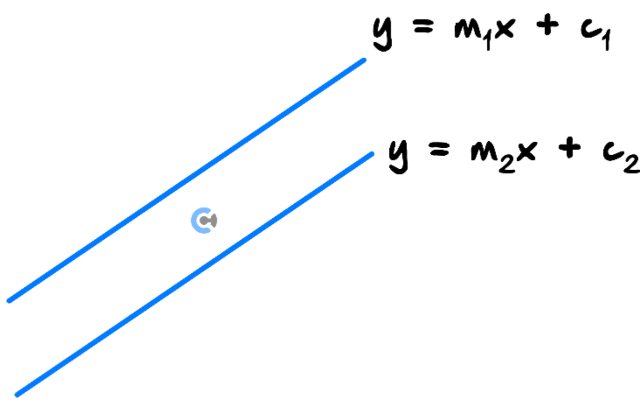


Vertical Distance $= y_2 - y_1$ where, $y_2 > y_1$.

- Find the difference between their y-values.

Sub-Section: Parallel and Perpendicular Lines

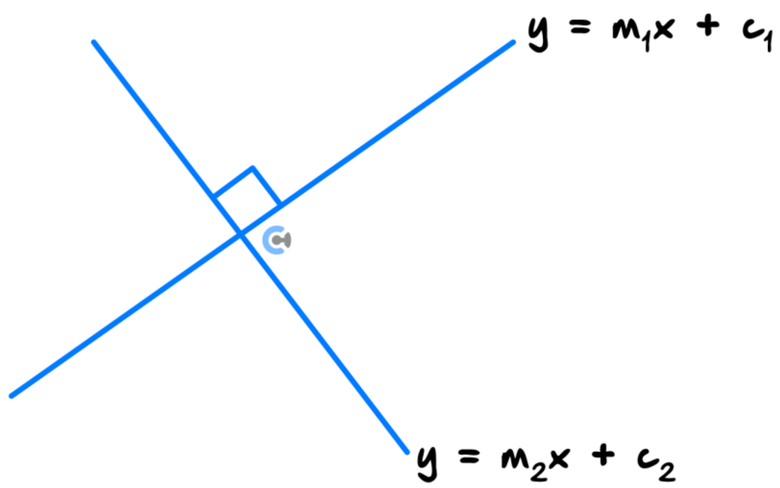
Parallel lines



- Parallel lines have the ________________ gradient.

$$m_1 = m_2$$

Perpendicular lines



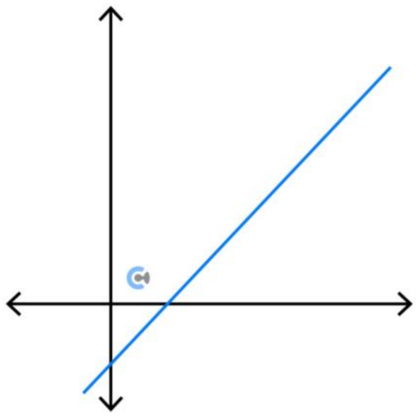
- A line that is perpendicular to another line has a gradient, which is the __________________________ of the gradient of the first line.

$$m_{\perp} = -\frac{1}{m}$$

Sub-Section: Angle between a Line and the x-axis

How do we find the angle between a line and the x-axis?

The angle between a line and the x-axis



- The angle between a line and the ______________ **direction of the x-axis** (anticlockwise) is given by:

$$\tan(\theta) = m$$

NOTE: Angles from the x-axis measured anticlockwise = ___________________ angles.

- Don't worry about it too much, it's just convention! (More on this in circular functions.)

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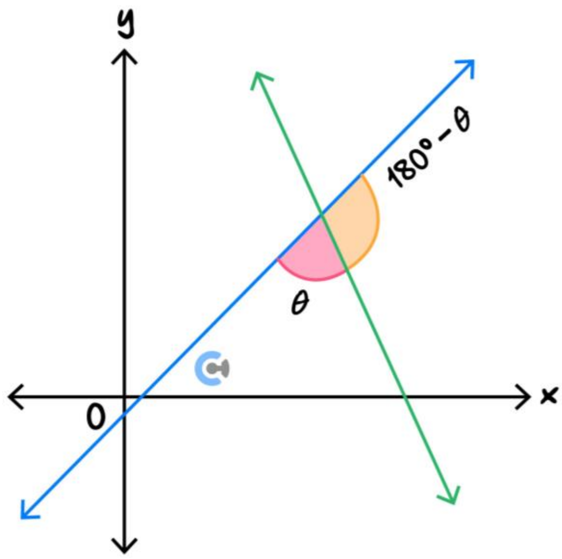
Sub-Section: Angle between the Two Lines

Slightly more complicated now!
How about an angle between two lines?

The acute angle between two lines



$\theta =$ ______________________

- **Alternatively:**

$$\tan(\theta) = \text{\_\_\_\_\_\_\_\_\_\_\_\_}$$

For your understanding, note that this formula is derived from the $\tan$ compound angle formula covered in SM12.

NOTE: $|x|$ just takes the positive value of x.

TIP: Make sure your CAS is in degrees.

Sub-Section: Finding Simultaneous Equations for Two Variables

Definition

Simultaneous linear equations

- Elimination method:
  - Add or subtract one equation from the other in order to ______________ one of the variables. Then have an equation in one variable that can be solved easily.
- Substitution method:
  - Make one of the variables the subject (generally x or y) and _________________ that value into the other equation.

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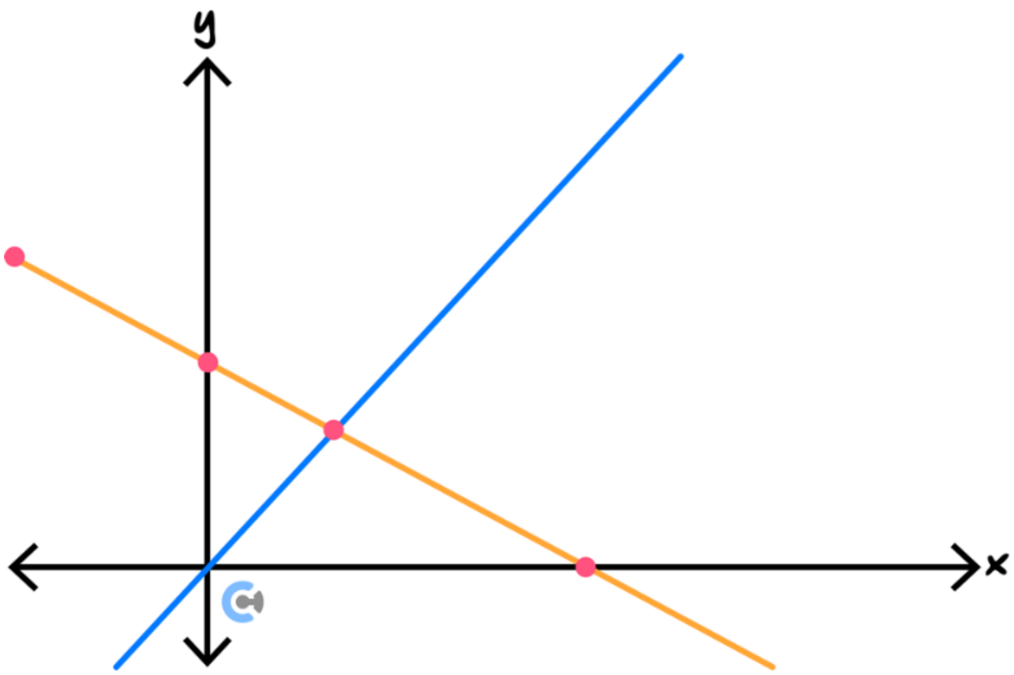
Sub-Section: Number of Solutions for Two Variables

What does the geometry look like for each number of solutions?

Exploration: Geometry of the number of solutions between linear graphs

- Unique solution:

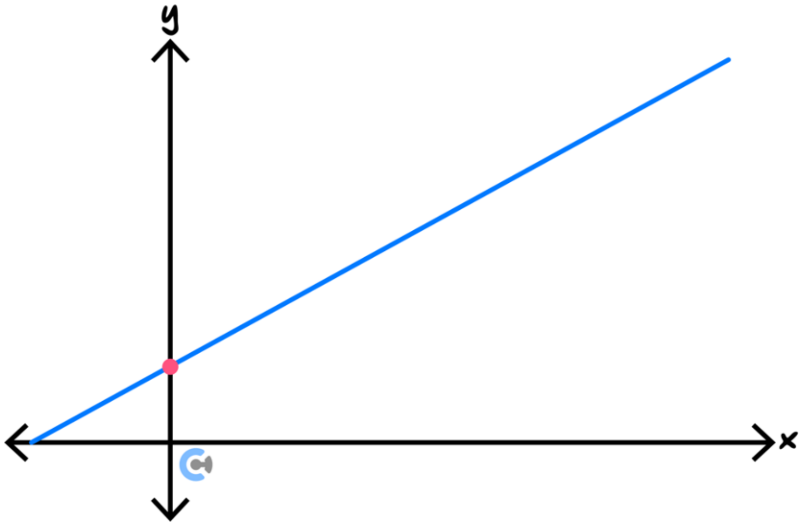
$$m_1 \neq m_2$$



They just need to have ________________________.

- **Infinite solutions:**

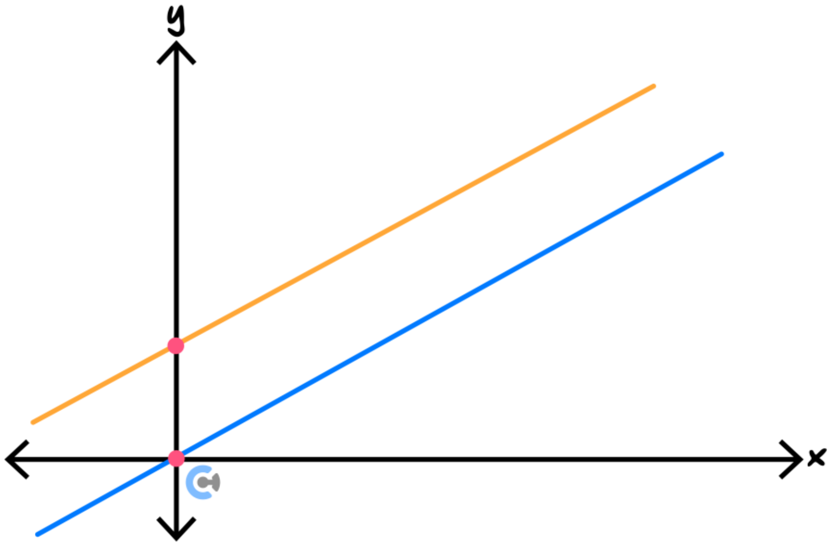
$$m_1 = m_2 \text{ and } c_1 = c_2$$



- They just need to have the same ________________ and the same _________________.
- In other words, they have to be the ____________________.

- **No solutions:**

$$m_1 = m_2 \text{ and } c_1 \neq c_2$$



- They need to have the _________________ but _______________ $+c$.
- They have to be two different ________________ lines.

<u>General solutions of simultaneous linear equations</u>

- Two linear equations are either:
  - The same line is expressed in a different form. In this case, they have ________________ solutions.
  - Unique lines which are parallel. In this case, they have _______________ solutions.
  - Unique lines which are not parallel. In this case, they have ____________________ solution.

TIP: It's a good idea to substitute your answer back into the equations to see if the criteria are met for each part.

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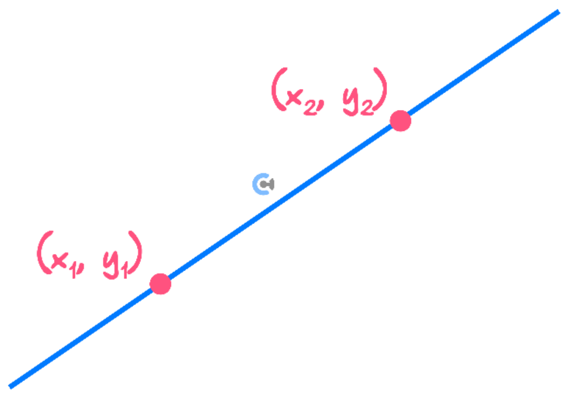
Sub-Section: Finding the Equation of the Line



- m: Gradient

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- $+c$: y-intercept

  - Substitute in any point to $y = mx + c$ equation.

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Sub-Section: Applying Midpoint to Find Reflected Points

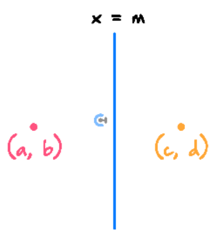
Finding reflections around horizontal and vertical axes

- Horizontal axis

- The ______________ changes for horizontal reflections.

$$\frac{b+d}{2} = m$$

- Vertical axis



- The ____________ changes for horizontal reflections.

$$\frac{a+c}{2} = m$$

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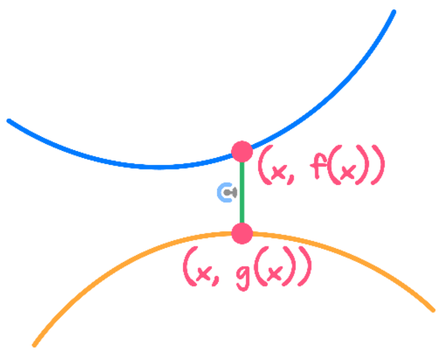
Sub-Section: Find Vertical Distance between Two Functions

Vertical Distance between Two Functions

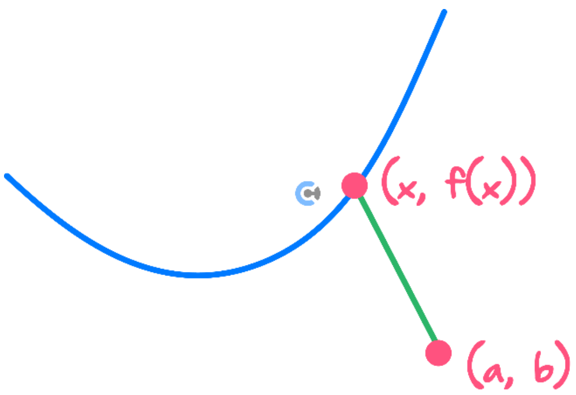


- Find the difference between the two y-values.

$$f(x) - g(x) \; where \; f \; is \; above \; g$$

Sub-Section: Finding Distance between a Point and a Function

Distance between a Function and a Point



- Find the distance between the point and $(x, function)$.

$$\textbf{Distance} = \sqrt{(x-a)^2 + (f(x) - b)^2}$$

Section B: Warmup

INSTRUCTION: 5 Minutes Writing.

Question 1

a. Find the equation of the line joining the points $(1,4)$ and $(5,6)$.

b. Find the reflection of the point $(2,4)$ about the line $y = 6$.

c. Find the vertical distance between the functions $f(x) = x^2 + 2$ and $g(x) = x - 1$ when $x = 2$.

d. Find the distance between the point $(4,6)$ and $f(x) = 2x - 6$ when $x = 5$.

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Section C: Exam 1 (23 Marks)

INSTRUCTION: 23 Marks. 30 Minutes Writing.

Question 2 (5 marks)

Consider the line segment AB with coordinates $A(1,2)$ and $B(7,8)$.

a. Find the midpoint of AB. (1 mark)

b. Find the equation of the line segment AB. (2 marks)

c. Find the equation of the perpendicular bisector of AB. (2 marks)

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Question 3 (4 marks)

Consider the linear equations:

$$(k+2)x + 4y = 6$$
$$3x + 2y = k - 1$$

a. For what value(s) of k, will the system have a unique solution? (2 marks)

b. For what value of k, will the system have infinitely many solutions? (1 mark)

c. Explain why the system will always have a solution. (1 mark)

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Question 4 (5 marks)

Consider the points $A(2,3)$ and $B(5,5)$.

a. Find the distance between points A and B. (1 mark)

b. The distance between point A and point $C(5, k)$ is 5. Find the possible value(s) of k. (2 marks)

c. Find the coordinates of the point D obtained by reflecting A in the line $x = -1$. (1 mark)

d. Find the coordinates of the point E obtained by reflecting B in the line $y = 2$. (1 mark)

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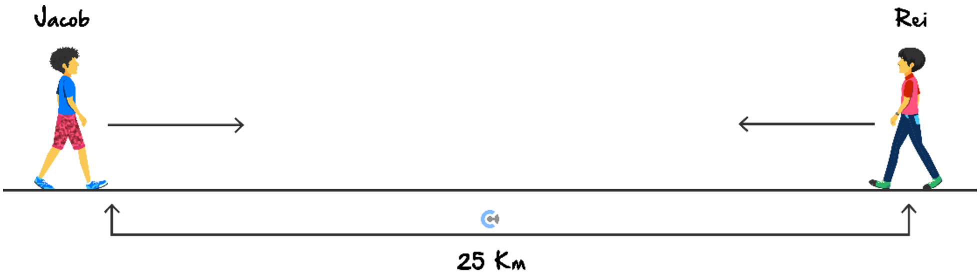
Question 5 (3 marks)

Jacob and Rei walk along the same walking trail to meet up. They start walking at the same time and walk in opposite directions towards each other and they are initially 25 km apart. Rei walks 2 km/h faster than Jacob. After walking for two hours Rei and Jacob are 1 km apart.



a. Determine Jacob and Rei's walking speed in km/h. (2 marks)

b. Determine how long it takes for them to meet up from when they start walking. (1 mark)

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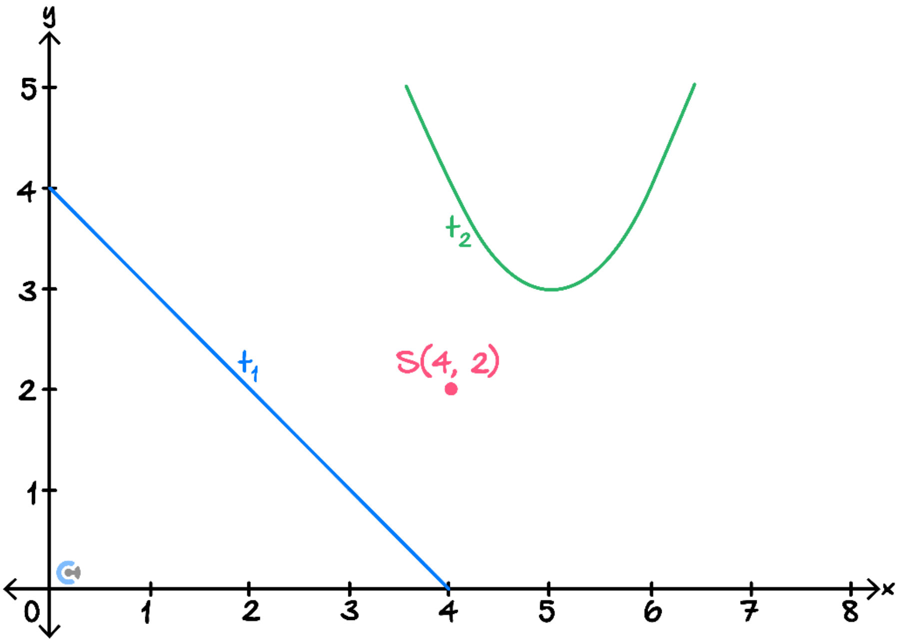
Question 6 (6 marks)

Sam is at his house at the point $S(4,2)$. There are two walking trails nearby, t_1 and t_2. t_1 is modelled by the line $y = 4 - x$ and t_2 is modelled by the parabola $y = (x-5)^2 + 3$. The situation is shown in the diagram below.



a. Find the distance from S to t_2, when $x = 5$. (2 marks)

b. Find the shortest horizontal distance between t_1 and t_2, when $y = 4$. (1 mark)

c. Find the equation of the line perpendicular to t_1, that passes through $S(4,2)$. (2 marks)

d. Hence, find the shortest distance between S and t_1. (1 mark)

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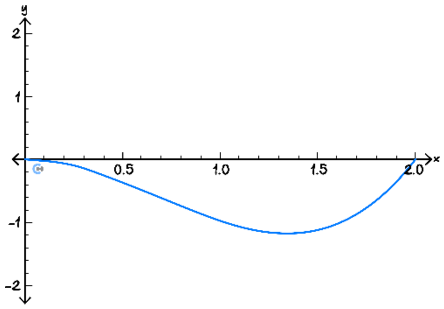
Section D: Tech Active Exam Skills

INSTRUCTION: 5 Minutes Writing.

Calculator Commands: Graphing

Mathematica

- Plot [function, {x, $x$min, $x$max}.
  Plot Range → {$y$min, $y$max}]
- Plot Range is optional but makes the scale appropriate for the question.

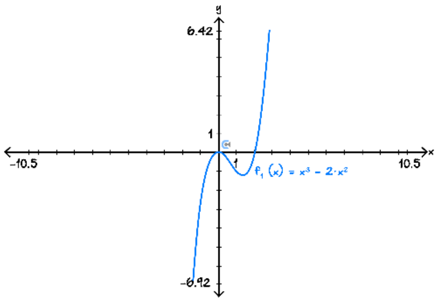


- Menu → 6 (Analyse) to find min/max x and y-intercepts.
- Restrict domain to $0 < x < 2$ use the bar can get it from ctrl+ = > < ≠ ≥ ≤ |.

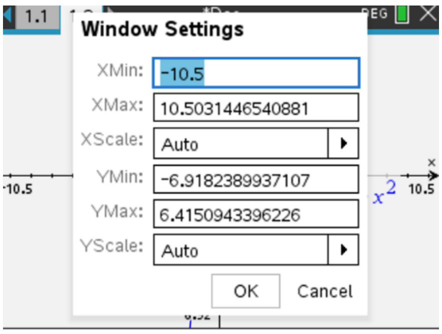
f1$(x)=x^3-2x^2|0<x<2$

TI-Nspire

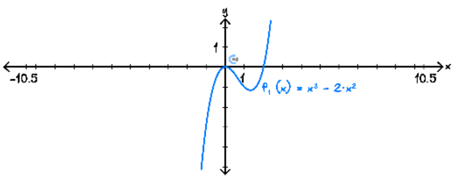
- Open a graph page and plot your function.



- Zoom settings: Menu → 4 (window/zoom) → 1 enter your x and y ranges.

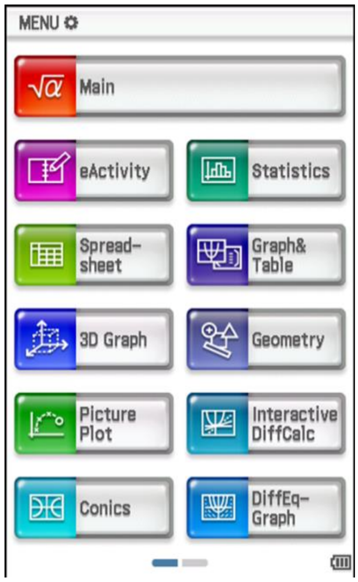


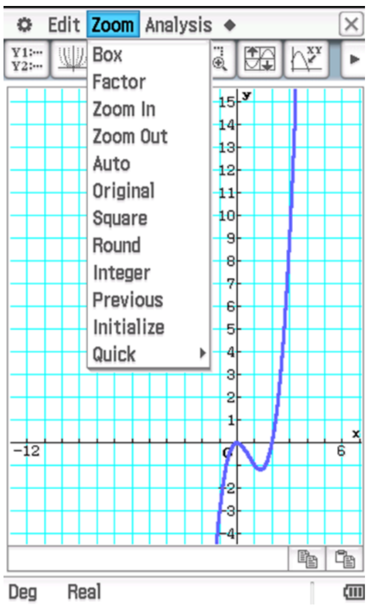
- Can also click the axis numbers on the graph and alter them directly.



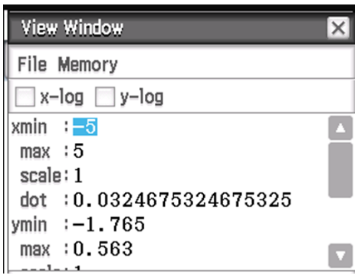
Casio Classpad

- Click Graph & Table, and enter the function.





- Analysis → G-Solve to find intercepts.
- Use this button to set the view window.



- Use | to restrict domain → Find it in Math 3.

$y1=x^3-2\cdot x^2 \mid 0<x<2$

<u>Calculator Commands:</u> Solving Equations

TI-Nspire

- Menu → 3 → 1

$\text{solve}(x^2-4\cdot x-9=0,x)$

$x=-(\sqrt{13}-2) \text{ or } x=\sqrt{13}+2$

Casio Classpad

- Action → Advanced → Solve

solve(x^2−4x−9=0, x)

$\{x=-\sqrt{13}+2, x=\sqrt{13}+2\}$

In[122]:= Solve[x^2 - 4 x - 9 == 0, x]

Out[122]= $\{\{x \to 2-\sqrt{13}\}, \{x \to 2+\sqrt{13}\}\}$

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Calculator Commands: Simultaneous Equations

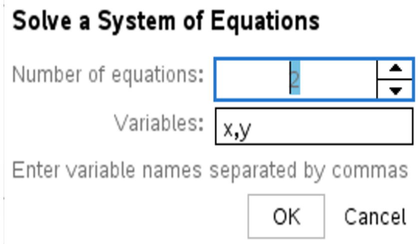
Mathematica

- Just do && between.
- Solve[equation&&equation , {var1, var2}]

```
In[128]:= Solve[2 x - 3 y == 16 && x + y == 3, {x, y}]
Out[128]= {{x → 5, y → -2}}
```

TI-Nspire

- Menu 3 7 1



$\text{solve}\left(\begin{cases}2\cdot x-3\cdot y=16\\ x+y=3\end{cases},\{x,y\}\right) \quad x=5 \text{ and } y=-2$

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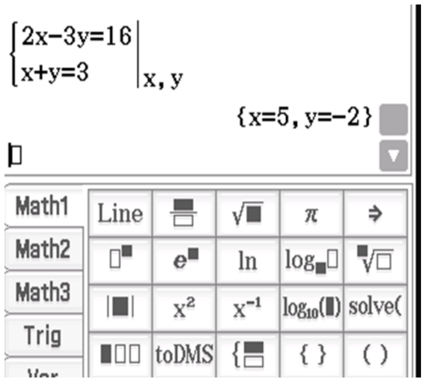
- Math1 → Click highlighted box→ Enter equations and variables you are solving for:



Question 7 Tech-Active.

Solve the equations $x + 4y = 16$ and $5x + 2y = 18$ for x and y.

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Calculator Commands: Finding the Angle between a Line and x-axis

Mathematica

In[124]:= ArcTan[2] / Degree // N

Out[124]= 63.4349

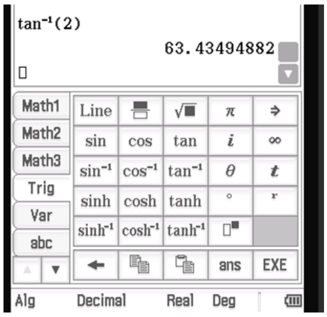
TI-Nspire

- Trig button. Check that you are in degrees.

$\tan^{-1}(2)$ 63.4349

Casio Classpad

- Keyboard →Trig. Change to decimals and degrees.



Calculator Commands: Finding the Angle between Two Lines

Mathematica

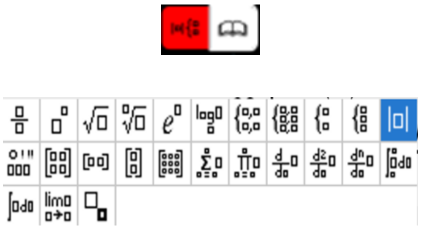
- Use the Abs[] function.

In[126]:= Abs[ArcTan[2] - ArcTan[1]] / Degree // N

Out[126]= 18.4349

TI-Nspire

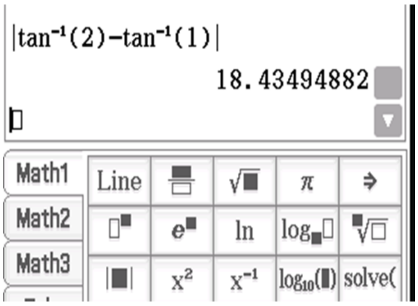
- Find the modulus sign.



$|\tan^{-1}(2)-\tan^{-1}(1)|$ 18.4349

Casio Classpad

- Modulus sign under Math1.



Space for Personal Notes

Question 8 Tech-Active.

Find the acute angle, correct to 2 decimal places, between the lines $y = 3x - 2$ and $y = x + 2$.

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Section E: Exam 2 (28 Marks)

INSTRUCTION: 28 Marks. 34 Minutes Writing.

Question 9 (1 mark)

The midpoint of PQ is $(5,3)$. If the coordinates of P are $(10,0)$, the coordinates of Q are:

A. $(0,-6)$

B. $(-1,3)$

C. $(2.5,1.5)$

D. $(0,6)$

Question 10 (1 mark)

The gradient of the line parallel to the line passing through $(1,2)$ and $(3,5)$ is:

A. 1

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. -1

Question 11 (1 mark)

The gradient of the line that makes an angle of $30°$ to the line which passes through $(-2,0)$ and $(-4,0)$ is:

A. $\frac{1}{\sqrt{3}}$

B. $\sqrt{3}$

C. 1

D. $-\sqrt{3}$

Question 12 (1 mark)

If $12x + 13y = 29$ and $13x + 12y = 21$, then $x + y$ is:

A. 4

B. -2

C. 2

D. -4

Question 13 (1 mark)

If the length of a rectangle is $5\ cm$ more than the breadth and if the perimeter of the rectangle is $40\ cm$, then the length and breadth of the rectangle will be (in cm):

A. 7.5 and 2.5.

B. 10 and 5.

C. 12.5 and 7.5.

D. 15.5 and 10.5.

Question 14 (1 mark)

Find the values of m for which the given system of equations has a unique solution:

$$mx - 2y = 9 \text{ and } 4x - y = 7$$

A. $m \neq 5$

B. $m \neq 10$

C. $m \neq 8$

D. $m \neq 2$

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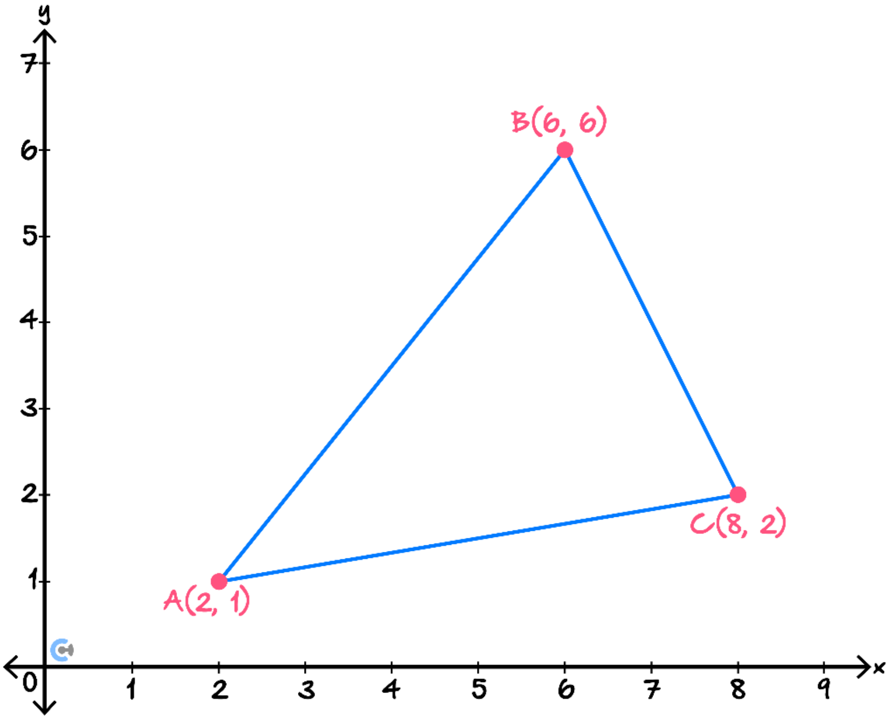
Question 15 (10 marks)

Consider the triangle with vertices $A(2,1)$, $B(6,6)$ and $C(8,2)$. The triangle is shown on the Cartesian plane below.



a. Find three equations of the lines that make up the triangle ABC. State the x-values that each line is valid for. (3 marks)

b.

i. Find the length of the line segment joining A and BC, which is perpendicular to BC. (2 marks)

ii. Hence, find the area of the triangle ABC. (2 marks)

The area of a triangle may be calculated using the formula $A = \frac{1}{2}bc\sin(\theta)$, where b and c are the lengths of two sides of the triangle with angle θ between them.

c. Find all internal angles of the triangle. Give your answer in degrees correct to two decimal places. (2 marks)

d. Use the formula $A = \frac{1}{2}bc\sin(\theta)$ to verify your answer for the triangle area from **b. ii.** (1 mark)

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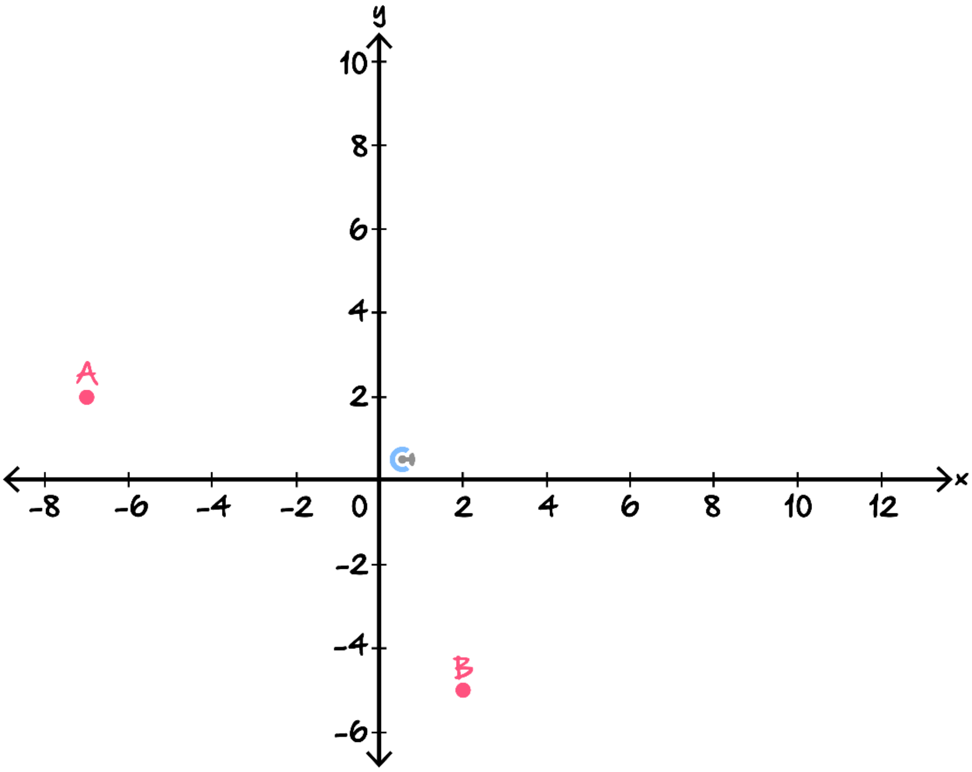
Question 16 (12 marks)

City planners are using a Cartesian plane to model locations for their new park.
Two points on the Cartesian plane have the coordinates $A(-7,2)$ and $B(2,-5)$ as shown on the Cartesian plane below.



a. Show that the gradient of AB is $-\frac{7}{9}$. (1 mark)

---

b. Hence, find the equation of AB, giving your answer in the form $ax + by + c = 0$, where $a, b, c \in R$. (2 marks)

---

c. Find the equation of the line that passes through A and is perpendicular to AB. (2 marks)

d. Find the distance between points A and B. (1 mark)

e. Find the possible coordinates of M are such that $\angle ABM = 90°$ and the distance between B and M is $\frac{1}{2}\sqrt{130}$. (2 marks)

f. A and B are both reflected about the line $y = mx + c$ to become the points C and D respectively. Find the values of m and c if $ABCD$ is a square, and c is a positive constant.

HINT: The line is parallel to AB. (3 marks)

g. Find the area of the triangle ABM. (1 mark)

Let's take a BREAK (Extension Stream)!

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Section F: Extension Exam 1 (13 Marks)

INSTRUCTION: 13 Marks. 18 Minutes Writing.

Question 17 (5 marks)

Consider the system of linear equations:

$$(k+2)x + 2y = 4$$

$$3x + (k-3)y = 2$$

a. For what value(s) of k, will the system have a unique solution? (2 marks)

b. For what value of k, will the system have infinitely many solutions? (2 marks)

c. For what value of k, will the system have no solution? (1 mark)

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Question 18 (3 marks)

The distance between the point $(4,7)$ and the function $f(x) = 2x - 1$ is $\sqrt{5}$ when $x = a$. Find the possible value(s) of a.

Question 19 (5 marks)

a. The point (p, q) is reflected about the lines $x = 2$ and then $y = 3$ to become the point $(4,5)$. Find the values of p and q. (1 mark)

b. The line with equation $y = 2x + 1$ is reflected about the line $x = 3$. Find the equation of the line after it has been reflected. (2 marks)

c. The point $(2,1)$ is reflected about a line $y = mx + c$ to become the point $(6,5)$. Find the values of m and c. (2 marks)

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Section G: Extension Exam 2 (20 Marks)

INSTRUCTION: 20 Marks. 24 Minutes Writing.

Question 20 (1 mark)

Consider the following pair of simultaneous equations:

$$2x + 2ay = 1$$
$$(3 - a)x + 2y = 2a - 1$$

For what value(s) of a do the equations have infinitely many solutions:

A. $a = 1$

B. $a = 2$

C. $a = 1,2$

D. $a \in R \backslash \{1,2\}$

Question 21 (1 mark)

The distance between the point $(3,2)$ and the line $y = 7 - x$ when $x = a$ is 2. The possible value(s) of a are:

A. 3,7

B. 3,4

C. 3 only

D. 3,5

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Question 22 (1 mark)

The point (3,2) is reflected about a line $y = mx + c$ to become the point (5,4). The values of m and c are:

A. $m = 1, c = -1$

B. $m = 1, c = 5$

C. $m = -1, c = 7$

D. $m = -1, c = 3$

Question 23 (1 mark)

When $x = a$ the vertical distance between the functions $f(x) = (x - 3)^2 + 2$ and $g(x) = x$ is 1.
All possible values of a are:

A. $a = 2{,}3$

B. $a = 3{,}4{,}5$

C. $a = 2{,}3{,}4{,}5$

D. $a = 2{,}3{,}4$

Question 24 (1 mark)

The lines $y = ax + 2$ and $y = bx + 3$ make an angle of 45° when they intersect. The relationship between a and b is:

A. $a = \frac{b+1}{b-1}$

B. $a = \frac{b-1}{b+1}$ or $a = -\frac{b+1}{1-b}$

C. $a = \frac{b-1}{1-b}$

D. $a = \frac{b+1}{1-b}$ or $a = \frac{b-1}{b+1}$

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Question 25 (15 marks)

Consider the points $A(6,6)$ and $C(14,10)$.

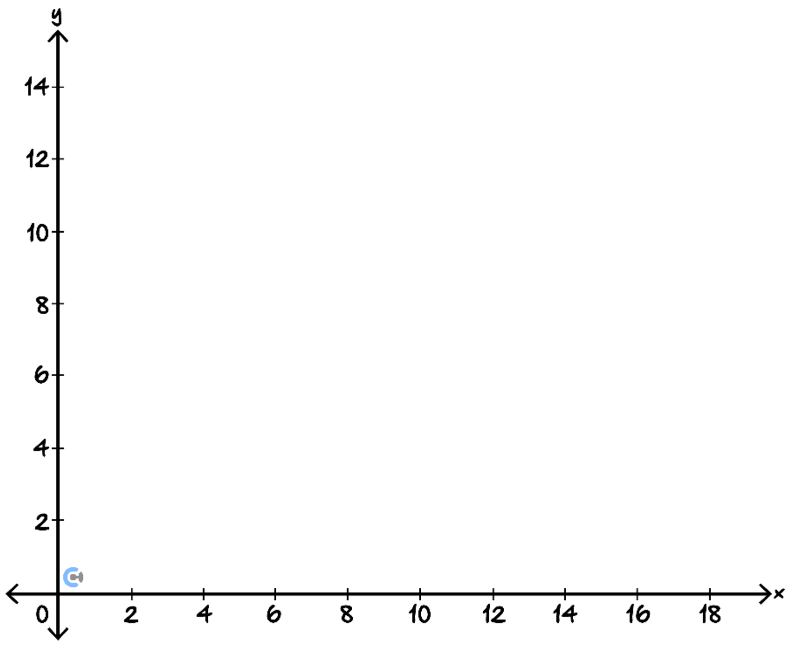
A parallelogram $ABCD$ with sides lengths $4\sqrt{2}$ and $4\sqrt{5}$ is formed.

The line segment AB has a gradient of -1 and the line segment AD has a gradient of 2.

It is known that point A has the smallest x-coordinate, point C has the largest x-coordinate and point B has the smallest y-coordinate of the parallelogram.

a. Show that B is at $(10,2)$ and D is at $(10,14)$. (3 marks)

b. Draw and label the parallelogram on the axes below. (2 marks)



c. Write down four equations for each of the line segments of the parallelogram. (2 marks)

d. Find the angles $\angle ABC$ and $\angle BAD$. Give your answer in degrees correct to two decimal places. (2 marks)

e. Find the shortest distance between AB and DC. (2 marks)

f. Hence, or otherwise, find the area of $ABCD$. (2 marks)

g. The points on the parallelogram are reflected on the line $x = 18$ to form a new parallelogram $EFGH$. Find the equations of all line segments that make up $EFGH$. (2 marks)

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