



Website: contoureducation.com.au | Phone: 1800 888 300

Email: hello@contoureducation.com.au

VCE Mathematical Methods ½
Circular Function I [0.16]
Workshop

Error Logbook:



New Ideas/Concepts	Didn't Read Question
Pg / Q #: _____ Notes:	Pg / Q #: _____ Notes:
Algebraic/Arithmetic/ Calculator Input Mistake	Working Out Not Detailed Enough
Pg / Q #: _____ Notes:	Pg / Q #: _____ Notes:

Section A: Recap

Radians and Degrees



$$1^c = \left(\frac{180}{\pi}\right)^o$$

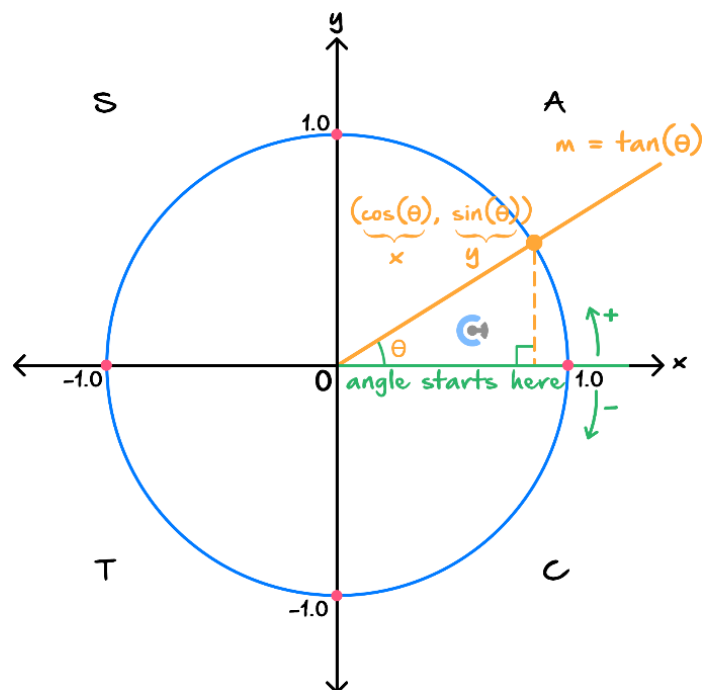
$$1^o = \left(\frac{\pi}{180}\right)^c$$

$$180^o = \pi^c$$

Unit Circle



➤ The unit circle is simply a circle of radius 1.



$$\sin(\theta) = y$$

$$\cos(\theta) = x$$

$$\tan(\theta) = \text{gradient}$$



Period of a Trigonometric Function

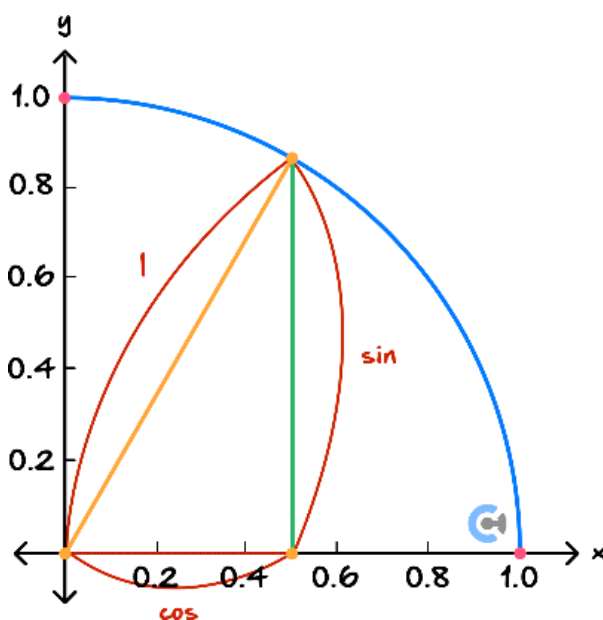
period of $\sin(nx)$ and $\cos(nx)$ functions $= \frac{2\pi}{n}$

period of $\tan(nx)$ functions $= \frac{\pi}{n}$

where $n = \text{coefficient of } x \text{ and } n > 0$



Pythagorean Identities



$$\sin^2(\theta) + \cos^2(\theta) = 1$$

► Can be used for finding one trigonometry function by using the other.

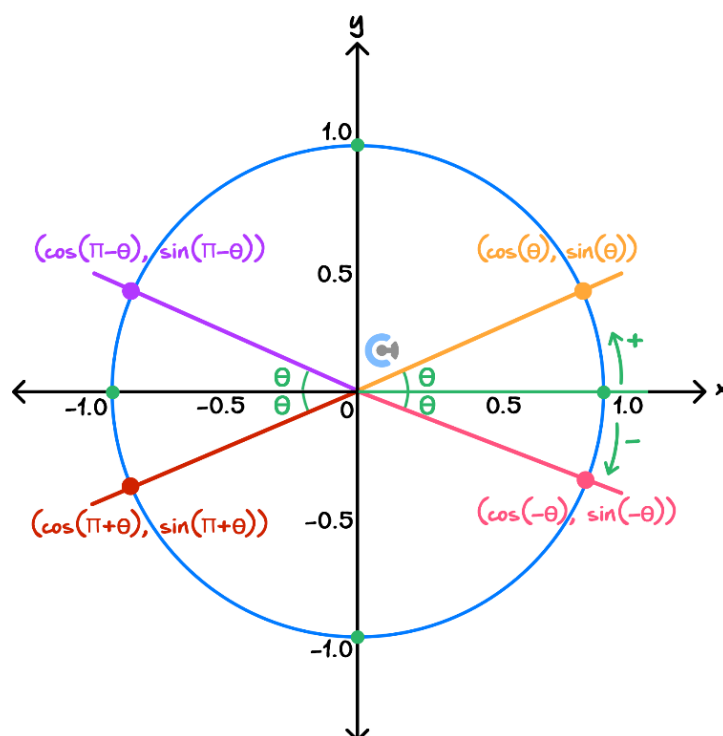
Space for Personal Notes



The Exact Values Table

x	0 (0°)	$\frac{\pi}{6}$ (30°)	$\frac{\pi}{4}$ (45°)	$\frac{\pi}{3}$ (60°)	$\frac{\pi}{2}$ (90°)
$\sin(x)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos(x)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan(x)$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined

Supplementary Relationships



➤ Simply look at the quadrant to find the correct sign.

🌀 Second Quadrant ($\pi - \theta$)

$$\cos(\pi - \theta) = -\cos(\theta)$$

$$\sin(\pi - \theta) = +\sin(\theta)$$

$$\tan(\pi - \theta) = -\tan(\theta)$$

 Third Quadrant ($\pi + \theta$)

$$\cos(\pi + \theta) = -\cos(\theta)$$

$$\sin(\pi + \theta) = -\sin(\theta)$$

$$\tan(\pi + \theta) = +\tan(\theta)$$

 Fourth Quadrant ($-\theta$)

$$\cos(-\theta) = +\cos(\theta)$$

$$\sin(-\theta) = -\sin(\theta)$$

$$\tan(-\theta) = -\tan(\theta)$$

Complementary Relationships

➤ Consider the quadrant for signs.

 First Quadrant ($\frac{\pi}{2} - \theta$)

$$\cos\left(\frac{\pi}{2} - \theta\right) = +\sin(\theta)$$

$$\sin\left(\frac{\pi}{2} - \theta\right) = +\cos(\theta)$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = +\frac{1}{\tan(\theta)}$$

 Second Quadrant ($\frac{\pi}{2} + \theta$)

$$\sin\left(\frac{\pi}{2} + \theta\right) = +\cos(\theta)$$

$$\cos\left(\frac{\pi}{2} + \theta\right) = -\sin(\theta)$$



$$\tan\left(\frac{\pi}{2} + \theta\right) = -\frac{1}{\tan(\theta)}$$

Third Quadrant $\left(\frac{3\pi}{2} - \theta\right)$

$$\sin\left(\frac{3\pi}{2} - \theta\right) = -\cos(\theta)$$

$$\cos\left(\frac{3\pi}{2} - \theta\right) = -\sin(\theta)$$

$$\tan\left(\frac{3\pi}{2} - \theta\right) = \frac{1}{\tan(\theta)}$$

Fourth Quadrant $\left(\frac{3\pi}{2} + \theta\right)$

$$\sin\left(\frac{3\pi}{2} + \theta\right) = -\cos(\theta)$$

$$\cos\left(\frac{3\pi}{2} + \theta\right) = +\sin(\theta)$$

$$\tan\left(\frac{3\pi}{2} + \theta\right) = -\frac{1}{\tan(\theta)}$$

► Steps:

1. Note complementary relationship by identifying a vertical angle $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$.
2. Equate to the opposite trigonometric function $\cos / \sin / \frac{1}{\tan(\theta)}$.
3. Determine the sign $(\pm)$ by considering the quadrant.

Space for Personal Notes



Supplementary v/s Complementary

Supplementary: $\text{trig}(\text{Horizontal Angle} \pm \theta)$

Complementary: $\text{trig}(\text{Vertical Angle} \pm \theta)$



Particular Solutions

- Solving trigonometric equations **for finite solutions.**
- **Steps:**
 1. Make the trigonometric function the subject.
 2. Find the necessary angle for one period.
 3. Solve for x by equating the necessary angles to the inside of the trigonometric functions.
 4. Add and subtract the period to find all other solutions in the domain.

Space for Personal Notes

Section B: Warm Up (13 Marks)

INSTRUCTION:

➤ **Regular: 13 Marks. 13 Minutes Writing.**

➤ **Extension: Skip.**



Question 1 (7 marks)

a. Find $\left(\frac{3\pi}{4}\right)^c$ in degrees. (1 mark)

135°

b. Find 150° in radians. (1 mark)

$5\pi/6$

c. Determine the period of $\sin(3x)$. (1 mark)

$2\pi/3$

d. Determine the period of $\tan\left(\frac{\pi x}{3}\right)$. (1 mark)

3

e. Evaluate $\sin\left(\frac{3\pi}{2}\right)$. (1 mark)

-1

f. Evaluate $\cos\left(-\frac{\pi}{6}\right)$. (1 mark)

$\sqrt{3}/2$

g. Evaluate $\tan\left(\frac{7\pi}{4}\right)$. (1 mark)

-1

Space for Personal Notes

Question 2 (6 marks)

Given that $\sin(x) = \frac{5}{13}$ and $\frac{\pi}{2} < x < \pi$, find:

a. $\cos(x)$. (2 marks)

-12/13

b. $\tan(x)$. (1 mark)

-5/12

c. $\cos\left(x + \frac{\pi}{2}\right)$. (1 mark)

-5/13

d. $\sin(2\pi - x)$. (1 mark)

-5/13

e. $\tan\left(\frac{\pi}{2} + x\right)$. (1 mark)

12/5

Space for Personal Notes

Section C: Exam 1 Questions (17 Marks)

INSTRUCTION:

- Regular: 17 Marks. 5 Minutes Reading. 25 Minutes Writing.
- Extension: 17 Marks. 5 Minutes Reading. 17 Minutes Writing.



Question 3 (5 marks)

- a. What value(s) can $\cos(x)$ take given that $\sin(x) = \frac{3}{5}$? (3 marks)

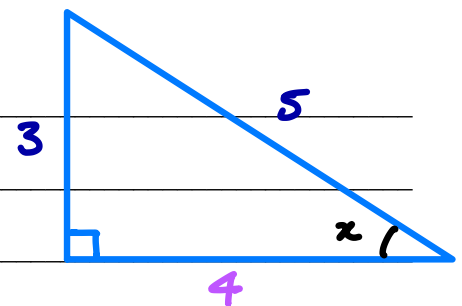
$$\sin^2(x) + \cos^2(x) = 1$$

$$\therefore \cos^2(x) = 1 - \left(\frac{3}{5}\right)^2$$

$$= 1 - \frac{9}{25}$$

$$= \frac{16}{25} \Rightarrow \cos(x) = +\frac{4}{5} \text{ if } x \text{ is in Q1}$$

$$\cos(x) = -\frac{4}{5} \text{ if } x \text{ is in Q2}$$



- b. Hence, find the possible value(s) of $\tan(x)$. (2 marks)

$$\text{If } x \text{ is in Q1: } \tan(x) = \frac{\left(\frac{3}{5}\right)}{\left(\frac{4}{5}\right)} = \frac{3}{4} "$$

$$\text{If } x \text{ is in Q2: } \tan(x) = \frac{\left(\frac{3}{5}\right)}{\left(-\frac{4}{5}\right)} = -\frac{3}{4} "$$

Space for Personal Notes

Question 4 (2 marks)

Given that the period of the function $\tan(n^2x)$ is 2. Find the value(s) of n .

$$P = \frac{\pi}{n^2} = 2$$

$$\therefore n^2 = \frac{\pi}{2} \Rightarrow n = \pm \sqrt{\frac{\pi}{2}} = \pm \frac{\sqrt{2\pi}}{2} //$$

Space for Personal Notes

Question 5 (7 marks)

Given that $\sin(\alpha) = m$, and $\cos(\beta) = 0.2$.

- a. Find the value of $\cos\left(\frac{\pi}{2} - \alpha\right)$. (2 marks)

$$= \sin(\alpha)$$

$$= m //$$

- b. Find the value of $\sin\left(\frac{3\pi}{2} - \beta\right)$. (2 marks)

Q3 $\Rightarrow$ $\ominus$

Comp

$$= -\cos(\beta)$$

$$= -0.2 //$$

- c. Find the value(s) of $\tan(\alpha)$. (3 marks)

$$\sin^2(\alpha) + \cos^2(\alpha) = 1$$

$$\cos^2(\alpha) = 1 - m^2$$

$$\cos(\alpha) = \pm \sqrt{1 - m^2}$$

$$\therefore \tan(\alpha) = \frac{m}{\sqrt{1 - m^2}} \text{ if } \alpha \text{ is in Q1 or Q3,}$$

or

$$\tan(\alpha) = \frac{-m}{\sqrt{1 - m^2}} \text{ if } \alpha \text{ is in Q2 or Q4,}$$

Space for Personal Notes

Question 6 (3 marks)

Consider the functions:

$$f(x) = \sin(nx) \quad \text{and} \quad g(x) = \cos(nx)$$

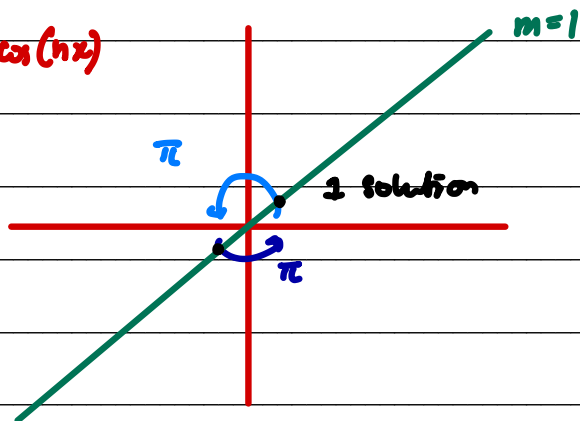
For what integer value of n will $f(x) = g(x)$ have exactly 6 solutions for, $x \in [0, 2\pi]$? Justify your answer.

$$\begin{aligned} \therefore \sin(nx) &= \cos(nx) \\ \div \cos(nx) & \quad \div \cos(nx) \\ \tan(nx) &= 1 \end{aligned}$$

$$\therefore p = \frac{\pi}{n}$$

every period there will be
1 solution.

6 solutions $\Rightarrow$ 6 periods



Space for Personal Notes

$$\frac{\text{Interval}}{6} = \frac{2\pi}{6} = \frac{\pi}{3} = \frac{\pi}{n} \Rightarrow \therefore n = 3 \text{ or } -3$$

Section D: Tech Active Exam Skills

Calculator Commands: Solving Trigonometric Functions



TI

solve(trig(..) = a, x) |
domain restriction

| is under control equal.

Casio

solve(trig(..) = a, x) |
domain restriction

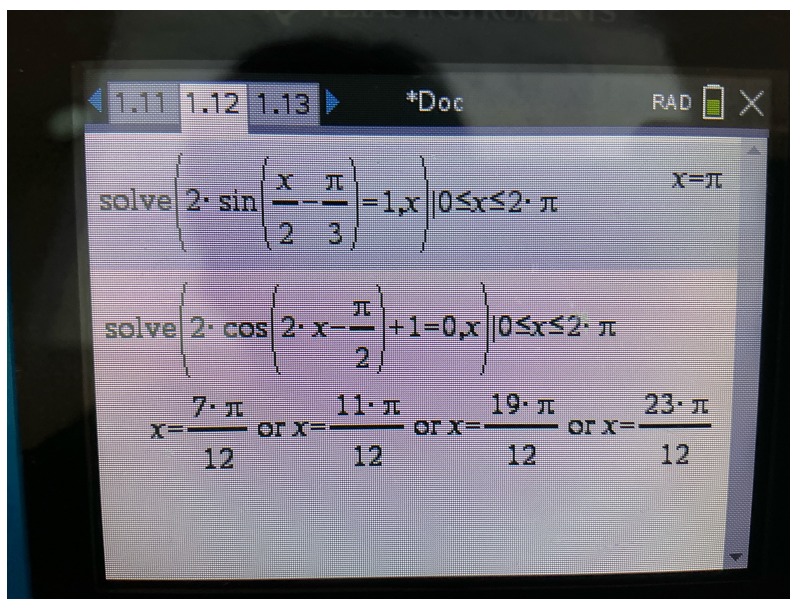
| is under maths 3.

Mathematica

Solve[trig[] == a &&
domain restriction, x]

Space for Personal Notes

$$2\cos\left(2x - \frac{\pi}{2}\right) + 1 = 0 \quad \text{for } x \in [0, 2\pi]$$



Section E: Exam 2 Questions (26 Marks)

INSTRUCTION:

- **Regular: 26 Marks. 5 Minutes Reading. 35 Minutes Writing.**
- **Extension: 26 Marks. 5 Minutes Reading. 26 Minutes Writing.**



Question 7 (1 mark)

$\frac{\pi}{2}$ radians in degrees is given by:

- A. 30°
- B. 90°**
- C. 15°
- D. 60°

Question 8 (1 mark)

For what values of k will $\sin(x + k) = \sin(x)$?

- A. $2n\pi, n \in \mathbb{Z}$**

*Sin repeats every $2\pi \Rightarrow$ General:
 $2\pi n, n \in \mathbb{Z}$*

- B. π
- C. 3π
- D. $\frac{\pi}{2}$

Space for Personal Notes

Question 9 (1 mark)

Given that $\sin(\alpha) = \frac{3}{5}$ and $\cos(\beta) = \frac{5}{13}$, with $\alpha \in (0, \frac{\pi}{2})$ and $\beta \in (\frac{3\pi}{2}, 2\pi)$. Evaluate,

$$\frac{\sin(\beta)}{\cos(\alpha)}$$

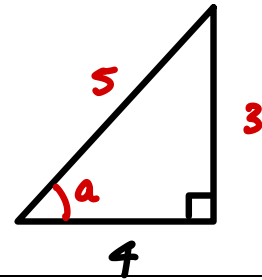
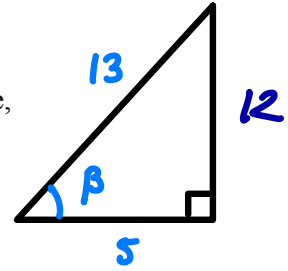
A. $\frac{13}{15}$

B. $-\frac{15}{13}$

C. $\frac{39}{25}$

D. $-\frac{25}{39}$

$$\begin{aligned} & \frac{-\frac{12}{13}}{+\frac{4}{5}} = \frac{-60}{52} \\ & = \frac{-30}{26} \\ & = -\frac{15}{13} \end{aligned}$$



Question 10 (1 mark)

What are the coordinates of the unit circle in terms of x ?

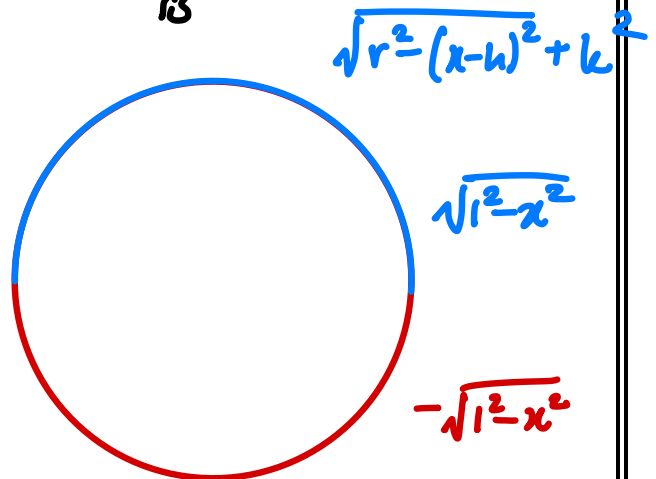
A. $(x, \sqrt{1-x^2})$

B. $(x, \pm\sqrt{1-x^2})$

C. $(\sqrt{1-x^2}, x)$

D. $(\sqrt{1-x^2}, -\sqrt{1-x^2})$

$$\begin{aligned} & (h, k) = (0, 0) \\ & \downarrow \\ & x^2 + y^2 = 1 \\ & y^2 = 1 - x^2 \\ & y = \pm\sqrt{1-x^2} \end{aligned}$$



Space for Personal Notes

Question 11 (1 mark)

The following equation has no real solutions:

$$\sin(n^2x) = \frac{\sqrt{5}}{2}, \quad 0 < x < 2\pi$$

Which of the following is the best explanation for why this is the case?

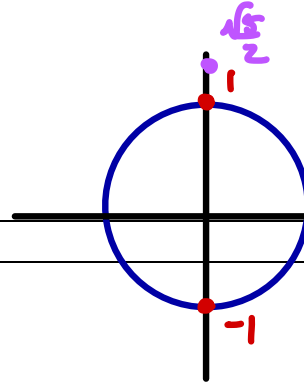
- A. We are not given the value of n .
- B. There are real solutions but they are not in the domain $x \in (0, 2\pi)$.
- C. The range of the sine function is $[-1, 1]$ but $\frac{\sqrt{5}}{2} > 1$.
- D. $\frac{\sqrt{5}}{2}$ is inside the domain $x \in (0, 2\pi)$.

$$\sqrt{5} \approx 2.2 \dots$$

$$\frac{2.2}{2} = 1.1$$

$$\frac{\sqrt{5}}{2} > \frac{\sqrt{4}}{2}$$

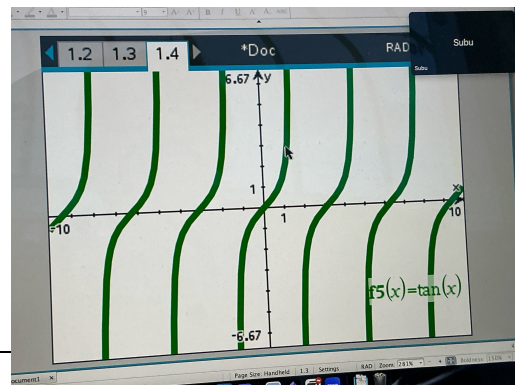
$$\frac{\sqrt{5}}{2} > 1$$



Question 12 (1 mark)

What is the range of $y = \tan(x)$?

- A. $\mathbb{R}$
- B. $\mathbb{R}^+$
- C. $[-1, 1]$
- D. $\mathbb{R} \setminus \left\{\frac{n\pi}{2}\right\}$



Question 13 (1 mark)

Solve the following equation for the given domain:

$$\sqrt{3}\tan\left(x - \frac{\pi}{2}\right) = 1, \quad x \in [0, \pi]$$

- A. $\frac{\pi}{3}$
- B. $\frac{\pi}{6}$
- C. $\frac{5\pi}{6}$
- D. $\frac{2\pi}{3}$

$$\tan\left(x - \frac{\pi}{2}\right) = \frac{1}{\sqrt{3}}$$

$$x - \frac{\pi}{2} = \frac{\pi}{6}$$

$$\therefore x = \frac{4\pi}{6} = \frac{2\pi}{3}$$

Question 14 (1 mark)

Why does $\tan(x)$ have a period of π ?

- A. It is asymptotic.
- B. Its values repeat every π radians.
- C. $\cos(x)$ and $\sin(x)$ has a period of π .
- D. Its period is not π .

Question 15 (1 mark)

Which of the following equations is true?

A. $\sin\left(\frac{\pi}{2} - x\right) = +\cos(x)$

B. $\sin\left(\frac{3\pi}{2} - x\right) = -\cos(x)$

C. $\cos\left(\frac{\pi}{2} + x\right) = -\sin(x)$

D. $\cos\left(\frac{\pi}{2} + x\right) = \sin(x)$

Interval = $n\pi$

Period = $\frac{2\pi}{n}$

No. of periods in interval = $\frac{\text{Interval}}{\text{Period}} = \frac{n\pi}{\left(\frac{2\pi}{n}\right)} = \frac{n^2}{2}$

Question 16 (1 mark)

How many x -intercepts will $\sin(nx)$ have over $(0, n\pi]$?

A. $\frac{2\pi}{n}n$

B. n^2

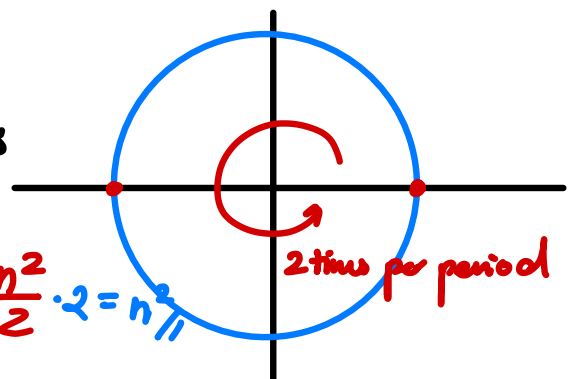
C. n

D. $n^2 + 1$

$\sin(nx) = 0$ in 1 period $\Rightarrow$ 2 times

$P = \frac{2\pi}{n} \Rightarrow$ No. of solns

$\frac{n\pi}{\left(\frac{2\pi}{n}\right)} = \frac{n\pi}{1} \cdot \frac{n}{2\pi} = \frac{n^2}{2} \cdot 2 = n^2$



Space for Personal Notes

no. of periods

$n=1:$

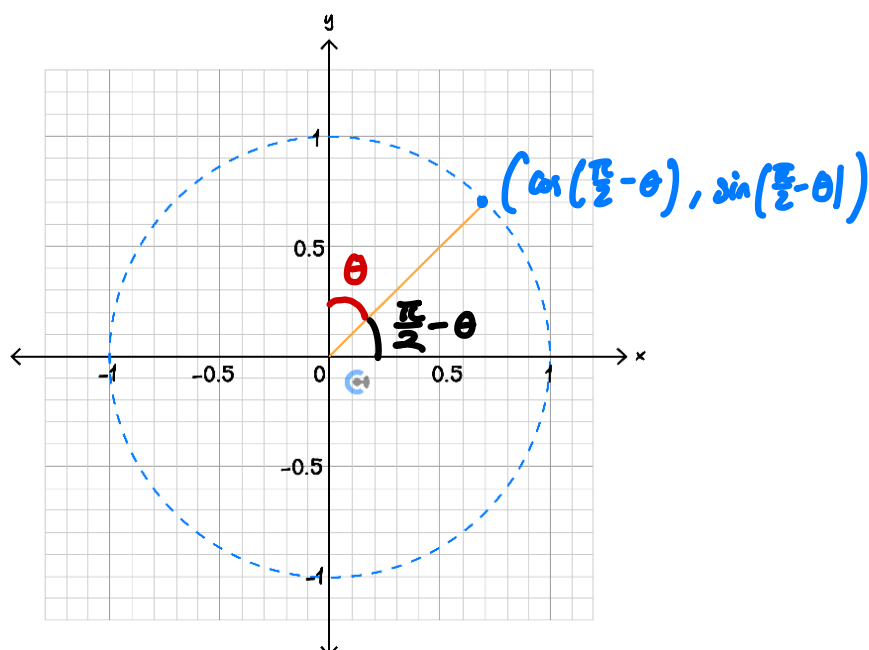
$\sin(x) \quad x \in (0, \pi] \Rightarrow x = \pi \quad (1)$

$n=2:$

$\sin(2x) \quad x \in (0, 2\pi] \Rightarrow 4 \text{ solutions}$

Question 17 (6 marks)

Consider the following unit circle:



- a. If the line makes an angle of, θ , with the y -axis. Express the coordinates of the unit circle in terms of θ . (2 marks)

$$x = \cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$y = \sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$$

$$\therefore (x, y) = (\sin \theta, \cos \theta)$$

- b. Find the coordinates in terms of y , for $x > 0$. (2 marks)

$$y = \cos \theta$$

$$x^2 + y^2 = 1$$

$$x^2 = 1 - y^2$$

$$x = \pm \sqrt{1 - y^2} \Rightarrow \therefore x = +\sqrt{1 - y^2}$$

$$\therefore (x, y) = (\sqrt{1 - y^2}, y)$$

c. Express $\tan(\theta)$ in terms of y , for $x > 0$. (2 marks)

$$\tan \theta = \frac{y}{x}$$

$$= \tan \theta = \frac{\sqrt{1-y^2}}{y} //$$

Space for Personal Notes

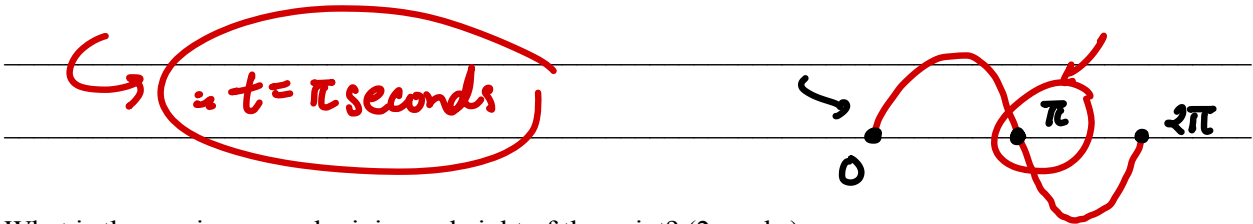
Question 18 (10 marks)

The height of a point on the pump of an oil rig relative to the ground can be modelled using the following function:

$$f(t) = 2\sin(t) - \sqrt{2}, \text{ for } t \geq 0$$

where $y = 0$ is the ground level and t is measured in seconds.

- a. How long does it take for the point to first return to its starting height? (1 mark)



- b. What is the maximum, and minimum height of the point? (2 marks)

Hint: sin and cos can only be between -1 and 1 .

Max height occurs when $\sin(t) = 1$: Max height = $2 - \sqrt{2}$ units //

Min height occurs when $\sin(t) = -1$: Min height = $-2 - \sqrt{2}$ units //

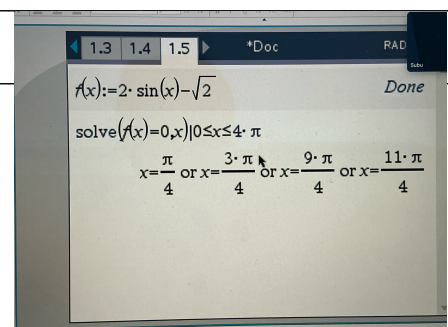
- c. i. For what values of $t \in [0, 4\pi]$, will the point be level with the ground? (3 marks)

Let $f(t) = 0$:

$$2\sin(t) - \sqrt{2} = 0$$

$$\sin(t) = \frac{\sqrt{2}}{2}$$

$\therefore t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ seconds //



- ii. Hence, state the values of $t \in [0, 4\pi]$ for which the point is above the ground level. (2 marks)

Solve $f(t) > 0$:

$\therefore t \in \left(\frac{\pi}{4}, \frac{3\pi}{4}\right) \cup \left(\frac{9\pi}{4}, \frac{11\pi}{4}\right)$

Handwritten solution for part ii showing the inequality $2 \cdot n16 \cdot \pi + \frac{\pi}{4} < x < 2 \cdot n16 \cdot \pi + \frac{3 \cdot \pi}{4}$ and the resulting intervals for t .

- d. The height of another point on the pump is modelled by $g(t) = \frac{\sin(t)}{\sqrt{2}} - \sqrt{2}$ instead. Can this point reach the ground level? Justify. (2 marks)

$\therefore$ Ground level $\Rightarrow y = 0$

$\hookrightarrow \in [-1, 1]$

Range $g(t) \in [-1 - \sqrt{2}, 1 - \sqrt{2}]$

$\sqrt{1 - \sqrt{2}} < 0$

$\therefore$ y-value is ALWAYS negative

it NEVER reaches ground level

Space for Personal Notes

Section F: Extension Exam 1 (9 Marks)

INSTRUCTION:

- Regular: Skip.
- Extension: 9 Marks. 10 Minutes Writing.



Question 19 (9 marks)

Solve the following trigonometric equations, giving all solutions in the given domain.

- a. Solve the equation $\sin(x) = \frac{1}{2}$ for $0 \leq x \leq 2\pi$. (2 marks)

$$2x = \frac{\pi}{6}, \pi - \frac{\pi}{6}$$

$$\therefore x = \frac{\pi}{12}, \frac{5\pi}{12}$$

$$\therefore P = \frac{2\pi}{2} = \frac{12\pi}{12}$$

$$D \in [0, \frac{24\pi}{12}]$$

$$\therefore x = \frac{\pi}{12}, \frac{13\pi}{12}, \frac{5\pi}{12}, \frac{17\pi}{12}$$

- b. Solve the equation $\tan^2(x) = 3$ for $0 \leq x < 2\pi$. (2 marks)

$$\therefore P = \pi = \frac{3\pi}{3} \quad D \in [0, \frac{6\pi}{3}]$$

$$\tan(x) = \sqrt{3} \quad \text{OR} \quad \tan(x) = -\sqrt{3}$$

$$\therefore x = \frac{\pi}{3}$$

$$\therefore x = \frac{2\pi}{3}$$

$$\therefore x = \frac{\pi}{3}, \frac{4\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}$$

$$\therefore x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

- c. Solve the equation $\overbrace{\sin(x)}^{\cos(x)} = \overbrace{\cos(x)}^{\sin(x)}$ for $0 \leq x < 2\pi$. (2 marks)

$$\tan(x) = 1$$

$$\therefore x = \frac{\pi}{4}$$

$$\therefore P = \frac{\pi}{4} = \frac{4\pi}{4}$$

$$D \in [0, 8\frac{\pi}{4})$$

$$\therefore x = \frac{\pi}{4}, \frac{5\pi}{4}$$

- d. Solve the equation $2\cos^2(x) - 3\cos(x) + 1 = 0$ for $-\pi \leq x \leq \pi$. (3 marks)

$$\text{Let } a = \cos(x) :$$

$$2a^2 - 3a + 1 = 0$$

$$(2a-1)(a-1) = 0$$

$$\therefore a = \frac{1}{2} \text{ or } a = 1$$

$$\cos(x) = \frac{1}{2}$$

$$Q1 \quad Q4$$

$$\therefore x = \frac{\pi}{3}, -\frac{\pi}{3}$$

$$\cos(x) = 1$$

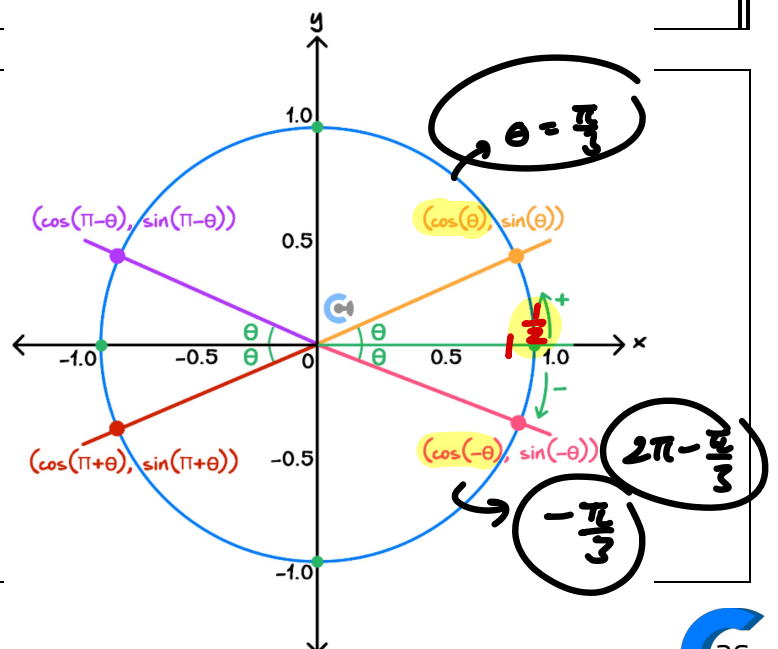
$$\therefore x = 0$$

$$\therefore P = \frac{2\pi}{1} = \frac{6\pi}{3}$$

$$D \in [-\frac{3\pi}{3}, \frac{3\pi}{3}]$$

$$\therefore x = -\frac{\pi}{3}, 0, \frac{\pi}{3}$$

Space for Personal Notes



Section G: Extension Exam 2 (12 Marks)

INSTRUCTION:

- Regular: Skip.
- Extension: 12 Marks. 15 Minutes Writing.



Question 20 (5 marks)

Consider the following two functions:

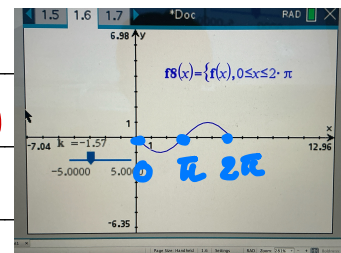
$$g(x) = \sin(x) \text{ and } f(x) = \cos(x - k), 0 \leq x \leq 2\pi$$

- a. For what value of k will $f(x)$ have three x -intercepts? For this value of k state the value(s) of x where $f(x)$ crosses the x -axis. Just provide one possible value for k . (2 marks)

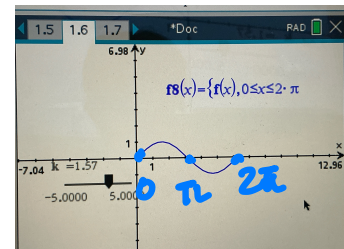
$$\cos(x-k) = 0$$

$$\therefore x = 0, \pi, 2\pi$$

$$\therefore k = -\frac{\pi}{2} \Rightarrow$$



$$\therefore k = \frac{\pi}{2} \Rightarrow$$

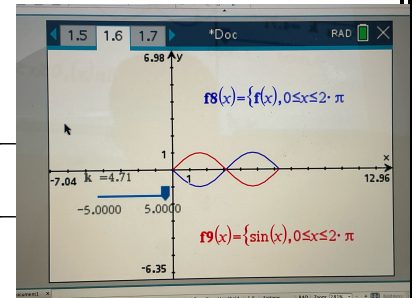


b. Suppose $k \in [0, 2\pi]$. Provide a value of k for which $f(x) = g(x)$ has:

i. 3 solutions. (1 mark)

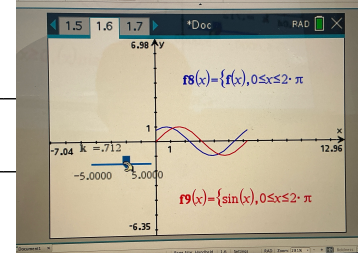
$$\cos(x-k) = \sin(x)$$

$k = -\frac{\pi}{2} \xrightarrow{+P} \therefore k = \frac{3\pi}{2}$



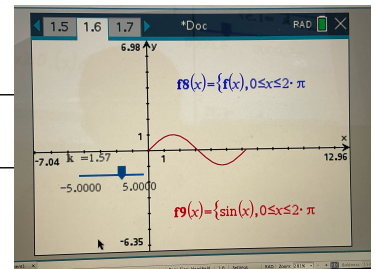
ii. 2 solutions. (1 mark)

$$\therefore k \in [0, 2\pi] \setminus \left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\}$$



iii. Infinitely many solutions. (1 mark)

$\therefore k = \frac{\pi}{2}$ →



Space for Personal Notes

Question 21 (7 marks)

The temperature $T(t)$ in degrees Celsius inside an office at time t hours after midnight is modelled by:

$$T(t) = 21 + 3 \cos\left(\frac{\pi}{6}(t - 4)\right),$$

$$\therefore P = \frac{2\pi}{(\frac{\pi}{6})} = 12$$

where $0 \leq t \leq 24$.

- a. State the maximum and minimum temperatures in the office, and the times at which they occur. (2 marks)

$$\text{Max Temp} = 24 \Rightarrow T(t) = 24$$

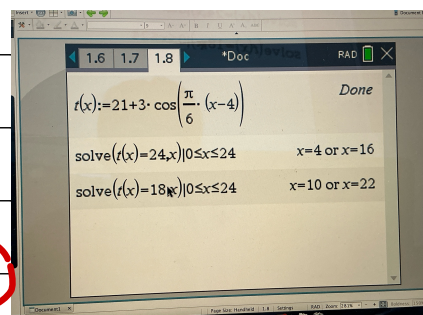
$$\text{Min Temp} = 18 \Rightarrow t = 4 \text{ OR } 16$$

b

$$\therefore T(t) = 18$$

$$\therefore t = 10 \text{ OR } 22$$

earliest



- b. Find the exact value of t for which the temperature is first 23°C . What time of day, to the nearest minute, does this t correspond to? (3 marks)

$$\text{Let } T(t) = 23:$$

$$21 + 3 \cos\left(\frac{\pi}{6}(t-4)\right) = 23$$

$$\therefore \cos\left(\frac{\pi}{6}(t-4)\right) = \frac{2}{3} \Rightarrow \frac{Q1, Q4}{0, -0}$$

$$\frac{\pi}{6}(t-4) = \cos^{-1}\left(\frac{2}{3}\right) \text{ OR } \frac{\pi}{6}(t-4) = -\cos^{-1}\left(\frac{2}{3}\right)$$

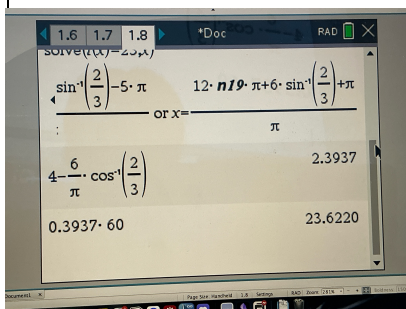
$$\frac{\pi}{6}(t-4) = \pm \cos^{-1}\left(\frac{2}{3}\right)$$

$$t-4 = \pm \frac{6}{\pi} \cos^{-1}\left(\frac{2}{3}\right)$$

$$t = 4 \pm \frac{6}{\pi} \cos^{-1}\left(\frac{2}{3}\right)$$

$$\Rightarrow \therefore t = 4 - \frac{6}{\pi} \cos^{-1}\left(\frac{2}{3}\right) \text{ (earliest)}$$

$$\Rightarrow \approx 2:24 \text{ AM}$$

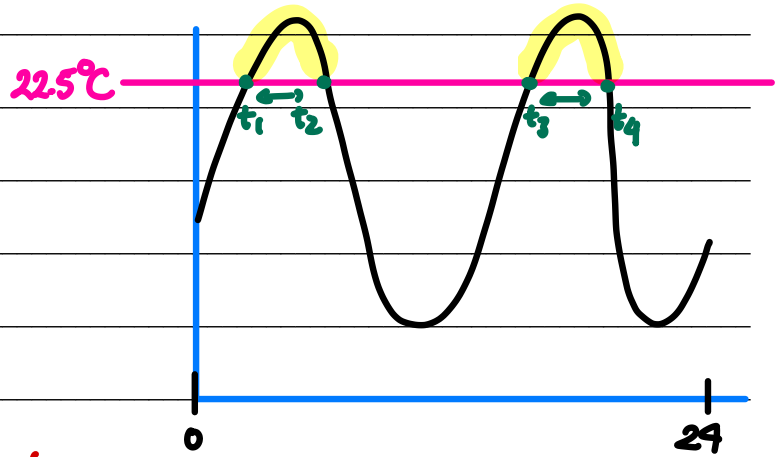


- c. What fraction of the day is the temperature above 22.5°C ? (A day starts at midnight and ends at midnight 24 hours later.) (2 marks)

$$\text{Fraction of Day} = \frac{(t_2 - t_1) + (t_4 - t_3)}{24}$$

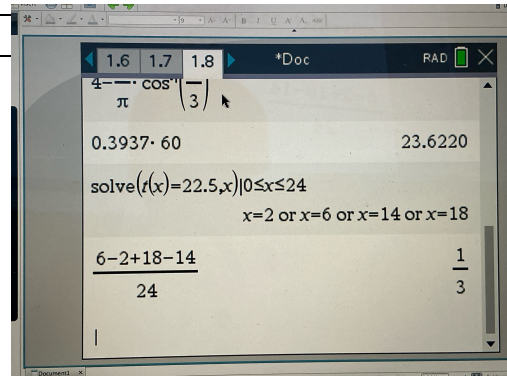
$$= \frac{1}{3}$$

$$\rightarrow = \frac{t_2 - t_1}{12} = \frac{1}{3}$$



Space for Personal Notes

fraction over 1
period = fraction
over ANY
period





Website: contoureducation.com.au | Phone: 1800 888 300 | Email: hello@contoureducation.com.au

VCE Mathematical Methods ½

Free 1-on-1 Support



Be Sure to Make The Most of These (Free) Services!

- Experienced Contour tutors (45 + raw scores, 99 + ATARs).
- For fully enrolled Contour students with up-to-date fees.
- After school weekdays and all-day weekends.

<u>1-on-1 Video Consults</u>	<u>Text-Based Support</u>
➤ Book via bit.ly/contour-methods-consult-2025 (or QR code below). ➤ One active booking at a time (must attend before booking the next).	➤ Message +61 440 138 726 with questions. ➤ Save the contact as "Contour Methods".

[Booking Link for Consults](https://bit.ly/contour-methods-consult-2025)
bit.ly/contour-methods-consult-2025



[Number for Text-Based Support](tel:+61440138726)
[+61 440 138 726](tel:+61440138726)