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VCE Mathematical Methods ½
AOS 3 Revision [0.15]
Workshop Solutions

Error Logbook:

New Ideas/Concepts	Didn't Read Question
Pg / Q #: ____________________ Notes:	Pg / Q #: ____________________ Notes:
Algebraic/Arithmetic/ Calculator Input Mistake	**Working Out Not Detailed Enough**
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Section A: Cheat Sheets

Cheat Sheet

[3.1.1] - Understand Probabilities in Terms of Favourable Outcomes From a Sample Space

[3.2.1] - Understand Probabilities in Terms of Favourable Outcomes From a Sample Space

- The sample space is the set of $\boxed{\text{all possible outcomes}}$ in an experiment.
- Total probability must always add to $\boxed{1}$.
- When there is some number of equally likely outcomes, the probability of a "successful" outcome can be calculated as:

$$\Pr(success) = \boxed{\frac{Number\ of\ successful\ outcomes}{Number\ of\ total\ outcomes}}$$

[3.1.2] - Basic Probability Operations. Use Venn Diagrams and/or Karnaugh Maps and Apply the Addition Rule

[3.2.2] - Basic Probability Operations. Use Venn Diagrams and/or Karnaugh Maps and Apply the Addition Rule

- If A and B are events, then we interpret the following operations as:
- $A \cup B = \boxed{A \text{ or } B}$
- $A \cap B = \boxed{A \text{ and } B}$
- $A' = \boxed{\text{not } A}$

$$\Pr(A') = \boxed{1 - \Pr(A)}$$

- Karnaugh Tables
  - We can represent probability problems using a Karnaugh Map.

	B	B'	
A	$\Pr(A \cap B)$	$\Pr(A \cap B')$	$\Pr(A)$
A'	$\Pr(A' \cap B)$	$\Pr(A' \cap B')$	$\Pr(A')$
	$\Pr(B)$	$\Pr(B')$	1

- The Addition Rule

$$\Pr(A \cup B) = \boxed{\Pr(A) + \Pr(B) - \Pr(A \cap B)}$$

[3.1.3] - Understand the Meaning Behind Independent and Mutually Exclusive Events

[3.2.3] - Understand the Meaning Behind Independent and Mutually Exclusive Events

- Mutually Exclusive Events
  - Two events A and B are **mutually exclusive** if they cannot occur at the same time.
  - Probability of both A and B happening together is $\boxed{\text{zero}}$.

$$\Pr(A \cap B) = \boxed{0}$$

- Independent Events
  - **Definition:** Independent events **do not affect** the likelihood of the other.
  - **Mathematically**:

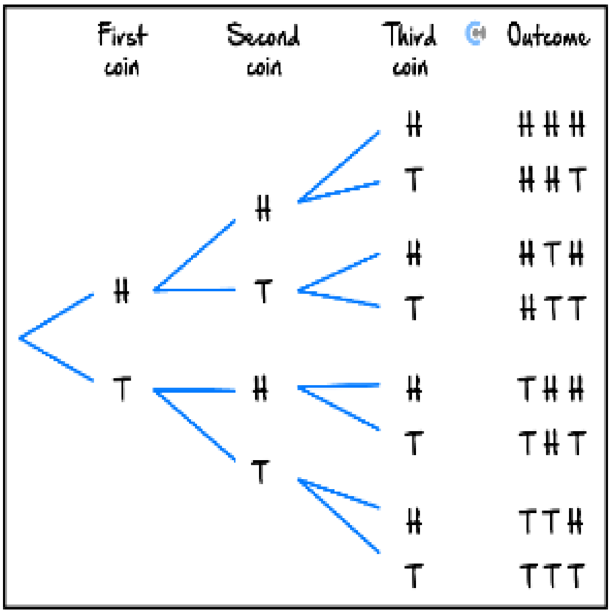
$$\Pr(A \cap B) = \boxed{\Pr(A) \times \Pr(B)}$$

Cheat Sheet

[3.1.4] – Understand Conditional Probability and Make Use of Tree Diagrams

[3.2.4] – Understand Conditional Probability and Make Use of Tree Diagrams

Tree Diagram



- Useful for $\boxed{\text{multiple sequence events}}$.
- To calculate the probability of a sequence, we $\boxed{\text{multiply}}$ the probabilities along the relevant branches.

Conditional Probability

- **Definition**: Probability of A given B.

$$\Pr(A|B) = \boxed{\frac{\Pr(A \cap B)}{\Pr(B)}}$$

Conditional Probability with Independent Events

$$\Pr(A|B) = \boxed{\Pr(A)}$$

- If A and B are independent, the given condition does **not** affect the probability of the event.

[3.3.1] – Find the Number of Permutations and Combinations

Box Diagram for Arrangements

- **Definition:** We can use it to write down the number of $\boxed{\text{arrangements}}$ for each position represented by each $\boxed{\text{box}}$.

Arrangement

- Generally:

Ways to arrange/order n many things for r spots $= \boxed{\frac{n!}{(n-r)!}}$

- We call this ${}^nP_r$.

$${}^nP_r = \boxed{\frac{n!}{(n-r)!}}$$

Selection

- Generally:

Ways to select r things from n many things $= \frac{{}^nP_r}{r!}$

- We call this ${}^nC_r$.

$${}^nC_r = \boxed{\frac{{}^nP_r}{r!} = \frac{n!}{r!\,(n-r)!}}$$

- Where $r =$ Number of selection spots.

Cheat Sheet

[3.3.2] – Find the Number of Permutations and Combinations with Restrictions/Composite

- **Composite Arrangements**
  - **Definition:** Occurs when an arrangement happens within another arrangement.
  - Steps:
    - Consider each group as $\boxed{\text{one}}$ object and find the arrangements.
    - Consider the $\boxed{\text{arrangements}}$ within the groups and multiply.
- **Arrangements with Restrictions**
  - **Definition:** The general principle to deal with restrictions is to:
    - Use a box diagram.
    - Fill in the number of options for the slot that has the restriction $\boxed{\text{first}}$.

[3.3.3] – Find Probabilities Using Counting Methods

- **Probability with Arrangements**

$$\Pr = \frac{n\,(Wanted\ Arrangements)}{\boxed{n\,(Total\ Arrangements)}}$$

- **Probability with Selections**

$$\Pr = \frac{n\,(Wanted\ Selections)}{\boxed{n\,(Total\ Selections)}}$$

[3.4.1] – Applying Pascal's Triangle and Symmetrical Properties of Combinations

- **Pascal's Triangle and ${}^nC_r$:**

  - A new entry in Pascal's triangle is found by adding the two entries $\boxed{\text{above it}}$ from the previous row.
- **Symmetrical Property:**

$${}^nC_r = \boxed{{}^nC_{n-r}}$$

[3.4.2] – Finding Selections of Any Size

- **Selection of Any Size:**

$${}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n = \boxed{2^n}$$

Section B: Exam 1 Questions (23 Marks)

INSTRUCTION:

- Regular: 23 Marks. 5 Minutes Reading. 35 Minutes Writing.
- Extension: 23 Marks. 5 Minutes Reading. 35 Minutes Writing.

Question 1 (3 marks)

Suppose that for events A and B, we know the following probabilities: $\Pr(A) = 0.3, \Pr(B) = 0.5$ and $\Pr(A \cup B) = 0.65$. Compute:

a. $\Pr(A')$. (1 mark) **[3.1.2]**

$$\boxed{\Pr(A') = 1 - \Pr(A) = 1 - 0.3 = 0.7 \quad (1\text{A})}$$

b. $\Pr(A' \cap B')$. (1 mark) **[3.1.2]**

$$\Pr(A' \cap B') = 1 - \Pr(A \cup B)$$
$$\Pr(A' \cap B') = 1 - 0.65 = 0.35 \quad (1\text{A})$$

c. $\Pr(A \cap B)$. (1 mark) **[3.1.2]**

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$
$$0.65 = 0.3 + 0.5 - \Pr(A \cap B)$$
$$\Pr(A \cap B) = 0.8 - 0.65 = 0.15 \quad (1\text{A})$$

Space for Personal Notes

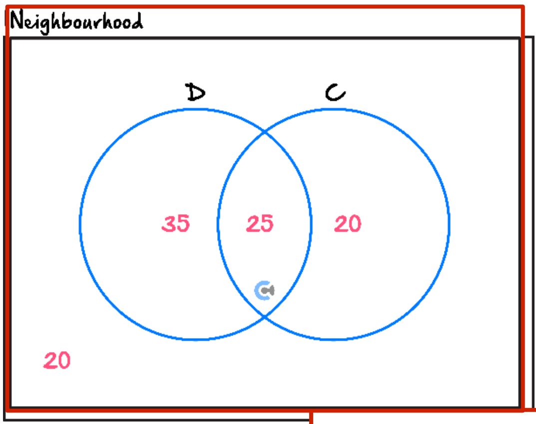
Question 2 (5 marks)

A neighbourhood survey was conducted with 100 residents. Of these 100:

- 60 own a dog.
- 45 own a cat.
- 20 own neither a cat nor a dog.

Let D be the event that a resident owns a dog and C be the event that they own a cat.

a. Fill in the Venn diagram showing the number of people in each of the four regions. (2 marks) **[3.1.2]**



(1M for 2 correct numbers, 1A all correct.)

b. If a resident is selected at random, find the following probabilities using your Venn diagram:

i. $\Pr(C \cap D)$. (1 mark) **[3.1.2]**

$$\boxed{\frac{25}{100} = \frac{1}{4} = 0.25}$$

ii. $\Pr(C \cup D)$. (1 mark) **[3.1.2]**

$$\boxed{\frac{80}{100} = \frac{4}{5} = 0.8}$$

iii. $\Pr(D|C')$. (1 mark) **[3.1.4]**

$$\boxed{\frac{35}{55} = \frac{7}{11}}$$

Question 3 (3 marks)

A company bottles fruit juice at two factories, Factory A and Factory B. 5% of the bottles from Factory A fail a quality check, and 15% of the bottles from Factory B fail the same check. In one hour, 40 bottles are produced from the Factory A and 60 bottles are produced from Factory B. At the end of the hour, one bottle is selected at random from all the bottles produced during that hour.

a. What is the probability that the selected bottle fails the quality check? Express your answer as a percentage. (2 marks) **[3.1.4]**

Let A be the event that the bottle is from Factory A, and B the event that it is from Factory B. Then:

$$\Pr(A) = \frac{40}{100} = \frac{2}{5}, \quad \Pr(B) = \frac{60}{100} = \frac{3}{5},$$
$$\Pr(F \mid A) = 0.05, \quad \Pr(F \mid B) = 0.15$$

We have that

$$\begin{aligned}\Pr(F) &= \Pr(A)\Pr(F \mid A) + \Pr(B)\Pr(F \mid B) \quad \text{(1M)}\\ &= \frac{2}{5} \times 0.05 + \frac{3}{5} \times 0.15\\ &= \frac{2}{5} \times \frac{1}{20} + \frac{3}{5} \times \frac{3}{20}\\ &= \frac{2}{100} + \frac{9}{100} = \frac{11}{100} \quad \text{(1A)}\end{aligned}$$

b. The selected bottle is found to have failed the quality check. What is the probability that it was bottled in Factory A? (1 mark) **[3.1.4]**

Use conditional probability:

$$\begin{aligned}\Pr(A \mid F) &= \frac{\Pr(A \cap F)}{\Pr(F)} = \frac{\Pr(A)\Pr(F \mid A)}{\Pr(F)}\\ &= \frac{\frac{2}{5} \times 0.05}{\frac{11}{100}} = \frac{\frac{2}{5} \times \frac{1}{20}}{\frac{11}{100}}\\ &= \frac{\frac{2}{100}}{\frac{11}{100}} = \frac{2}{11} \quad \text{(1A)}\end{aligned}$$

Space for Personal Notes

Question 4 (6 marks)

A bag contains 6 blue and 4 red marbles. Liz selects 3 marbles without replacement.

a. Find the probability that she selects exactly 1 blue marble. (2 marks) **[3.3.3]**

There are $\binom{6}{1}$ ways to choose 1 blue marble and $\binom{4}{2}$ ways to choose 2 red marbles. The total number of ways to choose 3 marbles from 10 is $\binom{10}{3}$.

$$\Pr(1 \text{ blue}) = \frac{\binom{6}{1}\cdot\binom{4}{2}}{\binom{10}{3}} = \frac{6\cdot 6}{120} = \frac{36}{120} = \frac{3}{10} \quad (1\text{M for numerator, 1A})$$

b. Find the probability that she selects more red marbles than blue. (2 marks) **[3.3.3]**

There are two favourable cases: (1 blue, 2 red) or (0 blue, 3 red). So,

$$\begin{aligned}\Pr(\text{more red than blue}) &= \frac{\binom{6}{1}\cdot\binom{4}{2}+\binom{6}{0}\cdot\binom{4}{3}}{\binom{10}{3}}\\ &= \frac{36+4}{120} = \frac{40}{120} = \frac{1}{3} \quad (1\text{M for the two cases, 1A})\end{aligned}$$

c. Liz repeats the process 4 times.
Find the probability that she never selects more red marbles than blue. (2 marks) **[3.3.3]**

$\Pr(\text{not more red than blue}) = \frac{2}{3}$ (1M).

Thus our desired probability is $\left(\frac{2}{3}\right)^4 = \frac{16}{81}$ (1A).

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Question 5 (6 marks)

A student council of 6 members are to organise different teams and committees. The council consists of 3 boys (Alex, Ben, Chris) and 3 girls (Dana, Emma, Freya).

a. In how many different ways can the council choose 2 members to attend a leadership conference? (1 mark) **[3.3.1]**

We are choosing 2 out of 6 with no order:

$$\binom{6}{2} = 15 \quad (1\text{A})$$

b. In how many ways can the council choose 2 members of different genders to attend the conference? (1 mark) **[3.3.2]**

There are 3 boys and 3 girls. Each mixed-gender pair is one boy and one girl:

$$3 \times 3 = 9 \text{ ways} \quad (1\text{A})$$

c. A pair is selected at random from all possible 2-member pairs. What is the probability that the pair is of different genders? (1 mark) **[3.3.3]**

Favourable $= 9$ mixed-gender pairs Total $= \binom{6}{2} = 15$

$$\Pr(\text{different gender}) = \frac{9}{15} = \frac{3}{5} \quad (1\text{A})$$

d. The student council wants to form subcommittees of any size (except empty). How many different subcommittees are possible? (2 marks) **[3.4.2]**

Each person can either be included or not: $2^6 = 64$ subsets. (1M)
Exclude the empty subset: $64 - 1 = 63$ subcommittees. (1A)

e. In how many different ways can the student council assign a Chair, a Vice-Chair and a Secretary from among the 6 members, if each person holds one role? (1 mark) **[3.3.1]**

This is a permutation of 3 positions from 6 people:

$$P(6,3) = 6 \times 5 \times 4 = 120 \quad (1A)$$

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Section C: Tech Active Exam Skills

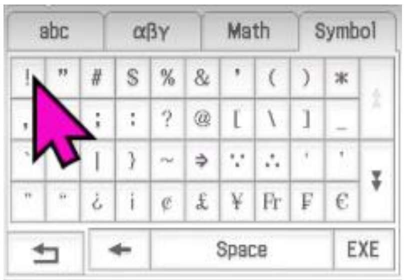
Calculator Commands: Factorial on Technology

- **Mathematica**
  - Exclamation Mark

    $x!$

- **TI-Nspire**
  - Menu 51

- **Casio Classpad**



Calculator Commands: Arrangements on Technology

- **Mathematica**
  - FactorialPower

```
FactorialPower[n, r]
```

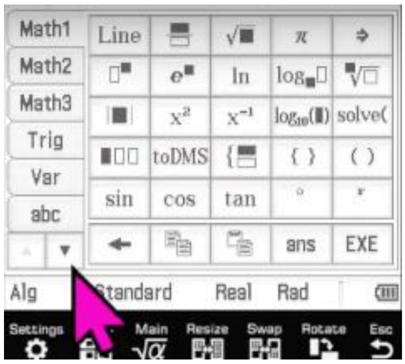
  - OR make your own:

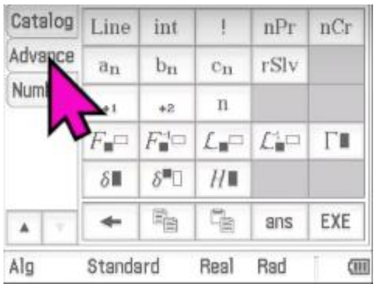
```
npr[n_, r_] := n ! / (n - r) !
```

- **TI-Nspire**
  - Menu 52

    $^{n}P_{r}(n, r)$

- **Casio Classpad**





$^{n}P_{r}(n, r)$

Space for Personal Notes

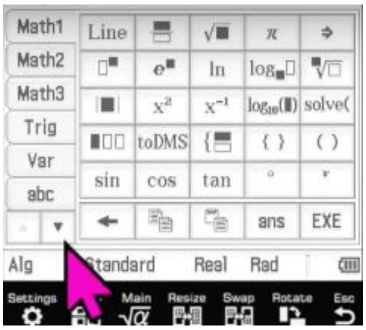
Calculator Commands: Combinations of Technology

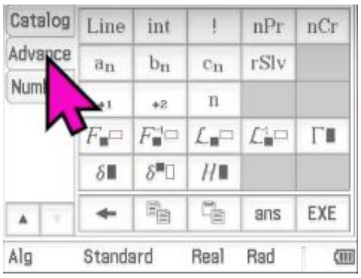
- **Mathematica**
  - Binomial $[n, r]$

- **TI-Nspire**
  - Menu 53

$${}^{n}C_{r}\ (n, r)$$

- **Casio Classpad**





$${}^{n}C_{r}\ (n, r)$$

Space for Personal Notes

Section D: Exam 2 Questions (31 Marks)

INSTRUCTION:

- Regular: 31 Marks. 5 Minutes Reading. 45 Minutes Writing.
- Extension: 31 Marks. 5 Minutes Reading. 31 Minutes Writing.

Question 6 (1 mark) **[3.1.1]**

A spinner is divided into 4 regions labelled A, B, C and D. Region A takes up half the spinner, while B, C and D are equal in size. The spinner is spun once. What is the probability of landing on either B or D?

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. $\frac{1}{3}$

D. $\frac{1}{6}$

$$\boxed{\frac{1}{6}+\frac{1}{6}=\frac{1}{3}}$$

Question 7 (1 mark) **[3.1.2]**

In a Methods class of 30 students, 16 study Chemistry, 18 study Biology, and 6 study neither. How many students study both Chemistry and Biology?

A. 4

B. 10

C. 6

D. 8

$$\boxed{16+18-24=10}$$

Space for Personal Notes

Question 8 (1 mark) **[3.1.3]**

Events A and B are such that $\Pr(A) = 0.4, \Pr(B) = 0.3$ and $\Pr(A \cap B) = 0$. Which of the following is true?

A. A and B are independent but not mutually exclusive.

B. A and B are mutually exclusive but not independent.

C. A and B are both independent and mutually exclusive.

D. A and B are neither independent nor mutually exclusive.

Question 9 (1 mark) **[3.1.4]**

A biased coin has a probability of 0.6 of landing heads. It is tossed twice. What is the probability that the second toss is heads, given that at least one toss is heads?

A. $\frac{3}{7}$

B. $\frac{5}{7}$

C. $\frac{9}{25}$

D. $\frac{17}{25}$

$$\Pr(HH) = 0.36, \Pr(HT) = 0.24, \Pr(TH) = 0.24, \Pr(TT) = 0.16$$

$$\text{Therefore } \frac{0.36 + 0.24}{0.84} = \frac{5}{7}$$

Question 10 (1 mark) **[3.3.1]**

How many different 3-digit numbers can be formed using the digits $2, 4, 6, 8, 0$ if repetition is not allowed and the number cannot begin with 0?

A. 48

B. 60

C. 64

D. 72

$$4 \times 4 \times 3 = 48$$

Space for Personal Notes

Question 11 (1 mark) **[3.3.2]**

From 6 books, 3 are selected to be arranged on a shelf, but two particular books must never be placed next to each other. How many such arrangements are possible?

A. 120

B. 96

C. 104

D. 84

Without restriction $3! \times \binom{6}{3} = 120$.

Two restricted books are selected together in 4 ways. Then they are next to each other in $2 \times 2 = 4$ ways. Thus $120 - 4 \times 4 = 104$

Question 12 (1 mark) **[3.3.3]**

A committee of 3 is chosen from 4 men and 5 women. What is the probability that the committee has more women than men?

A. $\frac{25}{42}$

B. $\frac{5}{9}$

C. $\frac{20}{56}$

D. $\frac{2}{3}$

$$\frac{\binom{4}{1}\binom{5}{2} + 1 \times \binom{5}{3}}{\binom{9}{3}}$$

Question 13 (1 mark) **[3.4.1]**

Which of the following is equal to ${}^{12}C_9$?

A. ${}^{9}C_3$

B. ${}^{15}C_6$

C. ${}^{12}C_3$

D. ${}^{11}C_4$

Space for Personal Notes

Question 14 (1 mark) **[3.4.2]**

A group of 7 friends are planning to form one team to play a game. The team must include at least one person, and any combination of the 7 friends can make up the team. How many different teams can they form?

A. 128

B. 127

C. 126

D. 124

$$\boxed{2^7 - 1 = 127}$$

Space for Personal Notes

Question 15 (10 marks)

Joseph has a set of 10 cards. Four of the cards have the number 3 printed on them, and the remaining six cards have the number 4 printed on them. He selects three cards at random, without replacement, and adds the numbers on the selected cards.

a. Show that the probability that the total sum of the numbers on the three selected cards is 11 is $\frac{1}{2}$. (3 marks) **[3.3.1] [3.3.3]**

Must have 2 cards with 4 and one card with 3. (1M)

This selction is possible in $\binom{4}{1} \times \binom{6}{2} = 60$ ways. (1M)

Cards can be selected in a total of $\binom{10}{3} = 120$ ways.

Thus the probability is $\frac{60}{120} = \frac{1}{2}$. (1A)

b. Complete the following table showing the probability that the three cards sum to the given number. (3 marks) **[3.1.1] [3.3.3]**

1A for each other than 1/2.

Sum	9	10	11	12
Probability	$\frac{1}{30}$	$\frac{3}{10}$	$\frac{1}{2}$	$\frac{1}{6}$

c. Let the event R be 'the sum of the numbers on the three selected cards is 11'. Let event S be 'the first card selected has the number 3 on it'. Determine whether events R and S are independent. Justify your answer. (3 marks) **[3.1.3]**

$\Pr(R) = 0.5$ and $\Pr(S) = 0.4$. (1M)

We have that $\Pr(R \cap S) = \Pr(3, 4, 4) = \frac{4}{10} \times \frac{6}{9} \times \frac{5}{8} = \frac{1}{6}$ (1M).

Not independent since $\Pr(R \cap S) = \frac{1}{6} \neq \Pr(R)\Pr(S) = \frac{1}{5}$. (1A)

d. Determine whether events R and S are mutually exclusive. Justify your answer. (1 mark) **[3.1.3]**

Not mutually exclusive since $\Pr(R \cap S) \neq 0$. (1A)

Space for Personal Notes

Question 16 (12 marks)

Rei and Subu are competing against each other in an Integration Bee. The probability of Rei winning the first round is 0.3. If Subu wins a round, the probability that he wins the next round is 0.5, but if he loses a round the probability that he wins the next round is only 0.1.

The match goes on for 4 rounds unless someone wins three rounds first and each round has a winner. It is possible for the match to end in a draw if both Rei and Subu win two rounds. All answers to this question should be exact unless stated otherwise.

a. Find the probability that Rei will win. (3 marks) **[3.1.4]**

Rei wins if:
- Rei wins 3 in a row: $\Pr(RRR) = 0.3 \cdot 0.9 \cdot 0.9$
- Rei loses the first, then wins next 3: $\Pr(SRRR) = 0.7 \cdot 0.5 \cdot 0.9 \cdot 0.9$
- Rei wins first, loses second, wins next two: $\Pr(RSRR) = 0.3 \cdot 0.1 \cdot 0.5 \cdot 0.9$
- Rei wins first two, loses third, wins fourth: $\Pr(RRSR) = 0.3 \cdot 0.9 \cdot 0.1 \cdot 0.5$

(1M for 4 cases, 1M for calculating probability of at least 2 cases correctly)

$$\begin{aligned}\Pr(\text{Rei wins}) &= 0.3 \cdot 0.9 \cdot 0.9 + 0.3 \cdot 0.9 \cdot 0.1 \cdot 0.5 + 0.3 \cdot 0.1 \cdot 0.5 \cdot 0.9 + 0.7 \cdot 0.5 \cdot 0.9 \cdot 0.9 \\ &= 0.243 + 0.0135 + 0.0135 + 0.2835 = 0.5535 \quad (1\text{A})\end{aligned}$$

b. Find the probability that Subu will win. (1 mark) **[3.1.4]**

Similar to above but now with Subu.

$$\begin{aligned}\Pr(\text{Subu wins}) &= 0.7 \cdot 0.5 \cdot 0.5 + 0.3 \cdot 0.1 \cdot 0.5 \cdot 0.5 + 0.7 \cdot 0.5 \cdot 0.1 \cdot 0.5 + 0.7 \cdot 0.5 \cdot 0.5 \cdot 0.1 \\ &= 0.2175 \quad (1\text{A})\end{aligned}$$

c. Let X be the number of rounds played until the match is finished.

i. Find $\Pr(X = 3)$. (2 marks) **[3.1.4]**

$\Pr(X = 3) = \Pr(RRR) + \Pr(SSS) = 0.3 \cdot 0.9 \cdot 0.9 + 0.7 \cdot 0.5 \cdot 0.5 = 0.418$ (1M use two cases, 1A).

ii. Find $\Pr(X = 4)$. (1 mark) **[3.1.4]**

$\Pr(X = 4) = 1 - 0.418 = 0.582.$ (1A)

d. Find the probability that Subu wins given that there are three rounds played. (2 marks) **[3.1.4]**

$$\Pr(S \mid 3 \text{ rounds}) = \frac{0.7 \times 0.5 \times 0.5}{0.418} = \frac{175}{418}. \text{ (1M conditional probability, 1A)}$$

The integration bee organisers decide that ending a match in a draw is not a satisfying outcome. If the match is a draw, a final deciding round is played to determine the winner.
Subu has nerves of steel and thrives under pressure, so if a deciding round is reached, the probability that Subu wins it is 0.95.

e. Find the probability that Subu wins his match against Rei. (2 marks) **[3.1.4]**

$\Pr(\text{5th round is played}) = 1 - 0.5535 - 0.2175 = 0.229.$ (1M)
Thus $\Pr(\text{Subu wins}) = 0.2175 + 0.229 \cdot 0.95 = 0.43505.$ (1A)

f. If the probability that Subu wins the match is 0.4. Find his probability of beating Rei in a deciding round. Give your answer correct to four decimal places. (1 mark) **[3.1.4]**

$$\boxed{0.2175 + 0.229 \times p = 0.4 \implies p = 0.7969}$$

Space for Personal Notes

Section E: Extension Exam 1 (10 Marks)

INSTRUCTION:

- Regular: Skip
- Extension: 10 Marks. 15 Minutes Writing.

Question 17 (5 marks)

On Sunday afternoons, Lena goes hiking with a probability of 0.25, visits a museum with a probability of 0.35, or stays at home. If she goes hiking, the probability that she spends more than \$50 on gear and travel is 0.7. If she visits the museum, the probability that she spends more than \$50 on tickets and food is 0.8. If she stays at home, she orders takeaway for \$10.

a. Find the probability that Lena visits the museum and spends less than \$50. (1 mark)

$$\begin{aligned}\Pr(\text{Museum and spend } < \$50) &= \Pr(\text{Museum}) \times \Pr(\text{Spend } < \$50 \mid \text{Museum}) \\ &= 0.35 \times (1-0.8) = 0.35 \times 0.2 = 0.07 \quad \text{(1A)}\end{aligned}$$

b. Given that she spends less than \$50, find the probability that she visited the museum. (4 marks)

Let M be the event "visits museum" and L be the event "spends less than \$50". We have that

$$\Pr(M \mid L) = \frac{\Pr(M \cap L)}{\Pr(L)} \quad \text{(1M)}$$

We already have:

$$\Pr(M \cap L) = 0.07$$

Now compute $\Pr(L)$, the total probability that she spends less than \$50:

$$\begin{aligned}\Pr(L) &= \Pr(\text{Hiking} \cap L) + \Pr(\text{Museum} \cap L) + \Pr(\text{Stays home} \cap L) \quad \text{(1M)} \\ &= \frac{1}{4} \times \frac{3}{10} + 0.07 + (1 - 0.25 - 0.35) \\ &= \frac{3}{40} + \frac{28}{400} + \frac{160}{400} \\ &= \frac{218}{400} \quad \text{(1A)}\end{aligned}$$

Therefore:

$$\Pr(M \mid L) = \frac{28/400}{218/400} = \frac{28}{218} = \frac{14}{109} \quad \text{(1A)}$$

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Question 18 (5 marks)

The digits 4,5,6,7,7,7 can be arranged to form many different 6-digit numbers.

a. How many distinct 6-digit numbers can be formed using all six digits? (2 marks)

$\frac{6!}{3!} = 6 \times 5 \times 4 = 120$ (1M for dividing by 3! or just doing $6 \times 5 \times 4$, 1A)

b. How many of these 6-digit numbers begin and end with an odd digit? (3 marks)

Case 1: Start with 5 end with 7. There are $\frac{4!}{2!} = 12$ ways (1M)
Case 2: Start with 7 end with 5. There are $\frac{4!}{2!} = 12$ ways.
Case 3: Start with 7 end with 7. There are $4! = 24$ ways. (1M)
Thus a total of $12 + 12 + 24 = 48$ numbers. (1A)

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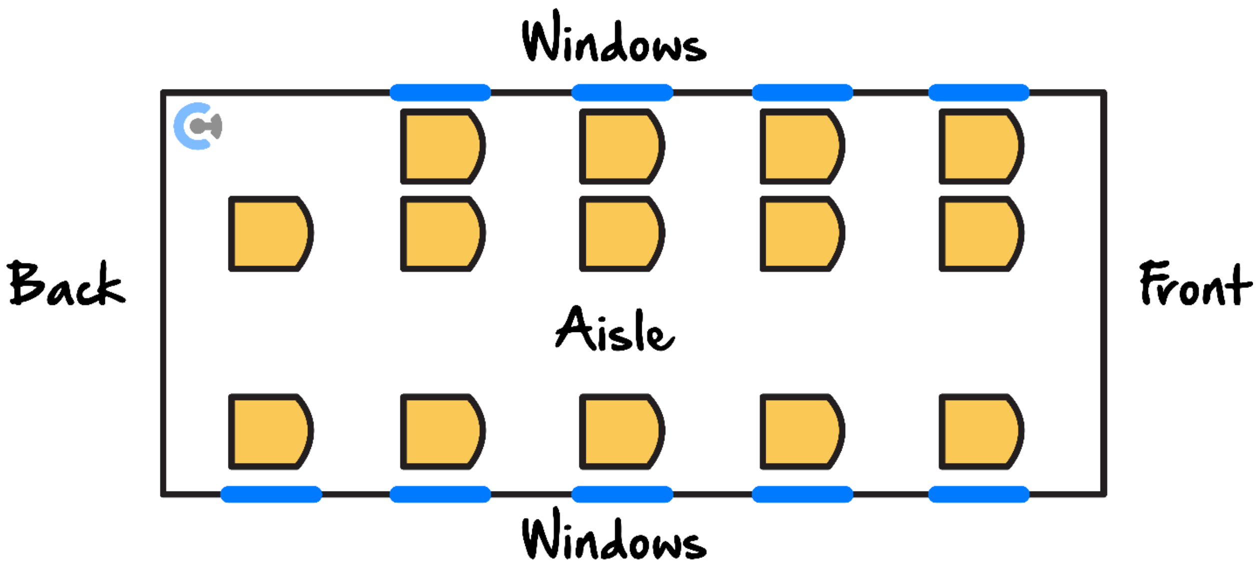
Section F: Extension Exam 2 (13 Marks)

INSTRUCTION:

- Regular: Skip
- Extension: 13 Marks. 20 Minutes Writing.

Question 19 (9 marks)

A small shuttle has 14 seats for passengers. The seats are arranged in 4 rows of 3 seats and a back row of 2 seats, as shown in the diagram below. 12 passengers board the shuttle.



a. How many different seating arrangements are there for the 12 passengers? Give your answer in the form $\frac{a!}{b}$ where a and b are positive integers. (1 mark)

$$\boxed{\frac{14!}{2!} = \frac{14!}{2}. \text{ (1A)}}$$

These 12 passengers include 2 married couples (Mr. and Mrs. Blue and Mr. and Mrs. Green), 5 students and 3 engineers.

b. The 3 engineers must sit in the front row. Each of the 5 students sit at a window seat. Mr. and Mrs. Blue sit in the same row on the same side of the aisle. Mr. and Mrs. Green also sit together in another row, also on the same side of the aisle. How many possible seating arrangements are there? (4 marks)

Engineers in $3! = 6$ ways. (1M)
There are 9 windows total but two are taken by the engineers and two are taken by the married couples. Thus only 5 windows left for 5 students. So students are arranged in $5! = 120$ ways. (1M)
Married couples have $(3 \times 2) \times (2 \times 2) = 24$ ways (1M).
Thus total ways is $6 \times 120 \times 24 = 17280$ ways. (1A)

c. If instead the 12 passengers are assigned seats at random, find the probability that Mrs. Blue is seated directly behind a student and Mrs. Green is seated in the front row. (4 marks)

Mrs Green is in the front row in 3 ways. (1M)
Then there are 10 seats that Mrs Blue can sit in that could be behind a student. So 10 options for where Mrs Blue sits. (1M)
There are 5 options for the student that sits in front of Mrs Blue.
Then remaining people can choose seat in $P(11, 9) = \frac{11!}{2}$. (1M)
Thus the overall probability is:

$$\frac{3 \times 10 \times 5 \times \frac{11!}{2}}{\frac{14!}{2}} = \frac{25}{364} \quad \text{(1A)}$$

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Question 20 (4 marks)

The word ARGENTINA contains nine letters, including the consonants R, G, N, T and the vowels A, E, I.

a. How many distinct arrangements can be formed using all nine letters? (1 mark)

$$\frac{9!}{2!2!} = 90720. \text{ (1A)}$$

b. How many of these arrangements begin with a consonant, followed by a vowel, then a consonant, and so on — alternating consonants and vowels throughout? (3 marks)

Arrange the 5 consonants in $\frac{5!}{2!} = 60$ ways. (1M)

Then we place vowels between the consonants in $\frac{4!}{2!} = 12$ ways. (1M)

Thus total ways is $60 \times 12 = 720$ ways. (1A)

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