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VCE Mathematical Methods ½

Combination & Permutation [0.14]

Workshop Solutions

Error Logbook:

New Ideas/Concepts	Didn't Read Question
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Algebraic/Arithmetic/ Calculator Input Mistake	**Working Out Not Detailed Enough**
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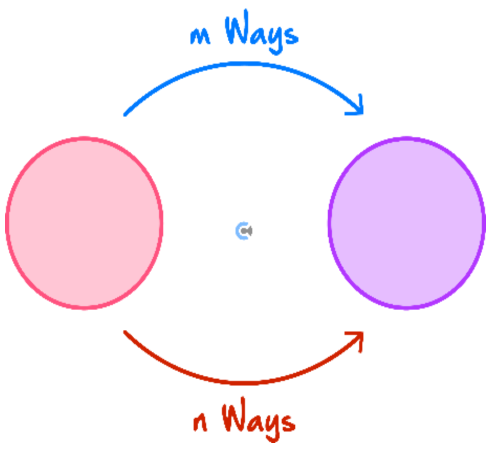
Section A: Recap

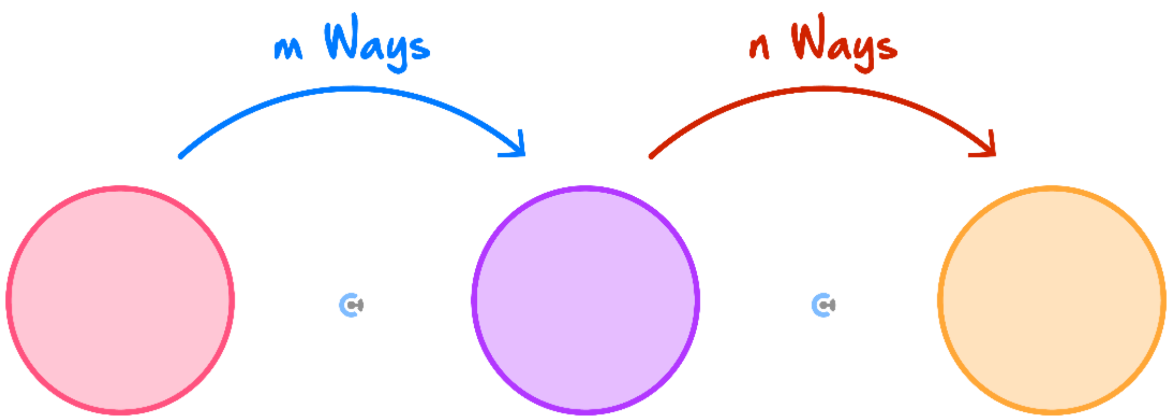
Addition Principle



- Associated with the use of the word "**OR**."

$$\text{Total Possibilities} = m + n$$

The Multiplication Principle



- Associated with the use of the word "**AND**."

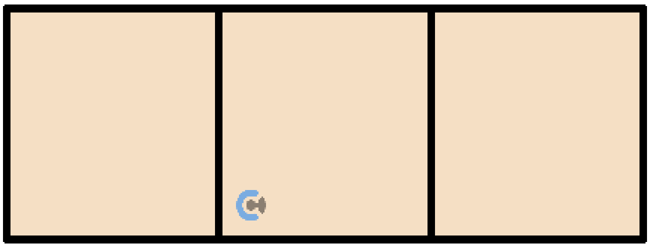
$$\text{Total Possibilities} = m \times n$$

Arrangements

- **Definition:** It is a study of a number of ways to arrange/order things.

Box Diagram for Arrangements

- **Definition:** We can use it to write down a number of arrangements for each position represented by each box.



Arrangement

- Generally:

$$\text{Ways to arrange/order } n \text{ many things for } r \text{ spots} = \frac{n!}{(n-r)!}$$

- We call this ${}^{n}P_{r}$.

$$ {}^{n}P_{r} = \frac{n!}{(n-r)!}$$

Composite Arrangements

- **Definition:** Occurs when an arrangement happens within another arrangement.
- **Steps:**
  - Consider each group as one object and find the arrangements.
  - Consider the arrangements within the groups and multiply.

Space for Personal Notes

Arrangements with Restrictions

- **Definition:** The general principle to deal with restrictions is to:
  - **Use the boxes.**
  - **Fill in the number of options for the slot that has the restriction first.**

Selection

- Generally:

$$\text{Ways to select } r \text{ things from } n \text{ many things} = \frac{{}^{n}P_{r}}{r!}$$

- We call this ${}^{n}C_{r}$,

$${}^{n}C_{r} = \frac{{}^{n}P_{r}}{r!} = \frac{n!}{r!\,(n-r)!}$$

- Where $r =$ number of selection spots.

Probability with Arrangements

$$\Pr = \frac{n\,(\text{Wanted Arrangements})}{n\,(\text{Total Arrangements})}$$

Probability with Selections

$$\Pr = \frac{n\,(\text{Wanted Selections})}{n\,(\text{Total Selections})}$$

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Section B: Warm Up (11 Marks)

INSTRUCTION:

- Regular: 11 Marks. 11 Minutes Writing.
- Extension: Skip

Question 1 (4 marks)

a. A student council needs to select a president and a vice president from a group of 6 students. In how many ways can these positions be filled? (1 mark)

$$\boxed{6 \times 5 = 30 \text{ ways. } (1A)}$$

b. A company needs to form a 3-member advisory committee from a group of 8 employees. In how many ways can this committee be chosen? (2 marks)

$$\boxed{8 \text{ choose } 3 = \frac{8!}{5!3!} = \frac{8 \times 7 \times 6}{6} = 56 \text{ ways. } (1M \text{ combinations, } 1A).}$$

c. In how many different ways can 5 distinct books be arranged in a row on a shelf? (1 mark)

$$\boxed{5! = 5 \times 4 \times 3 \times 2 = 60 \times 2 = 120 \text{ ways. } (1A)}$$

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Question 2 (4 marks)

Consider the following restrictions on forming groups and arrangements.

a. Five people (Alice, Bob, Charlie, Dave, and Eve) are to sit in a row, but Alice and Bob must sit next to each other. In how many different ways can they be seated? (2 marks)

Can treat Alice and Bob as one object. Then $4! = 24$ ways of arranging the group (1M). Now arrange within the group in $2! = 2$ ways.
Thus a total of $24 \times 2 = 48$ ways. (1A)

b. A club of 7 members want to form a 4-person team, but one particular member must be in the team. How many different teams can be formed? (2 marks)

One person is fixed on the team, thus we select 3 from 6. (1M)
$\frac{6!}{3!3!} = \frac{6 \times 5 \times 4}{6} = 20$ ways. (1A)

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Question 3 (3 marks)

A group of 10 students contain 4 boys and 6 girls. A committee of 3 students is chosen at random. What is the probability that the committee consists of exactly 2 girls and 1 boy?

Favourable outcomes $= \binom{4}{1} \times \binom{6}{2} = 4 \times 15 = 60$ (1M).

Total outcomes $= \binom{10}{3} = \frac{10 \times 9 \times 8}{3!} = \frac{720}{6} = 120.$ (1M)

Thus probability is $\frac{60}{120} = \frac{1}{2}$ (1A)

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Section C: Exam 1 Questions (18 Marks)

INSTRUCTION:

- Regular: 18 Marks. 3 Minutes Reading. 27 Minutes Writing.
- Extension: 18 Marks. 3 Minutes Reading. 18 Minutes Writing.

Question 4 (2 marks)

A men's department store sells 3 different suit jackets, 6 different shirts, 8 different ties, and 4 different pairs of pants. How many different suits consisting of a jacket, shirt, tie, and pants are possible?

$3 \times 6 \times 8 \times 4 = 576$ (1M multiplication, 1A)

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Question 5 (4 marks)

a. How many eight-digit numbers can be formed if the last number cannot be 1? (2 marks)

You can have repeated digits. 9 options for first digit and 9 options for last digit.

$$9^2 \times 10^6 = 81{,}000{,}000$$

(1M for 9^2, 1A)

b. How many 4-digit odd numbers can be formed if no digit can be repeated? (2 marks)

$$8 \times 8 \times 7 \times 5 = 2{,}240 \text{ (1A)}$$

{you have 5 options for last number 1,3,5,7,9 (1M, for the $\times 5$)
then, you need to make three-digit number with 9 remaining possible numbers. BUT, the first digit can't be zero. So, there are $8 \times 8 \times 7$ ways to make a three-digit number with 9 digits incl. zero.
So, answer is $8 \times 8 \times 7 \times 5$.}

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Question 6 (3 marks)

For the following question, leave your answer in terms of factorials.

In how many ways can we rearrange the letters in "MATHS IS FUN", if:

a. There are no restrictions? (1 mark)

There are 2 S.

$$\frac{10!}{2!} = \frac{10!}{2} \text{ (1A)}$$

b. The first and last letters must be vowels? (2 marks)

$3 \times 2 = 6$ ways for first and last to be vowel. (1M)

$\frac{8!}{2!} = \frac{8!}{2}$ ways for middle since 2 S.

Total ways $= 3 \times 8!$ (1A)

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Question 7 (3 marks)

For the following question, leave your answer in terms of ${}^{n}P_k$ or ${}^{n}C_k$.

You want to choose a committee of 5 people from 7 men and 8 women.

a. In how many ways can this be done? (1 mark)

In (a) it doesn't matter if the 5 people are men or women. So we are choosing 5 people from 15 and there are ${}^{15}C_5$ ways of doing this.

b. How many ways can this be done if you want more women than men on the committee? (2 marks)

In (b) we need either 3 women (and 2 men) or 4 women (and 1 man) or 5 women. So there are ${}^{8}C_3 \times {}^{7}C_2 + {}^{8}C_4 \times {}^{7}C_1 + {}^{8}C_5$ ways of doing this.

1M for summing different combinations.
1A

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Question 8 (2 marks)

Samuel is driving from Melbourne CBD to Glen Waverly, and he is blasting his Taylor Swift playlist on Spotify on the way home.

He has to choose three songs for the last few minutes of his drive. If there are nine songs that he feels are appropriate for that time slot, then how many ways can he choose and arrange to play three of those nine songs?

$$^9P_3 = 9 \times 8 \times 7 = 504$$

(1M, arrangement, 1A)

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Question 9 (4 marks)

Solve for n, if $n \in N$.

a. $\frac{(n-1)!}{(n-2)!} = 8$. (1 mark)

$$\boxed{n = 9}$$

b. $\frac{3(n+1)!}{(n-1)!} = 126$. (3 marks)

$$\frac{3(n+1)n(n-1)!}{(n-1)!} = 126 \text{ (1M)}$$

$$3n(n+1) = 126$$

$$3n^2 + 3n - 126 = 0$$

$$n^2 + n - 42 = 0 \text{ (1M for correct quadratic)}$$

$$(n+7)(n-6) = 0$$

$$n = -7, n = 6$$

$n = 6$ as $n \neq -7$ (1A)

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Section D: Exam 2 Questions (32 Marks)

INSTRUCTION:

- Regular: 32 Marks. 5 Minutes Reading. 48 Minutes Writing.
- Extension: 32 Marks. 5 Minutes Reading. 32 Minutes Writing.

Question 10 (1 mark)

How many different rearrangements are there of the letters in the word TATARS if the two A's are never adjacent?

A. 24

B. 120

C. 180

D. 220

In[81]:= $\frac{6!}{2!\times 2!}-\frac{5!}{2!}$

Out[81]= 120

Question 11 (1 mark)

How many four-digit numbers can be formed using the digits 1, 2, 3, 4, 5 and 6 at most once?

A. ${}^{6}C_4$

B. 4!

C. 6!

D. $6\times 5\times 4\times 3$

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Question 12 (1 mark)

A man is dealt 4 spade cards from an ordinary deck of 52 cards. If he is given five more random cards, what is the probability that none of them are spades?

A. $\frac{^{39}C_1}{^{48}C_5}$

B. $\frac{^{39}C_2}{^{48}C_5}$

C. $\frac{^{39}C_4}{^{48}C_5}$

D. $\frac{^{39}C_5}{^{48}C_5}$

Question 13 (1 mark)

Suppose there are 12 students, among whom are three students, Rei, Subu, and Liz. We want to send four students (chosen from the 12 students) to a convention. How many ways can this be done so that exactly two of the three are included?

A. 32

B. 64

C. 88

D. 108

$$\boxed{^3C_2 \times {^9C_2}}$$

Question 14 (1 mark)

What is the probability of picking a permutation of the word UTOPIA that begins and ends with a vowel from the set of all possible permutations of that word?

A. $\frac{1}{5}$

B. $\frac{1}{30}$

C. $\frac{2}{5}$

D. $\frac{3}{5}$

```
(*4 vowels and 6 letters total*)
In[164]:= 4*4!*3 / 6!
Out[164]= 2/5
```

Question 15 (1 mark)

In how many ways can four books be chosen from a collection of nine different books?

A. $4!$

B. $\frac{9!}{4!}$

C. 126

D. $9 \times 8 \times 7 \times 6$

```
In[165]:= Binomial[9, 4]

Out[165]= 126
```

Question 16 (1 mark)

In how many ways can four **identical** Contour Methods $1/2$ Workshop booklets and 3 identical Contour Specialist Maths $3/4$ Exams be arranged in a row?

A. 4×3

B. $\frac{7!}{4! \times 3!}$

C. $7! \times 3! \times 4!$

D. $4! \times 3!$

Question 17 (1 mark)

A standard deck of 52 playing cards with four suits (spades, hearts, clubs and diamonds) are shuffled and two cards are drawn at random, one after the other without replacement, what is the probability that the cards are of different suits?

A. $\frac{1}{12}$

B. $\frac{39}{51}$

C. $\frac{39}{52}$

D. $\frac{1}{52^2}$

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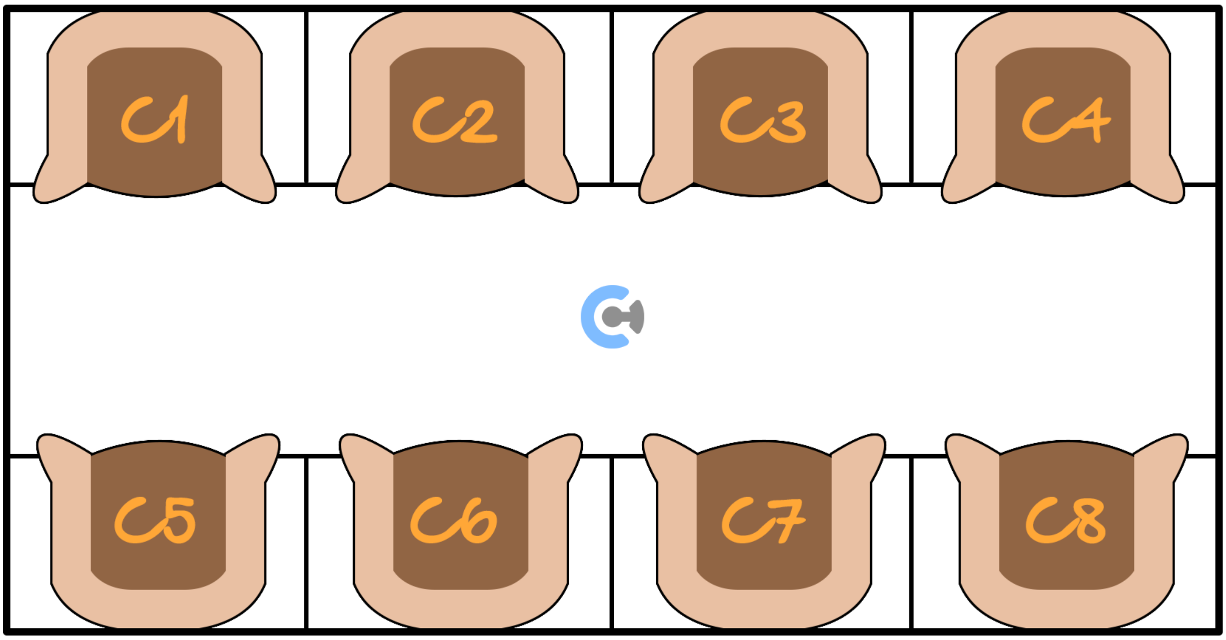
Question 18 (10 marks)

a. A railway carriage compartment consists of two bench seats facing each other, with each bench being able to accommodate four people.



i. How many ways can a party of eight people be seated in this compartment? (1 mark)

$$\text{Ways} = 8! = 40320$$

ii. How many ways can the eight passengers take their seats if Aiden and Alex must sit next to each other? (2 marks)

$$\text{Ways} = 2! \times (3 \times 2) \times 6! = 8640$$

1M for 6!
1A

b. There are thirteen boys and nine girls in the class. From this class, five students are randomly selected to form the class committee.

i. How many possibilities are there for the committee? (1 mark)

$$\text{Possibilities} = {}^{22}C_5 = 26334$$

ii. How many of the committees would contain at least one boy and one girl? (2 marks)

$$\text{Committees} = {}^{22}C_5 - {}^{13}C_5 - {}^{9}C_5 = 24921$$

1M for subtracting all boys and all girls.
1A

iii. The committee is to be seated around a table. In how many ways could the committee be seated? (1 mark)

$$\text{Ways} = 4! = 24$$

c. Consider the letters of **ZOOLOGICAL**.

i. How many distinct arrangements of all of the letters are possible? (1 mark)

$$\text{Arrangements} = \frac{10!}{3!2!} = 302400$$

ii. How many arrangements are possible if no letter **O** is placed next to another letter **O**? (2 marks)

Arrange other 7 letters first in:

$$\frac{7!}{2!} = 2520 \text{ ways (1M)}$$

Then put O between each of these letters in ${}^{8}C_3 = 56$ ways.
Thus $2520 \times 56 = 141120$ ways. (1A)

Question 19 (5 marks)

Michael buys five tickets in a raffle in which 20 tickets are sold. Three different tickets are to be drawn out without replacement for first, second, and third prizes. Find the probability that:

a. Michael wins all three prizes. (1 mark)

$\frac{5}{20} \times \frac{4}{19} \times \frac{3}{18} = \frac{1}{114}$ (1A)

b. Michael does not win a prize. (1 mark)

$\frac{15}{20} \times \frac{14}{19} \times \frac{13}{18} = \frac{91}{228}$ (1A)

c. Michael wins at least one prize. (1 mark)

$1 - \frac{91}{228} = \frac{137}{228}$ (1A)

d. Michael wins exactly one prize. (2 marks)

$$3 \times \frac{5}{20} \times \frac{15}{19} \times \frac{14}{18} = \frac{35}{76}$$

1M for 3 different ways.

1A.

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Question 20 (9 marks)

a. There are 10 children in your class but you can invite only 5 to your birthday party. How many different combinations of friends could you invite? Explain whether to use combinations or permutations. (2 marks)

${}^{10}C_5 = 252$ (1M, use combinations, 1A)

b. At a party, there are 75 people. Everybody shakes everybody's hand once. How many hands were shaken in total? (2 marks)

Hint: How many people are involved in shaking hands?

Need 2 people for a handshake to occur, thus,

$${}^{75}C_2 = 2775$$

(1M, recognise combinations where 2 are selected, 1A)

c. A postman has to deliver four letters to four different houses in a street. Unfortunately, the rain has erased the addresses, so he just distributes them randomly, one letter per house. What is the probability that every house gets the right letter? (2 marks)

There are $4! = 24$ ways to distribute letters (1M).
Only one scenario where all letters go to correct houses. Thus,

$\frac{1}{24}$. (1A)

d. In a lottery, you have to guess 6 out of 49 numbers.

What is the probability that you get all of them right? Give your answer in the form $a \times 10^{-b}$, where a is a number between 1 and 10 correct to 1 decimal place. (2 marks)

$^{49}C_6 = 13{,}983{,}816$ possible outcomes (1M)

prob$= \frac{1}{^{49}C_6} \approx 7.2 \times 10^{-8}$

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Section E: Extension Exam 1 (8 Marks)

INSTRUCTION:

- Regular: Skip
- Extension: 8 Marks. 2 Minutes Reading. 12 Minutes Writing.

Question 21 (8 marks)

a. For the following parts, express your answers in the form $\frac{a!}{b}$ for positive integers a and b. Find the number of different ways in which the 12 letters of the word STRAWBERRIES can be arranged,

i. If there are no restrictions. (2 marks)

There are 12 letters, 3R, 2E, 2S rest are unique.
Thus $\frac{12!}{3!2!2!} = \frac{12!}{24}$ (1M, dividing by 3!2!2!, 1A)

ii. If the 4 vowels A, E, E, I must all be together. (3 marks)

Vowels are arranged in $\frac{4!}{2!} = 12$ ways (1M).
Treat the vowels as one letter, then we are arranging 9 letters, where there are 3R and 2S.
Thus $\frac{9!}{3!2!} = \frac{9!}{12}$ ways. (1M for 9!).
Total ways is thus $\frac{9!}{12} \times 12 = 9! = \frac{9!}{1}$. (1A).

b. A team is being selected for an Arctic expedition.

i. A team of 4 explorers is chosen from a group of candidates. If the order of choosing is not taken into account, the number of ways to select the team is 3876. How many ways are there if the order of choosing is taken into account? Leave your answer in the form $a \times b$ for positive integers a and b. (1 mark)

$\boxed{3876 \times 4! = 3876 \times 24 \text{ (1A).}}$

ii. 4 explorers are chosen to go on the expedition. Each of these explorers can take 3 personal possessions with them. How many different ways can these possessions be arranged in a row if each explorer's possessions are kept together? Leave your answer in the form $a \times b^n$ for positive integers a, b and n. (2 marks)

$\boxed{4! \times (3!)^4 = 24 \times 6^4 \text{ (1M for } (3!)^4\text{, 1A)}}$

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Section F: Extension Exam 2 (17 Marks)

INSTRUCTION:

- Regular: Skip
- Extension: 17 Marks. 3 Minutes Reading. 25 Minutes Writing.

Question 22 (17 marks)

A bag contains 12 marbles, of which 5 are blue and 7 are red. A sample of 4 marbles is drawn without replacement.

a. In how many ways can any 4 marbles be selected from the 12 marbles? (1 mark)

$\binom{12}{4} = 495.$ (1A)

b. In how many ways can exactly 2 blue marbles and 2 red marbles be selected? (2 marks)

2 blue marbles from 5 in $\binom{5}{2} = 10$ ways (1M).

2 red marbles from 7 in $\binom{7}{2} = 21$ ways.

Thus $10 \times 21 = 210$ ways. (1A)

c. Hence, what is the probability that a randomly drawn set of 4 marbles consists of exactly 2 blue marbles and 2 red marbles? (1 mark)

$\frac{210}{495} = \frac{14}{33}$ (1A)

d. What is the probability that the sample contains at least 1 blue marble? (2 marks)

No blue in $\binom{7}{4} = 35$ ways. (1M)

Thus probability of at least 1 blue $= 1 - \frac{35}{495} = \frac{92}{99}$ (1A).

e. Suppose there is a bag with N total marbles, of which K are blue and N-K are red. A sample of n marbles is drawn at random. Using the logic from earlier parts, derive a general formula for the probability that the sample contains exactly k blue marbles. Leave your answer in terms of the binomial coefficient $\binom{n}{r} = {}^{n}C_{r}$. (2 marks)

$$\frac{\binom{K}{k} \times \binom{N-K}{n-k}}{\binom{N}{n}}$$

(1M for numerator, 1M for denominator)

Note: The probability you derived in the previous part follows a well-known probability distribution called the hypergeometric distribution.

Suppose we still have 5 blue marbles and 7 red marbles, however, the marbles are now randomly numbered 1 to 12 for identification.

f. How many different ways can we draw 4 marbles where: (2 marks)

- The lowest-numbered marble must be blue and
- The highest-numbered marble must be red?

Lowest number marble is blue in 5 ways and highest numbered marble is red in 7 ways. (1M).

Remaining two marbles can be drawn in $\binom{10}{2} = 45$ ways.

Thus total ways is $45 \times 5 \times 7 = 1575$ (1A).

g. Suppose now that 6 marbles are drawn and then arranged in a row. How many different arrangements are possible if: (2 marks)

- The lowest-numbered marble must be blue and,
- The highest-numbered marble must be red?

$$5 \times 7 \times \binom{10}{4} \times 4! = 176400 \text{ ways. } (1\text{M for } 4!,\ 1\text{A})$$

h. If 3 blue marbles are drawn in a row, the bag is refilled with an extra 3 red marbles before drawing the fourth marble. What is the probability that the last marble drawn is blue? Give your answer correct to three decimal places. (5 marks)

Case 1: We draw 3 blues in a row

$$\Pr(3 \text{ blue}) = \frac{5 \times 4 \times 3}{12 \times 11 \times 10} = \frac{1}{22} \ (1\text{M}).$$

$$\Pr(4\text{th blue} \mid 3 \text{ blue}) = \frac{1}{22} \times \frac{2}{12} = \frac{1}{132} \ (1\text{A})$$

Case 2: We dont draw 3 blues in a row.

$$\Pr(2B, 1R) = \frac{\binom{5}{2}\binom{7}{1}}{\binom{12}{3}} = \frac{7}{22}$$

$$\Pr(1B, 2R) = \frac{\binom{5}{1}\binom{7}{2}}{\binom{12}{3}} = \frac{21}{44}$$

$$\Pr(0B, 3R) = \frac{\binom{5}{0}\binom{7}{3}}{\binom{12}{3}} = \frac{7}{44}$$

(1M for any of these probabilities).

Then $\Pr(4\text{th blue} \mid \text{not } 3 \text{ blue in a row}) = \frac{3}{9} \times \frac{7}{22} + \frac{4}{9} \times \frac{21}{44} + \frac{5}{9} \times \frac{7}{44} = \frac{161}{396}$ (1A)

Thus the final probability

$$\Pr(4\text{th is blue}) = \frac{1}{22} \times \frac{2}{12} + \frac{21}{22} \times \frac{161}{396} = \frac{383}{968} = 0.396. \quad (1\text{A})$$

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