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VCE Mathematical Methods ½
Circular Function II [0.17]
Workshop

Error Logbook:

New Ideas / Concepts	Didn't Read Question
Pg / Q #: _____ Notes:	Pg / Q #: _____ Notes:
Algebraic / Arithmetic / Calculator Input Mistake	**Working Out Not Detailed Enough**
Pg / Q #: _____ Notes:	Pg / Q #: _____ Notes:

Section A: Recap

The Exact Values Table

x	$0\ (0^o)$	$\frac{\pi}{6}\ (30^o)$	$\frac{\pi}{4}\ (45^o)$	$\frac{\pi}{3}\ (60^o)$	$\frac{\pi}{2}\ (90^o)$
$\sin(x)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos(x)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan(x)$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined

Particular Solutions

- Solving trigonometric equations **for finite solutions**.
- **Steps:**
  1. Make the trigonometric function the subject.
  2. Find the necessary angle for one period.
  3. Solve for x by equating the necessary angles to the inside of the trigonometric functions.
  4. Add and subtract the period to find all other solutions in the domain.

General Solutions

- Solving infinite trigonometric equations.
- **Steps:**
  1. Make the trigonometric function the subject.
  2. Find the necessary angle for one period.
  3. Solve for x by equating the necessary angles to the inside of the trigonometric functions.
  4. Add $\text{period} \cdot n$ where $n \in Z$.

Multiple Forms of a General Solution

$$a + Period \cdot n = b + Period \cdot n$$

If the difference of a and b is a multiple of period.

General Solution with Domain Restriction

$$E.G \ \text{trig}\left(2x + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \text{ for } x \geq 0$$

- We can have infinite solutions for a restricted domain.
- The value of n is also restricted.

Hidden Quadratics

$$af(x)^2 + bf(x) + c = 0$$

Let $A = f(x)$

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Section B: Warm Up (13 Marks)

INSTRUCTION:

- Regular: 13 Marks. 15 Minutes Writing.
- Extension: Skip

Question 1 (6 marks)

Solve the following trigonometric equations over the specified domain:

a. $\sin(x) = -\frac{1}{2}$ for $x \in [0,2\pi]$. (2 marks)

b. $\cos(2x) = \frac{\sqrt{3}}{2}$ for $x \in [0,2\pi]$. (2 marks)

c. $\tan(3x) = -1$ for $x \in [0, \pi]$. (2 marks)

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Question 2 (4 marks)

Solve the following trigonometric equations:

a. $\sin(2x) = \frac{1}{\sqrt{2}}$. (2 marks)

b. $\tan\left(x + \frac{\pi}{4}\right) = \sqrt{3}$, for $x \geq 0$. (2 marks)

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Question 3 (3 marks)

a. Solve the quadratic equation $a^2 - 4a + 3 = 0$. (1 mark)

b. Hence, solve $\sin^2(x) - 4\sin(x) + 3 = 0$ for $x \in [0,4\pi]$. (2 marks)

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Section C: Exam 1 Questions (20 Marks)

INSTRUCTION:

- Regular: 20 Marks. 5 Minutes Reading. 30 Minutes Writing.
- Extension: 20 Marks. 5 Minutes Reading. 20 Minutes Writing.

Question 4 (3 marks)

Solve the following equation:

$$4\sin\left(2x-\frac{\pi}{3}\right)+1=3, \text{ for } x\in[-\pi,\pi]$$

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Question 5 (5 marks)

a. Find the general solution for the following equation: (3 marks)

$$\sin\left(2\left(x-\frac{\pi}{3}\right)\right)=\frac{1}{2}$$

b. Hence, state all the solutions that lie between 0 and 2π. (2 marks)

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Question 6 (3 marks)

a. Find the general solutions to the equation: (2 marks)

$$\sin(x) = \cos(x)$$

b. Hence, state the number of intersections between $\sin(x)$ and $\cos(x)$ where $x \in [0, 2\pi]$. (1 mark)

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Question 7 (4 marks)

Solve the following for x:

$$3 - 4\sin^2(x) + 4\cos(x) = 2$$

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Question 8 (5 marks)

a. Find the general solution, in terms of k, where $k \neq 0$ for: (2 marks)

$$\sin(kx) = 0$$

Let k be a positive integer.

b. Find the number of solutions (in terms of k) that exist for $x \in [-k\pi, k\pi]$. (3 marks)

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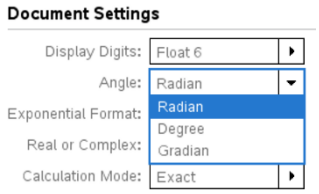
Section D: Tech Active Exam Skills

Calculator Commands: Degrees and Radians

TI

- Doc → 7 → 2



Casio

- Change at the bottom of the screen.

Mathematica

- In radians by default.
- Write "Degree."

In[27]:= `Sin[30 Degree]`

Out[27]= $\frac{1}{2}$

Calculator Commands: Solving trigonometric functions.

TI

- solve(trig(..) $= a, x$) | domain restriction.
- | is under control equal.

Casio

- solve(trig(..) $= a, x$) | domain restriction.
- | is under maths 3.

Mathematica

- Solve[trig[] == a && domain restriction, x].

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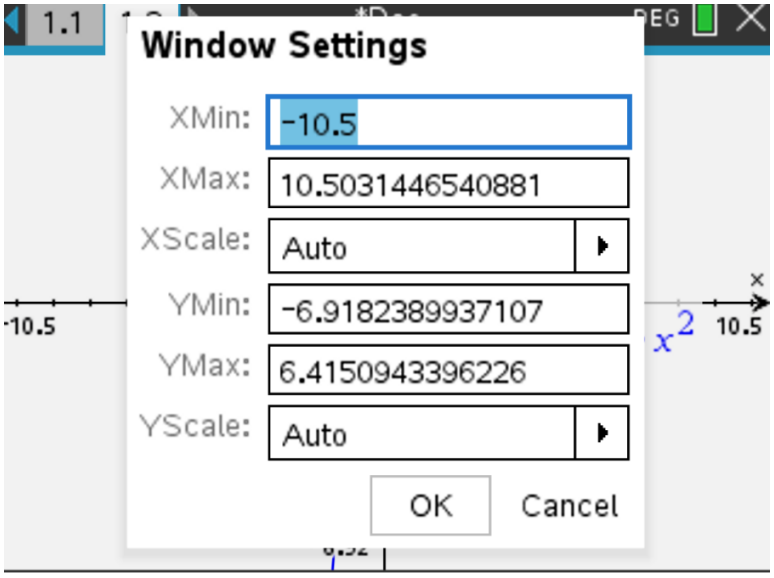
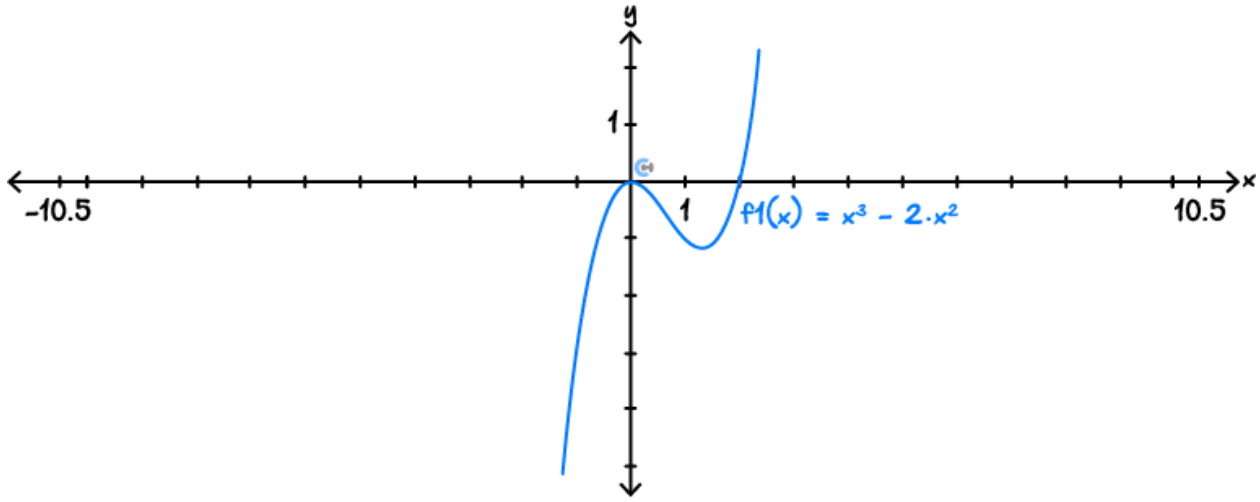
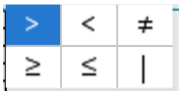
Calculator Commands: Graphing

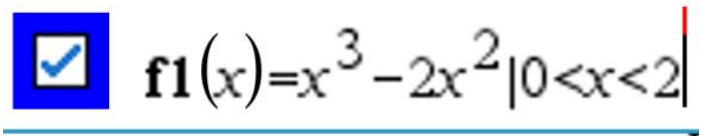
- Open a graph page and plot your function.
- Zoom settings: Menu → 4 (window / zoom) → 1 enter your x and y-ranges.



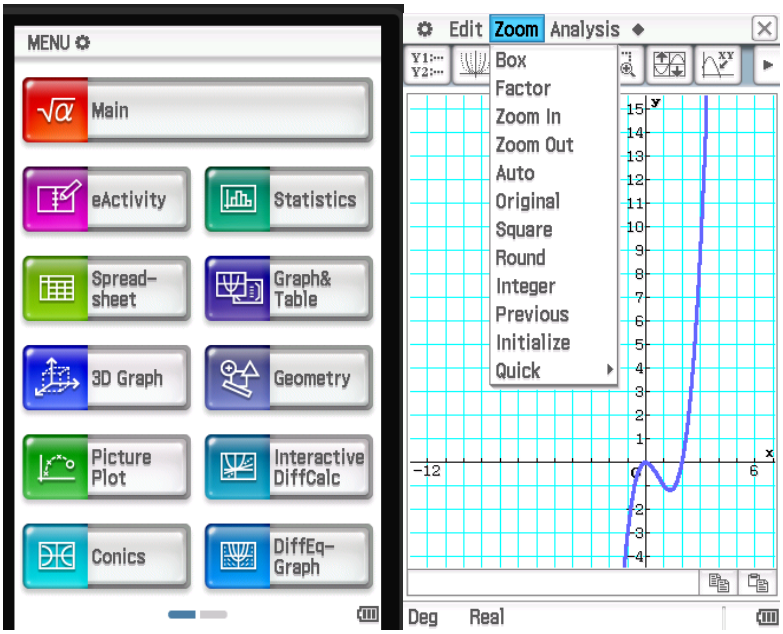
- Can also click the axis numbers on the graph and alter them directly.



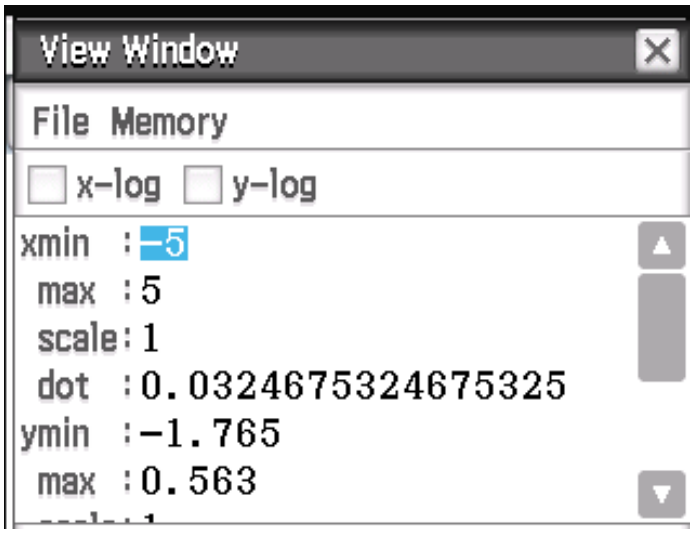
- Menu → 6 (Analyse) to find min / max x and y-intercepts.
- Restrict the domain to $0 < x < 2$, use the bar to get it from  ctrl+ =



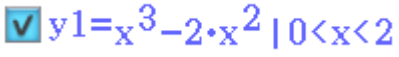
- **Casio:** Click graph & table, and enter the function.



- Analysis → G-Solve to find intercepts.
- Use this button  to set the view window.



- Use | to restrict the domain → find it in Math 3.



- **Mathematica:** Plot[function, $\{x, xmin, xmax\}$, PlotRange → $\{ymin, ymax\}$]
  - PlotRange is optional but can be used to make the scale appropriate for the question.

Section E: Exam 2 Questions (26 Marks)

INSTRUCTION:

- Regular: 26 Marks. 5 Minutes Reading. 37 Minutes Writing.
- Extension: 26 Marks. 5 Minutes Reading. 26 Minutes Writing.

Question 9 (1 mark)

Consider $g(x) = \cos(x)$. How many x-axis intercepts exist for $x \in [-\pi, \pi]$?

A. 0

B. 1

C. 2

D. 3

Question 10 (1 mark)

The graph of $y = 3\sin\left(x - \frac{\pi}{2}\right)$ is identical to:

A. $-3\cos(x)$

B. $3\sin(x)$

C. $3\cos(x)$

D. $3\sin(x)$

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Question 11 (1 mark)

Find the general solution for the equation:

$$2\cos\left(\frac{\pi}{4} - 2x\right) - 1 = 0$$

A. $\frac{(24n-1)\pi}{24}$, where $n \in \mathbb{Z}$

B. $\frac{(24n-1)\pi}{24}, \frac{(24n+7)\pi}{24}$, where $n \in \mathbb{Z}$

C. $\frac{(24n+1)\pi}{24}, \frac{(24n+7)\pi}{24}$, where $n \in \mathbb{Z}$

D. $\frac{(24n-1)\pi}{24}, \frac{(24n-7)\pi}{24}$, where $n \in \mathbb{Z}$

Question 12 (1 mark)

Which of the following equations is false?

A. $\sin\left(\frac{\pi}{4} + x\right) = \cos\left(\frac{\pi}{4} - x\right)$

B. $\tan(x) = \tan(\pi + x)$

C. $\cos\left(x - \frac{\pi}{2}\right) = -\sin(x)$

D. $\sin\left(\frac{\pi}{6} + x\right) = \cos\left(\frac{\pi}{3} + x\right)$

Question 13 (1 mark)

Which of the following is not the same as the rest, for $n \in Z$?

A. $6n + 1$

B. $6n - 5$

C. $6n - 1$

D. $6n - 11$

Question 14 (1 mark)

The expression $\sin^2(x) - \cos(x)$ can be written as:

A. $\cos^2(x) - \cos(x)$

B. $\cos^2(x) - \cos(x) + 1$

C. $-\cos^2(x) - \cos(x) + 1$

D. $\cos^2(x) - \cos(x) - 1$

Question 15 (1 mark)

Which of the following is an odd function? (An odd function has the property $f(-x) = -f(x)$).

A. $\sin\left(\frac{\pi}{2} - x\right)$

B. $\cos(x)$

C. $-\tan(x)$

D. $\tan(x) + 1$

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Question 16 (19 marks)

A vacuum cleaner has extremely fast rotating fan blades. To measure the speed of the blades, some engineering students kept track of a single point (A) on a blade, and plotted its motion. The following is the function modelling the point:

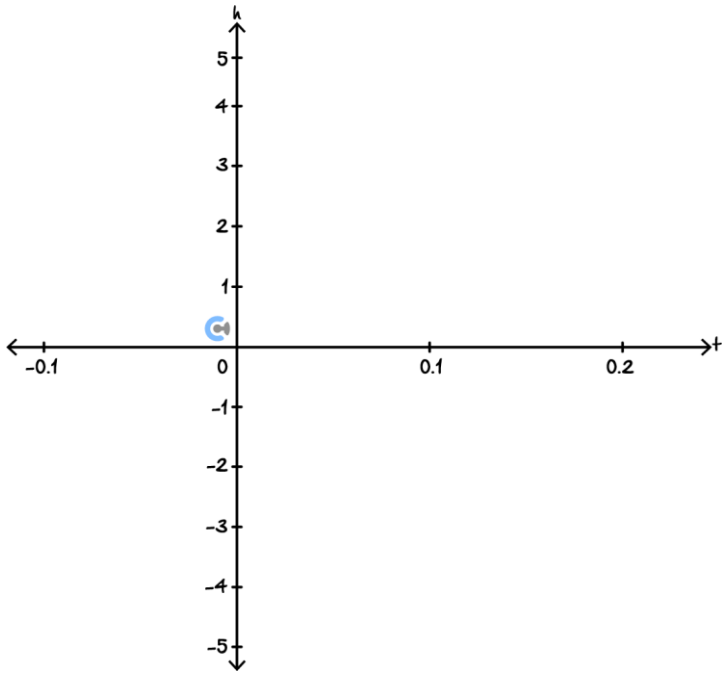
$$h = 4\sin(10\pi t)$$

where h is the height relative to the centre of the fan; a positive value means that the point is above the centre. t is measured in seconds.

a. What is the greatest relative height between the centre and the point A? (1 mark)

b. How many revolutions per minute is completed by the fan? (2 marks)

c. Use your calculator to help sketch the graph of h against t for the first $0.2\ s$, labelling all axes intercepts, and turning points. (3 marks)



d. Another point (B, on the same blade) is chosen. The relative height between this point and the centre is at most 2 units.

i. Given that the equation for the relative height of point B is, $h_{new} = A\sin(10\pi t)$, find the value of A. (1 mark)

ii. Find the gradient of the line that connects the origin to the **first** local maximum of h, and h_{new}. (3 marks)

iii. Given that a higher gradient corresponds to a higher vertical velocity, which fan point is rotating faster? (1 mark)

e. Measuring the time in seconds is impractical. Using transformations, convert t (in seconds) to t' (in centiseconds), where $1\ s\ =\ 100$ centiseconds (cs). (3 marks)

f. Another point is tracked, which is located on another blade. That blade's height can be modelled by the following function:

$$h_2\ =\ 4\cos(10\pi t)$$

where t is the time in seconds.

i. On the set of axes above, sketch h_2, for $0\ \leq\ t\ \leq\ 0.2$, label all axes intersections, and turning points. (2 marks)

ii. Hence, state the time at which $h_2\ =\ h, t\ \in\ [0, 0.2]$. (2 marks)

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Section F: Extension Exam 1 (12 Marks)

INSTRUCTION:

- Regular: Skip
- Extension: 12 Marks. 15 Minutes Writing.

Question 17 (4 marks)

Let $f: \mathbb{R} \to \mathbb{R}, f(x) = \cos\left(\frac{2\pi x}{3}\right)$.

a. Solve the equation $f(x) = -\frac{1}{\sqrt{2}}$ lines for $x \in [0, 5]$. (2 marks)

b. Let $g: \mathbb{R} \to \mathbb{R}, g(x) = 5f(x-1) + 3$.

Find the smallest positive value of x for which $g(x)$ is a minimum. (2 marks)

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Question 18 (5 marks)

a. Show that $\frac{\sin(x) - \cos(x)}{\sin(x) + \cos(x)} = \frac{\tan(x) - 1}{\tan(x) + 1}$. (1 mark)

b. Hence, solve the equation $\frac{\sin(x) - \cos(x)}{\sin(x) + \cos(x)} = \tan(x) - 1$ for $x \geq 0$. (4 marks)

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Question 19 (3 marks)

Let $f(x) = \tan(\pi x) - \sqrt{x}$.

Find how many solutions there are to $f(x) = 0$ for $x \in [0,10]$.

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Section G: Extension Exam 2 (12 Marks)

INSTRUCTION:

- Regular: Skip
- Extension: 12 Marks. 2 Minutes Reading. 16 Minutes Writing.

Question 20 (12 marks)

At a certain time of year, the depth of water, d metres, in the harbour at Azkaban is given by the rule:

$$d = 4 + 1.6\cos\left(\frac{\pi}{6}t\right)$$

where t is the time in hours after 4 AM.

a. At what time(s) does high tide occur for $t \in [0,24]$? (2 marks)

b. At what time(s) does low tide occur for $t \in [0,24]$? (2 marks)

A secure Ministry vessel transports prisoners from the mainland to Azkaban. The voyage from the Ministry port to Azkaban takes 40 minutes. The vessel only runs between the hours of 8 AM and 10 PM, and it can only enter and leave the Azkaban harbour if the depth of the water is at least 3 metres.

c. What is the earliest time the vessel can leave the Ministry port so that it arrives at Azkaban and can immediately enter the harbour? (3 marks)

d. The return voyage from Azkaban to the Ministry port also takes 40 minutes. The minimum time the vessel must spend docked at Azkaban is 10 minutes. The minimum time it must spend at the Ministry port is also 10 minutes.

i. What is the latest time the vessel can leave the Ministry port and return to the Ministry port in 90 minutes? (2 marks)

_____

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ii. How many complete prisoner transfer trips could the vessel make in one day? (3 marks)

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