Website: contoureducation.com.au | Phone: 1800 888 300
Email: hello@contoureducation.com.au

VCE Mathematical Methods ½
Circular Function Exam Skills [0.18]
Workshop

Error Logbook:

New Ideas/Concepts	Didn't Read Question
Pg / Q #: ____________ Notes:	Pg / Q #: ____________ Notes:
Algebraic/Arithmetic/ Calculator Input Mistake	**Working Out Not Detailed Enough**
Pg / Q #: ____________ Notes:	Pg / Q #: ____________ Notes:

Section A: Recap

The Exact Values Table

x	$0\ (0^o)$	$\frac{\pi}{6}\ (30^o)$	$\frac{\pi}{4}\ (45^o)$	$\frac{\pi}{3}\ (60^o)$	$\frac{\pi}{2}\ (90^o)$
$\sin(x)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos(x)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan(x)$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined

Particular Solutions

- Solving trigonometric equations **for finite solutions**.
- **Steps:**

QRAP

1. Make the trigonometric function the subject.
2. Find the necessary angle for one period.
3. Solve for x by equating the necessary angles to the inside of the trigonometric functions.
4. Add and subtract the period to find all other solutions in the domain.

Space for Personal Notes

Definition

General Solutions

- Solving infinite trigonometric equations.
- Steps:

1. Make the trigonometric function the subject.
2. Find the necessary angle for one period.
3. Solve for x by equating the necessary angles to the inside of the trigonometric functions.
4. Add $\boxed{\text{period} \cdot n}$ where $n \in Z$.

Multiple Forms of a General Solution

different starting point

$$a + \textit{Period} \cdot n = b + \textit{Period} \cdot n$$

If the difference of a and b is a multiple of period.

1, 3, 5, 7, 9

General Solution with Domain Restriction

$3+2k$, $7+2k$

$$\text{e.g. trig}\left(2x + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \text{ for } \boxed{x \geq 0}$$

infinite

$[0, \infty)$

- We can have infinite solutions for a restricted domain.
- The value of n is also restricted.

general

Space for Personal Notes

Definition

Period of a Trigonometric Function

$$\text{period of } \sin(nx) \text{ and } \cos(nx) \text{ functions} = \frac{2\pi}{n}$$

$$\text{period of } \tan(nx) \text{ functions} = \frac{\pi}{n}$$

where $n =$ coefficient of x and $n > 0$

Hidden Quadratics

$$af(x)^2 + bf(x) + c = 0$$

Let $A = f(x)$

Pythagorean Identity

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

Space for Personal Notes

Section B: Warm up Questions (13 Marks)

INSTRUCTION:

- Regular: 13 Marks, ~~15~~ 8 Minutes Writing.
- Extension: Skip

Question 1a, 2b, 3

↳ THEN do the rest

Question 1 (6 marks)

Solve the following trigonometric equations over the specified domain:

a. $\cos(x) = -\frac{1}{2}$ for $x \in [0, 2\pi]$. (2 marks)

quad 2, 3

ref $\frac{\pi}{3}$

period $\frac{2\pi}{1} = 2\pi$

angle $x = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$ $\cancel{x}$ x

b. $\sin(2x) = \frac{\sqrt{3}}{2}$ for $x \in [0, 2\pi]$. (2 marks)

quad 1, 2

ref $\frac{\pi}{3}$

period $\frac{2\pi}{2} = \pi$

angle

$2x = \frac{\pi}{3}, \pi - \frac{\pi}{3}$

$2x = \frac{\pi}{3}, \frac{2\pi}{3}$

$x = \frac{\pi}{6}, \frac{\pi}{3}$

$x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}$

$+\pi$ $+\pi$

c. $\tan(2x) = 1$ for $x \in \left[0, \frac{3\pi}{2}\right]$. (2 marks)

quad 1, ~~3~~

ref $\frac{\pi}{4}$

period $\frac{\pi}{2}$

angle

$2x = \frac{\pi}{4}$

$x = \frac{\pi}{8}$

$x = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}$

$+\frac{\pi}{2}$ $+\frac{\pi}{2}$

Space for Personal Notes

Question 2 (4 marks)

Solve the following trigonometric equations:

a. $\cos(3x) = -\frac{1}{2}$. (2 marks)

quad 2, 3

ref $\frac{\pi}{3}$

$3x = \pi - \frac{\pi}{3}, \ \pi + \frac{\pi}{3}$

$3x = \frac{2\pi}{3}, \frac{4\pi}{3}$

$x = \frac{2\pi}{9}, \frac{4\pi}{9}$

period : $\frac{2\pi}{3}$

$x = \frac{2\pi}{9} + \frac{2\pi}{3}n, \ n \in \mathbb{Z}$

$x = \frac{4\pi}{9} + \frac{2\pi}{3}n, \ n \in \mathbb{Z}$

b. $\tan\left(x - \frac{\pi}{6}\right) = -1$ for $x \geq 0$ (2 marks) $[0, \infty)$

quad 2, $\cancel{4}$

period $\frac{\pi}{1} = \pi$

ref $\frac{\pi}{4}$

angle $x - \frac{\pi}{6} = \pi - \frac{\pi}{4}$

$x - \frac{\pi}{6} = \frac{3\pi}{4}$

$x = \frac{9\pi}{12} + \frac{2\pi}{12}$

$x = \frac{11\pi}{12}$

$x = \frac{11\pi}{12} + \pi n, \ n \in \mathbb{Z}^+ \cup \{0\}$

Space for Personal Notes

Question 3 (3 marks)

Solve the equations $\tan^2\left(x - \frac{\pi}{3}\right) = \frac{1}{3}$.

period $= \frac{\pi}{1} = \pi$

$\tan\left(x - \frac{\pi}{3}\right) = \pm\sqrt{\frac{1}{3}}$

$\tan\left(x - \frac{\pi}{3}\right) = \sqrt{\frac{1}{3}}$ $\qquad$ $\tan\left(x - \frac{\pi}{3}\right) = -\sqrt{\frac{1}{3}}$

$\tan\left(x - \frac{\pi}{3}\right) = \frac{1}{\sqrt{3}} \frac{\times\sqrt{3}}{\times\sqrt{3}}$ $\qquad$ $\tan\left(x - \frac{\pi}{3}\right) = -\frac{1}{\sqrt{3}} \frac{\times\sqrt{3}}{\times\sqrt{3}}$

$\tan\left(x - \frac{\pi}{3}\right) = \frac{\sqrt{3}}{3}$ $\qquad$ $\tan\left(x - \frac{\pi}{3}\right) = -\frac{\sqrt{3}}{3}$

quad 1 $\qquad$ quad 2

ref $\frac{\pi}{6}$ $\qquad$ ref $\frac{\pi}{6}$

$x - \frac{\pi}{3} = ① \frac{\pi}{6}$ $\qquad$ $x - \frac{\pi}{3} = ② \pi - \frac{\pi}{6}$

Space for Personal Notes

$x - \frac{\pi}{3} = \frac{\pi}{6}$, $x - \frac{\pi}{3} = \frac{5\pi}{6}$

$x = \frac{\pi}{2} + \pi n$ $\qquad$ $x = \frac{7\pi}{6} + \pi n$ $\qquad$ $n \in \mathbb{Z}$

Section C: Exam 1 Questions (19 Marks)

INSTRUCTION:

- Regular: 19 Marks. ~~5~~ 3 Minutes Reading. ~~27~~ 22 Minutes Writing.
- Extension: 19 Marks. 5 Minutes Reading. 19 Minutes Writing.

Question 4 (2 marks)

Let α and β both be first quadrant angles. If $\sin(\alpha) = \frac{1}{4}$ and $\cos(\beta) = \frac{2}{3}$, find the value of $\sin(\beta) + \cos(\alpha)$.

O H A H ① ②

triangle method

$\sqrt{3^2-2^2} = \sqrt{5}$

$\sin(\beta) = \frac{\sqrt{5}}{3}$

$\sqrt{15}$

$\cos(\alpha) = \frac{\sqrt{15}}{4}$

$\sin\beta + \cos(\alpha) = \frac{\sqrt{5}}{3} + \frac{\sqrt{15}}{4}$

pythag identity

$\sin^2\beta + \cos^2\beta = 1$

$\sin^2\beta + \left(\frac{2}{3}\right)^2 = 1$

$\sin^2\beta = 1 - \frac{4}{9}$

$\sin^2\beta = \frac{5}{9}$

$\sin\beta = \pm\frac{\sqrt{5}}{3}$

quad 1

$\sin\beta = \frac{\sqrt{5}}{3}$

$\sin^2\alpha + \cos^2\alpha = 1$

$\left(\frac{1}{4}\right)^2 + \cos^2\alpha = 1$

$\cos^2\alpha = 1 - \frac{1}{16}$

$\cos^2\alpha = \frac{15}{16}$

$\cos\alpha = \pm\frac{\sqrt{15}}{4}$

$\cos\alpha = \frac{\sqrt{15}}{4}$

$\sin\beta + \cos\alpha = \frac{\sqrt{5}}{3} + \frac{\sqrt{15}}{4}$

Space for Personal Notes

Question 5 (5 marks)

a. Find the general solution for the following equation: (3 marks)

general

$$\cos\left(2\left(x+\frac{\pi}{6}\right)\right) = -\frac{1}{2}, \boxed{x \geq 0}$$

quad 2, 3

ref $\frac{\pi}{3}$

$2\left(x+\frac{\pi}{6}\right) = \pi-\frac{\pi}{3}, \pi+\frac{\pi}{3}$

$2\left(x+\frac{\pi}{6}\right) = \frac{2\pi}{3}, \frac{4\pi}{3}$

$x+\frac{\pi}{6} = \frac{\pi}{3}, \frac{2\pi}{3}$

$x = \frac{\pi}{6}, \frac{\pi}{2}$

period $= \frac{2\pi}{2} = \pi$

$x = \frac{\pi}{6} + \pi n$, $n \in \mathbb{Z}^+ \cup \{0\}$,

$x = \frac{\pi}{2} + \pi n$, $n \in \mathbb{Z}^+ \cup \{0\}$

b. Hence, find the sum of solutions for $x \in [0, 2\pi]$. (2 marks)

① $\frac{\pi}{6}, \frac{7\pi}{6}$

② $\frac{\pi}{2}, \frac{3\pi}{2}$

$\sum = \frac{\pi}{6} + \frac{7\pi}{6} + \frac{\pi}{2} + \frac{3\pi}{2}$

$= \frac{8\pi}{6} + 2\pi$

$= \frac{4\pi}{3} + \frac{6\pi}{3}$

$= \frac{10\pi}{3}$

Space for Personal Notes

Question 6 (4 marks)

Consider the function $f(x) = \sin(kx)$, where $k > 0$.

a. Find the value of k if $f(x)$ has a period of 5. (2 marks)

$\frac{2\pi}{k} = 5$

$k = \frac{2\pi}{5}$

b. Find the value(s) of k if $f(x) = 0$ has exactly 5 solutions on the interval $[0, 2\pi]$. (2 marks)

$0 \quad \frac{\pi}{k} \quad \frac{2\pi}{k} \quad \frac{3\pi}{k} \quad \frac{4\pi}{k} \quad \frac{5\pi}{k}$

$\frac{\pi}{k}$

period $\frac{2\pi}{k}$

① $\frac{4\pi}{k} \leq 2\pi$

$\frac{4\pi}{2\pi} \leq k$

$2 \leq k$

② $\frac{5\pi}{k} > 2\pi$

$\frac{5\pi}{2\pi} > k$

$\frac{5}{2} > k$

Space for Personal Notes

$2 \leq k < \frac{5}{2}$

Complementary (4.1)

Question 7 (4 marks)

a. Find the general solution to the equation: (3 marks)

$\tan = \frac{\sin}{\cos}$

$\cos \leftrightarrow \sin$

$\tan \leftrightarrow \frac{1}{\tan}$

$$\cos(3x) = \cos\left(3x - \frac{3\pi}{2}\right)$$

$= -\sin(3x)$

$\cos(3x) = -\sin(3x)$

$\div \cos(3x)$ $\div \cos(3x)$

$1 = -\frac{\sin(3x)}{\cos(3x)}$

$1 = -\tan(3x)$

$\tan(3x) = -1$

quad 2

ref $\frac{\pi}{4}$

$3x = \pi - \frac{\pi}{4}$

$3x = \frac{3\pi}{4}$

$x = \frac{\pi}{4} + \frac{\pi}{3}n, n \in \mathbb{Z}$

period $= \frac{\pi}{3}$

b. Hence, find how many points of intersection exist between $\cos(3x)$ ~~and~~ $\cos\left(3x - \frac{3\pi}{2}\right)$ on $[0, \pi]$. (1 mark)

↳ "how many sols for (a) is there from $[0, \pi]$"

$\frac{\pi}{4}$, $\frac{\pi}{4} + \frac{\pi}{3}$, $\frac{\pi}{4} + \frac{\pi}{3} + \frac{\pi}{3}$

$\frac{\pi}{4}$, $\frac{7\pi}{12}$, $\frac{11\pi}{12}$

3 sols

Space for Personal Notes

Question 8 (4 marks)

$\cos^2(\cdots) = 1 - \sin^2(\cdots)$

Solve the following:

no x^2 we can't change

$$8\sin^2\left(x-\frac{\pi}{3}\right)+2\boxed{\cos^2\left(x-\frac{\pi}{3}\right)}+3\sin\left(x-\frac{\pi}{3}\right)=5 \text{ for } x\in[-\pi,\pi]$$

$8\sin^2(x-\frac{\pi}{3})+2(1-\sin^2(x-\frac{\pi}{3}))+3\sin(x-\frac{\pi}{3})=5$

Let $A=\sin(x-\frac{\pi}{3})$

$8A^2+2(1-A^2)+3A=5$

$8A^2+2-2A^2+3A=5$

$6A^2+3A-3=0$

$2A^2+A-1=0$

$(2A-1)(A+1)=0$

$A=\frac{1}{2}, -1$

$\boxed{1}$ $\sin(x-\frac{\pi}{3})=\frac{1}{2}$

quad 1, 2 ref $\frac{\pi}{6}$

$x-\frac{\pi}{3}=\frac{\pi}{6}, \pi-\frac{\pi}{6}$

$x-\frac{\pi}{3}=\frac{\pi}{6}, \frac{5\pi}{6}$

$x=\cancel{\ }, -\frac{5\pi}{6}, \frac{\pi}{2}, \cancel{\frac{7\pi}{6}}$

2π

$\boxed{2}$ $\sin(x-\frac{\pi}{3})=-1$

$x-\frac{\pi}{3}=\frac{3\pi}{2}$

$x=\frac{3\pi}{2}+\frac{\pi}{3}$

$=\frac{9\pi+2\pi}{6}$

$=-\frac{\pi}{6}, \cancel{\frac{11\pi}{6}}$

-2π

Space for Personal Notes

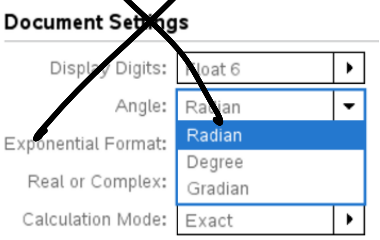
$x=-\frac{5\pi}{6}, \frac{\pi}{2}, -\frac{\pi}{6}$

Section D: Tech-Active Exam Skills

Calculator Commands: Degrees and Radians

TI

- Doc → 7 → 2



Casio

- Change at the bottom of the screen.

Mathematica

- In radians by default.
- Write "Degree."

In[27]:= `Sin[30 Degree]`

Out[27]= $\frac{1}{2}$

Calculator Commands: Solving Trigonometric Functions

TI

- solve$(\text{trig}(..) = a, x)$| domain restriction.
- | is under control equal.

Casio

- solve$(\text{trig}(..) = a, x)$ | domain restriction.
- | is under maths 3.

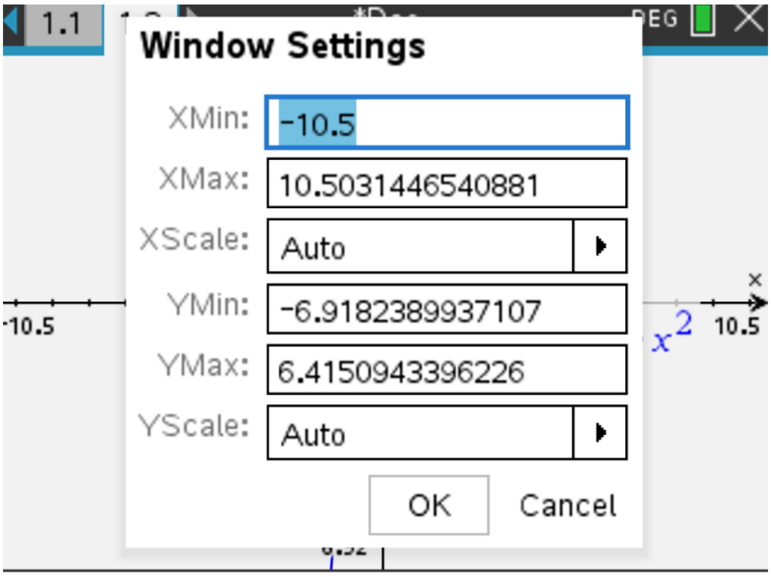
Mathematica

- Solve[trig[] $== a$ && domain restriction, x].

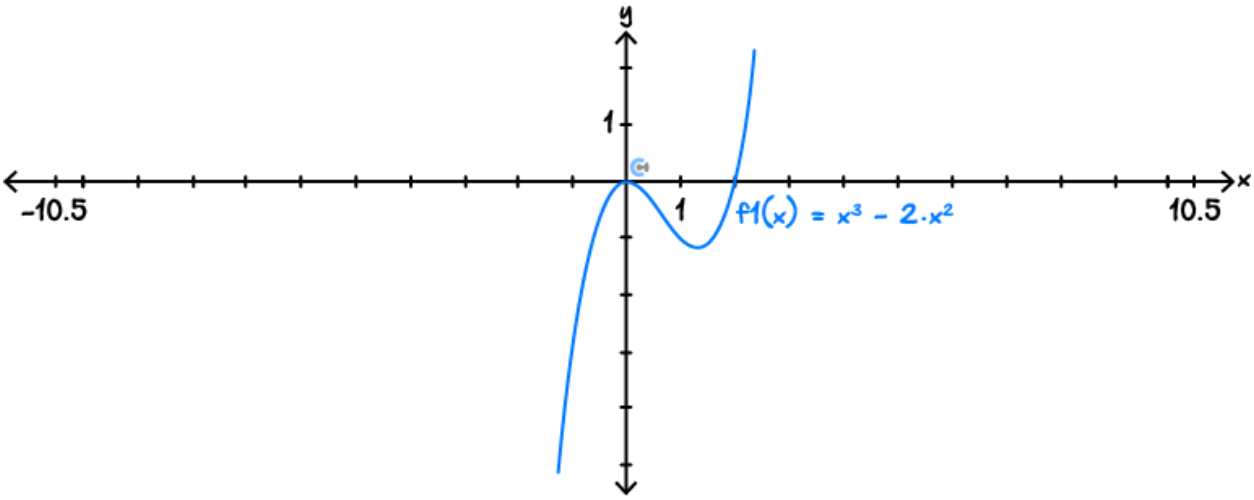
Space for Personal Notes

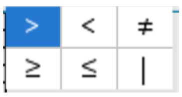
Calculator Commands: Graphing

- Open a graph page and plot your function.
- Zoom settings: Menu → 4 (window / zoom) → 1 enter your x and y-ranges.



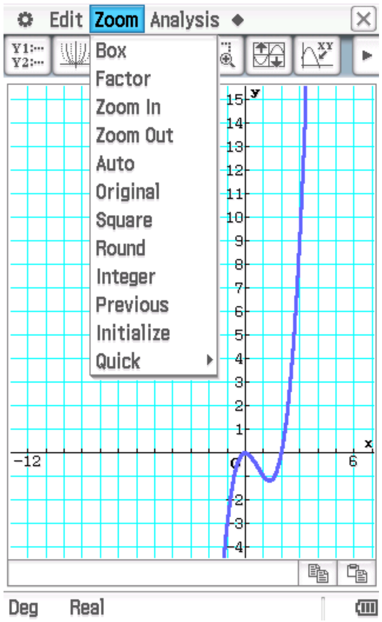
- Can also click the axis numbers on the graph and alter them directly.



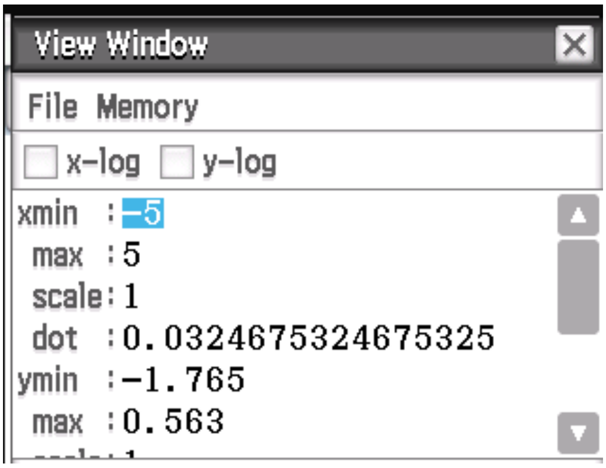
- Menu → 6 (Analyse) to find min / max x and y-intercepts.
- Restrict the domain to $0 < x < 2$, use the bar to get it from  ctrl+ =

$$\text{f1}(x)=x^3-2x^2|0<x<2$$

- **Casio:** Click graph & table, and enter the function.



  - Analysis → G-Solve to find intercepts.
  - Use this button  to set the view window.



  - Use | to restrict the domain → find it in Math 3.

$$y1=x^3-2\cdot x^2 \mid 0<x<2$$

- **Mathematica:** Plot[function, $\{x, xmin, xmax\}$, PlotRange → $\{ymin, ymax\}$]
  - PlotRange is optional but can be used to make the scale appropriate for the question.

Section E: Exam 2 Questions (24 Marks)

INSTRUCTION:

- Regular: 24 Marks. ~~5~~ 3 Minutes Reading. ~~33~~ 30 Minutes Writing.
- Extension: 24 Marks. 5 Minutes Reading. 24 Minutes Writing.

Question 9 (1 mark)

Let $f : \mathbb{R} \to \mathbb{R}, f(x) = 1 - 2\cos\left(\frac{2\pi x}{3}\right)$.

The period and range of this function are respectively:

$\frac{2\pi}{n}$

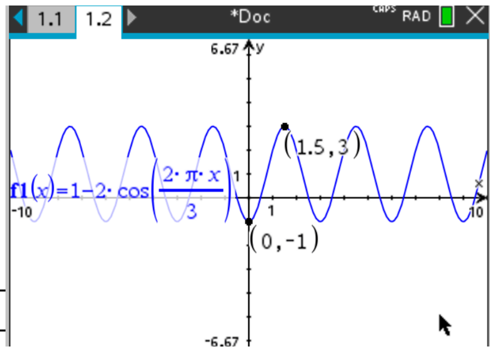
A. 3 and $[1, 3]$

B. 4 and $[-2, 2]$

C. 3 and $[-1, 3]$

D. 6π and $[-1, 1]$

$$\frac{2\cdot\pi}{\frac{2\cdot\pi}{3}} \quad 3$$



Question 10 (1 mark)

The solutions of the equation;

$$2\cos\left(2x - \frac{\pi}{4}\right) + 1 = 0$$

are:

A. $x = \frac{\pi(24k+11)}{24}$ or $x = \frac{\pi(24k-5)}{24}$, $k \in Z$

B. $x = \frac{\pi(12k+1)}{24}$ or $x = \frac{\pi(12k+5)}{24}$, $k \in Z$

C. $x = \frac{\pi(4k-3)}{24}$ or $x = \frac{\pi(4k+1)}{24}$, $k \in Z$

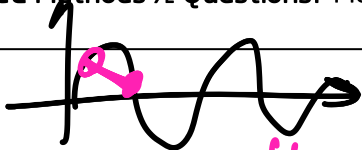
D. $x = \frac{\pi(4k+2)}{8}$, $k \in Z$

$$\text{solve}\left(2\cdot\cos\left(2\cdot x - \frac{\pi}{4}\right)+1=0, x\right)$$

$$x = \frac{(24\cdot n8-5)\cdot\pi}{24} \text{ or } x = \frac{(24\cdot n8+11)\cdot\pi}{24}$$

n n

Question 11 (1 mark)



Consider the function $f(x) = \sin(3x)$, where $x \in [0, \pi]$.

$y_1 = f(\frac{\pi}{6})$ $y_2 = f(\frac{5\pi}{6})$

The square of the length of the line segment joining the points on the graph for which $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$ is:

A. $\frac{25+\pi^2}{9}$

$\text{dist}^2 = \left(\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\right)^2$

B. $\frac{4\pi^2}{9}$

C. $\pi + 4$

D. 4

$\sqrt{\left(\frac{5\cdot\pi}{6}-\frac{\pi}{6}\right)^2-\left(f\left(\frac{5\cdot\pi}{6}\right)-f\left(\frac{\pi}{6}\right)\right)^2}$ $\quad \frac{2\cdot\pi}{3}$

$\left(\frac{2\cdot\pi}{3}\right)^2$ $\quad \frac{4\cdot\pi^2}{9}$

Question 12 (1 mark)

The sum of the solutions to the equation $\sin(2x) = \frac{1}{2}$ over the interval $[-\pi, d]$ is $\frac{\pi}{12}$.

The value of d could be:

A. $\frac{\pi}{3}$

B. $\frac{5\pi}{6}$

−ve +ve

C. $\frac{7\pi}{6}$

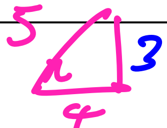
D. $\frac{15\pi}{4}$

$\text{solve}\left(\sin(2\cdot x)=\frac{1}{2},x\right)|-\pi\le x\le\frac{7\cdot\pi}{6}$

$x=\frac{-11\cdot\pi}{12}$ or $x=\frac{-7\cdot\pi}{12}$ or $x=\frac{\pi}{12}$ or $x=\frac{5\cdot\pi}{12}$ or

Space for Personal Notes

Question 13 (1 mark)



$\rightarrow \sin(x) = \frac{O}{H} = -\frac{3}{5}$

Let $\cos(x) = \frac{4}{5}$ and $\sin^2(y) = \frac{9}{25}$, where $x, y \in \left[\frac{3\pi}{2}, 2\pi\right]$. quad 4

The value of $\sin(x) + \cos^2(y)$ is:

$\sin^2(y) + \cos^2(y) = 1$

$\cos^2(y) = 1 - \sin^2(y)$

$= 1 - \frac{9}{25}$

$= \frac{16}{25}$

$-\frac{3}{5} + \frac{16}{25}$

A. $-\frac{23}{125}$

B. $-\frac{1}{5}$

C. $\frac{1}{25}$

D. $\frac{1}{5}$

Question 14 (1 mark)

quad 1/3 quad 1 quad 1

Given that $\tan(\alpha) = d$ where $d > 0$ and $0 < \alpha < \frac{\pi}{2}$, the sum of the solutions to $\tan(2x) = d$ where $0 < x < \frac{3\pi}{4}$, in terms of α, is:

Ref angle : α

angle

$2x = \alpha$

$x = \frac{\alpha}{2}$ $\frac{\alpha}{2} + \frac{\pi}{2}$

$\frac{\pi}{2}$

A. $\frac{\pi + \alpha}{2}$

B. 2α

C. $\frac{\pi}{2} + \alpha$

D. $\frac{3(\pi+\alpha)}{2}$

$\frac{\alpha}{2} + \left(\frac{\alpha}{2} + \frac{\pi}{2}\right)$

$= \alpha + \frac{\pi}{2}$

Question 15 (1 mark)

Consider the function $f : [-a\pi, a\pi] \to \mathbb{R}$, $f(x) = \sin(ax)$, where a is a positive integer.

The number of solutions to $f(x) = -1$ is always equal to:

$2a\pi$

↳ trial error.

A. a

B. $2a$

C. 4

D. a^2

period: $\frac{2\pi}{a}$

$\frac{\text{dist } 2a\pi}{\text{period } \frac{2\pi}{a}} = a^2$

Question 16 (9 marks)

a. During a particular day at the Contour office, the temperature inside the building between 10: : 00 AM and 7: 30 PM fluctuates so that t hours after 10: 00 AM, the temperature T°C is given by:

$$T = 20 + 5\sin\left(\frac{\pi t}{6}\right)$$

$f(x) = x^2$

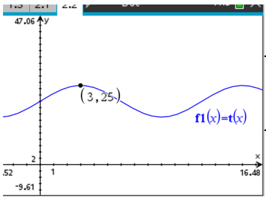
dom $\underline{f}$: $x \in R$

i. State the implied domain for T. (1 mark)

$t \in [0, 9.5]$

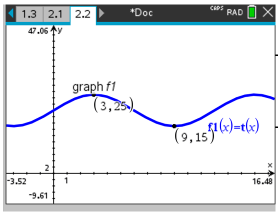
ii. State the maximum temperature and the time it occurs. (1 mark)

3 hrs after 10am



25°C at 1 pm

iii. State the minimum temperature and the time it occurs. (1 mark)



15°C at 7pm

b.

i. Calculate the temperature in the building at 11: 30 AM. Give your answer correct to 1 decimal place. (1 mark)

$t = 1.5$

$T(1.5) = 23.5\,°C$

ii. At what time does the temperature first reach 24°C? (2 marks) to the nearest min

solve$(t(x)=24, x) \mid 0 \le x \le 2$	$x=1.771$
0.7710034118052· 60	46.2602

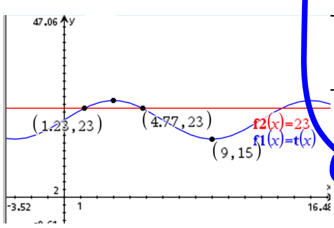
1 hr 46 min after

11:46 am

When the temperature reaches 23°C, an air conditioner in only the boardroom is switched on, and it is switched off when the temperature in the rest of the building falls below 23°C. Assume that the air conditioning in the board room does not affect the temperature in the rest of the building.

c. The air conditioner in the boardroom costs $5 per hour to run. Determine the cost of running the air conditioner in the boardroom during the day. Give your answer correct to the nearest cent. (3 marks)

cost $= 5 \times$ no. hours temp $\geq 23\,°C$



hours $= 4.77 - 1.23 = 3.54$

cost $= 5 \times 3.54$

$= \$17.71$

Space for Personal Notes

Question 17 (8 marks)

The height above ground level of a particular carriage on a Ferris wheel is given by:

$$h = 22 - 20\cos\left(\frac{\pi}{10}t\right)$$

where h is the height in metres above the ground and t is the time in minutes after boarding the ride.

a. Calculate how far above the ground the carriage is initially. (1 mark)

$2m$

b. Calculate how high the carriage will be after 5 minutes. (1 mark)

$h(t):=22-20\cdot\cos\left(\frac{\pi\cdot t}{10}\right)$	*Done*
$h(0)$	2
$h(5)$	22

$\rightarrow 22m$

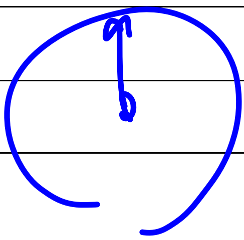
duration

c. Determine what fraction of a revolution the Ferris wheel will complete in a 10-minute time interval. (1 mark)

$\text{period} = \frac{2\pi}{n} = 2\pi \div \frac{\pi}{10} = 20$

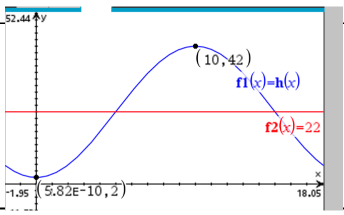
$\text{frac} = \frac{10\text{ min}}{20\text{ min}} = \frac{1}{2}$

d. The carriage is attached by strong steel radial spokes to the centre of the circular wheel. State the length of a radial spoke. (1 mark)

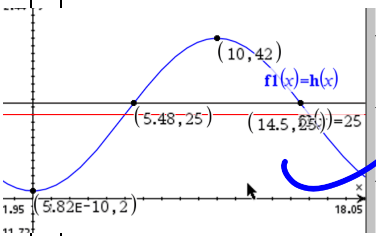


max − middle $= 42 - 22$

$= 20\text{ m}$



e. Calculate the total amount of time, to the nearest second, in one revolution, that the carriage is higher than 25 metres above the ground. (2 marks)



$t = 5.479,\ 14.521$

$\times 60$

duration $= 9.04145$ min

$= 9$ min 2 secs

f. The operator doubles the speed of the Ferris wheel. Write a new equation for the height h in terms of t. (2 marks)

$\frac{1}{2}$ period

$h(t) = 22 - 20\cos(nt)$

original : 20 min

new : 10 mins

$\frac{2\pi}{n} = 10$

$n = \frac{2\pi}{10} = \frac{\pi}{5}$

$h(t) = 22 - 20\cos\left(\frac{\pi}{5}t\right)$

Space for Personal Notes

Section F: Extension Exam 1 (10 Marks)

INSTRUCTION:

- Regular: Skip
- Extension: 10 Marks. 2 Minutes Reading. 15 Minutes Writing.

Question 18 (3 marks)

Solve the following equation,

$$5\cos\left(2x + \frac{\pi}{4}\right) - 2 = 1, \text{for } x \in [-\pi, \pi]$$

You will need to use the inverse cos function $\cos^{-1}$ in your answer.

Hint: $\sqrt{2} \approx 1.41$

Space for Personal Notes

Question 19 (2 marks)

Show that:

$$1 + \tan^2(x) = \frac{1}{\cos^2(x)}$$

Space for Personal Notes

Question 20 (5 marks)

Consider the function $f(x) = \sin\left(n^2 x - \frac{\pi}{2}\right)$.

a. Find the value(s) of n if $f(x)$ has a period of 3π. (2 marks)

b. Now only consider when $n > 0$. Find the value(s) of n such that $f(x) = 0$ has 6 solutions on the interval $[0, 2\pi]$. (3 marks)

Space for Personal Notes

Section G: Extension Exam 2 (15 Marks)

INSTRUCTION:

- Regular: Skip
- Extension: 15 Marks. 2 Minutes Reading. 16 Minutes Writing.

Question 21 (15 marks)

In a greenhouse, the temperature is carefully controlled to ensure optimal growth conditions for a rare species of orchid. The temperature T°C varies according to the rule:

$$T = 25 - 2\cos\left(\frac{\pi t}{12}\right)$$

where t is the time **in minutes** since the heating cycle began.

a. State the range of temperature in the greenhouse. (1 mark)

b. Determine the time after the cycle begins when the temperature reaches its maximum value. (1 mark)

c. How many complete heating cycles occur in one hour? (2 marks)

d. If the heating system runs continuously for 2 hours, what is the temperature at the end of this time? (2 marks)

e. Express the rule for the temperature in terms of a sine function. Write your answer in the form $a\sin(b(t-c))+d$. Where a, c, d are positive integers and b is a real number. (2 marks)

After observing fluctuations in external weather conditions, the greenhouse technician decides to adjust the temperature program. The goal is to start the heating cycle 2 minutes later than usual and make it run twice as fast to respond more quickly to rapid temperature changes.

f. Write a rule for the image of T under these changes. (2 marks)

g. Let $T_1(t) = 25 - 2\cos\left(\frac{\pi t}{12}\right)$ and define a new function $T_2(t) = T_1(6t - 12)$. State the transformations that map the graph of T_1 to the graph of T_2. (2 marks)

Weather conditions fluctuate again. The technician decides to change the heating cycle to following the rule:

$$T_3(t) = k - 2\cos\left(\frac{\pi t}{12}\right)$$

h. Find the value of k if, over one complete cycle, the temperature is above $26°$ for 60% of the time. Give your answer correct to three decimal places. (3 marks)

Space for Personal Notes

CONTOUREDUCATION

Website: contoureducation.com.au | **Phone:** 1800 888 300 | **Email:** hello@contoureducation.com.au

VCE Mathematical Methods ½

Free 1-on-1 Support

Be Sure to Make the Most of These (Free) Services!

- Experienced Contour tutors ($45+$ raw scores, $99+$ ATARs).
- For fully enrolled Contour students with up-to-date fees.
- After school weekdays and all-day weekends.

1-on-1 Video Consults	**Text-Based Support**
➤ Book via bit.ly/contour-methods-consult-2025 (or QR code below). ➤ One active booking at a time (must attend before booking the next).	➤ Message +61 440 138 726 with questions. ➤ Save the contact as "Contour Methods".

Booking Link for Consults
bit.ly/contour-methods-consult-2025

Number for Text-Based Support
+61 440 138 726