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Email: hello@contoureducation.com.au

VCE Mathematical Methods ½
Circular Function I [0.16]
Workshop

Error Logbook:



New Ideas/Concepts	Didn't Read Question
Pg / Q #: _____ Notes:	Pg / Q #: _____ Notes:
Algebraic/Arithmetic/ Calculator Input Mistake	Working Out Not Detailed Enough
Pg / Q #: _____ Notes:	Pg / Q #: _____ Notes:

Section A: Recap

Radians and Degrees



$$1^c = \left(\frac{180}{\pi}\right)^o$$

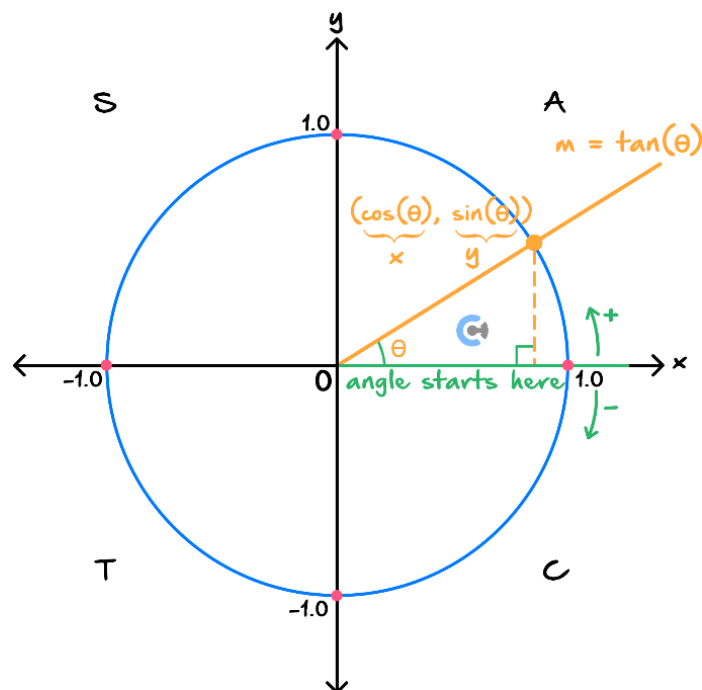
$$1^o = \left(\frac{\pi}{180}\right)^c$$

$$180^o = \pi^c$$

Unit Circle



- The unit circle is simply a circle of radius 1.



$$\sin(\theta) = y$$

$$\cos(\theta) = x$$

$$\tan(\theta) = \text{gradient}$$



Period of a Trigonometric Function

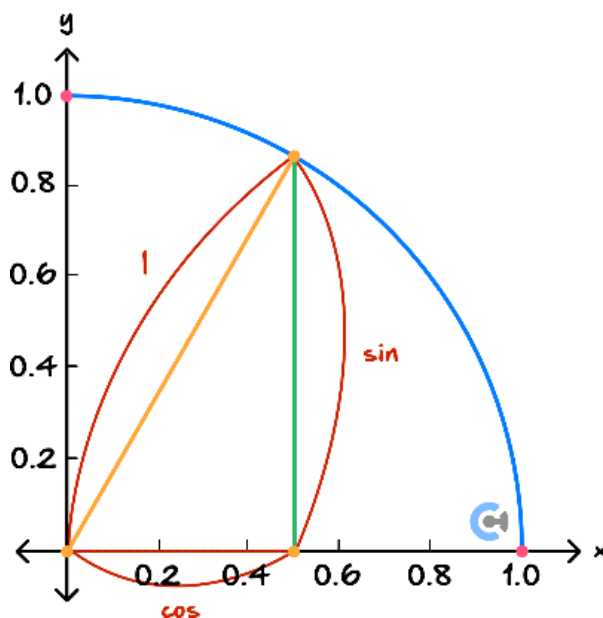
period of $\sin(nx)$ and $\cos(nx)$ functions $= \frac{2\pi}{n}$

period of $\tan(nx)$ functions $= \frac{\pi}{n}$

where $n = \text{coefficient of } x \text{ and } n > 0$



Pythagorean Identities



$$\sin^2(\theta) + \cos^2(\theta) = 1$$

► Can be used for finding one trigonometry function by using the other.

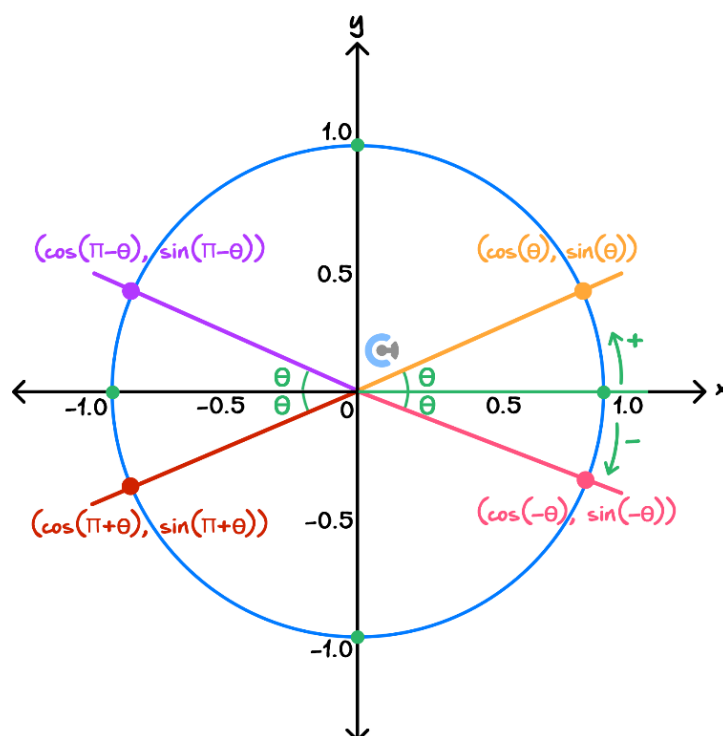
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The Exact Values Table

x	0 (0°)	$\frac{\pi}{6}$ (30°)	$\frac{\pi}{4}$ (45°)	$\frac{\pi}{3}$ (60°)	$\frac{\pi}{2}$ (90°)
$\sin(x)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos(x)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan(x)$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined

Supplementary Relationships



➤ Simply look at the quadrant to find the correct sign.

 **Second Quadrant ($\pi - \theta$)**

$$\cos(\pi - \theta) = -\cos(\theta)$$

$$\sin(\pi - \theta) = +\sin(\theta)$$


$$\tan(\pi - \theta) = -\tan(\theta)$$

 Third Quadrant ($\pi + \theta$)

$$\cos(\pi + \theta) = -\cos(\theta)$$

$$\sin(\pi + \theta) = -\sin(\theta)$$

$$\tan(\pi + \theta) = +\tan(\theta)$$

 Fourth Quadrant ($-\theta$)


$$\cos(-\theta) = +\cos(\theta)$$

$$\sin(-\theta) = -\sin(\theta)$$

$$\tan(-\theta) = -\tan(\theta)$$

Complementary Relationships


➤ Consider the quadrant for signs.

 First Quadrant ($\frac{\pi}{2} - \theta$)

$$\cos\left(\frac{\pi}{2} - \theta\right) = +\sin(\theta)$$

$$\sin\left(\frac{\pi}{2} - \theta\right) = +\cos(\theta)$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = +\frac{1}{\tan(\theta)}$$


 Second Quadrant ($\frac{\pi}{2} + \theta$)

$$\sin\left(\frac{\pi}{2} + \theta\right) = +\cos(\theta)$$

$$\cos\left(\frac{\pi}{2} + \theta\right) = -\sin(\theta)$$




$$\tan\left(\frac{\pi}{2} + \theta\right) = -\frac{1}{\tan(\theta)}$$

 Third Quadrant $\left(\frac{3\pi}{2} - \theta\right)$

$$\sin\left(\frac{3\pi}{2} - \theta\right) = -\cos(\theta)$$

$$\cos\left(\frac{3\pi}{2} - \theta\right) = -\sin(\theta)$$

$$\tan\left(\frac{3\pi}{2} - \theta\right) = \frac{1}{\tan(\theta)}$$

 Fourth Quadrant $\left(\frac{3\pi}{2} + \theta\right)$

$$\sin\left(\frac{3\pi}{2} + \theta\right) = -\cos(\theta)$$

$$\cos\left(\frac{3\pi}{2} + \theta\right) = +\sin(\theta)$$

$$\tan\left(\frac{3\pi}{2} + \theta\right) = -\frac{1}{\tan(\theta)}$$

► **Steps:**

1. Note complementary relationship by identifying a vertical angle $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$.
2. Equate to the opposite trigonometric function $\cos / \sin / \frac{1}{\tan(\theta)}$.
3. Determine the sign (\pm) by considering the quadrant.

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Supplementary v/s Complementary

Supplementary: $\text{trig}(\text{Horizontal Angle} \pm \theta)$

Complementary: $\text{trig}(\text{Vertical Angle} \pm \theta)$



Particular Solutions

- Solving trigonometric equations **for finite solutions**.
- **Steps:**
 1. Make the trigonometric function the subject.
 2. Find the necessary angle for one period.
 3. Solve for x by equating the necessary angles to the inside of the trigonometric functions.
 4. Add and subtract the period to find all other solutions in the domain.

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Section B: Warm Up (13 Marks)

INSTRUCTION:

➤ **Regular: 13 Marks. 13 Minutes Writing.**

➤ **Extension: Skip.**



Question 1 (7 marks)

a. Find $\left(\frac{3\pi}{4}\right)^c$ in degrees. (1 mark)

b. Find 150° in radians. (1 mark)

c. Determine the period of $\sin(3x)$. (1 mark)

d. Determine the period of $\tan\left(\frac{\pi x}{3}\right)$. (1 mark)

e. Evaluate $\sin\left(\frac{3\pi}{2}\right)$. (1 mark)

f. Evaluate $\cos\left(-\frac{\pi}{6}\right)$. (1 mark)

g. Evaluate $\tan\left(\frac{7\pi}{4}\right)$. (1 mark)

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Question 2 (6 marks)

Given that $\sin(x) = \frac{5}{13}$ and $\frac{\pi}{2} < x < \pi$, find:

a. $\cos(x)$. (2 marks)

b. $\tan(x)$. (1 mark)

c. $\cos\left(x + \frac{\pi}{2}\right)$. (1 mark)

d. $\sin(2\pi - x)$. (1 mark)

e. $\tan\left(\frac{\pi}{2} + x\right)$. (1 mark)

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Section C: Exam 1 Questions (17 Marks)

INSTRUCTION:



- **Regular: 17 Marks. 5 Minutes Reading. 25 Minutes Writing.**
- **Extension: 17 Marks. 5 Minutes Reading. 17 Minutes Writing.**

Question 3 (5 marks)

- a. What value(s) can $\cos(x)$ take given that $\sin(x) = \frac{3}{5}$? (3 marks)

- b. Hence, find the possible value(s) of $\tan(x)$. (2 marks)

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Question 4 (2 marks)

Given that the period of the function $\tan(n^2x)$ is 2. Find the value(s) of n .

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Question 5 (7 marks)

Given that $\sin(\alpha) = m$, and $\cos(\beta) = 0.2$.

- a.** Find the value of $\cos\left(\frac{\pi}{2} - \alpha\right)$. (2 marks)

- b.** Find the value of $\sin\left(\frac{3\pi}{2} - \beta\right)$. (2 marks)

- c.** Find the value(s) of $\tan(\alpha)$. (3 marks)

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Question 6 (3 marks)

Consider the functions:

$$f(x) = \sin(nx) \quad \text{and} \quad g(x) = \cos(nx)$$

For what integer value of n will $f(x) = g(x)$ have exactly 6 solutions for, $x \in [0, 2\pi]$? Justify your answer.


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Section D: Tech Active Exam Skills




Calculator Commands: Solving Trigonometric Functions

➤ TI

 `solve(trig(..) = a, x) |`
domain restriction


 | is under control equal.

➤ Casio

 `solve(trig(..) = a, x) |`
domain restriction

 | is under maths 3.

➤ Mathematica

 `Solve[trig[] == a &&`
domain restriction, x]

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Section E: Exam 2 Questions (26 Marks)

INSTRUCTION:



- **Regular: 26 Marks. 5 Minutes Reading. 35 Minutes Writing.**
- **Extension: 26 Marks. 5 Minutes Reading. 26 Minutes Writing.**

Question 7 (1 mark)

$\frac{\pi}{2}$ radians in degrees is given by:

- A. 30°
- B. 90°
- C. 15°
- D. 60°

Question 8 (1 mark)

For what values of k will $\sin(x + k) = \sin(x)$?

- A. $2n\pi, n \in \mathbb{Z}$
- B. π
- C. 3π
- D. $\frac{\pi}{2}$

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Question 9 (1 mark)

Given that $\sin(\alpha) = \frac{3}{5}$ and $\cos(\beta) = \frac{5}{13}$, with $\alpha \in \left(0, \frac{\pi}{2}\right)$ and $\beta \in \left(\frac{3\pi}{2}, 2\pi\right)$. Evaluate,

$$\frac{\sin(\beta)}{\cos(\alpha)}$$

A. $\frac{13}{15}$

B. $-\frac{15}{13}$

C. $\frac{39}{25}$

D. $-\frac{25}{39}$

Question 10 (1 mark)

What are the coordinates of the unit circle in terms of x ?

A. $(x, \sqrt{1-x^2})$

B. $(x, \pm\sqrt{1-x^2})$

C. $(\sqrt{1-x^2}, x)$

D. $(\sqrt{1-x^2}, -\sqrt{1-x^2})$

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Question 11 (1 mark)

The following equation has no real solutions:

$$\sin(n^2x) = \frac{\sqrt{5}}{2}, \quad 0 < x < 2\pi$$

Which of the following is the best explanation for why this is the case?

- A. We are not given the value of n .
- B. There are real solutions but they are not in the domain $x \in (0, 2\pi)$.
- C. The range of the sine function is $[-1, 1]$ but $\frac{\sqrt{5}}{2} > 1$.
- D. $\frac{\sqrt{5}}{2}$ is inside the domain $x \in (0, 2\pi)$.

Question 12 (1 mark)

What is the range of $y = \tan(x)$?

- A. R
- B. R^+
- C. $[-1, 1]$
- D. $R \setminus \left\{\frac{n\pi}{2}\right\}$

Question 13 (1 mark)

Solve the following equation for the given domain:

$$\sqrt{3}\tan\left(x - \frac{\pi}{2}\right) = 1, \quad x \in [0, \pi]$$

- A. $\frac{\pi}{3}$
- B. $\frac{\pi}{6}$
- C. $\frac{5\pi}{6}$
- D. $\frac{2\pi}{3}$

Question 14 (1 mark)

Why does $\tan(x)$ have a period of π ?

- A. It is asymptotic.
- B. Its values repeat every π radians.
- C. $\cos(x)$ and $\sin(x)$ has a period of π .
- D. Its period is not π .

Question 15 (1 mark)

Which of the following equations is true?

- A. $\sin\left(\frac{\pi}{2} - x\right) = -\cos(x)$
- B. $\sin\left(\frac{3\pi}{2} - x\right) = \cos(x)$
- C. $\cos\left(\frac{\pi}{2} + x\right) = -\sin(x)$
- D. $\cos\left(\frac{\pi}{2} + x\right) = \sin(x)$

Question 16 (1 mark)

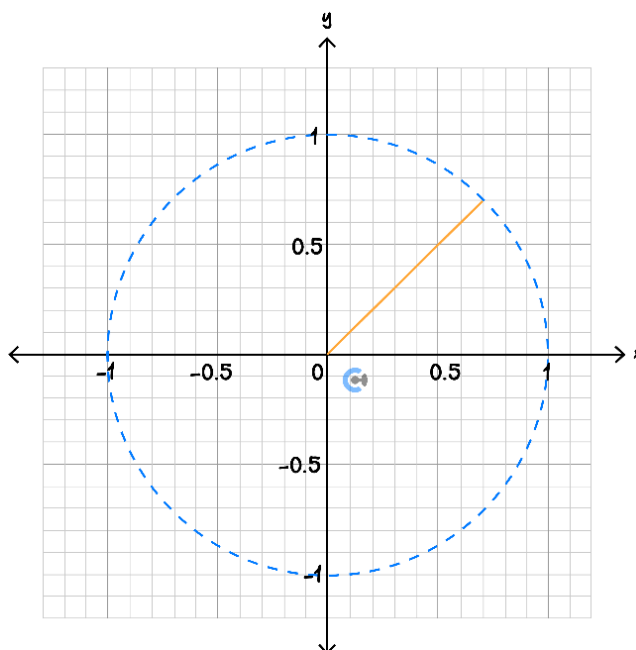
How many x -intercepts will $\sin(nx)$ have over $(0, n\pi]$?

- A. $\frac{2\pi}{n}n$
- B. n^2
- C. n
- D. $n^2 + 1$

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Question 17 (6 marks)

Consider the following unit circle:



- a. If the line makes an angle of, θ , with the **y-axis**. Express the coordinates of the unit circle in terms of θ . (2 marks)

- b. Find the coordinates in terms of y , for $x > 0$. (2 marks)

c. Express $\tan(\theta)$ in terms of y , for $x > 0$. (2 marks)

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Question 18 (10 marks)

The height of a point on the pump of an oil rig relative to the ground can be modelled using the following function:

$$f(t) = 2\sin(t) - \sqrt{2}, \text{ for } t \geq 0$$

where $y = 0$ is the ground level and t is measured in seconds.

- a.** How long does it take for the point to first return to its starting height? (1 mark)

- b.** What is the maximum, and minimum height of the point? (2 marks)

Hint: sin and cos can only be between -1 and 1 .

- c.**
- i.** For what values of $t \in [0, 4\pi]$, will the point be level with the ground? (3 marks)

ii. Hence, state the values of $t \in [0, 4\pi]$ for which the point is above the ground level. (2 marks)

d. The height of another point on the pump is modelled by $g(t) = \sin(t) - \sqrt{2}$ instead. Can this point reach the ground level? Justify. (2 marks)

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Section F: Extension Exam 1 (9 Marks)

INSTRUCTION:

➤ **Regular: Skip.**

➤ **Extension: 9 Marks. 10 Minutes Writing.**



Question 19 (9 marks)

Solve the following trigonometric equations, giving all solutions in the given domain.

a. Solve the equation $\sin(2x) = \frac{1}{2}$ for $0 \leq x \leq 2\pi$. (2 marks)

b. Solve the equation $\tan^2(x) = 3$ for $0 \leq x < 2\pi$. (2 marks)

c. Solve the equation $\sin(x) = \cos(x)$ for $0 \leq x < 2\pi$. (2 marks)

d. Solve the equation $2 \cos^2(x) - 3 \cos(x) + 1 = 0$ for $-\pi \leq x \leq \pi$. (3 marks)

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Section G: Extension Exam 2 (12 Marks)

INSTRUCTION:

- Regular: Skip.
- Extension: 12 Marks. 15 Minutes Writing.



Question 20 (5 marks)

Consider the following two functions:

$$g(x) = \sin(x) \text{ and } f(x) = \cos(x - k), 0 \leq x \leq 2\pi$$

- a. For what value of k will $f(x)$ have three x -intercepts? For this value of k state the value(s) of x where $f(x)$ crosses the x -axis. Just provide one possible value for k . (2 marks)

b. Suppose $k \in [0, 2\pi]$. Provide a value of k for which $f(x) = g(x)$ has:

i. 3 solutions. (1 mark)

ii. 2 solutions. (1 mark)

iii. Infinitely many solutions. (1 mark)

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Question 21 (7 marks)

The temperature $T(t)$ in degrees Celsius inside an office at time t hours after midnight is modelled by:

$$T(t) = 21 + 3 \cos\left(\frac{\pi}{6}(t - 4)\right),$$

where $0 \leq t \leq 24$.

- a.** State the maximum and minimum temperatures in the office, and the times at which they occur. (2 marks)

- b.** Find the **exact** value of t for which the temperature is first 23°C . What time of day, to the nearest minute, does this t correspond to? (3 marks)

- c. What fraction of the day is the temperature above 22.5°C ? (A day starts at midnight and ends at midnight 24 hours later.) (2 marks)

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