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VCE Mathematical Methods ½
Probability Exam Skills [0.13]
Workshop Solutions

Error Logbook:

New Ideas/Concepts	Didn't Read Question
Pg / Q #: ______ Notes:	Pg / Q #: ______ Notes:
Algebraic/Arithmetic/ Calculator Input Mistake	**Working Out Not Detailed Enough**
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Section A: Recap

Definition

Sample Space (ε)

- The set of all possible outcomes in an experiment.
- For tossing two coins in a row, the sample space is:

$$\varepsilon = \{HH, HT, TH, TT\}$$

- For rolling a standard 6-sided dice, the sample space is:

$$\varepsilon = \{1, 2, 3, 4, 5, 6\}$$

- Total probability adds up to 1.

Definition

Calculating Probabilities for Equally Likely Outcomes

- When there is some number of equally likely outcomes, the probability of a "successful" outcome can be calculated as:

$$\Pr(success) = \frac{\text{number of successful outcomes}}{\text{number of total outcomes}}$$

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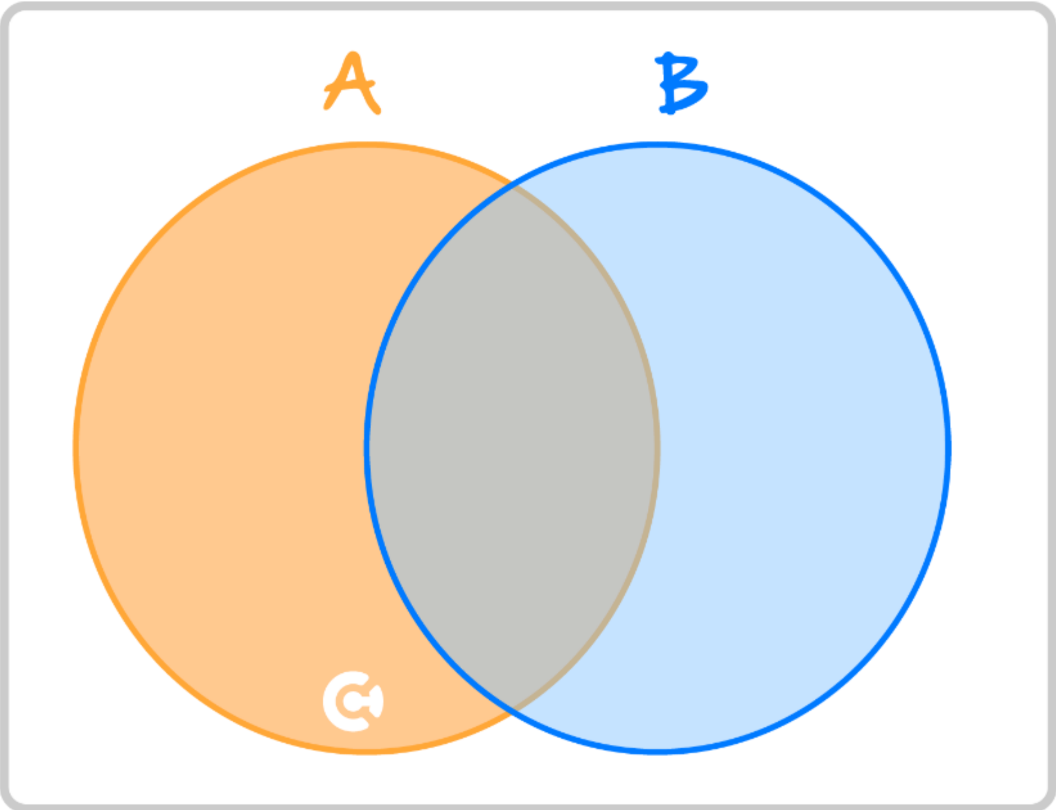
Definition

Union, Intersection, and Complement

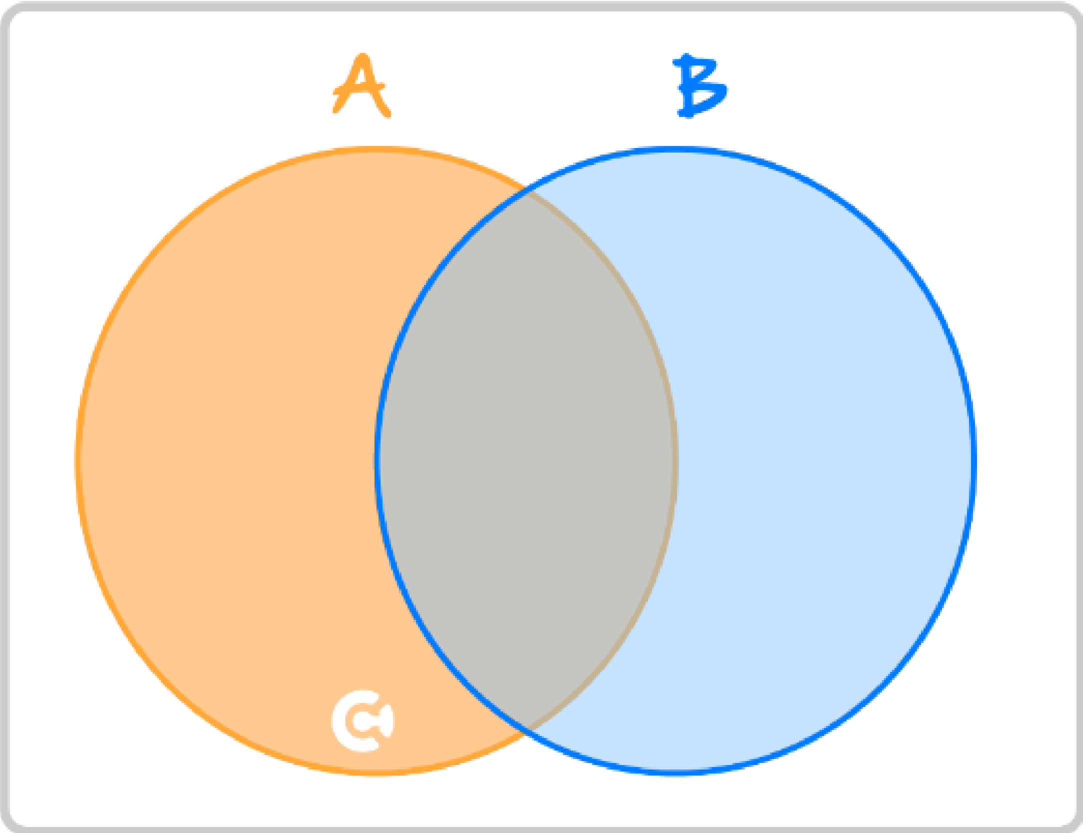


- $\boldsymbol{A \cup B =}$
  - Union of two events aka "or".
  - Equivalent to either event A **OR** event B **OR** BOTH occurring.
- $\boldsymbol{A \cap B =}$
  - Intersection of two events aka "and".
  - Equivalent to both event A **AND** event B occurring.
- $\boldsymbol{A' =}$
  - A complement aka "not".
  - Equivalent to event A NOT occurring.
  - E.g. if $A =$ dice rolled a 6, then $A' =$ dice rolled anything except a 6.

$$\mathbf{Pr}(A') = \mathbf{1} - \mathbf{Pr}(A)$$

Definition

Venn Diagram



- A Venn diagram is useful to visualise the two events.

Karnaugh Tables

- We can also represent probability problems using a Karnaugh map:

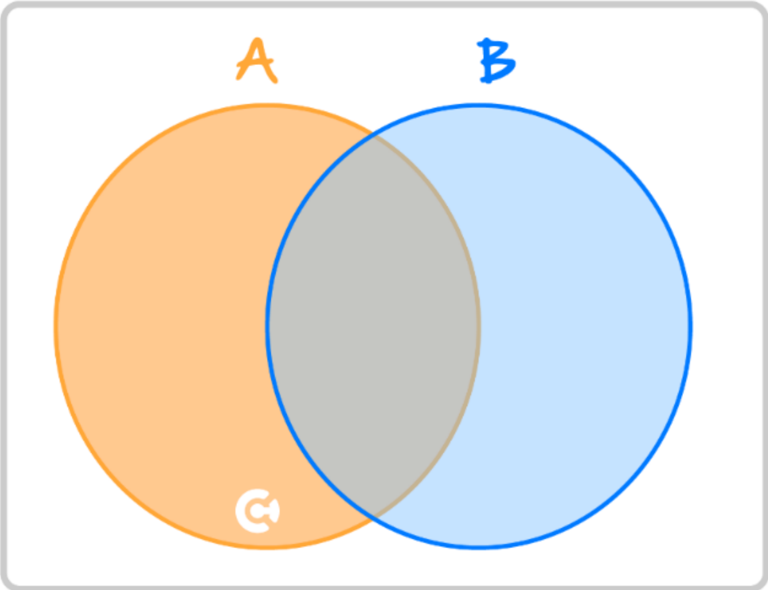
	B	B'	
A	$\Pr(A \cap B)$	$\Pr(A \cap B')$	$\Pr(A)$
A'	$\Pr(A' \cap B)$	$\Pr(A' \cap B')$	$\Pr(A')$
	$\Pr(B)$	$\Pr(B')$	1

  - The rows and columns add up to the last cell value.

- Remember the **total** probability must always add to 1.

Definition

The Addition Rule



- When we add the probabilities of A and B, we count the outcomes contained in $A \cap B$ **twice.**
- So, we must subtract one of them to get the probability of $A \cup B$:

$$\mathbf{Pr}(A \cup B) = \mathbf{Pr}(A) + \mathbf{Pr}(B) - \mathbf{Pr}(A \cap B)$$

Definition

Mutually Exclusive Events

- Two events A and B are **mutually exclusive** if they cannot occur at the same time.
- The probability of both A and B happening together is zero:

$$\mathbf{Pr}(A \cap B) = 0$$

Definition

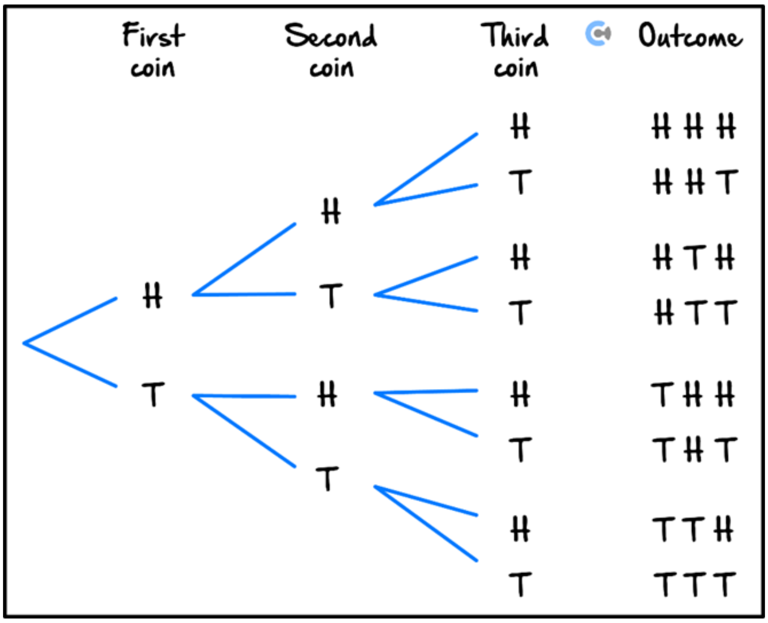
Independent Events

- Two events A and B are independent when the occurrence of one **does not affect** the likelihood of the other.
- When two events are independent:

$$\mathbf{Pr}(A \cap B) = \mathbf{Pr}(A) \times \mathbf{Pr}(B)$$

Tree Diagram

- Useful for multiple sequence events.
- To calculate the probability of a sequence, we **multiply the probabilities along the relevant branches.**
- For instance, the following tree diagram shows the outcomes of three successive coin tosses.



Conditional Probability

- Probability of A given B:

$$\mathbf{Pr}(\boldsymbol{A}|\boldsymbol{B}) = \frac{\mathbf{Pr}(\boldsymbol{A} \cap \boldsymbol{B})}{\mathbf{Pr}(\boldsymbol{B})}$$

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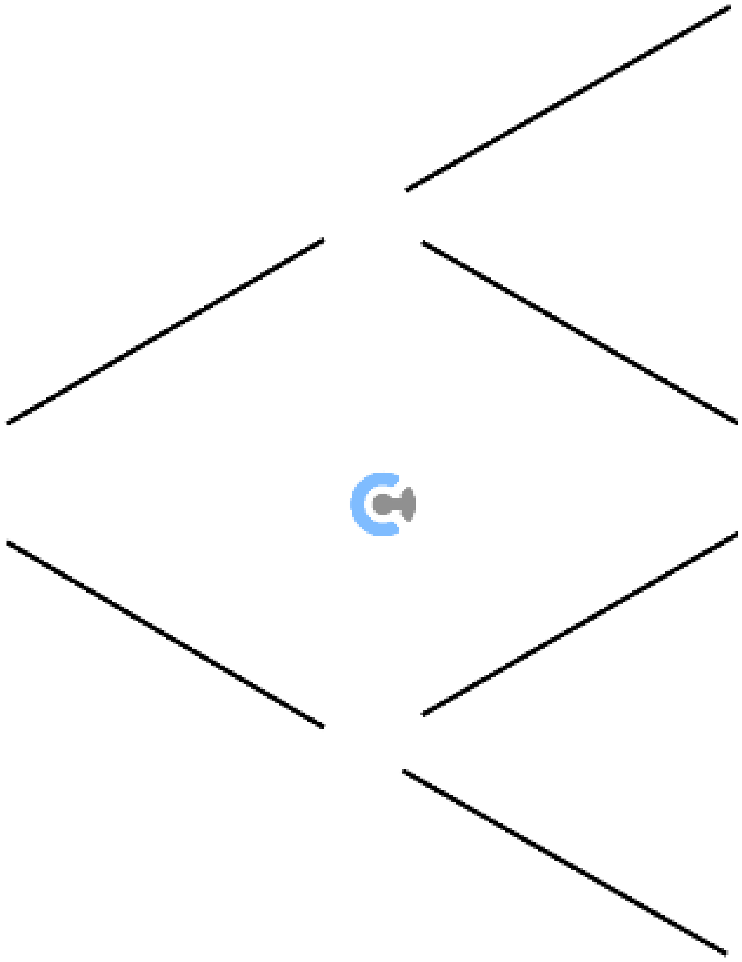
Tree Diagram for Condition Probability



- Tree diagram is perfect for conditional probability as each branch is a conditional probability.

$$\text{Each branch} = \mathbf{Pr(Leaf|Root)}$$

Conditional Probability with Independent Events

$$\mathbf{Pr}(A|B) = \mathbf{Pr}(A)$$

- If A and B are independent, the given condition does **not** affect the probability of the event.

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Section B: Exam 1 Questions (24 Marks)

INSTRUCTION:

- Regular: 24 Marks. 5 Minutes Reading. 35 Minutes Writing.
- Extension: 24 Marks. 5 Minutes Reading. 24 Minutes Writing.

Question 1 (8 marks)

Consider events A and B.

It is known that $\Pr(B) = \frac{1}{3}$, $\Pr(A|B) = \frac{2}{3}$ and $\Pr(A|B') = \frac{3}{8}$.

a. Find $\Pr(A \cap B)$. (2 marks)

$\Pr(A \cap B) = \Pr(A \mid B) \times \Pr(B)$ (1M)
$= \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$ (1A)

b. Find $\Pr(A \cap B')$. (1 mark)

$\Pr(A \cap B') = \Pr(A \mid B') \times \Pr(B')$
$= \frac{3}{8} \times \frac{2}{3} = \frac{6}{24} = \frac{1}{4}$ (1A)

c. Find $\Pr(A)$. (2 marks)

$\Pr(A) = \Pr(A \cap B) + \Pr(A \cap B')$ (1M)
$= \frac{2}{9} + \frac{1}{4} = \frac{17}{36}$ (1A)

d. Find $\Pr(A \cup B)$. (2 marks)

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B) \text{ (1M)} = \frac{1}{3} + \frac{17}{36} - \frac{2}{9} = \frac{7}{12} \text{ (1A)}$$

e. State whether events A and B are independent. (1 mark)

Not independent since $\Pr(A)\Pr(B) \neq \Pr(A \cap B)$.

Question 2 (6 marks)

Two fair dice, one blue and one white, are thrown simultaneously. Each die is numbered from 1 to 6.

Find the probability that:

a. Two sixes are thrown. (1 mark)

Since each die has a $\frac{1}{6}$ probability of rolling a six, the probability of rolling two sixes is:

$$P(\text{two sixes}) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36} \quad \text{(1A)}$$

b. Exactly one three is thrown. (1 mark)

The probability of rolling a 3 on the blue die and not on the white die is:

$$P(\text{blue} = 3, \text{ white} \neq 3) = \frac{1}{6} \times \frac{5}{6} = \frac{5}{36} \quad \text{(1M)}$$

Similarly, the probability of rolling a 3 on the white die and not on the blue die is:

$$P(\text{white} = 3, \text{ blue} \neq 3) = \frac{5}{36}$$

Adding both cases:

$$P(\text{exactly one three}) = \frac{5}{36} + \frac{5}{36} = \frac{10}{36} = \frac{5}{18} \quad \text{(1A)}$$

c. The sum of the numbers thrown is less than seven. (2 marks)

The valid dice rolls are:

$$\{(1,1),(1,2),(1,3),(1,4),(1,5),(2,1),(2,2),(2,3),(2,4),$$
$$(3,1),(3,2),(3,3),(4,1),(4,2),(5,1)\}$$

There are 15 successful outcomes out of 36, so:

$$P(\text{sum} < 7) = \frac{15}{36} = \frac{5}{12} \quad \text{(1A)}$$

d. A two or a five is thrown. (2 marks)

The probability of rolling a 2 on at least one die:

$$P(\text{blue} = 2) + P(\text{white} = 2) - P(\text{both} = 2)$$

$$= \frac{6}{36} + \frac{6}{36} - \frac{1}{36} = \frac{11}{36} \quad \text{(1M)}$$

Similarly, the probability of rolling a 5 on at least one die:

$$P(\text{blue} = 5) + P(\text{white} = 5) - P(\text{both} = 5)$$

$$= \frac{6}{36} + \frac{6}{36} - \frac{1}{36} = \frac{11}{36}$$

Since a 2 and a 5 can both appear together, we use:

$$P(2 \text{ or } 5) = \frac{11}{36} + \frac{11}{36} - \frac{2}{36} = \frac{20}{36} = \frac{10}{18} = \frac{5}{9} \quad \text{(1A)}$$

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Question 3 (7 marks)

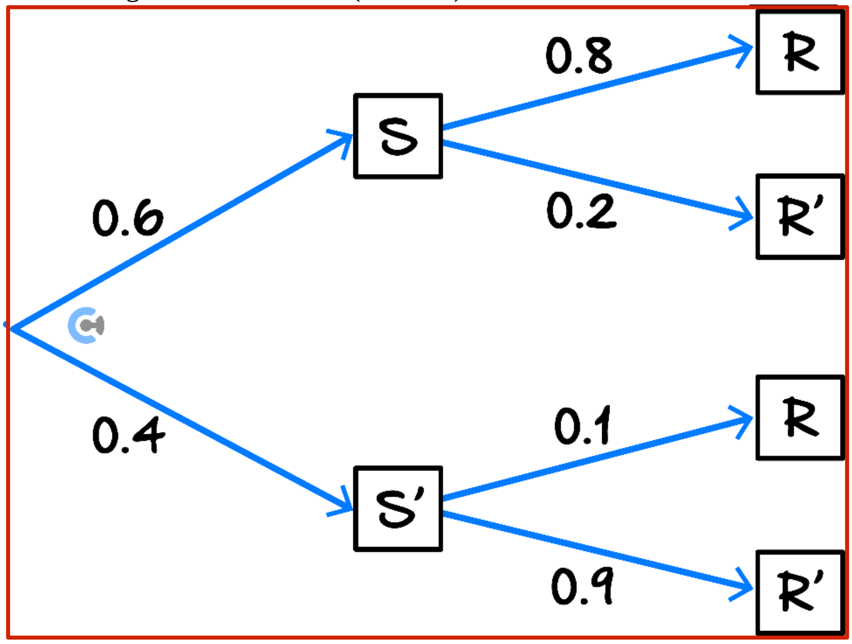
If Sam goes to the casino on the weekend, there is an 80% chance that Rohan will go with him.

If Sam does not go to the casino on the weekend, there is a 10% chance that Rohan will still go to the casino.

Sam goes to the casino on 60% of weekends.

a. Let S be the event that Sam goes to the casino on the weekend and let R be the event that Rohan goes to the casino on the weekend.

Draw a tree diagram showing this information. (3 marks)



b. Find the probability that Rohan goes to the casino on any weekend. (2 marks)

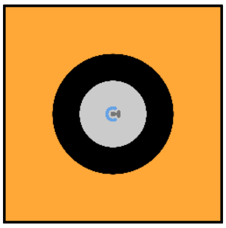
$\Pr(R) = \Pr(R \cap S) + \Pr(R \cap S')$ (1M)
$= 0.6 \times 0.8 + 0.4 \times 0.1 = 0.48 + 0.04 = 0.52$ (1A)

c. Find the probability that on a particular weekend, Rohan went to the casino, Sam did not go to the casino. (1 mark)

$0.4 \times 0.1 = 0.04$

Question 4 (3 marks)

The square target shown has sides of length 2 metres. Inside the square is a black circle of radius 1 metre, and a grey circle of radius 0.4 metres. Suppose that a dart is thrown at the target, and it is equally likely to hit any part of the target.



Find the probability that the dart hits the black region.

Large circle has area π and small grey circle has area $0.4^2\pi = 0.16\pi$ (1M).
Thus the black region has area $\pi - 0.16\pi = 0.84\pi$. (1M)
The probability of hitting the black region is therefore

$$\frac{0.84\pi}{4} = 0.21\pi \quad (1A)$$

Section C: Exam 2 Questions (28 Marks)

INSTRUCTION:

- Regular: 28 Marks. 5 Minutes Reading. 40 Minutes Writing.
- Extension: 28 Marks. 5 Minutes Reading. 28 Minutes Writing.

Question 5 (1 mark)

For two events, A and B, $\Pr(A) = 0.6$, $\Pr(B) = 0.8$, and $\Pr(A \cap B) = 0.5$. Find $\Pr(A \cup B)$.

A. 0.4

B. 0.8

C. 0.9

D. 1

Question 6 (1 mark)

A box contains ten red balls and six yellow balls. When two balls are randomly selected from the box without replacement, the probability that both balls have the same colour is:

A. $\frac{1}{8}$

B. $\frac{5}{14}$

C. $\frac{3}{8}$

D. $\frac{1}{2}$

```
In[1]:= 10 / 16 * 9 / 15 + 6 / 16 * 5 / 15
Out[1]= 1/2
```

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Question 7 (1 mark)

The probability of rolling an odd number or a multiple of 2 using a 6-sided die whose faces are numbered $2, 3, 4, 4, 5, 6$ is:

A. 1

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. $\frac{3}{4}$

Question 8 (1 mark)

If events A and B are mutually exclusive, then it is always true that:

A. $\Pr(A|B) = \Pr(B)$

B. $\Pr(A|B) = \Pr(A)$

C. $\Pr(A|B) = \Pr(A) + \Pr(B)$

D. $\Pr(A|B) = 0$

Question 9 (1 mark)

If for two events A and B, $\Pr(A) = 0.35, \Pr(B) = 0.6$ and $\Pr(A|B) = 0.2$, then $\Pr(A \cap B)$ is closest to:

A. 0.35

B. $\frac{12}{35}$

$$0.2 \times 0.6 = 0.12$$

C. $\frac{6}{35}$

D. 0.12

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Question 10 (1 mark)

Which of the following alternatives gives the correct values of c and d in the probability table below?

	B	B'	
A	0.4		0.7
A'	c		d
		0.4	

A. $c = 0.2$ and $d = 0.3$

B. $c = 0.3$ and $d = 0.2$

C. $c = 0.6$ and $d = 0.3$

D. $c = 0.7$ and $d = 0.4$

Question 11 (1 mark)

On a game show, there are sums of money hidden in 6 suitcases. Only two of the suitcases have money, the other suitcases are empty. If a contestant chooses two suitcases without replacement, their probability of choosing at least one suitcase with a prize is:

A. $\frac{2}{5}$

B. $\frac{5}{9}$

C. $\frac{8}{15}$

D. $\frac{3}{5}$

```
In[9]:= 2/6 * 1/5 + 2/6 * 4/5 + 4/6 * 2/5
Out[9]= 3/5

In[10]:= 1 - (4/6 * 3/5)
Out[10]= 3/5
```

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Question 12 (1 mark)

Rod buys an early morning coffee at a certain cafe 85% of the time. Kate does the same 60% of the time, independently of whether Rod does. The probability that only one of them buys an early morning coffee on a particular morning is:

A. 0.57

B. 0.51

C. 0.43

D. 0.62

```
In[11]:= 0.85 * 0.4 + 0.15 * 0.6
Out[11]= 0.43
```

Question 13 (10 marks)

There are 200 lawyers in a large company. Of these 200:

- 150 have been promoted to the status of Junior Partner.
- 110 are women.
- 30 are not women and are not Junior Partners.

a. What is the probability that a lawyer chosen at random is:

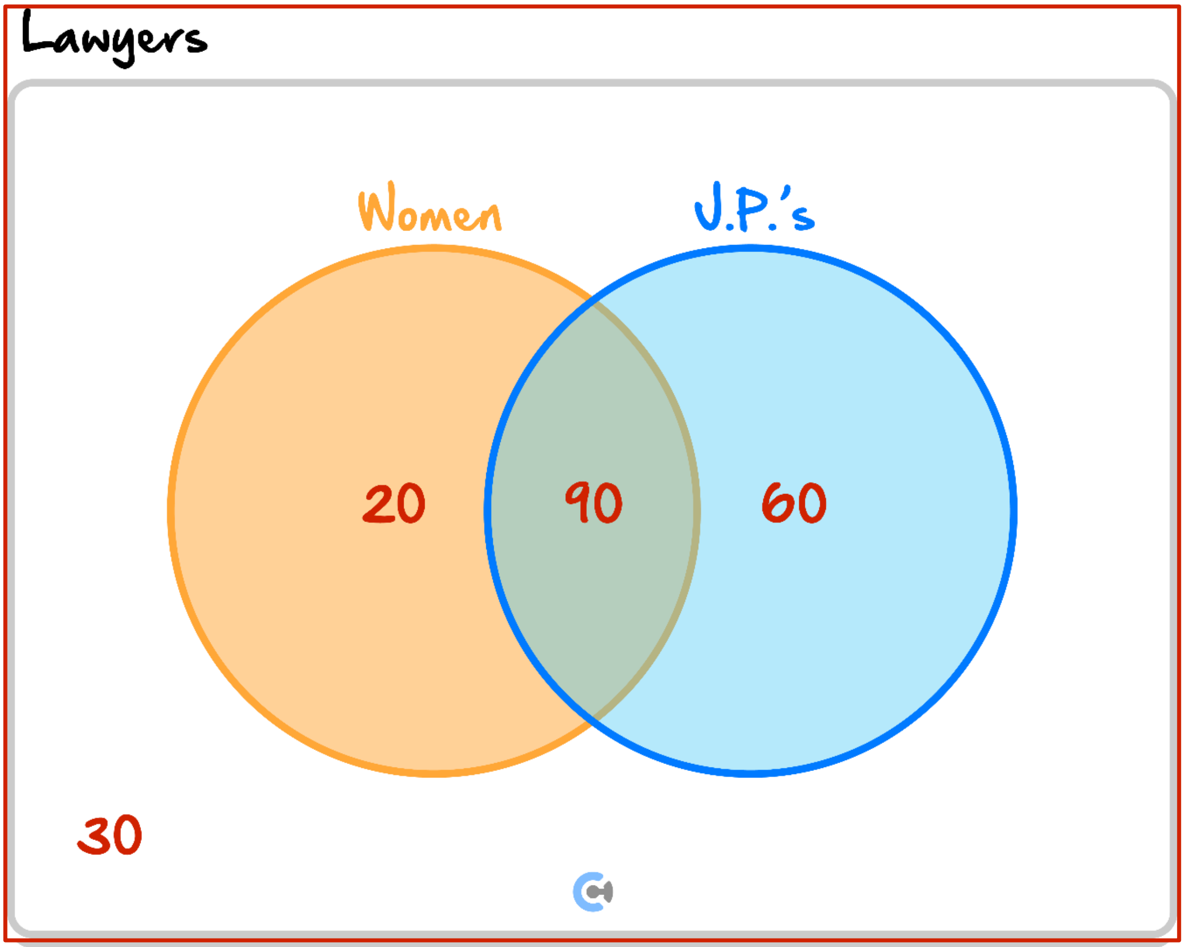
i. not a woman? (1 mark)

$$\frac{90}{200} = \frac{9}{20} = 0.45 \text{ (1A)}$$

ii. Not a Junior Partner? (1 mark)

$$\frac{50}{200} = \frac{1}{4} = 0.25 \text{ (1A)}$$

b. Fill in the Venn diagram showing the number of people in the four regions. (3 marks)



(1M for 1 correct number, 1M for 3 correct numbers, 1A all correct)

c. If a lawyer is to be chosen at random, find the following probabilities using your Venn diagram.

i. $\Pr(Woman \cap J.P)$. (1 mark)

$$\frac{90}{200} = \frac{9}{20} = 0.45 \text{ (1A)}$$

ii. $\Pr(Woman \cup J.P)$. (1 mark)

$$\frac{170}{200} = \frac{17}{20} = 0.85 \text{ (1A)}$$

iii. $\Pr(Not\ woman \cup not\ J.P)$. (1 mark)

$$\frac{11}{20}$$

d. What is the probability that a **Junior Partner** chosen at random is:

i. Not a woman? (1 mark)

$$\frac{60}{150} = \frac{2}{5} = 0.4 \text{ (1A)}$$

ii. A woman? (1 mark)

$$\frac{90}{150} = \frac{3}{5} = 0.6 \text{ (1A)}$$

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Question 14 (10 marks)

Ava has a ten-week holiday at the end of the school year. Each week during the holidays, Ava and five of her friends go shopping at Chadstone.

Ava's sister, Emily, is also on holiday for the first four weeks. The probability that Emily will drive Ava to Chadstone in any given week depends on whether or not she drove her the previous week.

If Emily drives her one given week, then the probability that she does so the next week is 0.4. If Emily does not drive her one given week, then the probability that she does the next week is 0.7.

a. Suppose Emily drives Ava to Chadstone in the first week. (Assume this to be the case when answering all parts below).

i. What is the probability that Emily does not drive Ava to Chadstone in the second week? (1 mark)

Let E_i be the event that Emily drives Ava in week i.
$\Pr(E_2' \mid E_1) = 0.6$ (1A)

ii. What is the probability that Emily will not drive Ava to Chadstone in either of the next two weeks? (1 mark)

$\Pr(E_2' \cap E_3' \mid E_1) = 0.6 \times 0.3 = 0.18$ (1A)

iii. What is the probability that Emily drives Ava to Chadstone in exactly one of the next two weeks? (2 marks)

$\Pr(E_2' \cap E_3 \mid E_1) + \Pr(E_2 \cap E_3' \mid E_1)$ (1M)
$= 0.6 \times 0.7 + 0.4 \times 0.6 = 0.66$ (1A)

iv. If Emily drives Ava to Chadstone on exactly one of the next two weeks, what is the probability that she drives her on the second of these weeks only? (2 marks)

Let O be the event Emily is driven on exactly one of the next two weeks. $\Pr(O) = 0.66$.

$\Pr(\text{driven on second only}) = \dfrac{\Pr(E_2' \cap E_3 \mid E_1)}{\Pr(O)}$ (1M)

$= \dfrac{0.42}{0.66} = \dfrac{7}{11}$ (1A)

Ava and her friends either go shopping at Chadstone in the morning or the afternoon. If they go in the morning one week, then the probability that they will decide to go again in the morning the following week is 0.45. If they go in the afternoon one week, then the probability that they will go in the afternoon the next week is 0.4.

b. If the friends go shopping in the afternoon one week, what is the probability that they go in the morning the following week? (1 mark)

$1 - 0.4 = 0.6$ (1A)

c. Suppose the friends go shopping in the morning in the first week of the holidays. What is the probability that they also go in the morning in the fourth week of the holidays? Give your answer correct to 2 decimal places (3 marks)

Let A_i be the event they go shopping in the afternoon on the ith week given that they went shopping in the morning on the first week.
Let M_i be the event they go shopping in the morning on the ith week given that they went shopping in the morning on the first week.

$\Pr(M_4) = \Pr(A_2 \cap A_3 \cap M_4) + \Pr(A_2 \cap M_3 \cap M_4) + \Pr(M_2 \cap A_3 \cap M_4) + \Pr(M_2 \cap M_3 \cap M_4)$ (1M, or tree diagram)

$= (0.55 \times 0.4 \times 0.6) + (0.55 \times 0.6 \times 0.45) + (0.45 \times 0.55 \times 0.6) + (0.45 \times 0.45 \times 0.45)$ (1M, 4 correct probabilities)

$= 0.520125$

Section D: Extension Exam 1 (9 Marks)

INSTRUCTION:

- Regular: Skip
- Extension: 9 Marks. 2 Minutes Reading. 15 Minutes Writing.

Question 15 (9 marks)

A group of boys love dice games and during the holidays they intend to relax by inventing and playing games. George wants to create a game that involves rolling two unbiased, four-sided dice, with faces numbered 1-4.

a. Let X be a random variable that is equal to the sum of the numbers that are rolled on the two dice.

i. What is the size of the sample space when two four-sided dice are rolled? (1 mark)

$4^2 = 16$ (1A)

ii. Complete the table below describing the probability of the two dice summing to a certain number x. (2 marks)

x	2	3	4	5	6	7	8
$\Pr(X = x)$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{1}{8}$	$\frac{1}{16}$

b. If $A = \{\text{sum of numbers rolled on the two dice is greater than } 6\}$ and

$B = \{\text{the number rolled on each die is the same}\}$, find:

i. Find $\Pr(A)$. (1 mark)

$$\Pr(A) = \frac{3}{16} \text{ (1A)}$$

ii. Find $\Pr(B)$. (1 mark)

$$\Pr(B) = \frac{1}{4} \text{ (1A)}$$

iii. Find $\Pr(A \cap B)$. (1 mark)

$$\Pr(A \cap B) = \Pr(4, 4) = \frac{1}{16} \text{ (1A)}$$

c. Determine if events A and B are:

i. Mutually exclusive, giving reasons. (1 mark)

$$\Pr(A \cap B) \neq 0$$

$\therefore A$ and B are not mutually exclusive (1A)

ii. Independent, giving reasons. (2 marks)

$$\Pr(A) \times \Pr(B) = \frac{3}{16} \times \frac{1}{4} = \frac{3}{64} \text{(1M)}$$

$$\neq \Pr(A \cap B)$$

Therefore A and B are not independent (1A)

d. George thinks it would be more exciting if prizes were involved in his game.

It costs \$3.00 to play the game, which involves rolling the two dice and finding the sum of the numbers rolled. The prizes are \$8.00 if a sum of more than 6 is obtained and \$1.00 if a sum of 6 or less is obtained.

i. What is the expected gain/loss for this game? Leave your answer as a fraction of a dollar.

$$\text{average gain} = 5\left(\frac{3}{16}\right) + (-2)\left(\frac{13}{16}\right) \text{ (1M)}$$

$$= -\frac{11}{16}$$

Therefore expect to lose $\frac{11}{16}$ dollars.

ii. Is this game fair? Explain.

No, not a fair game since on average you expect to lose money.

Space for Personal Notes

Section E: Extension Exam 2 (11 Marks)

INSTRUCTION:

- Regular: Skip
- Extension: 12 Marks. 2 Minutes Reading. 13 Minutes Writing.

Background Information

- In the game of Dungeons & Dragons, it is common to roll multiple dice and sum their results. For example, rolling $\mathbf{3d6}$ means rolling a **six-sided die three times** and summing the results. This forms a probability distribution where different sums have different likelihoods of occurring.

- Mathematicians use **moment-generating functions ($\mathbf{MGFs}$)** to find closed formulas for the probability of obtaining a given sum. The probability of obtaining a total sum $X = x$ can be found using the formula:

$$P(X = x) = [e^{xt}]\big(f(t)\big)$$

Where:

$$f(t) = \frac{1}{k^n}\left(\sum_{m=1}^{k} e^{mt}\right)^n$$

Represents the generating function for rolling n dice each with k sides. The notation $[e^{xt}]\big(f(t)\big)$ indicates taking the coefficient in front of the term e^{xt}.

- For example, for rolling $\mathbf{3d6}$ and computing $P(X = 14)$, the moment generating function is expanded as:

$$f(t) = \frac{1}{6^3}(e^t + e^{2t} + e^{3t} + e^{4t} + e^{5t} + e^{6t})^3$$

$$= \frac{e^{3t}}{216} + \frac{e^{4t}}{72} + \frac{e^{5t}}{36} + \frac{5e^{6t}}{108} + \frac{5e^{7t}}{72} + \frac{7e^{8t}}{72} + \frac{25e^{9t}}{216} + \frac{e^{10t}}{8} + \frac{e^{11t}}{8} + \frac{25e^{12t}}{216}$$

$$+ \frac{7e^{13t}}{72} + \frac{5e^{14t}}{72} + \frac{5e^{15t}}{108} + \frac{e^{16t}}{36} + \frac{e^{17t}}{72} + \frac{e^{18t}}{216}$$

- The probability of obtaining a sum of 14 is then found by extracting the coefficient of e^{14t}:

$$P(X = 14) = [e^{14t}](f(t)) = \frac{5}{72} \approx 0.06944.$$

- Also, note that $\Pr(X = 3) = \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$ as we would expect!

Question 16 (11 marks)

Subu is designing a board game where players roll **four-sided dice** and sum the results. Subu needs help analysing the distribution of rolling $\mathbf{4d4}$ (four four-sided dice).

Let X be the random variable that is the sum obtained when rolling $\mathbf{4d4}$.

a.

i. List all possible sums that can be obtained from rolling $\mathbf{4d4}$. (1 mark)

$\{4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$ (1A)

ii. Determine the size of the sample space when rolling $\mathbf{4d4}$. (1 mark)

$4^4 = 256$ (1A)

b. Using the generating function:

$$f(t) = \frac{1}{4^4}(e^t + e^{2t} + e^{3t} + e^{4t})^4$$

i. Expand $f(t)$ to express it as a sum of exponentials. (1 mark)

$$f(t) = \frac{e^{4t}}{256} + \frac{e^{5t}}{64} + \frac{5e^{6t}}{128} + \frac{5e^{7t}}{64} + \frac{31e^{8t}}{256} + \frac{5e^{9t}}{32} + \frac{11e^{10t}}{64} + \frac{5e^{11t}}{32} + \frac{31e^{12t}}{256} + \frac{5e^{13t}}{64} + \frac{5e^{14t}}{128} + \frac{e^{15t}}{64} + \frac{e^{16t}}{256}$$

(1A)

In[114]:= $\frac{1}{4\text{^}4}$ (Exp[t] + Exp[2 t] + Exp[3 t] + Exp[4 t])^4 // Expand

Out[114]= $\frac{e^{4t}}{256} + \frac{e^{5t}}{64} + \frac{5 e^{6t}}{128} + \frac{5 e^{7t}}{64} + \frac{31 e^{8t}}{256} + \frac{5 e^{9t}}{32} + \frac{11 e^{10t}}{64} + \frac{5 e^{11t}}{32} + \frac{31 e^{12t}}{256} + \frac{5 e^{13t}}{64} + \frac{5 e^{14t}}{128} + \frac{e^{15t}}{64} + \frac{e^{16t}}{256}$

ii. Hence, determine $\Pr(X = 10)$. (1 mark)

$$\Pr(X = 10) = \frac{11}{64}$$

c. Find the average sum obtained when rolling $\mathbf{4d4}$. (2 marks)

One roll has an average of $\frac{1+2+3+4}{4} = 2.5$ (1M).

Therefore average of **4d4** is $2.5 \times 4 = 10$ (1A)

In[119]:= $\frac{1}{256} * 4 + \frac{1}{64} * 5 + \frac{5}{128} * 6 + \frac{5}{64} * 7 + \frac{31}{256} * 8 + \frac{5}{32} * 9 + \frac{11}{64} * 10 + \frac{5}{32} * 11 + \frac{31}{256} * 12 + \frac{5}{64} * 13 + \frac{5}{128} * 14 + \frac{1}{64} * 15 + \frac{1}{256} * 16$

Out[119]= 10

In[118]:= $\left(\frac{1+2+3+4}{4}\right) * 4$

Out[118]= 10

d. Subu wants to design a game where rolling a sum greater than 12 earns a player \$10 while rolling 12 or less earns \$4. The cost to play is \$6 per turn.

i. Calculate the expected profit/loss per game. Give your answer to the nearest cent. (3 marks)

$\Pr(\text{more than } 12) = \frac{5}{64} + \frac{5}{128} + \frac{1}{64} + \frac{1}{256} = \frac{35}{256}$ (1M).

So $\Pr(12 \text{ or less}) = \frac{221}{256}$.

Therefore expected profit $= 4 \times \frac{35}{256} + (-2) \times \frac{221}{256} = -1.1796875$ (1M, for the 4 and -2).

Therefore expect to lose \$1.18 per game.

ii. Subu's students complain that this game is not fair because they will lose money on average. Find how much the students should demand the game to cost to play if the game is fair. Give your answer to the nearest cent. (2 marks)

Let the game cost $\$k$ to play. We require that

$$(10 - k) \times \frac{35}{256} + (4 - k) \times \frac{221}{256} = 0 \quad \text{(1M)}$$

$\implies k = \frac{617}{128} \approx 4.82$

Game should cost \$4.82 to play. (1A)

```
In[129]:= Solve[(10 - k) * 35/256 + (4 - k) * 221/256 == 0, k] // N
Out[129]= {{k → 4.82031}}

In[130]:= Solve[(10 - k) * 35/256 + (4 - k) * 221/256 == 0, k]
Out[130]= {{k → 617/128}}

In[131]:= 6 - 1.1796875
Out[131]= 4.82031
```

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