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VCE Mathematical Methods ½
Circular Function II [0.17]
Workshop Solutions

Error Logbook:

New Ideas / Concepts	Didn't Read Question
Pg / Q #: ____________________ Notes:	Pg / Q #: ____________________ Notes:
Algebraic / Arithmetic / Calculator Input Mistake	**Working Out Not Detailed Enough**
Pg / Q #: ____________________ Notes:	Pg / Q #: ____________________ Notes:

Section A: Recap

The Exact Values Table

x	$0\ (0^o)$	$\frac{\pi}{6}\ (30^o)$	$\frac{\pi}{4}\ (45^o)$	$\frac{\pi}{3}\ (60^o)$	$\frac{\pi}{2}\ (90^o)$
$\sin(x)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos(x)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan(x)$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined

Particular Solutions

- Solving trigonometric equations **for finite solutions**.
- **Steps:**
  1. Make the trigonometric function the subject.
  2. Find the necessary angle for one period.
  3. Solve for x by equating the necessary angles to the inside of the trigonometric functions.
  4. Add and subtract the period to find all other solutions in the domain.

General Solutions

- Solving infinite trigonometric equations.
- **Steps:**
  1. Make the trigonometric function the subject.
  2. Find the necessary angle for one period.
  3. Solve for x by equating the necessary angles to the inside of the trigonometric functions.
  4. Add $\text{period} \cdot n$ where $n \in Z$.

Multiple Forms of a General Solution

$$a + Period \cdot n = b + Period \cdot n$$

If the difference of a and b is a multiple of period.

General Solution with Domain Restriction

$$E.G \text{ trig}\left(2x + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \text{ for } x \geq 0$$

- We can have infinite solutions for a restricted domain.
- The value of n is also restricted.

Hidden Quadratics

$$af(x)^2 + bf(x) + c = 0$$

$$\text{Let } A = f(x)$$

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Section B: Warm Up (13 Marks)

INSTRUCTION:

- Regular: 13 Marks. 15 Minutes Writing.
- Extension: Skip

Question 1 (6 marks)

Solve the following trigonometric equations over the specified domain:

a. $\sin(x) = -\frac{1}{2}$ for $x \in [0,2\pi]$. (2 marks)

$$x = \frac{7\pi}{6}, \frac{11\pi}{6} \quad [\mathbf{1A\ each}]$$

b. $\cos(2x) = \frac{\sqrt{3}}{2}$ for $x \in [0,2\pi]$. (2 marks)

$$2x = \frac{\pi}{6}, \frac{11\pi}{6}. \quad [\mathbf{1M}]$$

$$x = \frac{\pi}{12}, \frac{11\pi}{12}.$$

The period is π. So all solutions are

$$x = \frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12} \quad [\mathbf{1A}]$$

c. $\tan(3x) = -1$ for $x \in [0, \pi]$. (2 marks)

$3x = \frac{3\pi}{4}$ **[1M]**

$x = \frac{\pi}{4}$

Period is $\frac{\pi}{3} = \frac{4\pi}{12}$, so all solutions are

$$x = \frac{\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12} \quad \textbf{[1A]}$$

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Question 2 (4 marks)

Solve the following trigonometric equations:

a. $\sin(2x) = \frac{1}{\sqrt{2}}$. (2 marks)

$$2x = \frac{\pi}{4} + 2n\pi, \frac{3\pi}{4} + 2n\pi \quad [\mathbf{1M}]$$

Thus

$$x = \frac{\pi}{8} + n\pi, \frac{3\pi}{8} + n\pi, n \in \mathbb{Z} \quad [\mathbf{1A}]$$

b. $\tan\left(x + \frac{\pi}{4}\right) = \sqrt{3}$, for $x \geq 0$. (2 marks)

$$x + \frac{\pi}{4} = \frac{\pi}{3} + n\pi \quad [\mathbf{1M}]$$

$$x = \frac{\pi}{12} + n\pi.$$

So now considering the domain

$$x = \frac{\pi}{12} + n\pi, n \in \mathbb{Z}^+ \cup \{0\} \quad [\mathbf{1A}]$$

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Question 3 (3 marks)

a. Solve the quadratic equation $a^2 - 4a + 3 = 0$. (1 mark)

$$\boxed{(a-3)(a-1)=0}$$

b. Hence, solve $\sin^2(x) - 4\sin(x) + 3 = 0$ for $x \in [0,4\pi]$. (2 marks)

Have $\sin(x) = 1$ or $\sin(x) = 3$. The second is not possible.
Thus $\sin(x) = 1 \implies x = \frac{\pi}{2}, \frac{5\pi}{2}$.

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Section C: Exam 1 Questions (20 Marks)

INSTRUCTION:

- Regular: 20 Marks. 5 Minutes Reading. 30 Minutes Writing.
- Extension: 20 Marks. 5 Minutes Reading. 20 Minutes Writing.

Question 4 (3 marks)

Solve the following equation:

$$4\sin\left(2x - \frac{\pi}{3}\right) + 1 = 3, \text{ for } x \in [-\pi, \pi]$$

We have

$$\begin{aligned}\sin\left(2x - \frac{\pi}{3}\right) &= \frac{1}{2}\\ 2x - \frac{\pi}{3} &= \frac{\pi}{6}, \frac{5\pi}{6} \quad [1M]\\ 2x &= \frac{\pi}{2}, \frac{7\pi}{6}\\ x &= \frac{\pi}{4}, \frac{7\pi}{12} \quad [1M]\end{aligned}$$

The period is π so we can add/subtract π to these answers to get all soluions in the domain.
$x = -\frac{3\pi}{4}, -\frac{5\pi}{12}, \frac{\pi}{4}, \frac{7\pi}{12}$. [1A]

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Question 5 (5 marks)

a. Find the general solution for the following equation: (3 marks)

$$\sin\left(2\left(x-\frac{\pi}{3}\right)\right)=\frac{1}{2}$$

We have

$$\sin\left(2\left(x-\frac{\pi}{3}\right)\right)=\frac{1}{2} \implies 2\left(x-\frac{\pi}{3}\right)=\frac{\pi}{6}+2n\pi,\ \frac{5\pi}{6}+2n\pi \quad [1M]$$

Solving:

$$\begin{aligned} x-\frac{\pi}{3}&=\frac{\pi}{12}+n\pi,\quad \frac{5\pi}{12}+n\pi \quad [1M]\\ x&=\frac{\pi}{12}+\frac{\pi}{3}+n\pi=\frac{5\pi}{12}+n\pi\\ x&=\frac{5\pi}{12}+n\pi,\quad \frac{9\pi}{12}+n\pi=\frac{3\pi}{4}+n\pi \end{aligned}$$

So the general solutions are:

$$x=\frac{5\pi}{12}+n\pi,\quad x=\frac{3\pi}{4}+n\pi, n\in\mathbb{Z} \quad [1A]$$

b. Hence, state all the solutions that lie between 0 and 2π. (2 marks)

We take values of $k=0$ and $k=1$ to find values in $[0,2\pi]$:

From $x=\frac{5\pi}{12}+k\pi$:

$$k=0 \implies x=\frac{5\pi}{12},\quad k=1 \implies x=\frac{5\pi}{12}+\pi=\frac{17\pi}{12}$$

From $x=\frac{3\pi}{4}+k\pi$:

$$k=0 \implies x=\frac{3\pi}{4},\quad k=1 \implies x=\frac{3\pi}{4}+\pi=\frac{7\pi}{4}$$

So the solutions in $[0,2\pi]$ are:

$$x=\frac{5\pi}{12},\ \frac{3\pi}{4},\ \frac{17\pi}{12},\ \frac{7\pi}{4}$$

[1M for two correct, 1A all four solutions]

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Question 6 (3 marks)

a. Find the general solutions to the equation: (2 marks)

$$\sin(x) = \cos(x)$$

Divide both sides by $\cos(x)$ (assuming $\cos(x) \neq 0$):

$$\sin(x) = \cos(x) \implies \tan(x) = 1 \implies x = \frac{\pi}{4} + n\pi, \quad n \in \mathbb{Z}$$

[1M for $\tan(x) = 1$, **1A**]

b. Hence, state the number of intersections between $\sin(x)$ and $\cos(x)$ where $x \in [0, 2\pi]$. (1 mark)

From the general solution $x = \frac{\pi}{4} + n\pi$, we check values of n that give $x \in [0,\ 2\pi]$:

$$n = 0 \implies x = \frac{\pi}{4}$$
$$n = 1 \implies x = \frac{5\pi}{4}$$

Only two such values lie in the interval, so the number of solutions is:

2 **[1A]**

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Question 7 (4 marks)

Solve the following for x:

$$3 - 4\sin^2(x) + 4\cos(x) = 2$$

Use the identity $\sin^2(x) = 1 - \cos^2(x)$:

$$\begin{aligned} 3 - 4(1 - \cos^2(x)) + 4\cos(x) &= 2 \\ 3 - 4 + 4\cos^2(x) + 4\cos(x) &= 2 \\ -1 + 4\cos^2(x) + 4\cos(x) &= 2 \\ 4\cos^2(x) + 4\cos(x) - 3 &= 0 \quad [\mathbf{1M}] \end{aligned}$$

Let $a = \cos(x)$.

$$\begin{aligned} 4a^2 + 4a - 3 &= 0 \\ (2a-1)(2a+3) &= 0 \\ a &= \frac{1}{2}, -\frac{3}{2} \quad [\mathbf{1M}] \end{aligned}$$

So $\cos(x) = \frac{1}{2}, -\frac{3}{2}$. But $\cos(x) = -\frac{3}{2}$ is not valid. [**1M**]

$$\cos(x) = \frac{1}{2} \implies x = \pm\frac{\pi}{3} + 2n\pi, \quad n \in \mathbb{Z}$$

$$\text{Final answer: } x = 2n\pi \pm \frac{\pi}{3}, \quad \text{where } n \in \mathbb{Z} \quad [\mathbf{1A}]$$

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Question 8 (5 marks)

a. Find the general solution, in terms of k, where $k \neq 0$ for: (2 marks)

$$\sin(kx) = 0$$

We solve:

$$\sin(kx) = 0 \implies kx = n\pi, \quad n \in \mathbb{Z} \quad [\mathbf{1M}]$$
$$\implies x = \frac{n\pi}{k}, \quad n \in \mathbb{Z} \text{ and } k \neq 0 \quad [\mathbf{1A}]$$

Let k be a positive integer.

b. Find the number of solutions (in terms of k) that exist for $x \in [-k\pi, k\pi]$. (3 marks)

There is always a solution at $x = 0$ and the period is $\frac{2\pi}{k}$ **[1M]** .

For $x \in [0, k\pi]$, there are $\frac{k^2}{2}$ complete periods $\left(\frac{2\pi}{k} \times \frac{k^2}{2} = k\pi\right)$.

Each period has two solutions, so for $x \in (0, k\pi]$ there are k^2 solutions. **[1M]**

By symmetry and including the solution at $x = 0$, we have a total of $2k^2 + 1$ solutions. **[1A]**

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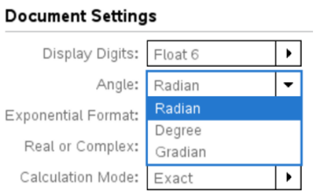
Section D: Tech Active Exam Skills

Calculator Commands: Degrees and Radians

TI

- Doc → 7 → 2



Casio

- Change at the bottom of the screen.

Mathematica

- In radians by default.
- Write "Degree."

In[27]:= **Sin[30 Degree]**

Out[27]= $\frac{1}{2}$

Calculator Commands: Solving trigonometric functions.

TI

- solve$(\text{trig}(..) = a, x)$ | domain restriction.
- | is under control equal.

Casio

- solve$(\text{trig}(..) = a, x)$ | domain restriction.
- | is under maths 3.

Mathematica

- Solve[trig[] $== a$ && domain restriction, x].

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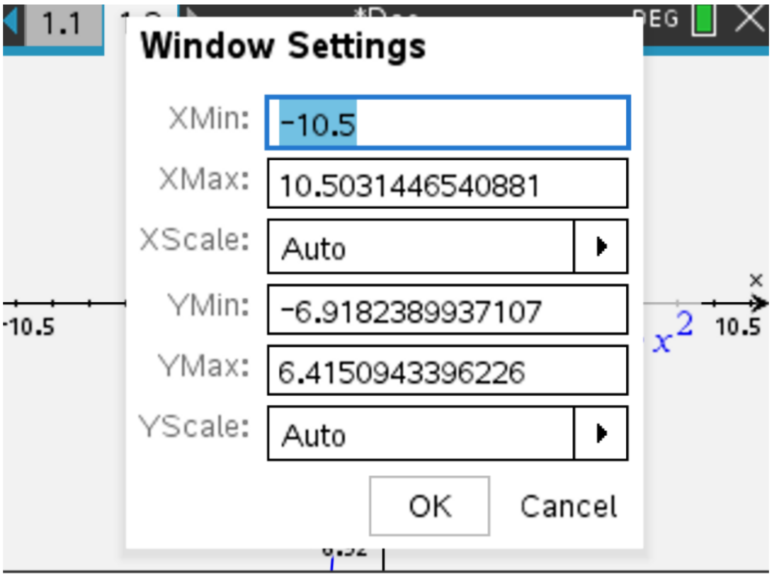
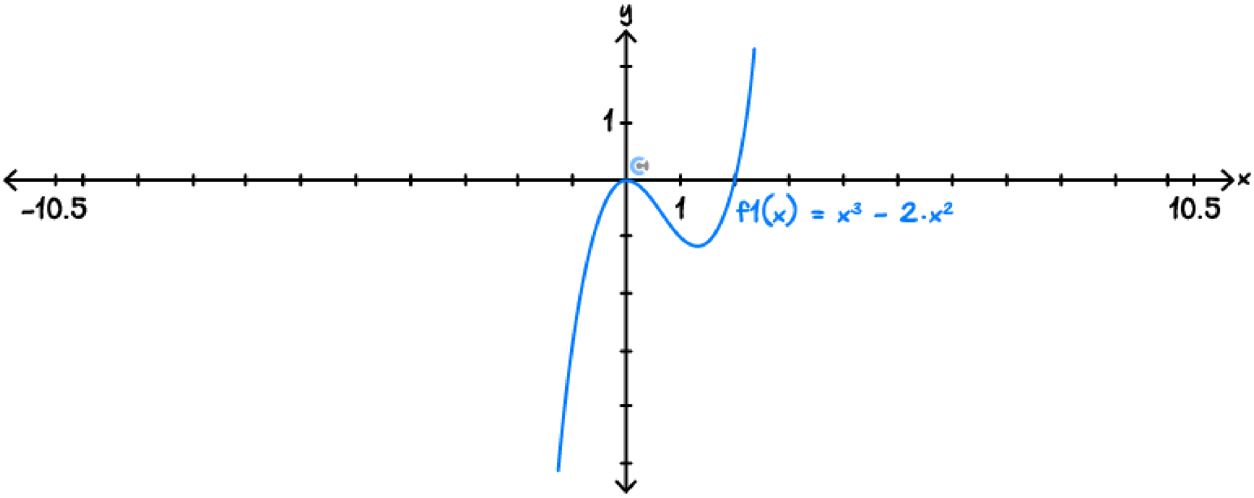
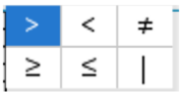
<u>Calculator Commands:</u> Graphing

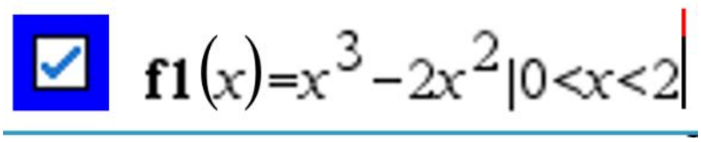
- Open a graph page and plot your function.
- Zoom settings: Menu → 4 (window / zoom) → 1 enter your x and y-ranges.



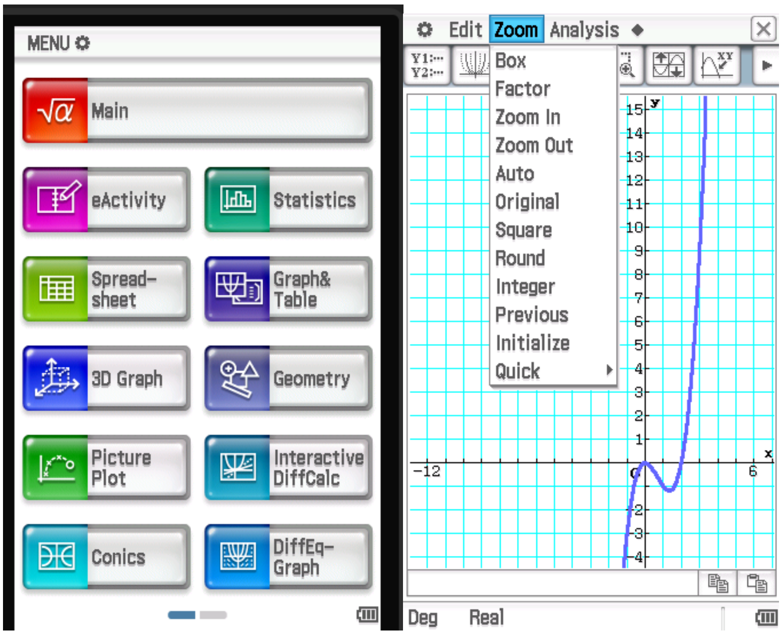
- Can also click the axis numbers on the graph and alter them directly.



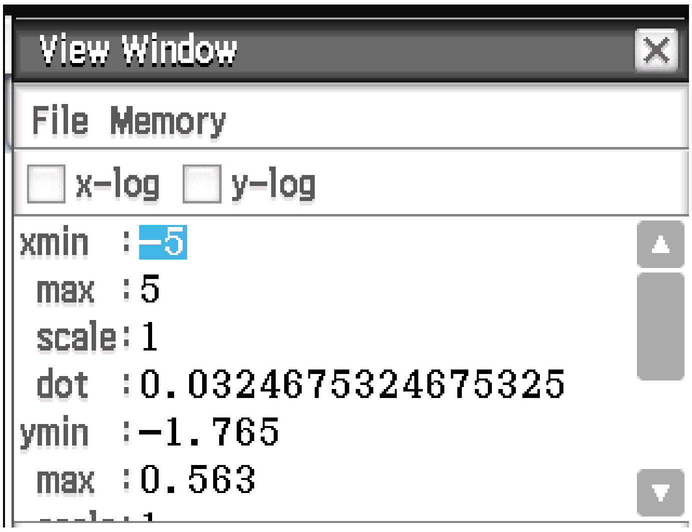
- Menu → 6 (Analyse) to find min / max x and y-intercepts.
- Restrict the domain to $0 < x < 2$, use the bar to get it from  ctrl+ =



- **Casio:** Click graph & table, and enter the function.



- Analysis → G-Solve to find intercepts.
- Use this button  to set the view window.



- Use | to restrict the domain → find it in Math 3.

- **Mathematica:** Plot[function, $\{x, xmin, xmax\}$, PlotRange → $\{ymin, ymax\}$]
  - PlotRange is optional but can be used to make the scale appropriate for the question.

Section E: Exam 2 Questions (26 Marks)

INSTRUCTION:

- Regular: 26 Marks. 5 Minutes Reading. 37 Minutes Writing.
- Extension: 26 Marks. 5 Minutes Reading. 26 Minutes Writing.

Question 9 (1 mark)

Consider $g(x) = \cos(x)$. How many x-axis intercepts exist for $x \in [-\pi, \pi]$?

A. 0

B. 1

C. 2

D. 3

$$\boxed{x = \pm\frac{\pi}{2}}$$

Question 10 (1 mark)

The graph of $y = 3\sin\left(x - \frac{\pi}{2}\right)$ is identical to:

A. $-3\cos(x)$

B. $3\sin(x)$

C. $3\cos(x)$

D. $3\sin(x)$

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Question 11 (1 mark)

Find the general solution for the equation:

$$2\cos\left(\frac{\pi}{4} - 2x\right) - 1 = 0$$

A. $\frac{(24n-1)\pi}{24}$, where $n \in \mathbb{Z}$

B. $\frac{(24n-1)\pi}{24}, \frac{(24n+7)\pi}{24}$, where $n \in \mathbb{Z}$

C. $\frac{(24n+1)\pi}{24}, \frac{(24n+7)\pi}{24}$, where $n \in \mathbb{Z}$

D. $\frac{(24n-1)\pi}{24}, \frac{(24n-7)\pi}{24}$, where $n \in \mathbb{Z}$

Question 12 (1 mark)

Which of the following equations is false?

A. $\sin\left(\frac{\pi}{4} + x\right) = \cos\left(\frac{\pi}{4} - x\right)$

B. $\tan(x) = \tan(\pi + x)$

C. $\cos\left(x - \frac{\pi}{2}\right) = -\sin(x)$

D. $\sin\left(\frac{\pi}{6} + x\right) = \cos\left(\frac{\pi}{3} + x\right)$

Question 13 (1 mark)

Which of the following is not the same as the rest, for $n \in Z$?

A. $6n + 1$

B. $6n - 5$

C. $6n - 1$

D. $6n - 11$

Question 14 (1 mark)

The expression $\sin^2(x) - \cos(x)$ can be written as:

A. $\cos^2(x) - \cos(x)$

B. $\cos^2(x) - \cos(x) + 1$

C. $-\cos^2(x) - \cos(x) + 1$

D. $\cos^2(x) - \cos(x) - 1$

Question 15 (1 mark)

Which of the following is an odd function? (An odd function has the property $f(-x) = -f(x)$).

A. $\sin\left(\frac{\pi}{2} - x\right)$

B. $\cos(x)$

C. $-\tan(x)$

D. $\tan(x) + 1$

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Question 16 (19 marks)

A vacuum cleaner has extremely fast rotating fan blades. To measure the speed of the blades, some engineering students kept track of a single point (A) on a blade, and plotted its motion. The following is the function modelling the point:

$$h = 4\sin(10\pi t)$$

where h is the height relative to the centre of the fan; a positive value means that the point is above the centre. t is measured in seconds.

a. What is the greatest relative height between the centre and the point A? (1 mark)

The maximum value of $\sin(\theta)$ is 1, so:

$$h = 4\sin(10\pi t) \implies \max(h) = 4$$

So the greatest relative height is 4 units. **[1A]**

b. How many revolutions per minute is completed by the fan? (2 marks)

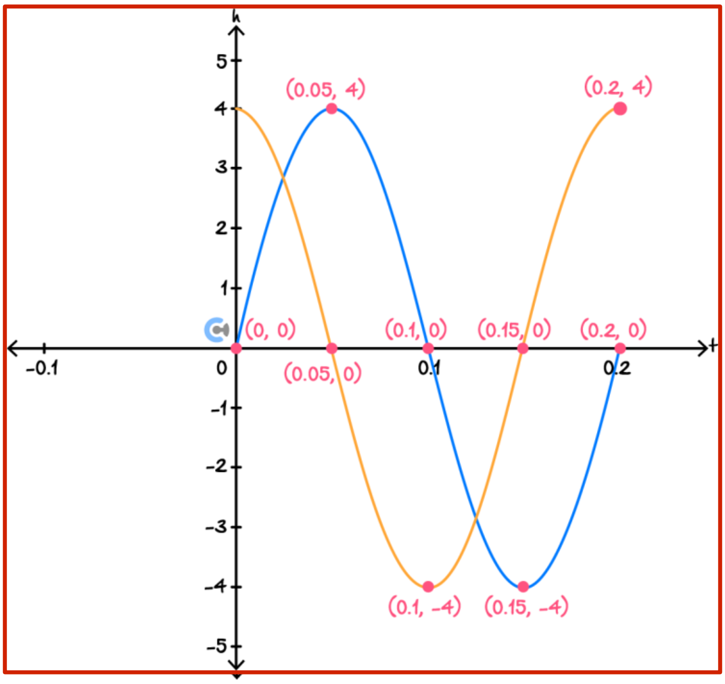
The period of the sine function is:

$$\text{Period} = \frac{2\pi}{10\pi} = \frac{1}{5} \text{ seconds per revolution} \quad \textbf{[1M]}$$

$$\text{Revolutions per minute} = \frac{60}{\frac{1}{5}} = 300 \quad \textbf{[1A]}$$

c. Use your calculator to help sketch the graph of h against t for the first 0.2 s, labelling all axes intercepts, and turning points. (3 marks)

1M shape, 1M turning points, 1M intercepts.



d. Another point (B, on the same blade) is chosen. The relative height between this point and the centre is at most 2 units.

i. Given that the equation for the relative height of point B is, $h_{new} = A\sin(10\pi t)$, find the value of A. (1 mark)

$h_{\text{new}} = 2\sin(10\pi t)$
So $A = 2$. **[1A]**

ii. Find the gradient of the line that connects the origin to the **first** local maximum of h, and h_{new}. (3 marks)

For h, the first max occurs at a quarter-period:

$$t = \frac{1}{20} \quad \textbf{[1M]}, \quad h = 4 \implies \text{gradient} = \frac{4-0}{1/20} = 80 \quad \textbf{[1A]}$$

For h_{new}, max $= 2$:

$$\text{gradient} = \frac{2-0}{1/20} = 40 \quad \textbf{[1A]}$$

iii. Given that a higher gradient corresponds to a higher vertical velocity, which fan point is rotating faster? (1 mark)

Point A has a greater gradient (80), so point A is rotating faster. **[1A]**

e. Measuring the time in seconds is impractical. Using transformations, convert t (in seconds) to t' (in centiseconds), where $1\ s = 100$ centiseconds (cs). (3 marks)

Let $t' = 100t \implies t = \frac{t'}{100}$ **[1M]**

Substitute into the original function:

$$h = 4\sin\left(10\pi \cdot \tfrac{t'}{100}\right) \quad \textbf{[1M]}$$

$$= 4\sin\left(\frac{\pi t'}{10}\right)$$

So:

$$h(t') = 4\sin\left(\frac{\pi t'}{10}\right) \quad \textbf{[1A]}$$

f. Another point is tracked, which is located on another blade. That blade's height can be modelled by the following function:

$$h_2 = 4\cos(10\pi t)$$

where t is the time in seconds.

i. On the set of axes above, sketch h_2, for $0 \leq t \leq 0.2$, label all axes intersections, and turning points. (2 marks)

1M shape, 1M intercepts and turning points

ii. Hence, state the time at which $h_2 = h, t \in [0, 0.2]$. (2 marks)

Solve $4\sin(10\pi t) = 4\cos(10\pi t) \implies \tan(10\pi t) = 1$

$$10\pi t = \frac{\pi}{4} + n\pi \implies t = \frac{1}{40} + \frac{n}{10}$$

For $t \in [0,\ 0.2]$, we get:

$$t = 0.025,\ 0.125 \quad \textbf{[1M]}$$

Then $h = 4\sin(10\pi t) = \pm 2\sqrt{2}$

So:

$$(0.025,\ 2\sqrt{2}),\quad (0.125,\ -2\sqrt{2}) \quad \textbf{[1A]}$$

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Section F: Extension Exam 1 (12 Marks)

INSTRUCTION:

- Regular: Skip
- Extension: 12 Marks. 15 Minutes Writing.

Question 17 (4 marks)

Let $f : \mathbb{R} \to \mathbb{R}, f(x) = \cos\left(\frac{2\pi x}{3}\right)$.

a. Solve the equation $f(x) = -\frac{1}{\sqrt{2}}$ lines for $x \in [0, 5]$. (2 marks)

$\frac{2\pi x}{3} = \frac{3\pi}{4}, \frac{5\pi}{4}$ **[1M]**

Thus $x = \frac{9}{8}, \frac{15}{8}$. Function has period $\frac{2\pi}{2\pi/3} = 3$.

So all solutions are

$$x = \frac{9}{8}, \frac{15}{8}, \frac{33}{8}, \frac{39}{8} \quad \textbf{[1A]}$$

b. Let $g : \mathbb{R} \to \mathbb{R}, g(x) = 5f(x-1) + 3$.

Find the smallest positive value of x for which $g(x)$ is a minimum. (2 marks)

f has minimum when $\frac{2\pi}{3}x = \pi \implies x = \frac{3}{2}$ **[1M]** .

Translation 1 unit to the right.

So g has a minimum when $x = \frac{5}{2}$. **[1A]**

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Question 18 (5 marks)

a. Show that $\frac{\sin(x) - \cos(x)}{\sin(x) + \cos(x)} = \frac{\tan(x) - 1}{\tan(x) + 1}$. (1 mark)

Divide everything in numerator and denominator by $\cos(x)$.

$$\frac{\frac{\sin(x)}{\cos(x)} - 1}{\frac{\sin(x)}{\cos(x)} + 1} = \frac{\tan(x) - 1}{\tan(x) + 1} \quad \textbf{[1A]}$$

b. Hence, solve the equation $\frac{\sin(x) - \cos(x)}{\sin(x) + \cos(x)} = \tan(x) - 1$ for $x \geq 0$. (4 marks)

Let $t = \tan(x)$ then we have

$$\frac{t-1}{t+1} = t - 1 \quad \textbf{[1M]}$$

$$(t-1) = (t-1)(t+1)$$

$$(t-1)(1-(t+1)) = 0$$

$$t(t-1) = 0$$

So $\tan(x) = 0$ or $\tan(x) = 1$ **[1M]**

Thus $x = n\pi$ or $x = \frac{\pi}{4} + n\pi$ for $n \in \mathbb{Z}^+ \cup \{0\}$. **[1A solutions, 1A correct restriction of n]**

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Question 19 (3 marks)

Let $f(x) = \tan(\pi x) - \sqrt{x}$.

Find how many solutions there are to $f(x) = 0$ for $x \in [0,10]$.

Consider $\tan(\pi x) = \sqrt{x}$.
The period of $\tan(\pi x)$ is 1. **[1M]**
There are 10 periods over the domain, which will give 10 solutions. **[1M]**
However on the interval $[0, 1]$ there are 2 solutions since $\tan(0) = \sqrt{0} = 0$.
So in total we have 11 solutions **[1A]** .

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Section G: Extension Exam 2 (12 Marks)

INSTRUCTION:

- Regular: Skip
- Extension: 12 Marks. 2 Minutes Reading. 16 Minutes Writing.

Question 20 (12 marks)

At a certain time of year, the depth of water, d metres, in the harbour at Azkaban is given by the rule:

$$d = 4 + 1.6\cos\left(\frac{\pi}{6}t\right)$$

where t is the time in hours after 4 AM.

a. At what time(s) does high tide occur for $t \in [0,24]$? (2 marks)

High tide occurs when $\cos\left(\frac{\pi}{6}t\right) = 1$ **[1M]** .

This occurs at $\frac{\pi}{6}t = 0, 2\pi, 4\pi, \cdots \Rightarrow t = 0, 12, 24, \ldots$.

Within $t \in [0, 24]$, the high tides occur at $t = 0, 12, 24$.

So high tide occurs at **4 a.m. and 4 p.m. and 4 a.m. the next day** **[1A]**

b. At what time(s) does low tide occur for $t \in [0,24]$? (2 marks)

Low tide occurs when $\cos\left(\frac{\pi}{6}t\right) = -1$. **[1M]**

This occurs when $\frac{\pi}{6}t = \pi, 3\pi, 5\pi, \cdots \Rightarrow t = 6, 18, \ldots$.

Within $[0, 24]$, low tide occurs at $t = 6$ and $t = 18$.

So low tide occurs at **10 a.m. and 10 p.m.** **[1A]**

A secure Ministry vessel transports prisoners from the mainland to Azkaban. The voyage from the Ministry port to Azkaban takes 40 minutes. The vessel only runs between the hours of 8 AM and 10 PM, and it can only enter and leave the Azkaban harbour if the depth of the water is at least 3 metres.

c. What is the earliest time the vessel can leave the Ministry port so that it arrives at Azkaban and can immediately enter the harbour? (3 marks)

Let arrival time be t, so depature is at $t - \frac{2}{3}$ hours.
We need $d(t) \geq 3$. **[1M]**

Solve $d(t) = 3$ to get $t = 4.289, 7.71, 16.289, 19.711$. **[1M]**
So could depart when $t = 3.62$ or $t = 7.04$
But vessel is not running when $t = 3.62$ (out of hours) and if it departs at say $t = 4$ then cannot immediately enter harbor.
So $t = 7.04$, which is 11 : 03 am. **[1A]**

d. The return voyage from Azkaban to the Ministry port also takes 40 minutes. The minimum time the vessel must spend docked at Azkaban is 10 minutes. The minimum time it must spend at the Ministry port is also 10 minutes.

i. What is the latest time the vessel can leave the Ministry port and return to the Ministry port in 90 minutes? (2 marks)

From the previous calculations the latest time we can leave from Azkaban is $t = 16.289$ **[1M]** .
Takes a minimum of 50 minutes $= 5/6$ hours from leaving ministry port to leaving Azkaban port.
Therefore leave Ministry port at $16.289 - 5/6 = 15.4561$, so $7:27$ pm. **[1A]**

ii. How many complete prisoner transfer trips could the vessel make in one day? (3 marks)

First trip can depart at $11:03$ am ($t = 7.04$) and last trip must depart before $7:27$ pm ($t = 15.456$). **[1M]**
Each round trip takes 100 minutes $= \frac{5}{3}$ hours. **[1M]**
So can complete $\left\lfloor \frac{15.456 - 7.04}{5/3} \right\rfloor + 1 = 5 + 1 = 6$ full trips. **[1A]**
(there were 5 trips completed in the interval $7.04 \leq t \leq 15.456$ and then the final trip.)

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