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VCE Mathematical Methods ½
Graphs of Circular Function [0.19]
Workshop

Error Logbook:



New Ideas/Concepts	Didn't Read Question
<p>Pg / Q #: _____</p> <p>Notes:</p>	<p>Pg / Q #: _____</p> <p>Notes:</p>
Algebraic/Arithmetic/ Calculator Input Mistake	Working Out Not Detailed Enough
<p>Pg / Q #: _____</p> <p>Notes:</p>	<p>Pg / Q #: _____</p> <p>Notes:</p>

Section A: Recap

The Exact Values Table

x	0 (0°)	$\frac{\pi}{6}$ (30°)	$\frac{\pi}{4}$ (45°)	$\frac{\pi}{3}$ (60°)	$\frac{\pi}{2}$ (90°)
$\sin(x)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos(x)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan(x)$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined

Particular Solutions

➤ Solving trigonometric equations for finite solutions.

➤ Steps:

1. Make the trigonometric function the subject.
2. Find the necessary angle for one period.
3. Solve for x by equating the necessary angles to the inside of the trigonometric functions.
4. Add and subtract the period to find all other solutions in the domain.

quad angle
Q R A P
ref \pm period

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→ numbers



General Solutions

- Solving infinite trigonometric equations.
- Steps:
 1. Make the trigonometric function the subject.
 2. Find the necessary angle for one period.
 3. Solve for x by equating the necessary angles to the inside of the trigonometric functions.
 4. Add period $\times n$ where $n \in \mathbb{Z}$.



Period of a Trigonometric Function

Period of $\sin(nx)$ and $\cos(nx)$ functions = $\frac{2\pi}{n}$

Period of $\tan(nx)$ functions = $\frac{\pi}{n}$

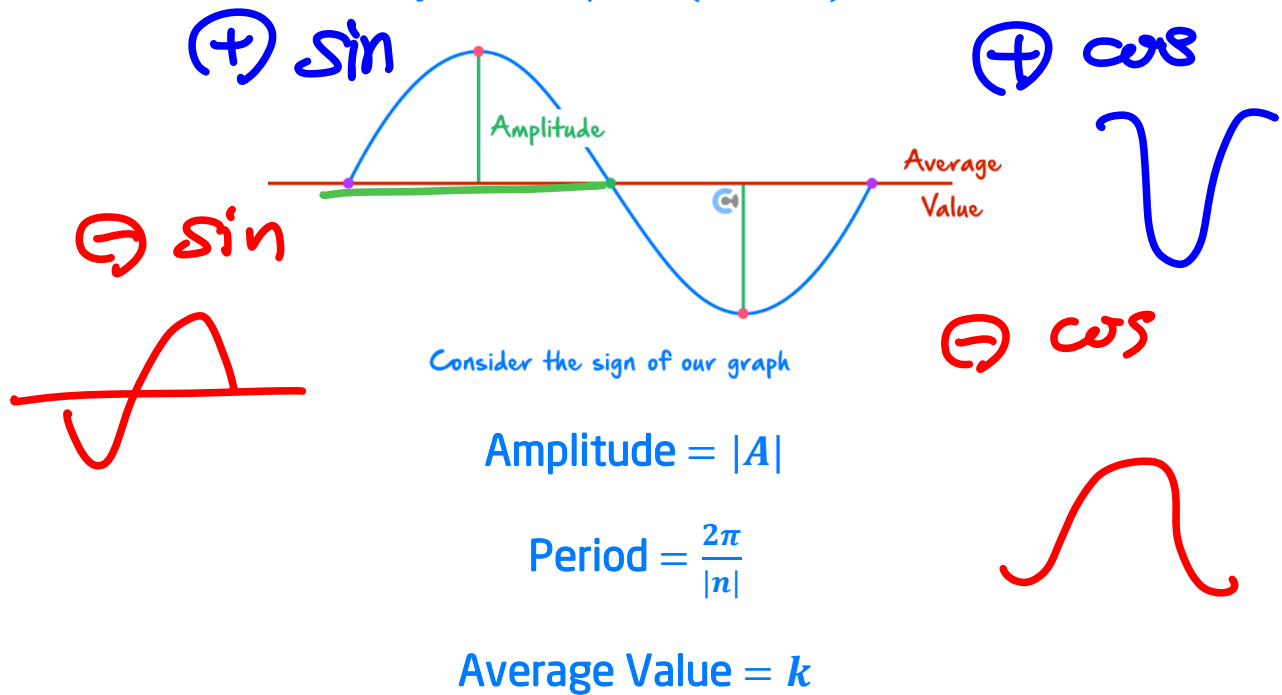
Where $n = \text{coefficient of } x \text{ and } n > 0$

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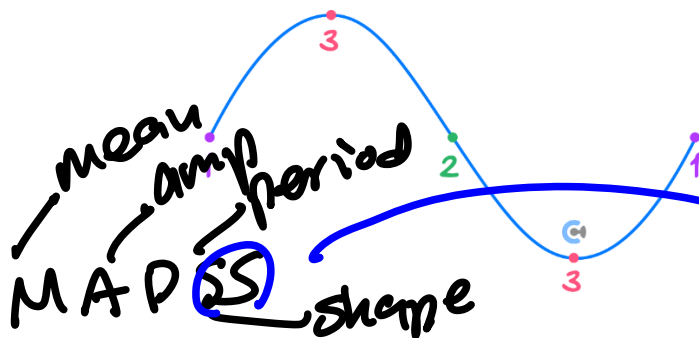


Amplitude, Period and Average Value

For $y = A \sin/\cos (nx + b) + k$



Graphing of sin and cos Functions



1. Identify Amplitude, Period, Mean Value and Positive/Negative Shape.

2. Create a "mini-version" of the graph you are about to draw.

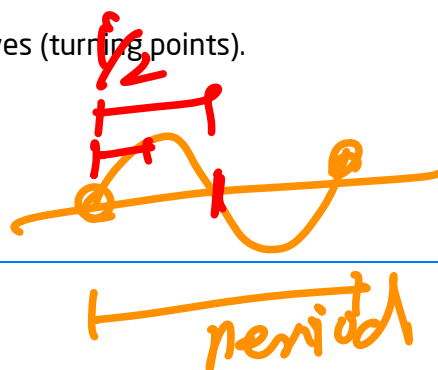
3. Start plotting the function from when the angle = 0.

4. Draw the start and end of the periods, and plot the halves (turning points).

5. Find any x -intercepts. $\Rightarrow y=0$

6. Join all the points!

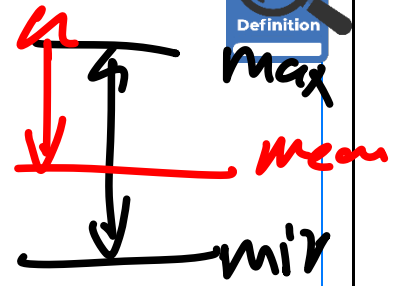
starting point



Finding the Rule

$$\text{Amplitude (A)} = \frac{\text{max} - \text{min}}{2}$$

$$\text{Average (k)} = \frac{\text{max} + \text{min}}{2}$$



Definition

Average
top & bottom

Fraction of Period

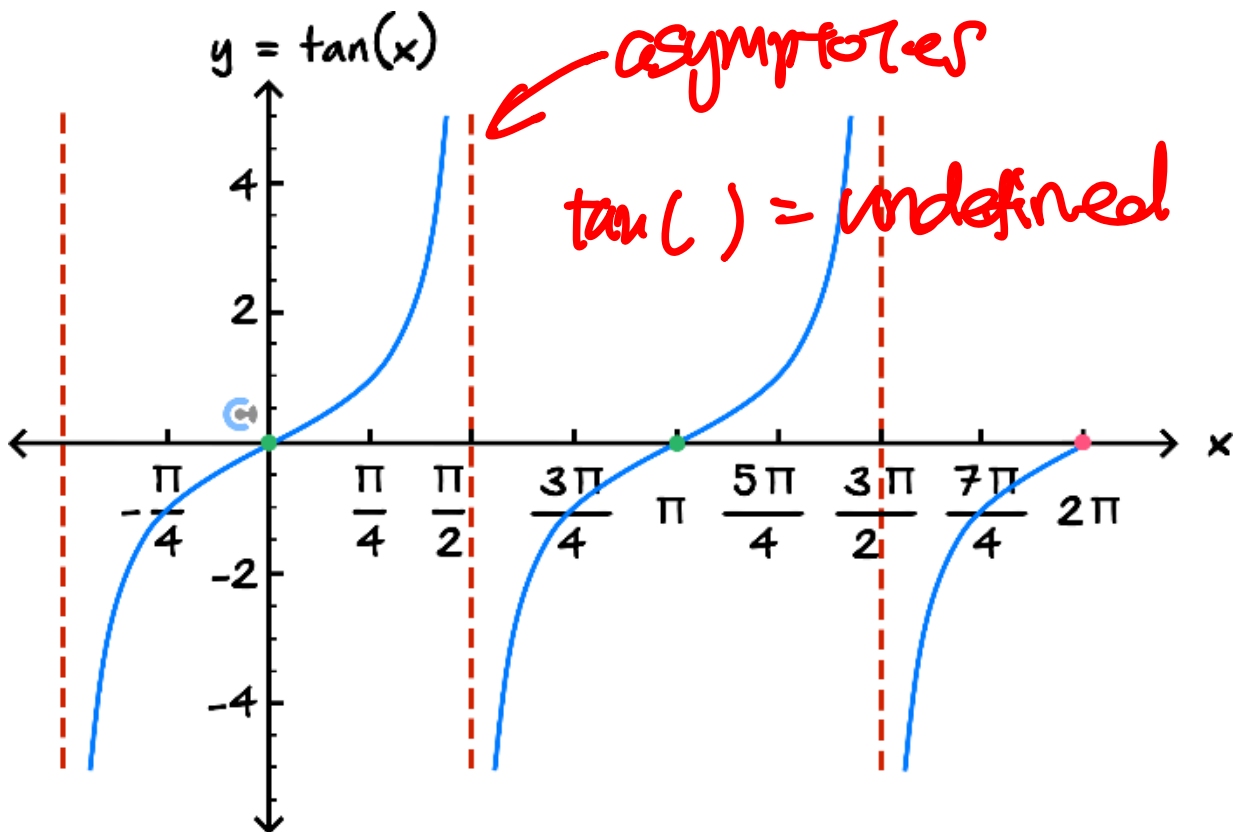
$$\text{Fraction of Period} = \left(\frac{\text{Duration}}{\text{Period}} \right) \quad 8/24 = 1/3$$

$$\% \text{ of Period} = \frac{\text{Duration}}{\text{Period}} \times 100\%$$



Definition

Graph of Tangent



Definition



Steps for Sketching tan Functions

1. Identify:

• The period = $\frac{\pi}{n}$.

2. Find the vertical asymptotes by solving for angle = $\frac{\pi}{2}$.

$$\tan(\quad)$$

$$(\quad) = \frac{\pi}{2}$$

3. Find other vertical asymptotes within the domain by adding the period to the answer from the previous step.

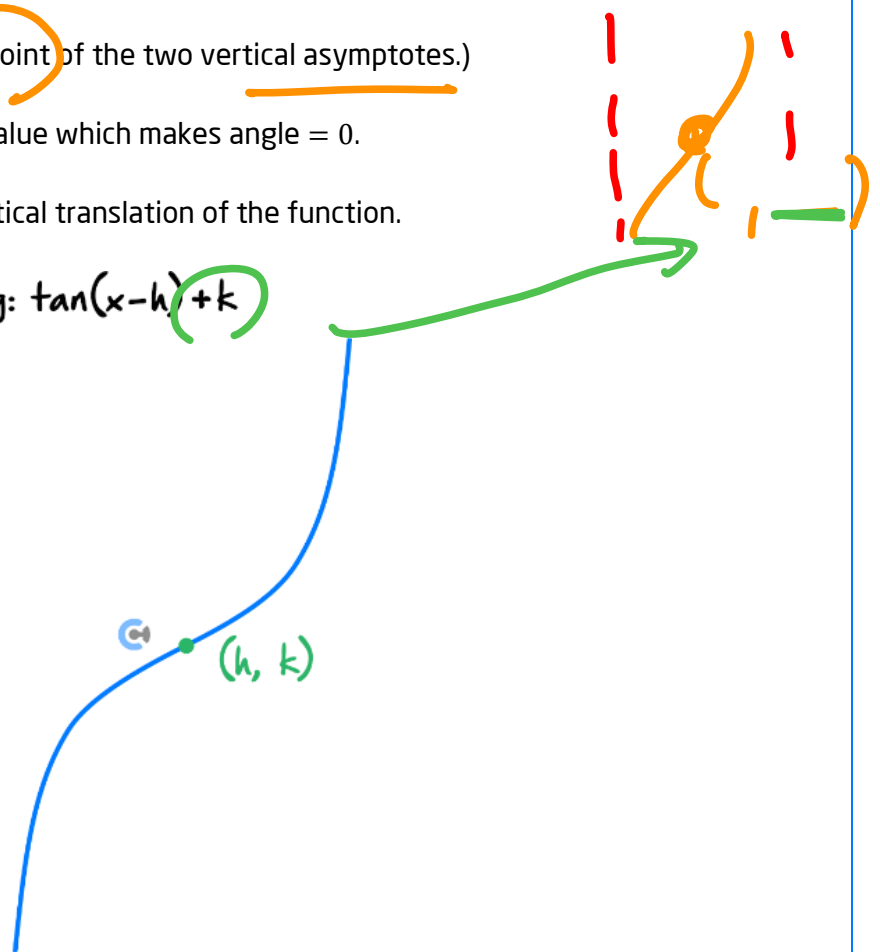
• For instance, for $\tan\left(2x - \frac{\pi}{3}\right)$, solve $2x - \frac{\pi}{3} = \frac{\pi}{2}$ for x .

4. Plot the inflection point (h, k) . (Midpoint of the two vertical asymptotes.)

• x -value of inflection point = x -value which makes angle = 0.

• y -value of inflection point = vertical translation of the function.

eg: $\tan(x-h)+k$



5. Find any x -intercepts.

6. Sketch a "cubic-like" shape.

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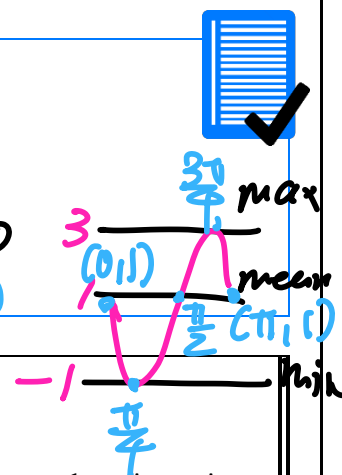
Section B: Warm Up (6 Marks)

INSTRUCTION:

➤ Regular: 6 Marks. 9 Minutes Writing.

➤ Extension: Skip

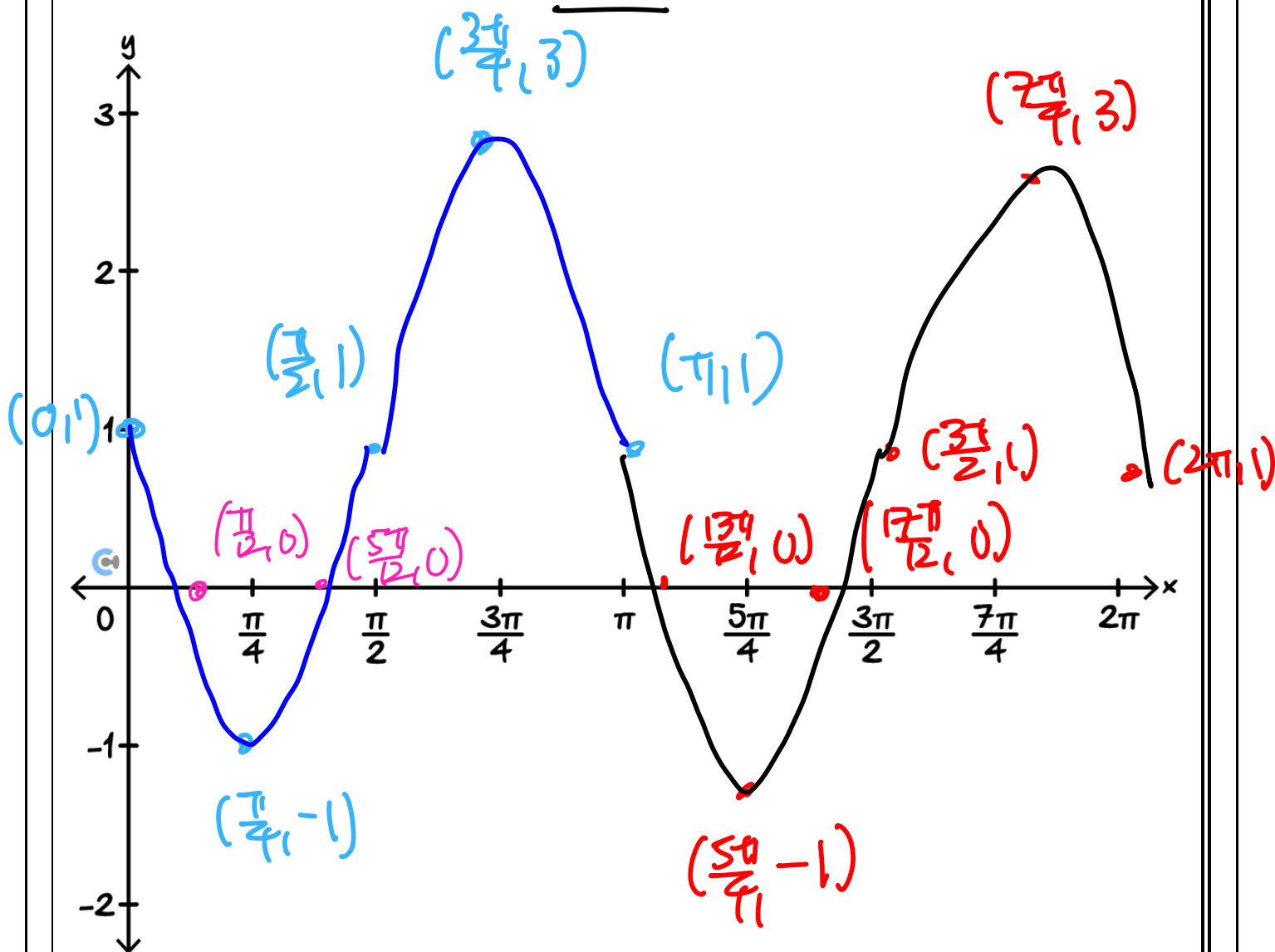
① MAPSS
1 2 π θ \rightarrow $(2x) = 0$
 $\frac{2\pi}{1} = \frac{2\pi}{2} = \pi$
 $x=0$



Question 1 (3 marks)

Sketch the following function on the set of axes below, labelling all axes intercepts, endpoints, and turning points with their coordinates.

$$f(x) = 1 - 2 \sin(2x), x \in [0, 2\pi]$$



2-int

$$0 = 1 - 2 \sin(2x)$$

$$\sin(2x) = \frac{1}{2}$$

Quad: 1, 2

Ref: $\frac{\pi}{6}$

$$2x = \textcircled{1} \frac{\pi}{6}, \textcircled{2} \frac{\pi}{6}$$

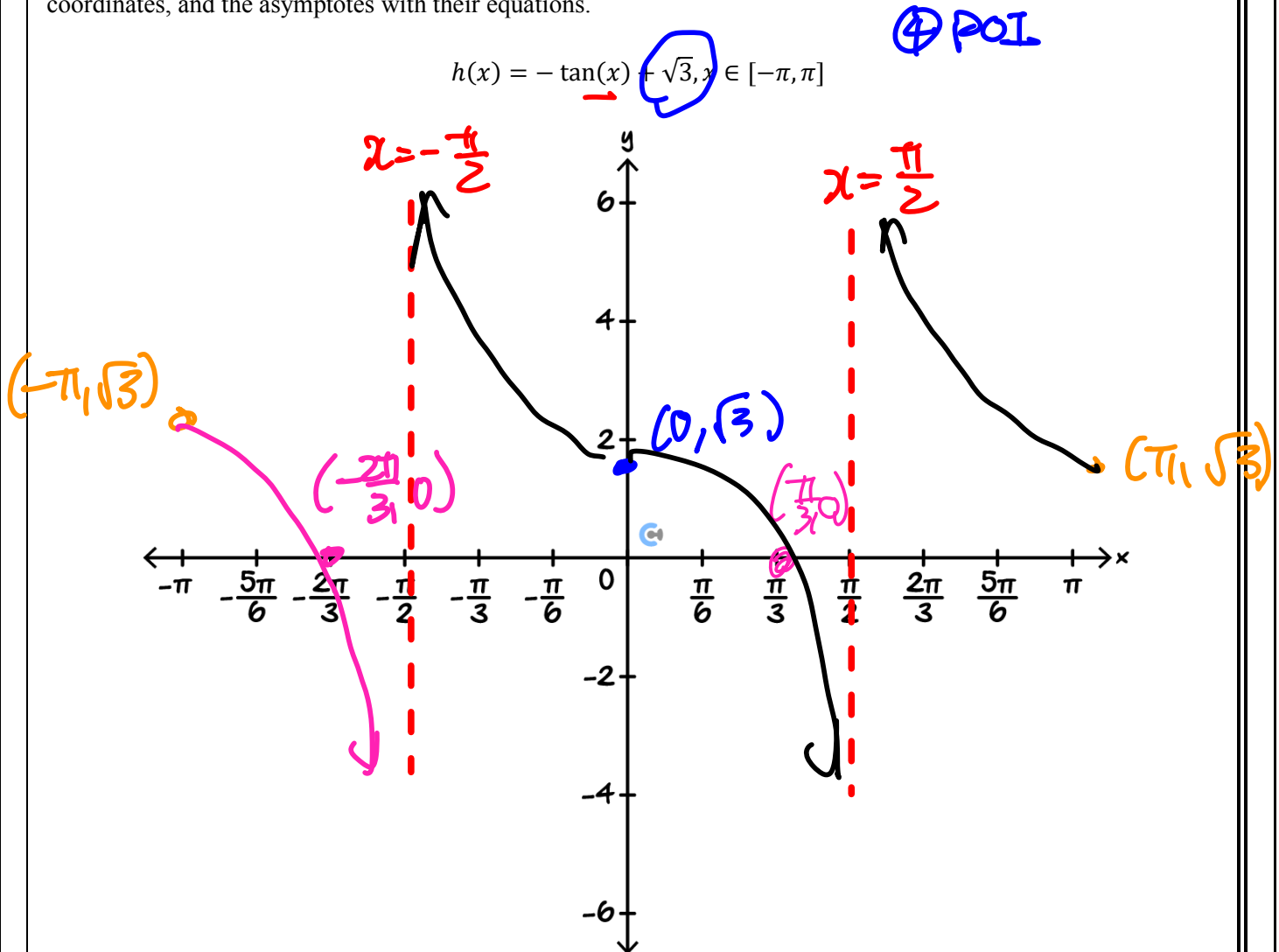
$$2x = \frac{\pi}{6}, \frac{5\pi}{6} \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$$

① period = $\frac{\pi}{n} = \pi$ ② VNG asymptote
 $(x) = \frac{\pi}{2}$

Question 2 (3 marks)

Sketch the following function on the set of axes below, labelling all axes intercepts and endpoints with their coordinates, and the asymptotes with their equations.

$h(x) = -\tan(x) + \sqrt{3}, x \in [-\pi, \pi]$



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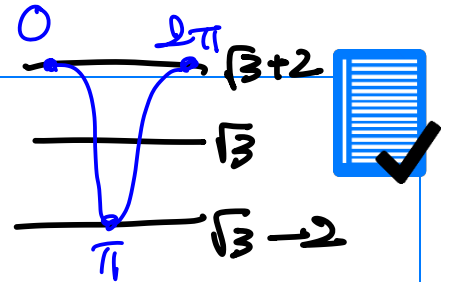
$x = \frac{\pi}{3}$
 $0 = -\tan(x) + \sqrt{3}$
 $\tan(x) = \sqrt{3}$
 $x = \frac{\pi}{3}$
quad ①
ref $\frac{\pi}{3}$

Section C: Exam 1 Questions (19 Marks)

INSTRUCTION:

➤ Regular: 19 Marks. 5 Minutes Reading. 27 Minutes Writing.

➤ Extension: 19 Marks. 5 Minutes Reading. 19 Minutes Writing.



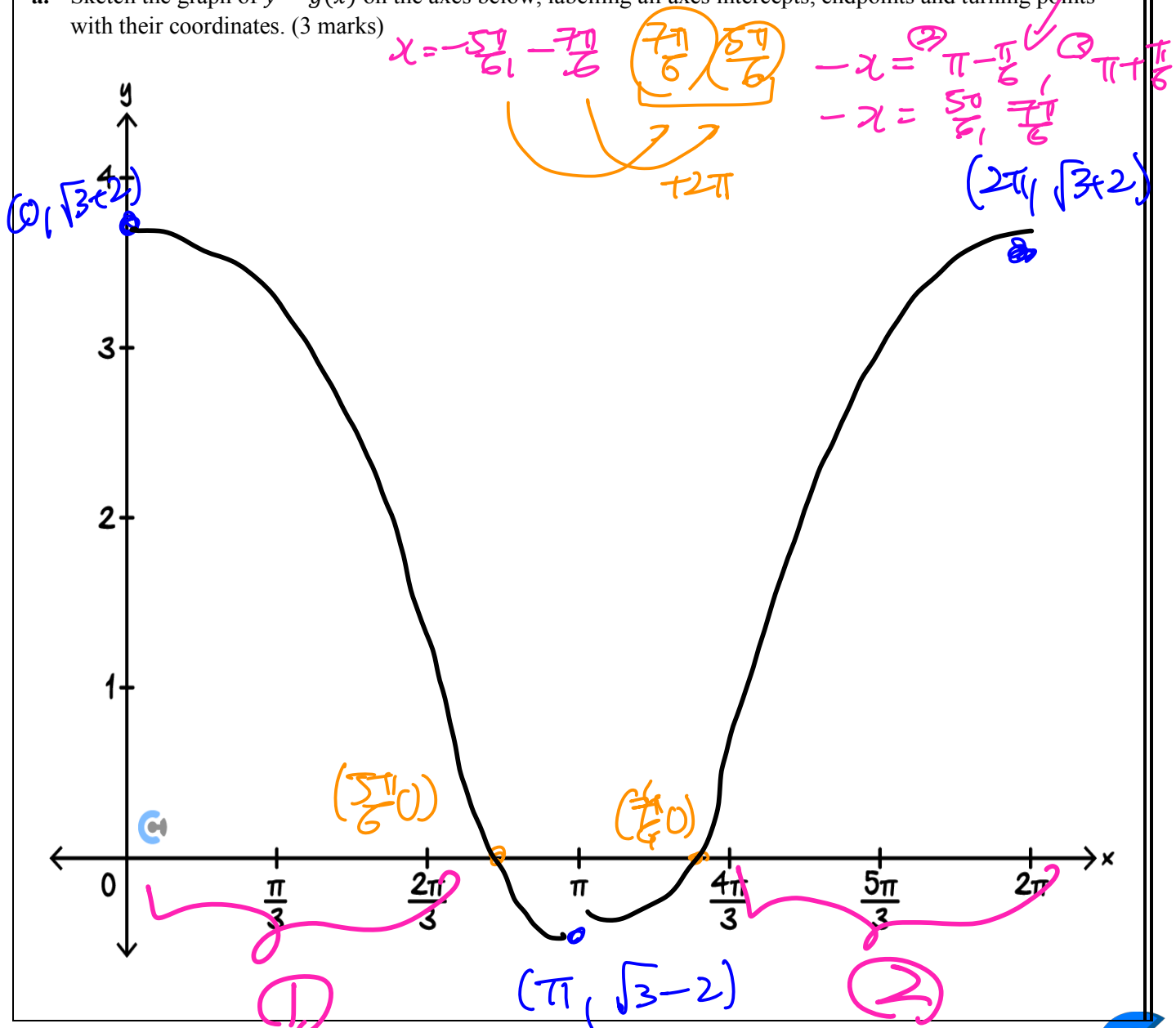
MA P SS $\rightarrow (-x) = 0$
 $\sqrt{3} - 2 \sin(\pi) = 0$
 $x = 0$

Question 3 (5 marks)

Consider the function:

$g(x) = \sqrt{3} + 2 \cos(-x), x \in [0, 2\pi]$

- a. Sketch the graph of $y = g(x)$ on the axes below, labelling all axes intercepts, endpoints and turning points with their coordinates. (3 marks)



b. What fraction of the graph of $g(x)$ lies above the x -axis? (2 marks)

① $\frac{5\pi}{6}$ ② $2\pi - \frac{7\pi}{6} = \frac{5\pi}{6}$

duration = $\frac{5\pi}{6} + \frac{5\pi}{6} = \frac{5\pi}{3}$

frac = $\frac{\text{duration}}{\text{period}} = \frac{\frac{5\pi}{3}}{2\pi} = \frac{5\pi}{6\pi} = \frac{5}{6}$

Question 4 (3 marks)

A trigonometric function with rule $f(x) = a \sin(bx) + c$, satisfies the following properties:

➤ $\text{ran} = [-4, 6]$.

➤ $f(0) = 1$.

➤ The period is π .

$\frac{2\pi}{b} = \text{period}$
 ↑ ↑ ↑
 amp period mean

Find a possible rule, for the function f , that satisfies all of these properties.

$a, b, c \in \mathbb{R}^+$

mean = $\frac{6-4}{2} = 1$
 amp = max - mean = 5

$\frac{2\pi}{b} = \pi \Rightarrow b = \frac{2\pi}{\pi} = 2$

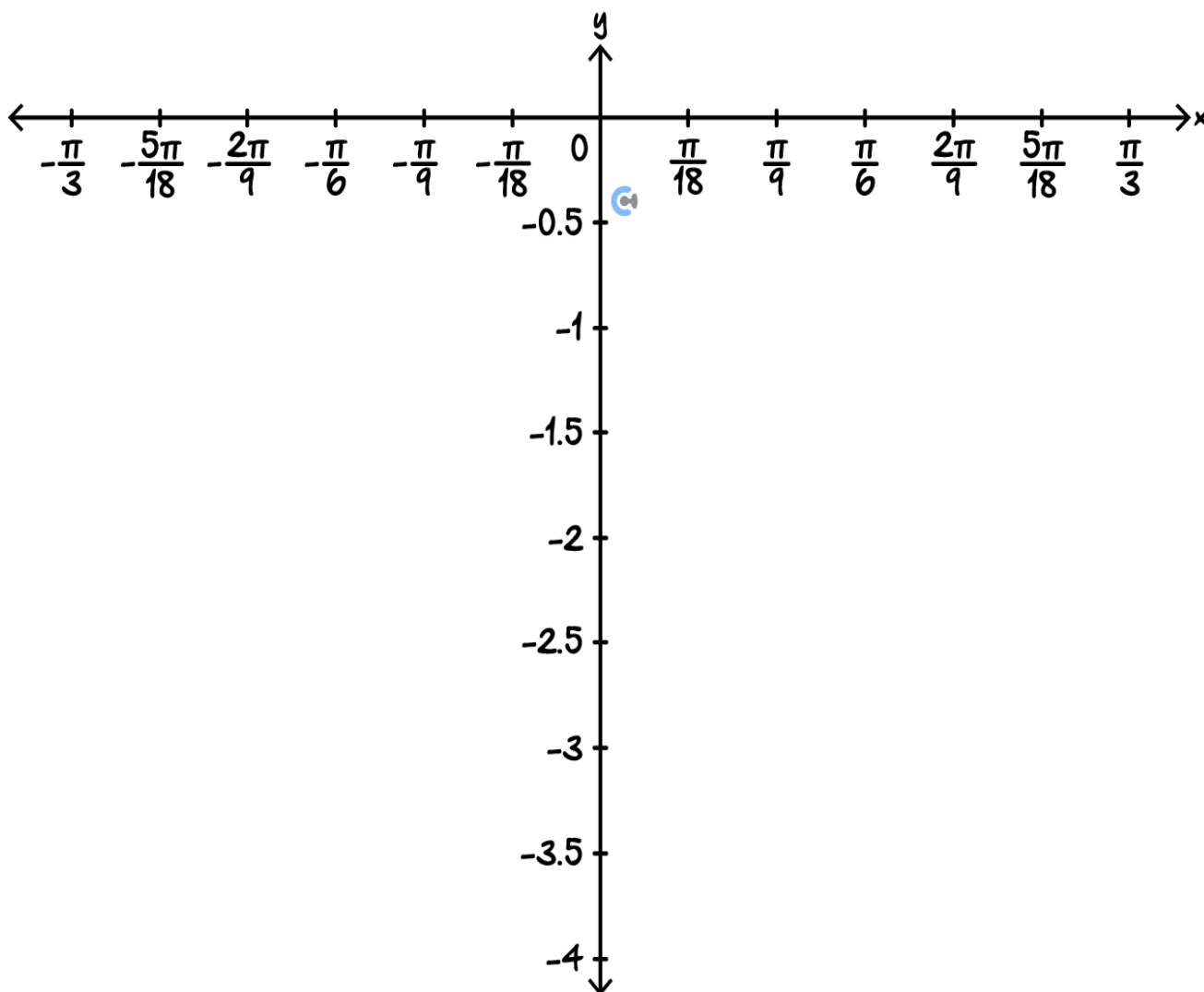
$y = 5 \sin(2x) + 1$

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Question 5 (6 marks)

- a. Sketch the following on the axes below, labelling all axes intercepts, endpoints, and turning points with their coordinates. (4 marks)

$$y = 2 \sin\left(\frac{\pi}{3} - 3x\right) - \sqrt{3}, x \in \left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$$



- b. What fraction of the graph lies **below** the x -axis? (2 marks)

Question 6 (5 marks)

- a. Sketch the following on the axes below, labelling all intercepts, and endpoints with their coordinates, and all asymptotes with their equations. (3 marks)

2-101

$$0 = 1 - \tan\left(\frac{\pi}{3} - x\right)$$

$$\tan\left(\frac{\pi}{3} - x\right) = 1$$

$$\frac{\pi}{3} - x = \frac{\pi}{4}$$

$$x = \frac{\pi}{3} - \frac{\pi}{4}$$

$$x = \frac{\pi}{12}$$

y-int

$$y = 1 - \tan\left(\frac{\pi}{3}\right) = 1 - \sqrt{3}$$

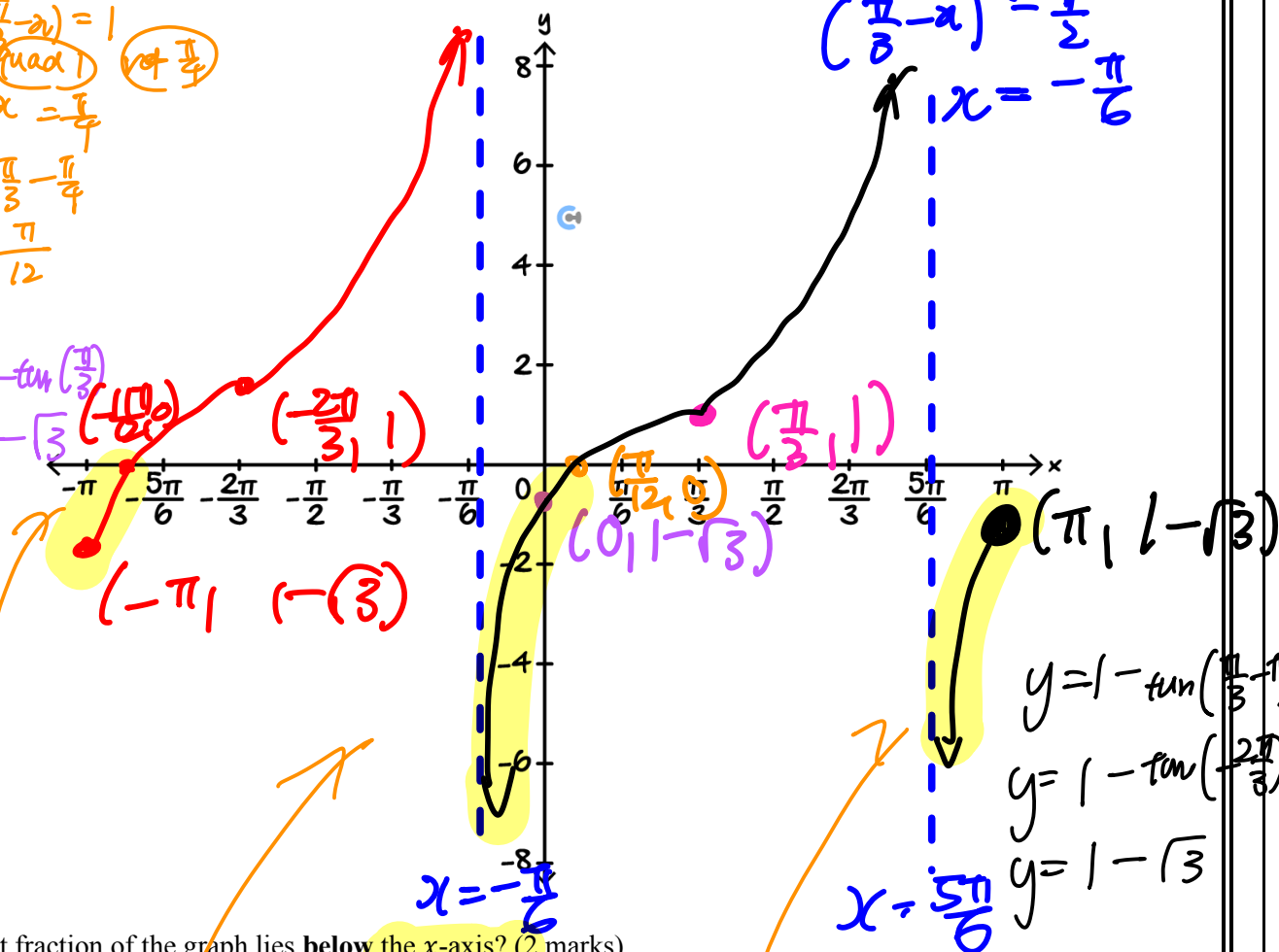
$$y = 1 - \tan\left(\frac{\pi}{3} - x\right), x \in [-\pi, \pi]$$

① period $\frac{\pi}{1} = \pi$

② asymptote

$$\left(\frac{\pi}{3} - x\right) = \frac{\pi}{2}$$

$$x = -\frac{\pi}{6}$$



- b. What fraction of the graph lies below the x-axis? (2 marks)

$$-\frac{\pi}{6} \rightarrow \frac{\pi}{12}$$

$$\frac{5\pi}{6} \rightarrow \pi$$

$$\text{duration} = \frac{\pi}{12} - \left(-\frac{\pi}{6}\right) = \frac{\pi}{4}$$

$$\text{duration} = \frac{\pi}{6}$$

$$-\pi \rightarrow -\frac{11\pi}{12}$$

$$\text{duration} = \frac{\pi}{12}$$

$$\text{total duration} = \frac{\pi}{4} + \frac{\pi}{6} + \frac{\pi}{12}$$

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$$\text{frac} = \frac{\text{duration}}{\text{graph}} = \frac{\pi}{2} \div 2\pi = \frac{1}{4}$$

Section D: Tech Active Exam Skills

Calculator Commands: Degrees and Radians

TI

Doc → 7 → 2

Document Settings

Display Digits:	Float 6
Angle:	Radian
Exponential Format:	Radian
Real or Complex:	Degree
Calculation Mode:	Exact

Casio

Change at the bottom of the screen.

Alg	Decimal	Real	Rad
-----	---------	------	-----

Mathematica

In radians by default.

Write "Degree."

In[27]:= Sin[30 Degree]

Out[27]= $\frac{1}{2}$



Calculator Commands: Solving trigonometric functions.

TI

solve(trig(..) = a, x) | domain restriction.

| is under control equal.

Casio

solve(trig(..) = a, x) | domain restriction.

| is under maths 3.

Mathematica

Solve[trig[] == a && domain restriction, x].



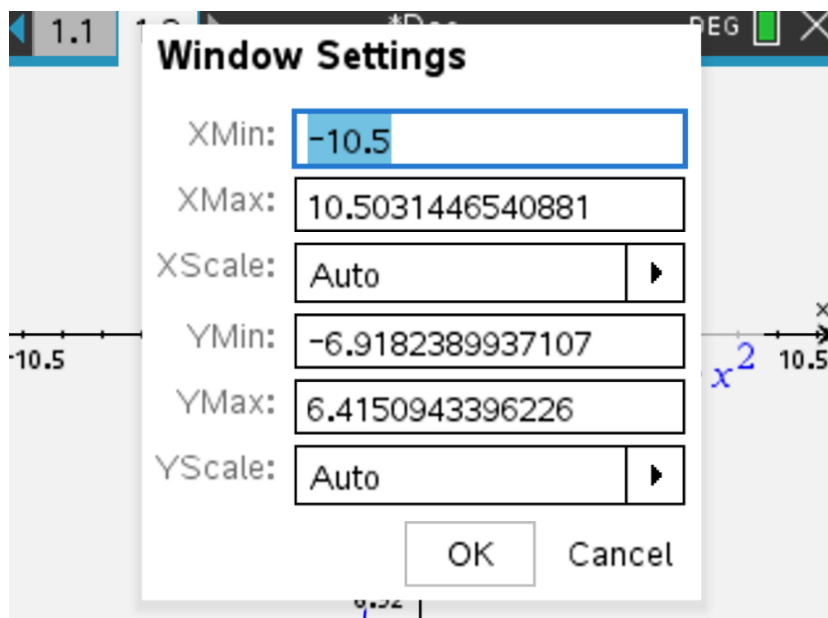
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Solve (sin(x) = 1/2, x) | 0 ≤ x ≤ 2π

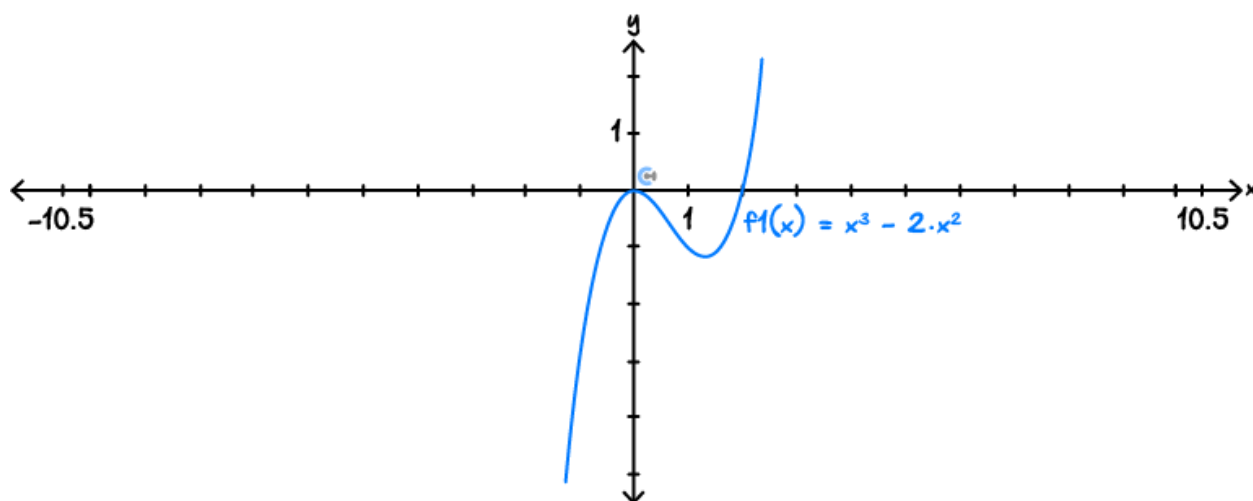


Calculator Commands: Graphing

- Open a graph page and plot your function.
- Zoom settings: Menu → 4 (window / zoom) → 1 enter your x and y -ranges.



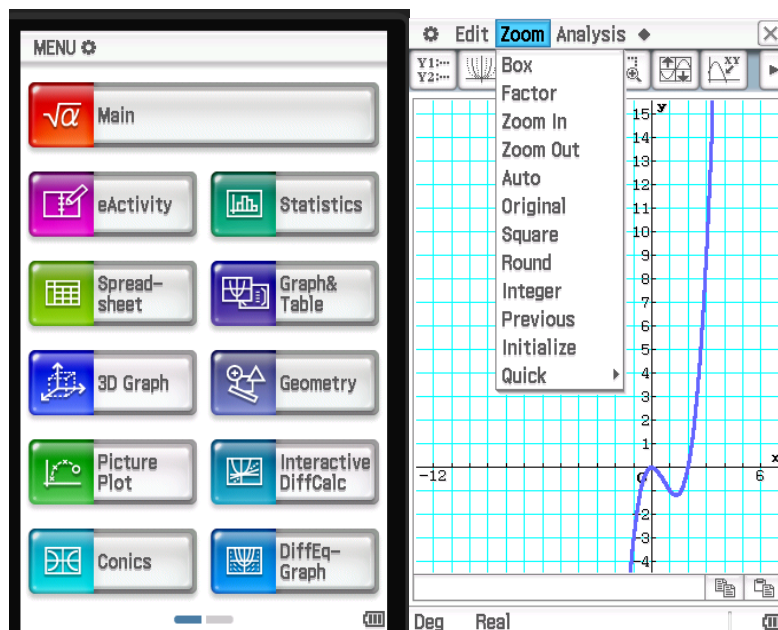
- Can also click the axis numbers on the graph and alter them directly.




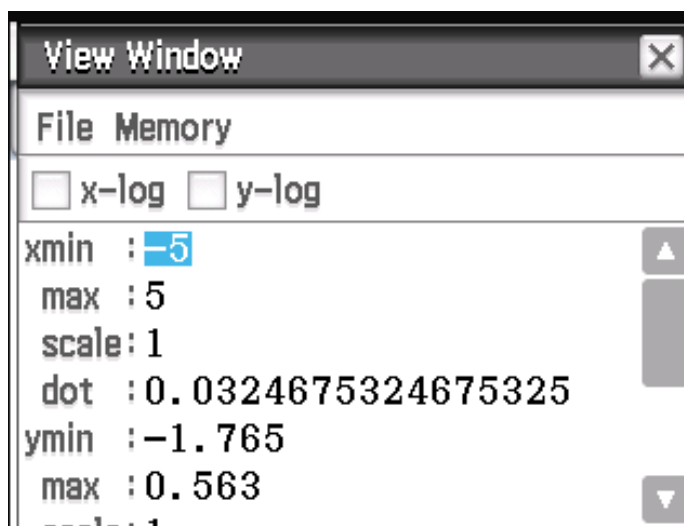
- Menu → 6 (Analyse) to find *min* / *max* x and y -intercepts.
- Restrict the domain to $0 < x < 2$, use the bar to get it from ctrl+ =

☒ $f1(x) = x^3 - 2x^2 | 0 < x < 2 |$

- **Casio:** Click graph & table, and enter the function.




- Analysis → G-Solve to find intercepts.
- Use this button  to set the view window.



- Use | to restrict the domain → find it in Math 3.

$$\checkmark y1=x^3-2\cdot x^2 \mid 0<x<2$$

- **Mathematica:** `Plot[function, {x, xmin, xmax}, PlotRange → {ymin, ymax}]`

 PlotRange is optional but can be used to make the scale appropriate for the question.

Section E: Exam 2 Questions (30 Marks)

INSTRUCTION:

- **Regular: 30 Marks. 5 Minutes Reading. 45 Minutes Writing.**
- **Extension: 30 Marks. 5 Minutes Reading. 30 Minutes Writing.**



Question 7 (1 mark)

What is the amplitude and period for the following function:

$$f(x) = 1 - 2 \sin\left(4x - \frac{\pi}{2}\right)$$

- A. 2, 4
- B. $2, 2\pi$
- C. $2, \frac{\pi}{4}$
- D. $2, \frac{\pi}{2}$

$$\frac{2\pi}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$$

Question 8 (1 mark)

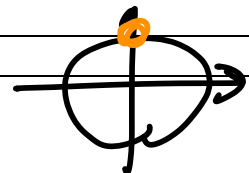
The general equation for all the asymptotes of $y = \tan(x)$, can be found by solving for x in the equation:

- A. $\sin(x) = 0$
- B. $\cos(x) = 1$
- C. $\sin(x) = 1$
- D. $\tan(x) = 1$

$$\tan(x) = \text{Undefined}$$

$$\frac{\sin(x)}{\cos(x)} = \text{Undefined}$$

$$\cos(x) = 0$$



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Question 9 (1 mark)

Which of the following is false?

- A. The amplitude of a trigonometric function does not depend on the dilation along the x -axis.
- B. Some trigonometric functions can always satisfy the equation: $f(x) = k, k \in \mathbb{R}$.
- C. The tangent function is defined for the domain $x \in \mathbb{R}$.
- D. The sine function is defined for the domain $x \in \mathbb{R}$.

← NO. HAS asymptotes

Question 10 (1 mark)

Which of the following has a range of $[-3, 2]$?

- A. ~~2.5 cos(x)~~
- B. ~~3 cos(x)~~
- C. 2.5 cos(x) - 0.5
- D. ~~2.5 sin(x) + 0.5~~

← max
↑
min

$$\text{mean} = \frac{-3+2}{2} = -\frac{1}{2}$$

Question 11 (1 mark)

What is the period and average value of the following trigonometric function?

$$y = \sin\left(\frac{\pi}{2}x + 2\right) - 1$$

- A. $\pi, -1$
- B. $4\pi, 1$
- C. $4, -1$ ✓
- D. $1, -1$

$$\begin{aligned} \text{period} &= \frac{2\pi}{\frac{\pi}{2}} = 2\pi \div \frac{\pi}{2} \\ &= 2\pi \times \frac{2}{\pi} = 4 \end{aligned}$$

Space for Personal Notes

Question 12 (1 mark)

What is the equation for the asymptotes of the following function?

$$f(x) = \tan\left(2x - \frac{\pi}{3}\right)$$

A. $x = \frac{n\pi}{2} + \frac{\pi}{12}$

B. $x = \frac{n\pi}{2} - \frac{\pi}{12}$

C. $x = \frac{(6n-1)\pi}{12}$

D. $x = \frac{(6n+1)\pi}{12}$

$\frac{\pi}{2}$

$$\left(2x - \frac{\pi}{3}\right) = \frac{\pi}{2}$$

$$2x = \frac{5\pi}{6}$$

more;

$$x = \frac{5\pi}{12} + \frac{\pi}{2}n$$

period

Question 13 (1 mark)

What fraction of the graph $y = \sin(x)$ lies above the x -axis?

A. $\frac{1}{2}$

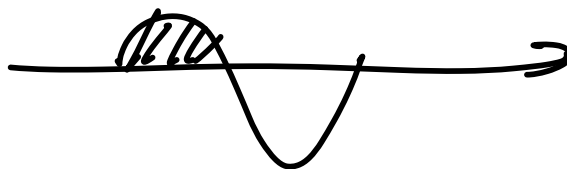
B. $\frac{1}{3}$

C. $\frac{2}{3}$

D. $\frac{1}{6}$

$-\frac{\pi}{2}$

$$x = -\frac{\pi}{2} + \frac{\pi}{2}n$$



Question 14 (1 mark)

Which of the following is not the same as the rest, for $n \in \mathbb{Z}$?

A. $(6n - 7)\pi$

B. $(6n + 1)\pi$

C. $(6n - 1)\pi$

D. $(6n - 5)\pi$

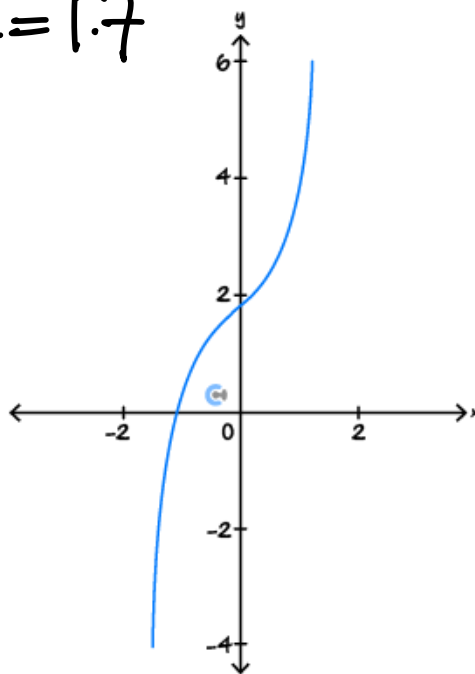
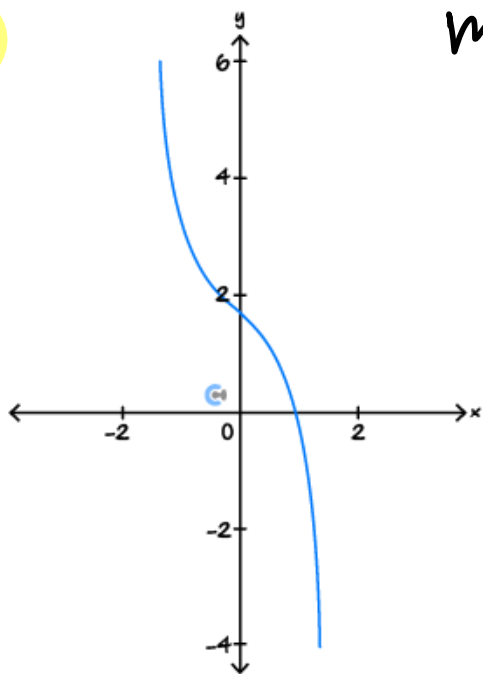
Question 15 (1 mark)

Which of the following best represents a part of the graph of:

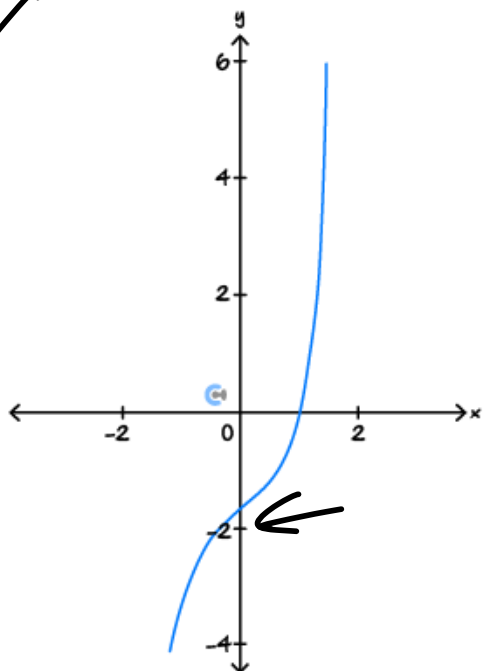
$y = \sqrt{3} - \tan(x)$

mean = 1.7

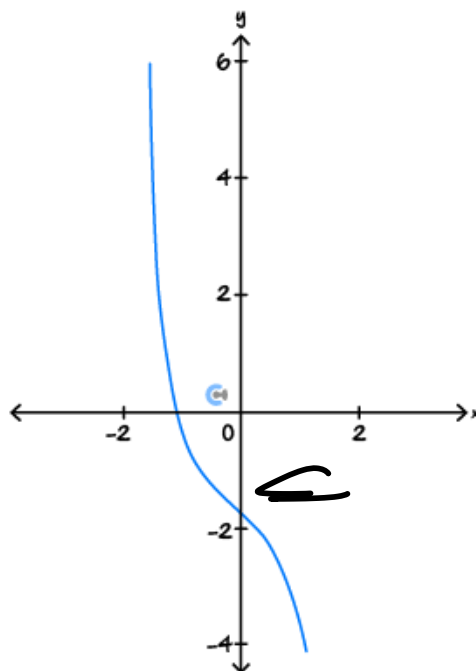
A.



C.

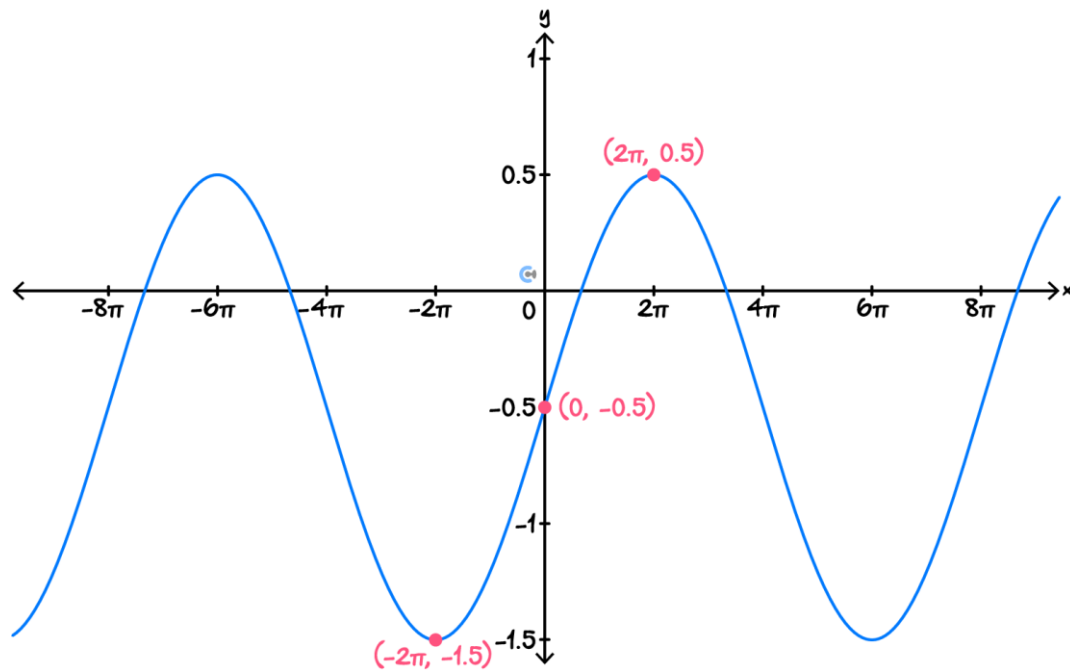


D.



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Question 16 (1 mark)



Which of the following functions could represent the graph above?

A. $y = 0.5 \sin\left(\frac{x}{4}\right) - 1$

B. $y = -\sin\left(\frac{x}{8}\right) + 0.5$

C. $y = \sin\left(\frac{x}{8}\right) - 0.5$

D. $y = \sin\left(\frac{x}{4}\right) - 0.5$

Sketch on CAS

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Question 17 (20 marks)

The height of a carriage above the ground on a Ferris wheel can be modelled by the following equation:

h in metres

$$h(t) = 2 \cos\left(\frac{\pi}{4}t + \frac{\pi}{3}\right) + 3$$

where $t \geq 0$ is the amount of time passed in minutes.

- a. How long does it take for a carriage to go around the Ferris wheel once? (1 mark)

find the period

$$2\pi \div \frac{\pi}{4} = 2\pi \times \frac{4}{\pi} = 8 \text{ mins}$$

- b. What are the maximum height and minimum height of a carriage in the Ferris wheel? (2 marks)

$$\begin{aligned} \text{mean} &= 3 \\ \text{amp} &= 2 \end{aligned}$$

$$\text{max} = 3 + 2 = 5 \text{ m}$$

$$\text{min} = 3 - 2 = 1 \text{ m}$$

- ★ Rewrite $h(t)$ in terms of the sine function. (2 marks)

$$\sin(\theta) = \cos\left(\frac{\pi}{2} - \theta\right) \quad (\text{complementary})$$

$$\frac{\pi}{2} - \theta = \frac{\pi}{4}t + \frac{\pi}{3}$$

$$\theta = -\frac{\pi}{4}t + \frac{\pi}{6}$$

$$\Rightarrow h(t) = 2 \sin\left(-\frac{\pi}{4}t + \frac{\pi}{6}\right) + 3$$

- d. A carriage can only take passengers when the Ferris wheel is at its lowest point. find the time t , in minutes, when this occurs. (3 marks)

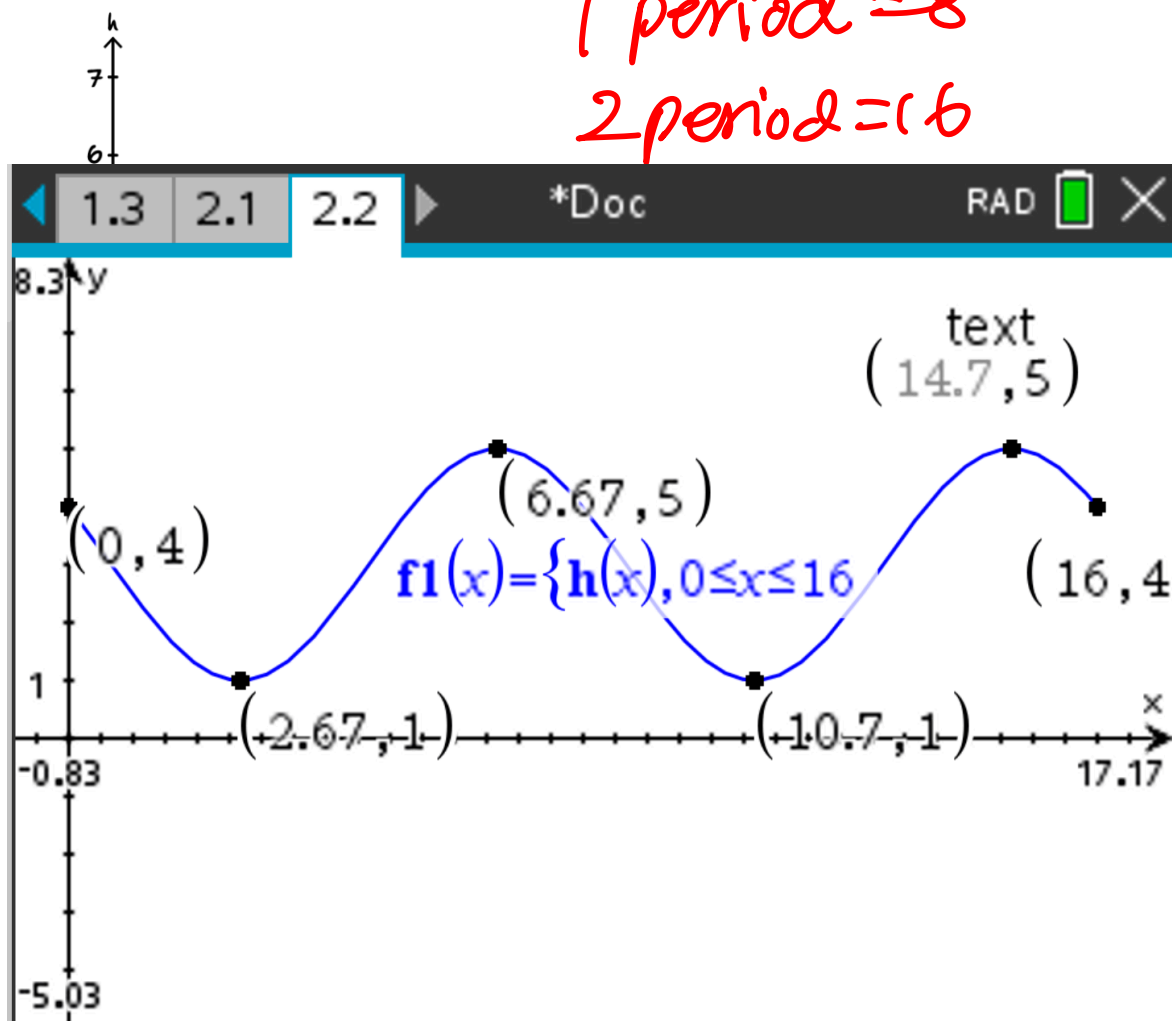
solve($h(t)=1, t$)

$$t = \frac{8 \cdot (3n - 2)}{3}$$

$$h(t) = 1$$

$$t = \frac{8(3n-2)}{3} \text{ min, } n \in \mathbb{Z}^+$$

- e. Sketch the graph of $h(t)$, for the first 2 periods on the set of axes below, labelling all turning points and axes-intercepts. (4 marks)



f. A good view of the city can be seen when the carriage is more than 2 m above the ground.

i. For what fraction of the ride can passengers get to see a nice view? (2 marks)

$$\text{frac} = \frac{\text{duration}}{\text{period}}$$

$$h(t) = 2, \quad t \in [0, 8] \quad \text{1 period}$$

$$t = \frac{4}{3}, 4$$

$$\text{helow duration} = 4 - \frac{4}{3} = \frac{12}{3} - \frac{4}{3} = \frac{8}{3}$$

$$\text{helow frac} = \frac{8}{3} \div 8 = \frac{1}{3} \quad \text{above } \frac{2}{3}$$

ii. At what times is the carriage at exactly 2 m and travelling upwards? (2 marks)

$$h(t) = 2$$

$$t = 4 + 8n, \quad n \in \mathbb{Z}^+ \cup \{0\}$$

iii. At what times is the carriage at exactly 2 m and travelling downwards? (2 marks)

$$t = \frac{4}{3} + 8n, \quad n \in \mathbb{Z}^+ \cup \{0\}$$

- g. To make the ride more exciting, the Ferris wheel has been reprogrammed to follow a new path such that the height can be modelled by:

$$h_{\text{new}}(t) = 2 \cos\left(\frac{\pi t}{4} + \frac{\pi}{3}\right) - \sin\left(\frac{\pi t}{3} + \frac{\pi}{3}\right) + 4$$

What is the total time for one ride (one ride is one complete cycle) now? (2 marks)

period = $2\pi \div \frac{\pi}{4} = 8$

$\text{LCM} = 24$

$\text{period} = 2\pi \div \frac{\pi}{3} = 6$

both repeat every 24 mins

Space for Personal Notes

Section F: Extension Exam 1 (16 Marks)

INSTRUCTION:

➤ Regular: Skip

➤ Extension: 16 Marks. 2 Minutes Reading. 20 Minutes Writing.

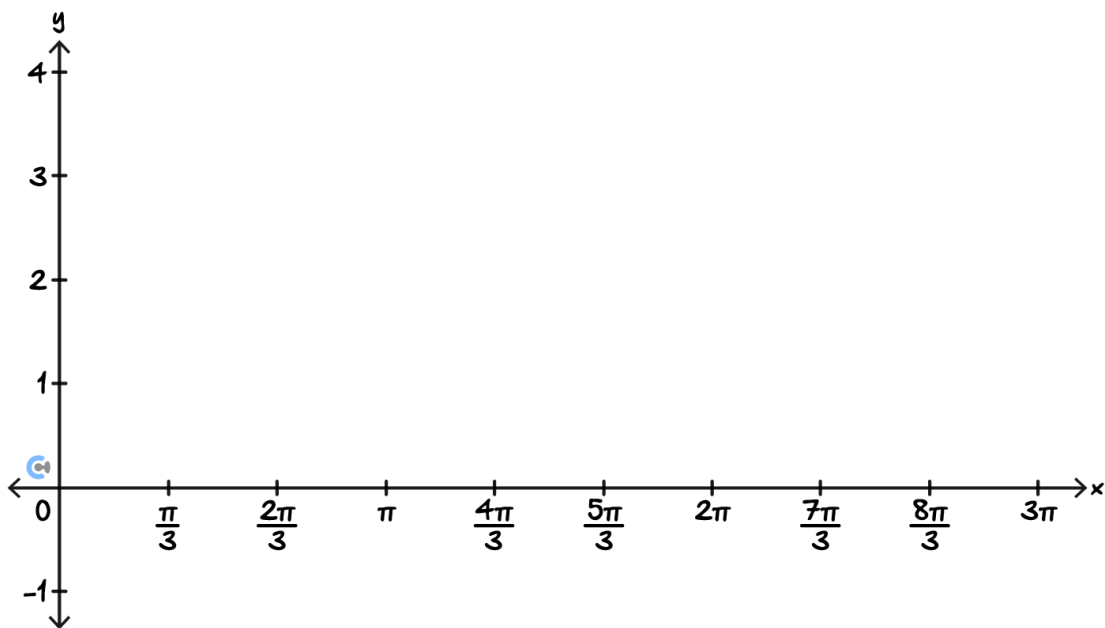


Question 18 (7 marks)

Consider the following function:

$$f(x) = 2 \sin\left(\frac{x}{2} - \frac{\pi}{3}\right) + 1, x \in [0, 3\pi]$$

- a. Sketch $f(x)$ on the set of axes below. Label all axes intercepts and turning points with their coordinates. (3 marks)



- b. What fraction of $f(x)$ lies **above** the x -axis? (1 mark)

c. Consider another function,

$$g: R \rightarrow R, g(x) = 2 \sin\left(\frac{x}{2} - \frac{\pi}{3}\right) + 1.$$

What fraction of $g(x)$ lies **above** the x -axis? (1 mark)

d. Now consider the function

$$h: R \rightarrow R, h(x) = g(x) + k$$

Find the value of k such that $\frac{1}{3}$ of $h(x)$ lies **above** the x -axis. (2 marks)

Space for Personal Notes

Question 19 (9 marks)

Consider the function:

$$f(x) = 2 \sin^2(x) - \cos(x) - 1$$

- a.** Solve the equation $f(x) = 0$. (3 marks)

- b.** Consider the function:

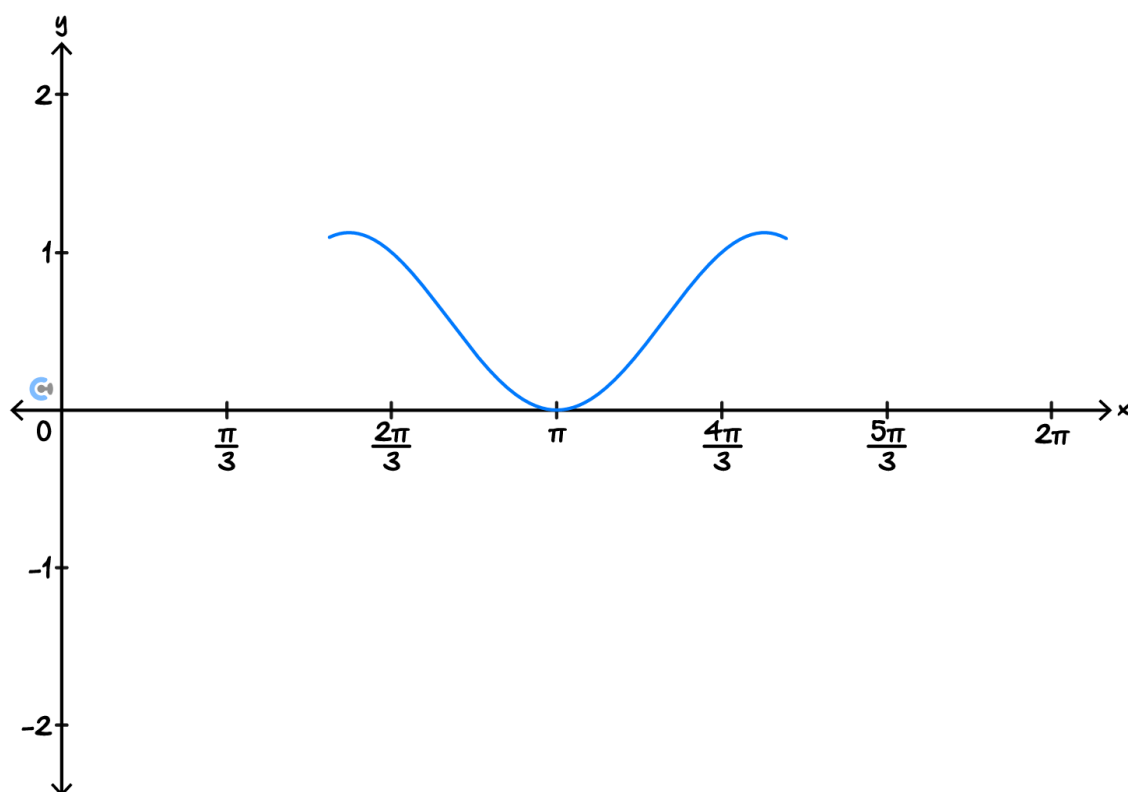
$$g: [-1, 1] \rightarrow \mathbb{R}, g(x) = -2x^2 - x + 1$$

- i.** Find the range of g . (2 marks)

ii. Hence, state the range of f . (1 mark)

c. Part of the graph of $y = f(x)$ is shown on the axes below.

Use your answers to the previous parts to sketch the rest of the graph of $y = f(x)$ for $x \in [0, 2\pi]$. Label any axes intercepts and endpoints with coordinates. (3 marks)



Space for Personal Notes

Section G: Extension Exam 2 (15 Marks)

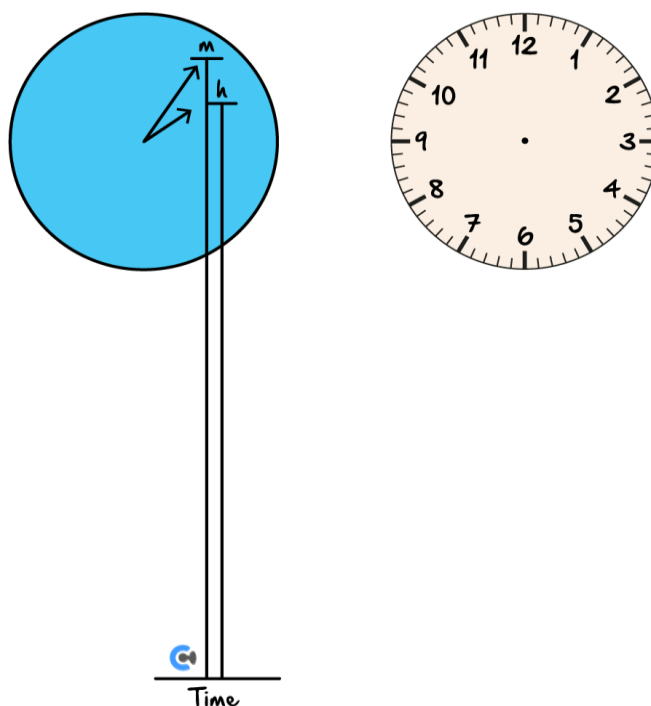
INSTRUCTION:

➤ Regular: Skip

➤ Extension: 15 Marks. 2 Minutes Reading. 18 Minutes Writing.



Question 20 (15 marks)



The height of the tip of a minute hand of a clock on the wall is modelled by the following equation:

$$m(t) = 2.3 + 0.4 \sin\left(\frac{\pi}{6} - 120\pi t\right)$$

where $m(t)$ is the height t hours after a certain moment.

a. How far above the ground in the centre of the clock? (1 mark)

b. How long does it take for the minute hand to go around the clock once? (1 mark)

c.

i. What was the initial height of the minute hand? (1 mark)

ii. Determine the initial trajectory. That is, is the minute hand moving down or up? (1 mark)

iii. After how many seconds is the minute hand first at the same height that it started at? (2 marks)

iv. Hence, determine the digit that the minute hand was pointing to when $t = 0$. (1 mark)

- d. For what fraction of the time is the minute hand above 2.5 *m*. (1 mark)

- e. The height of the tip of the hour hand can be modelled by the following:

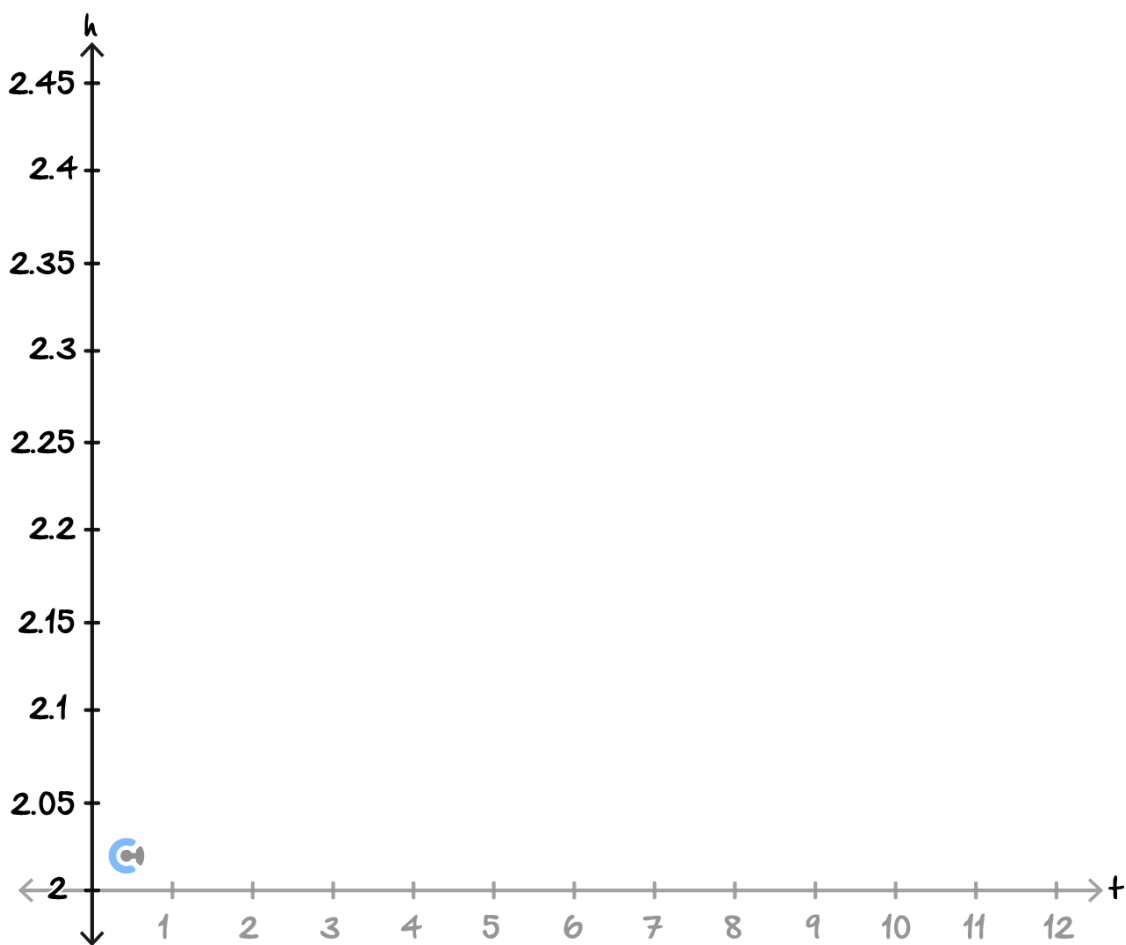
$$h(t) = 2.3 + 0.15 \sin\left(\frac{\pi}{6} - at\right)$$

where $h(t)$ is the height t **hours** after a certain moment.

- i. What is the length of the hour hand? (1 mark)

- ii. Find the value of a . (1 mark)

- f. On the set of axes below, draw one period of $h(t)$. Label all turning points and endpoints. (3 marks)



- g. Find the height H metres, such that the hour hand of the clock is **above** this value exactly 1 hour over a full cycle. Give your answer correct to three decimal places. (2 marks)



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VCE Mathematical Methods ½

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