Website: contoureducation.com.au | Phone: 1800 888 300
Email: hello@contoureducation.com.au

VCE Mathematical Methods ½
Probability [0.12]
Workshop

Error Logbook:

New Ideas/Concepts	Didn't Read Question
Pg / Q #: ____________________ Notes:	Pg / Q #: ____________________ Notes:
Algebraic/Arithmetic/ Calculator Input Mistake	**Working Out Not Detailed Enough**
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Section A: Recap

Definition

Sample Space (ε)

- The set of all possible outcomes in an experiment.
- For tossing two coins in a row, the sample space is:

$$\varepsilon = \{HH, HT, TH, TT\}$$

- For rolling a standard 6-sided dice, the sample space is:

$$\varepsilon = \{1, 2, 3, 4, 5, 6\}$$

- Total probability adds up to 1.

Definition

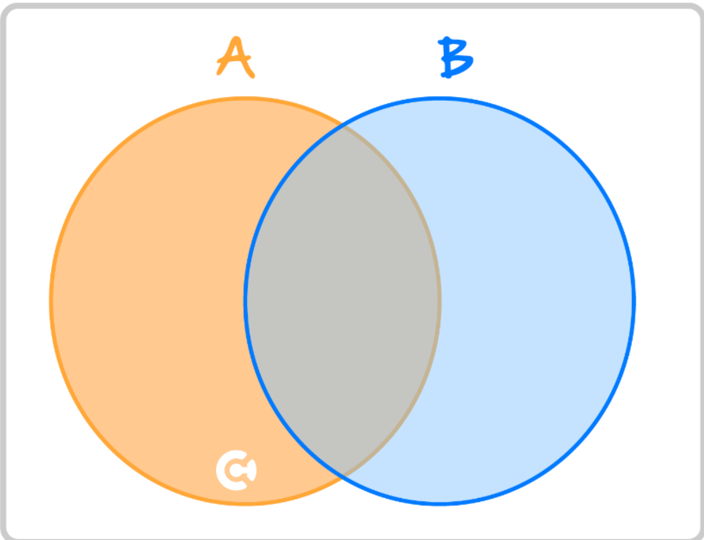
Calculating Probabilities for Equally Likely Outcomes

- When there is some number of equally likely outcomes, the probability of a "successful" outcome can be calculated as:

$$\Pr(success) = \frac{number\ of\ successful\ outcomes}{number\ of\ total\ outcomes}$$

Definition

Union, Intersection, and Complement

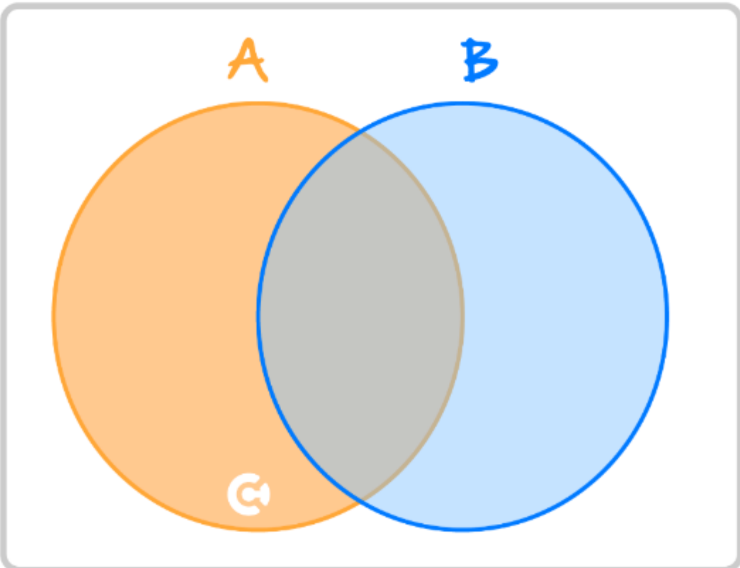


- $A \cup B =$
  - Union of two events aka “or”.
  - Equivalent to either event A **OR** event B **OR** BOTH occurring.
- $A \cap B =$
  - Intersection of two events aka “and”.
  - Equivalent to both event A **AND** event B occurring.
- $A' =$
  - A complement aka “not”.
  - Equivalent to event A NOT occurring.
  - E.g. if $A =$ dice rolled a 6, then $A' =$ dice rolled anything except a 6.

$$\mathbf{Pr(A') = 1 - Pr(A)}$$

Definition

<u>Venn Diagram</u>



- Venn diagram is useful to visualise the two events.

Karnaugh Tables

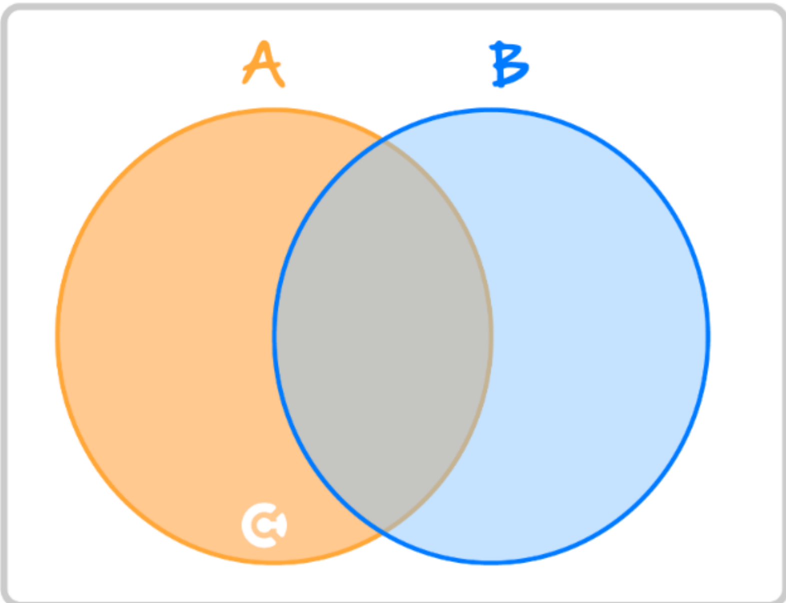
- We can also represent probability problems using a Karnaugh map.

	$\boldsymbol{B}$	$\boldsymbol{B'}$	
$\boldsymbol{A}$	$\Pr(A \cap B)$	$\Pr(A \cap B')$	$\mathbf{Pr}(\boldsymbol{A})$
$\boldsymbol{A'}$	$\Pr(A' \cap B)$	$\Pr(A' \cap B')$	$\mathbf{Pr}(\boldsymbol{A'})$
	$\mathbf{Pr}(\boldsymbol{B})$	$\mathbf{Pr}(\boldsymbol{B'})$	$\mathbf{1}$

  - The rows and columns add up to the last cell value.

- Remember the **total** probability must always add to 1.

The Addition Rule



- When we add the probabilities of A and B, we count the outcomes contained in $A \cap B$ **twice.**

- So, we must subtract one of them to get the probability of $A \cup B$.

$$\mathbf{Pr}(\boldsymbol{A} \cup \boldsymbol{B}) = \mathbf{Pr}(\boldsymbol{A}) + \mathbf{Pr}(\boldsymbol{B}) - \mathbf{Pr}(\boldsymbol{A} \cap \boldsymbol{B})$$

Definition

Mutually Exclusive Events

- Two events A and B are **mutually exclusive** if they cannot occur at the same time.
- The probability of both A and B happening together is zero.

$$\Pr(A \cap B) = 0$$

Definition

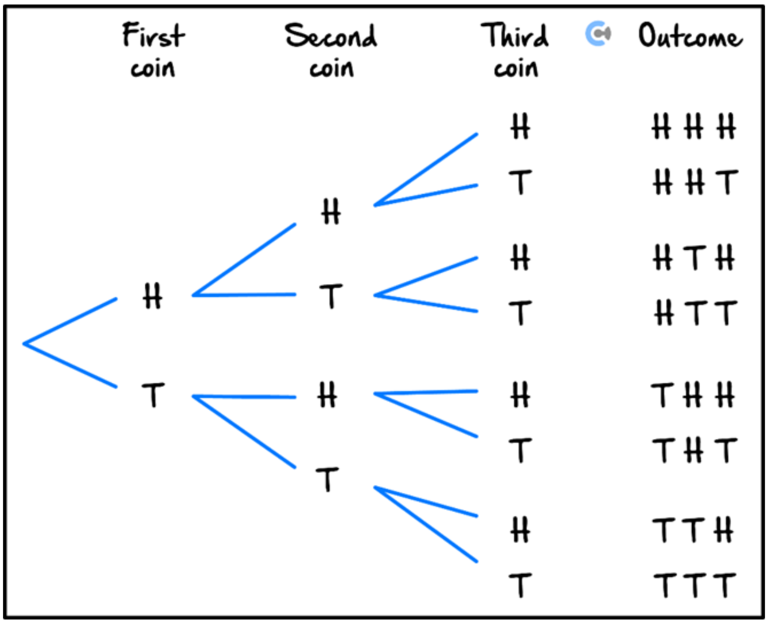
Independent Events

- Two events A and B are independent when the occurrence of one **does not affect** the likelihood of the other.
- When two events are independent:

$$\Pr(A \cap B) = \Pr(A) \times \Pr(B)$$

Tree Diagram

- Useful for multiple sequence events.
- To calculate the probability of a sequence, we **multiply the probabilities along the relevant branches.**
- For instance, the following tree diagram shows the outcomes of three successive coin tosses.



<u>Conditional Probability</u>

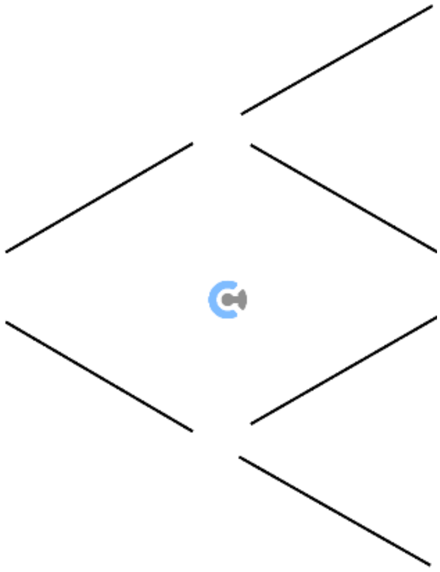
Definition

- Probability of A given B:

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

<u>Tree Diagram for Condition Probability</u>

Definition



- Tree diagram is perfect for conditional probability as each branch is conditional probability.

$$\text{Each branch} = \textbf{Pr}\ (\textbf{Leaf}|\textbf{Root})$$

<u>Conditional Probability with Independent Events</u>

Definition

$$\Pr(A|B) = \Pr(A)$$

- If A and B are independent, the given condition does **not** affect the probability of the event.

Space for Personal Notes

Section B: Warm Up (11 Marks)

INSTRUCTION:

- Regular: 11 Marks. 10 Minutes Writing.
- Extension: Skip

Question 1 (6 marks)

A bag contains 5 red balls, 3 blue balls, and 2 green balls. A ball is randomly drawn from the bag.

a. What is the probability that the ball drawn is red? (1 mark)

Pr(red)=5/10=1/2

b. What is the probability that the ball drawn is not blue? (1 mark)

Pr(not blue)=7/10

Once a ball has been drawn it is **not** replaced.

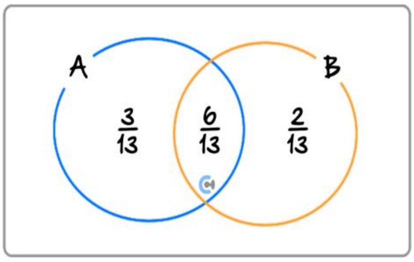
c. What is the probability that a red ball is drawn first and a blue ball is drawn second? (2 marks)

1/2×3/9=1/6

d. Find the probability that the second ball drawn is red, given that the first ball drawn is green. (2 marks)

Now have 5 red balls, 3 blue balls, 1 green ball. Therefore 5/9.
Or using conditional probability formula
Pr(second red ┤| first green)=(2/10×5/9)/(2/10)=5/9

Question 2 (5 marks)



The figure above shows the probability sample space for two events A and B, summarised by a Venn diagram. Determine the following probabilities:

a. At least one of the events A, B occurs. (1 mark)

11/13

b. At most one of the events A, B occurs. (1 mark)

7/13

c. Only event B occurs. (1 mark)

2/13

d. Exactly one of the events A, B occurs. (1 mark)

5/13

e. Are the events A and B independent? (1 mark)

Not independent since $\Pr(A \cap B) = \frac{6}{13} \neq \Pr(A)\Pr(B) = \frac{9}{13} \times \frac{8}{13}$

Section C: Exam 1 Questions (17 Marks)

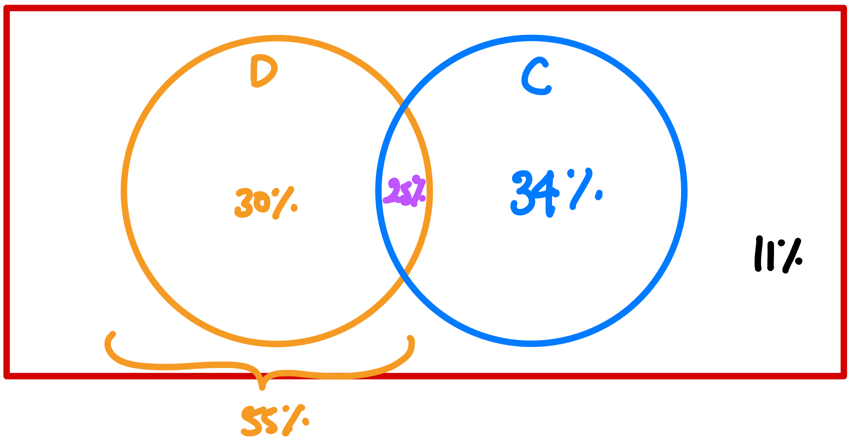
INSTRUCTION:

- Regular: 17 Marks. 5 Minutes Reading. 25 Minutes Writing.
- Extension: 17 Marks. 5 Minutes Reading. 17 Minutes Writing.

Question 3 (5 marks)

Of the families in a village, 55% have dogs and 25% have dogs and cats. Of the families in this village, 11% have neither a dog nor a cat.

a. Illustrate this information in a fully completed Venn diagram. (2 marks)



b. Find the percentage of families that do not have cats. (1 mark)

$\Pr(C') = 41\%$ //

c. Determine the probability that a family picked at random will own dogs or cats, but not both. (2 marks)

$\Pr(\text{Not Both}) = \Pr(D \cap C') + \Pr(C \cap D')$

$= 30\% + 34\%$

$= 64\%$ //

Question 4 (6 marks)

Events A and B are such that $\Pr(A) = \frac{1}{3}$ and $\Pr(B) = \frac{3}{5}$.

a. If $\Pr(A \cup B) = \frac{4}{5}$, determine whether A and B are independent, mutually exclusive or neither. (2 marks)

Addition Rule $\Rightarrow$ $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\frac{4}{5} = \frac{1}{3} + \frac{3}{5} - \Pr(A \cap B)$

$\Pr(A \cap B) = \frac{1}{3} + \frac{3}{5} - \frac{4}{5} = \frac{1}{3} - \frac{1}{5} = \frac{5-3}{15}$

$= \frac{2}{15} \neq \frac{1}{3} \cdot \frac{3}{5}$

$\therefore \Pr(A \cap B) \neq \Pr(A) \cdot \Pr(B)$ & $\Pr(A \cap B) \neq 0$

$\therefore$ A and B are neither independent nor ME.

b. If $\Pr(A \cup B) = 1 - p$, find the value of p, when,

i. A and B are independent. (2 marks)

$\Pr(A \cap B) = \Pr(A) \cdot \Pr(B) = \frac{1}{3} \cdot \frac{3}{5} = \frac{1}{5}$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$1 - p = \frac{1}{3} + \frac{3}{5} - \frac{1}{5}$

$1 - p = \frac{5 + 6}{15}$

$1 - p = \frac{11}{15} \Rightarrow \therefore p = \frac{4}{15}$

ii. A and B are mutually exclusive. (2 marks)

$\Pr(A \cap B) = 0$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$1 - p = \frac{1}{3} + \frac{3}{5} - 0$

$1 - p = \frac{5 + 9}{15}$

$1 - p = \frac{14}{15}$

$\therefore p = \frac{1}{15}$

Space for Personal Notes

Question 5 (6 marks)

Two fair dice are rolled. Determine the probability that:

a. At least one dice lands on a 4. (1 mark)

$Pr(\geq \text{One } 4) = 1 - Pr(\text{No } 4s)$

$= 1 - \frac{5}{6} \cdot \frac{5}{6} = 1 - \frac{25}{36} = \frac{11}{36}$ //

b. The two dice roll different numbers. (1 mark)

(1,1) (2,2) (3,3) ... (6,6)

$Pr(\text{Diff No.s}) = 1 - Pr(\text{Same No.})$

$= 1 - \frac{6}{36} \Rightarrow 1 - \frac{1}{6} = \frac{5}{6}$ //

c. At least one dice lands on a 4, given that the two dice roll different numbers. (2 marks)

$$Pr(4 \mid \text{Diff No.s}) = \frac{Pr(\geq \text{one } 4 \cap \text{Diff No.s})}{Pr(\text{Diff No.s})} = \frac{\left(\frac{10}{36}\right)}{\left(\frac{5}{6}\right)}$$

$$= \frac{\cancel{10}^{2}}{\cancel{36}_{3\,\cancel{6}}} \cdot \frac{\cancel{6}}{\cancel{5}}$$

$= \frac{1}{3}$ //

Consider the event of a single six-sided dice being rolled.

d. Determine the probability that a 4 was rolled, given that an even number has been rolled. (2 marks)

$$Pr(4 \mid \text{Even}) = \frac{Pr(4 \cap \text{Even})}{Pr(\text{Even})} = \frac{1}{3}$$ //

Space for Personal Notes

Section D: Exam 2 Questions (33 Marks)

INSTRUCTION:

- Regular: 33 Marks. 5 Minutes Reading. 45 Minutes Writing.
- Extension: 33 Marks. 5 Minutes Reading. 33 Minutes Writing.

=

Question 6 (1 mark)

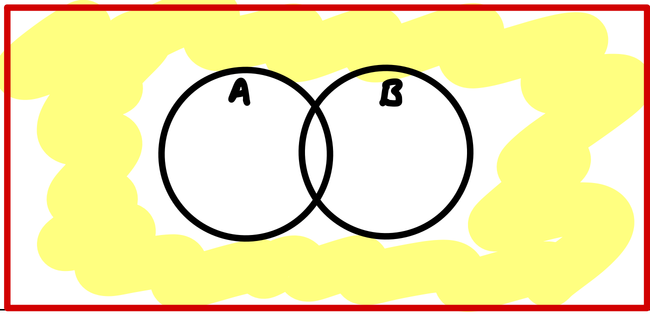
For events A and B, if $\Pr(A' \cap B') = 0.34$ then $\Pr(A\ U\ B)'$ is equal to:

A. 0.66

B. 0.17

C. 0.34

D. 0.33



Question 7 (1 mark)

For events A and B, $\Pr(A) = p$, $\Pr(B) = \frac{1}{4}$ and $\Pr(AUB) = \frac{2}{7}$. The probability of the intersection of A and B is equal to:

A. $p - \frac{1}{28}$

B. $1 - p$

C. $p + \frac{1}{4}$

D. $p + \frac{1}{28}$

Addition Rule $\Rightarrow \Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\frac{2}{7} = p + \frac{1}{4} - \Pr(A \cap B)$

$\therefore \Pr(A \cap B) = p + \frac{1}{4} - \frac{2}{7}$

$= p + \frac{7-8}{28}$

$= p - \frac{1}{28}$

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Question 8 (1 mark)

The letters of the words CONTOUR EDUCATION are placed at random in a row. The probability of the first letter being a vowel is:

A. $\frac{3}{7}$

B. $\frac{9}{16}$

C. $\frac{1}{2}$

D. $\frac{7}{16}$

$Pr(\text{Vowel}) = \frac{8}{16} = \frac{1}{2}$

Question 9 (1 mark)

The probability of passing the Contour MM12 Exam is 0.6, and the probability of your parents being happy with your result is 0.3. Given that the probability of passing the exam and your parents being happy with your result is 0.25, find $\Pr(\textit{Not Passing and Parents being Unhappy})$.

A. 0.4

B. 0.35

C. 0.25

D. 0.7

	P	P'	
H	0.25		0.3
H'	0.35	0.35	0.7
	0.6	0.4	1

Question 10 (1 mark)

Two identical 20-sided dice are rolled. What is the probability that the same number will appear on each of them?

A. $\frac{2}{7}$

B. $\frac{1}{20}$

C. $\frac{13}{20}$

D. $\frac{3}{20}$

$Pr(\{1,1\} + \{2,2\} \ldots \{20,20\}) = 20 \cdot \left(\frac{1}{20} \cdot \frac{1}{20}\right)$

$= \frac{1}{20}$ //

Question 11 (1 mark)

A bag contains 2 brown and 6 white socks. Ram pulls out two socks. What is the probability that both socks are of the same colour?

A. $\frac{3}{7}$

B. $\frac{5}{8}$

C. $\frac{4}{7}$

D. $\frac{9}{16}$

B: $\frac{2}{8} \cdot \frac{1}{7} = \frac{2}{56}$
$+$
W: $\frac{6}{8} \cdot \frac{5}{7} = \frac{30}{56}$

$= \frac{32}{56} = \frac{16}{28} = \frac{8}{14} = \frac{4}{7}$ //

Question 12 (1 mark)

What is the probability of getting the number 2 at least once in a regular die if it is rolled 6 times?

A. $1 - \left(\frac{5}{6}\right)^6$

B. $1 - \left(\frac{1}{6}\right)^6$

C. $\left(\frac{5}{6}\right)^6$

D. None of the above.

$Pr(\geq \text{one } 2) = 1 - Pr(\text{No } 2\text{s})$

$= 1 - \left(\frac{5}{6}\right)^6$ //

Question 13 (1 mark)

$\rightarrow Pr(A \cap B) = 0$

Events A and B are said to be mutually exclusive if:

~~**A.**~~ $P(A \cap B) = P(A)P(B)$

~~**B.**~~ $P(A \cup B) = 0$

~~**C.**~~ $P(A) = P(B)$

D. $P(A) + P(B) = P(A \cup B)$

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Question 14 (1 mark) 4R 3B

An urn contains 7 balls of which four are red and three are black. Two balls are drawn at random. What is the probability that they are of different colours?

A. $\frac{1}{5}$

B. $\frac{2}{7}$

C. $\frac{4}{7}$

D. $\frac{3}{7}$

$Pr(RB) + Pr(BR) = \frac{4}{7} \cdot \frac{3}{6} + \frac{3}{7} \cdot \frac{4}{6}$

$= \frac{12}{42} + \frac{12}{42} = \frac{24}{42}$

$= \frac{12}{21} = \frac{4}{7}$

Question 15 (1 mark)

12 cards are numbered from 1 to 12. If one card is drawn at random, what is the probability that the number on the card is not a prime number?

A. $\frac{1}{2}$

B. $\frac{7}{12}$

C. $\frac{1}{12}$

D. $\frac{5}{12}$

$Pr(\text{No Prime}) = 1 - Pr(\text{Prime})$

$= 1 - \frac{\{2, 3, 5, 7, 11\}}{12}$

$= 1 - \frac{5}{12} = \frac{7}{12}$ //

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Question 16 (11 marks)

R = rain, M = malfunctioning

In Mr. Amitav's neighbourhood, there is an 18% chance of rain on any given day. It is also known that the probability that Mr. Amitav's garage door malfunctions on any given day is 0.09. Mr. Amitav believes that the probability of both rain and garage door malfunction occurring on any given day is $\frac{81}{5000}$.

a. Show that the incidence of rain and garage door malfunction are independent events. (2 marks)

$\Pr(A \cap B) = \Pr(A) \cdot \Pr(B)$

$\Pr(R \cap M) = \frac{81}{5000}$

$\Pr(R) \cdot \Pr(M) = \frac{18}{100} \times \frac{9}{100} = \frac{162}{10{,}000} = \frac{81}{5000}$

$\therefore$ R and M are independent

Assume that the incidence of rain or no rain on any given day does not affect the chance of rain on the following day. Assume the same for garage door malfunction. For the following, give your answer correct to 3 decimal places..

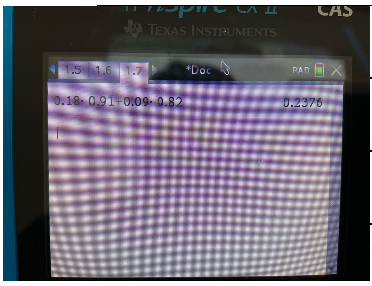
b. Mr. Amitav considers a day to be 'poor' if it either rains or his garage door malfunctions, and 'terrible' if it rains and his garage door malfunctions.

i. What is the probability that any given day is 'poor' but not 'terrible' by Mr. Amitav's appraisal? (3 marks)

$\Pr(\text{Poor}) = \Pr(R \cap M') + \Pr(M \cap R')$

$= 0.18 \times 0.91 + 0.09 \times 0.82$

≈ 0.238



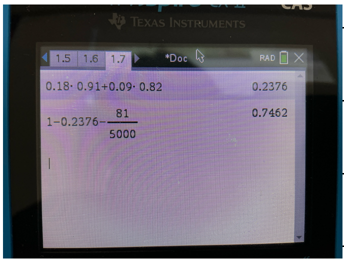
ii. A day is considered to be 'good' if it is not 'poor' or 'terrible'. → (3 d.p.)

What is the probability that any given day is a 'good' day for Mr. Amitav? (3 marks)

$\Pr(\text{Good}) = 1 - \Pr(\text{Poor}) - \Pr(\text{Terrible})$

$= 1 - 0.2376 - \frac{81}{5000}$

$= 0.746$ //

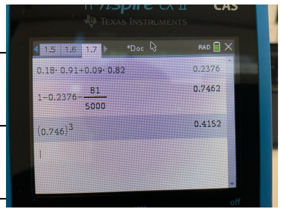


→ (3 d.p.)

iii. Calculate the probability of Mr. Amitav having three consecutive 'good' days. (1 mark)

$\Pr(3 \text{ Consecutive Good Days}) = (0.746)^3$

≈ 0.415 //



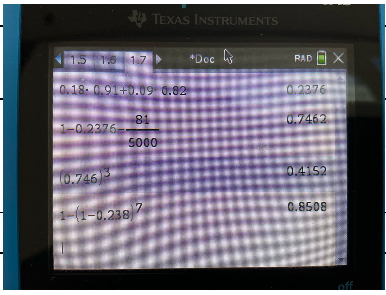
→ (3 d.p.)

iv. Calculate the probability that Mr. Amitav has at least one poor day in a week. (2 marks)

$\Pr(\geq 1 \text{ Poor Day In a Week}) = 1 - \Pr(\text{No Poor Days In a Week})$

$= 1 - (1 - 0.238)^7$

$= 0.851$ //



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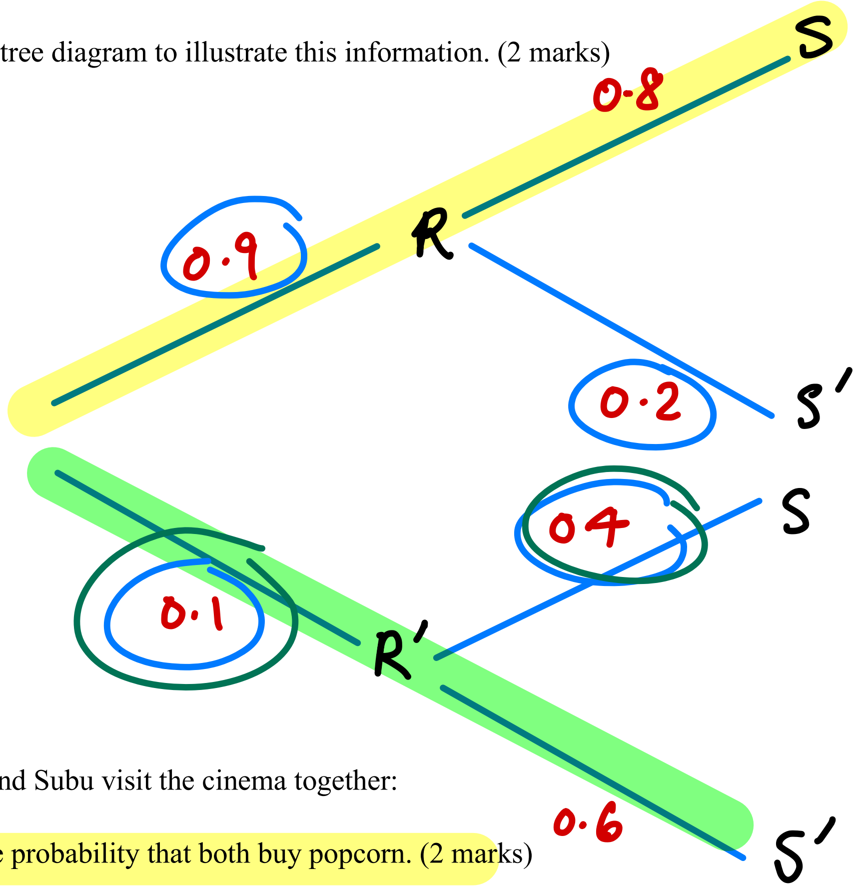
Question 17 (12 marks)

Rei and Subu often go to the cinema together. On such visits, there is a probability of 0.9 that Rei will buy popcorn. If Rei buys popcorn, the probability that Subu will also buy popcorn is 0.8. If he does not, then Subu's probability of buying popcorn is reduced to 0.4.

a. Construct a tree diagram to illustrate this information. (2 marks)



b. When Rei and Subu visit the cinema together:

i. Find the probability that both buy popcorn. (2 marks)

$\Pr(R \cap S) = 0.72$

ii. Show that the probability that neither buys popcorn is 0.06. (2 marks)

$\Pr(R' \cap S') = 0.1 \times 0.6 = 0.06$

iii. Find the probability that exactly one of them buys popcorn. (2 marks)

$\hookrightarrow$: $Pr(\text{One buying Popcorn}) = Pr(R \cap S') + Pr(S \cap R')$

$= 0.9 \times 0.2 + 0.4 \times 0.1$

$= 0.18 + 0.04$

$= 0.22$ //

c. Joseph sometimes joins Rei and Subu on their cinema visits. On these occasions, the probability that Joseph buys popcorn is 0.45 if both Rei and Subu buy popcorn and 0.35 if exactly one of Rei and Subu buy popcorn.

Find the probability that when Rei, Subu, and Joseph visit the cinema together.

i. All three buy popcorn. (2 marks)

$Pr(R \cap S \cap J) = 0.72 \times 0.45$

$= 0.324$ //

ii. Subu and Joseph buy popcorn, but Rei does not. (2 marks)

$\hookrightarrow$ $Pr(S \cap J \cap R') = 0.04 \times 0.35$

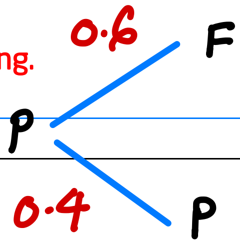
$= 0.014$ //

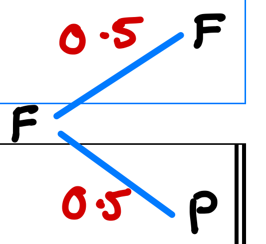
Space for Personal Notes

Section E: Extension Exam 1 (9 Marks)

INSTRUCTION:

- Regular: Skip
- Extension: 9 Marks. 2 Minutes Reading. 12 Minutes Writing.





Question 18 (3 marks)

Every Friday, Alex goes to one of two local gyms - FitZone or PowerHouse.

If he goes to FitZone one Friday, the probability that he goes to PowerHouse the next Friday is 0.5. If he goes to PowerHouse one Friday, the probability that he goes to FitZone the next Friday is 0.6.

On any given Friday, the gym he visits depends only on the one he went to on the previous Friday.

If Alex goes to PowerHouse one Friday, what is the probability that he visits PowerHouse on exactly two of the next three Fridays?

Let F = Fitzone & P = Powerhouse

$\Pr(P \text{ on } 2 \text{ of next } 3 \text{ fridays}) = \Pr(PPF) + \Pr(PFP) + \Pr(FPP)$

$= 0.4 \times 0.4 \times 0.6 + 0.4 \times 0.6 \times 0.5 + 0.6 \times 0.5 \times 0.4$

$P \rightarrow$

$= 0.096 + 0.12 + 0.12$

$= 0.336$ OR $\frac{42}{125}$ //

Space for Personal Notes

Question 19 (6 marks)

A coffee shop sells boxes of assorted muffins.

A box contains 12 muffins. There are only four types of muffins in the box. They are:

- Banana muffin, with chocolate chips.
- Banana muffin, without chocolate chips.
- Plain muffin, with chocolate chips.
- Plain muffin, without chocolate chips.

B = banana muffins

C = Chocolate Chips

It is known that, in the box:

- $\frac{5}{12}$ of the muffins are banana muffins.
- $\frac{1}{3}$ of the muffins have chocolate chips.
- $\frac{1}{6}$ of the muffins are banana muffins with chocolate chips.

a. A muffin is chosen at random from the box. Find the probability that it is a plain muffin without chocolate chips. (1 mark)

$B' \cap C'$

$\Pr(B' \cap C') = \frac{5}{12}$ //

	B	B'	
C	2	2	4
C'	3	5	8
	5	7	12

b. The 12 muffins in the box are randomly allocated to two new boxes, Box X and Box Y. Each new box contains 6 muffins. One of the two new boxes is chosen at random and then a muffin from that box is chosen at random.

$7-p$ = No. of plain muffins in Y

Let p be the number of plain muffins in Box X.

$$Pr(Y|B) = \frac{Pr(Y\cap B)}{Pr(B)} = \frac{p-1}{5}$$

Find the probability, in terms of ~~b~~ p, that the muffin comes from Box Y given that it is a banana muffin. (2 marks)

	B	B'	
Y	$p-1$	$7-p$	6
Y'	$6-p$	p	6
	5	7	12

$p-1 \geq 0$

$p \geq 1$

$p \geq 0$

c. State the smallest value that p can take. (1 mark)

$p-1 \geq 0 \Rightarrow \therefore p \geq 1$

Min $p = 1$ //

d. The coffee shop owner knows that exactly $\frac{1}{2}$ of their customers will order a drink and food.

Five customers are selected at random and their orders are inspected.

$OOOOD = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}$

$OOOOO = \left(\frac{1}{2}\right)^5$

Find the exact probability that at least 4 of these customers ordered a drink and food. (2 marks)

$Pr(\geq 4 \text{ ppl ordered}) = Pr(4 \text{ ordering}) + Pr(5 \text{ ordering})$

$= 5\cdot\left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^5$

$= 6\cdot\left(\frac{1}{2}\right)^5 = \frac{6}{32} = \frac{3}{16}$ //

Section F: Extension Exam 2 (12 Marks)

First 2 pages only!

INSTRUCTION:

- Regular: Skip
- Extension: 12 Marks. 2 Minutes Reading. 16 Minutes Writing.

Question 20 (12 marks)

In this question, we will explore a classic probability problem generally known as the "Birthday Problem".

A group of people is randomly selected, and we are interested in determining the probability that at least two of them share the same birthday. Assume that there are 365 days in a year and that each person's birthday is equally likely to fall on any of these 365 days.

a. Let A_n be the event that in a group of size n, no two people share the same birthday.

i. Calculate $\Pr(A_2)$. (1 mark)

$\Pr(A_2) = \frac{365}{365} \cdot \frac{364}{365} = \frac{364}{365}$ //

$\frac{1}{365^n} \cdot \frac{365!}{(365-n)!}$

ii. Calculate $\Pr(A_3)$. (1 mark)

$\Pr(A_3) = 1 \cdot \frac{364}{365} \cdot \frac{363}{365} = \frac{132132}{133225}$

b. Using the results from **part a.**, or otherwise, derive a general formula for the probability that all n people have different birthdays. Use the factorial function $n! = 1 \times 2 \ldots \times n$ in your answer. (2 marks)

$$Pr(A_n) = \frac{1}{365^n} \cdot \frac{365 \cdot 364 \cdots\cdots}{(365-n)!}$$

$$= \frac{365!}{365^n \cdot (365-n)!}$$

c. Let B_n be the event that at least 2 out of n people have the same birthday.

Hence, write down a general formula for the probability that at least two people have the same birthday in a group of n people. (1 mark)

Pr(No 2 ppl same bday)

$$Pr(B_n) = 1 - \frac{365!}{365^n (365-n)!}$$

$= Pr(\geq 2$ ppl has same bday$)$

d. Calculate the probability that in a group of 10 people, all have different birthdays. Give your answer correct to three decimal places. (1 mark)

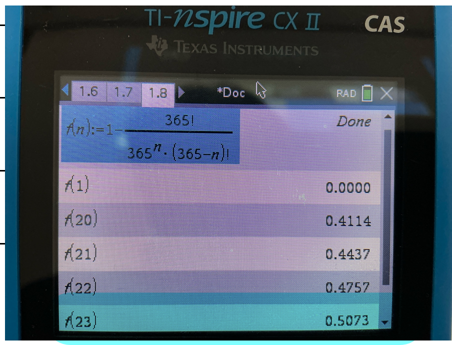
$Pr(A_{10}) = 0.883$ //

e. Determine the smallest number of people, n, such that the probability of at least two people sharing a birthday is greater than 50%. (2 marks)

$Pr(B_n) \geq 0.5$

Trial & Error

$\therefore n \approx 23$ ppl



Let e be Euler's constant (approximately 2.718). The approximation $e^x \approx 1 + x$ is very good when the absolute value of x, x is close to zero.

(Where to find e on CAS: TI: same as where you find π, Casio under Math1, Mathematica, use capital E.)

f. Use the approximation $e^{-\frac{n}{365}} \approx 1 - \frac{n}{365}$ to show that $\Pr(B_n) \approx 1 - e^{-\frac{n(n-1)}{730}}$. (2 marks)

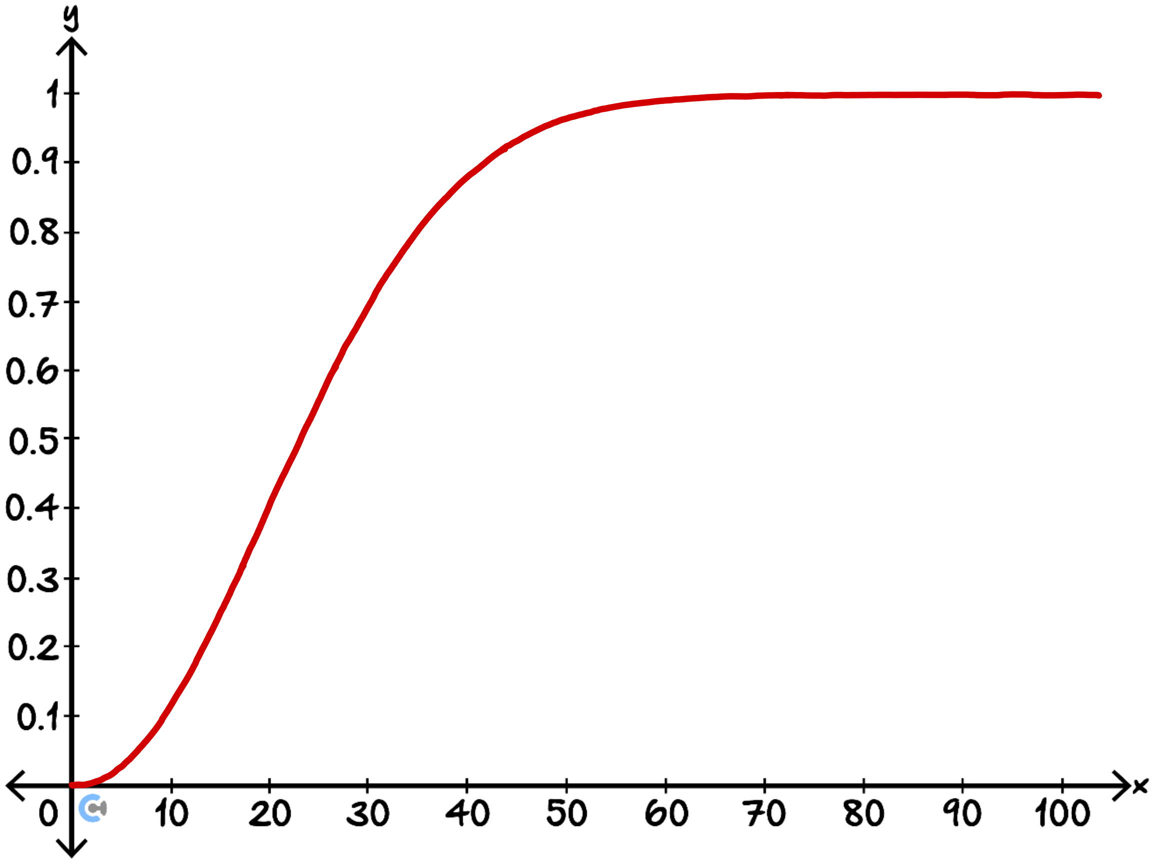
$\Pr(A_n) = 1 \times \left(1 - \frac{1}{365}\right) \times \left(\frac{2}{365}\right) \times \dots \times \left(\frac{n-1}{365}\right)$

$= e^{\frac{-1}{365}} \times e^{\frac{-2}{365}} \times \dots \times e^{\frac{n-1}{365}}$

$= e^{-\frac{(1+2+3 \dots n-1)}{365}} = e^{\frac{-n(n-1)}{730}}$

$\hookrightarrow \therefore \Pr(B_n) = 1 - e^{\frac{-n(n-1)}{730}}$ //

g. Sketch the function $f(n) = 1 - e^{-\frac{n(n-1)}{730}}$, on the axes below. (2 marks)



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VCE Mathematical Methods ½

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