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VCE Mathematical Methods ½
AOS 2 Revision [0.11]
Workshop

Error Logbook:

New Ideas/Concepts	Didn't Read Question
Pg / Q #: _____ Notes:	Pg / Q #: _____ Notes:
Algebraic/Arithmetic/ Calculator Input Mistake	**Working Out Not Detailed Enough**
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Section A: Cheat Sheet

Cheat Sheet

[2.1.1] - Sketch And Find The Rule Of Hyperbola Functions

Rectangular Hyperbola

$$y = \frac{a}{x-h} + k$$

Steps for sketching:

1. Find the horizontal and vertical _____ and plot them on the axis.
2. Find the x- and y- _____ and plot on the axes (if they exist).
3. Identify the _____ of the graph by considering any reflections and sketch the curve.

Finding the Equation of a Hyperbola from its Graph

We generally need three facts about the hyperbola.

$$y = \frac{a}{x-h} + k$$

Steps

1. Look for the _____.
2. Sub in a _____ to find the value of a.

[2.1.2] - Sketch And Find The Rule Of Truncus Functions

Truncus

$$y = \frac{a}{(x-h)^2} + k$$

Steps for sketching:

1. Find the horizontal and vertical _____ and plot them on the axis.
2. Find the x- and y- _____ and plot on the axes (if they exist).
3. Identify the _____ of the graph by considering any reflections and sketch the curve.

Finding the Equation of a Truncus from its Graph

We generally need three facts about the Truncus.

$$y = \frac{a}{(x-h)^2} + k$$

Steps

1. Look for the _____.
2. Sub in a _____ to find the value of a.

Cheat Sheet

[2.1.3] - Sketch And Find The Rule Of Root Functions

Square Root Functions

$$y = a\sqrt{b(x-h)} + k$$

Steps for sketching

1. Find the start point.
2. Find the x- and y- intercepts and plot on the axes (if they exist).
3. Identify the shape of the graph by considering any reflections and sketch the curve.

Finding the Equation of a Root Function from its Graph

We generally need three facts about the root function.

$$y = a\sqrt{\pm(x-h)} + k$$

Steps

1. Look for the starting point (h,k).
2. Sub in a point to solve the value of a.

[2.1.4] - Sketch And Find The Rule Of Semicircles And Circles

Circles

$$(x-h)^2 + (y-k)^2 = r^2$$
$$where\ r > 0$$

- Centre: (h,k)
- Radius: r

Steps

1. Find the centre of the circle.
2. Find the radius of the circle.
3. Find axes intercepts (if they exist).
4. Identify the shape of the graph and sketch the curve.

Semicircles

$$y = \pm\sqrt{r^2 - (x-h)^2} + k$$

$$x = \pm\sqrt{r^2 - (y-k)^2} + h$$

Steps for finding the rule of circles and semicircles

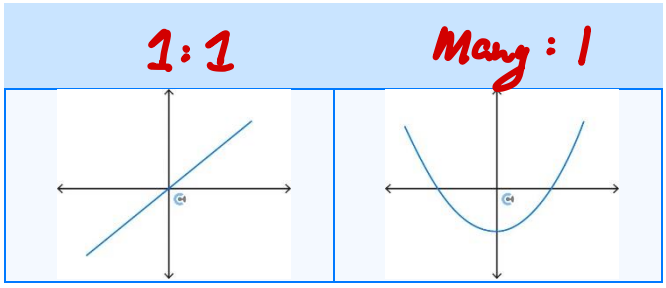
1. Identify the centre, (h,k).
2. Identify the radius, r.

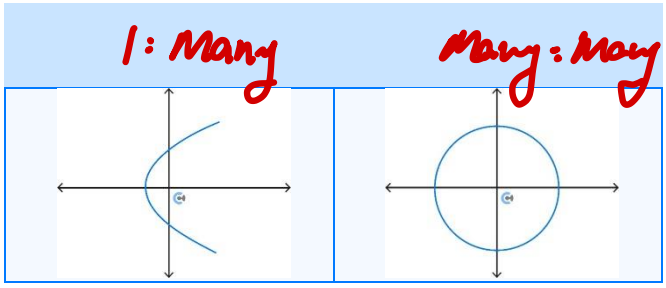
Cheat Sheet

[2.1.5] - Identify The Type Of Relations And Identify Whether The Relation Is A Function

Types of Relations

- There are four types of relations:





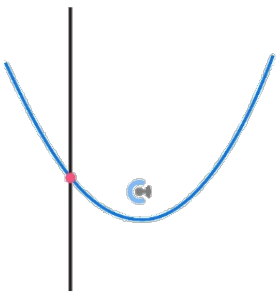
Functions

$$y = f(x)$$

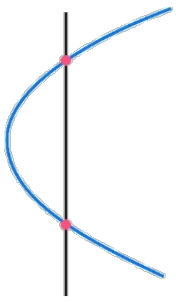
- Functions are relations that make one y-value at any given x-value.

Vertical Line Test

- **Definition**: Tells apart between functions and non-function relations.



Passes : Function

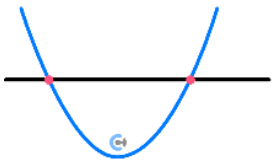


Fails : Not function

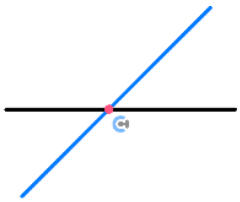
Every function only intersects a vertical line once.

Horizontal Line Test

- **Definition**: Tells apart between many to one and one to one functions. (And relations.)



Fails: Many to one



Passes: One to one

- One-to-one function hits **any** horizontal line drawn at most once.

Cheat Sheet

[2.2.1] – Find Domain and Range of Functions

- **Interval Notation**
  - Parentheses (non-inclusive):

$$x \in (a, b) \Rightarrow a < x < b$$

  - Square brackets [inclusive]:

$$x \in [a, b] \Rightarrow a \leq x \leq b$$

- **Maximal Domain**
  - Inside of a log must be ___>0___.
  - Inside of a root must be ___≥ 0___.
  - Denominator ___$\neq 0$___.

[2.2.2] – Sketch and Find the Domain and Range of Hybrid Functions

- **Piecewise (Hybrid) Functions**
  - Series of functions.

$$h(x) = \begin{cases} f(x), & Domain_1 \\ g(x), & Domain_2 \end{cases}$$

  - When we have an x intercept for one graph, sum graph intersects the other graph.
  - $Domain_1$ and $Domain_2$ represent the x values for which the two functions are ___defined___.
  - The two domains do not have to join!

[2.2.3] - Find the Rule, Domain, Range, and Intersections Between Inverse Functions

f needs to be ___1:1___ for f^{-1} to exist.

- Domain of the inverse function equals to ___range of original___ and vice versa.
- Symmetrical around ___$y=x$___.
- For intersections of inverses, we can equate the function to ___x___.

[2.3.1] - Restrict Domain Such That The Inverse Function Exists

- A function must be ___1:1___ for the inverse function to exist.

[2.3.2] - Figure Out Possible Rule of a Graph

- If the question is Tech-Active, make the ___scale___ of your graph the same as the question.
- Get to the correct answer through ___elimination___.

[2.3.3] - Solve Number of Solution Problems Graphically

- Solutions to $f(x) = k$ are the intersection points between ___$y=f(x)$___ and ___$y=k$___.

[2.4.1] - Applying x' and y' Notation to Find Transformed Points, Find the Interpretation of Transformations and Altered Order of Transformations

- The transformed point is called the ___image___ and is denoted by ___(x', y')___.
- The dilation factor is ___multiplied___ to the original coordinate.
- Reflection makes the original coordinates the ___negative___ of their original values.
- Translation ___adds___ a unit to the original coordinate.
- Transformations should be interpreted when ___x' & y'___ are isolated.
- The order of transformation follows the ___BODMAS___ order.
- To change the order of transformations, we either ___factorise OR expand___

[2.4.2] - Find Transformed Functions

- To transform the function, replace its ___old x & y___ with the new one.

Cheat Sheet

[2.4.3] - Find Transformations From Transformed Function (Reverse Engineering)

- To find the transformations, simply equate the LHS & RHS after separating the transformations of x and y.

[2.5.1] - Apply Quick Method to Find Transformations

- For applying transformations in the quick method:
  - Apply everything for x in the opposite direction, including the order!
- For interpreting transformations in the quick method:
  - Read everything for x in the opposite direction, including the order!

[2.5.2] - Find Opposite Transformations

- Order is reversed.
- All transformations are in the opposite direction.

[2.5.3] - Apply Transformations of Functions to Find Their Domain, Range, Transformed Points

- Everything moves together as a function.
- **Steps:**
  1. Find the transformation between two functions.
  2. Apply the same transformations to domain, range, and points.

[2.5.4] - Find Transformations of the Inverse Functions $f(x)$

- **Steps:**
  1. Find the transformations between the two original functions.
  2. Inverse the transformations found in **1**.

(Swap x & y for)

[2.5.5] - Find Multiple Transformations for the Same Functions

- Same transformations can be done for x or y by either putting it in or out of the $f(\quad)$ (x / y)
- Commonly, look for basic algebra, index, and log laws.

Section B: Exam 1 Questions (27 Marks)

INSTRUCTION:

- Regular: 27 Marks. 5 Minutes Reading. 40 Minutes Writing.
- Extension: 27 Marks. 5 Minutes Reading. 27 Minutes Writing.

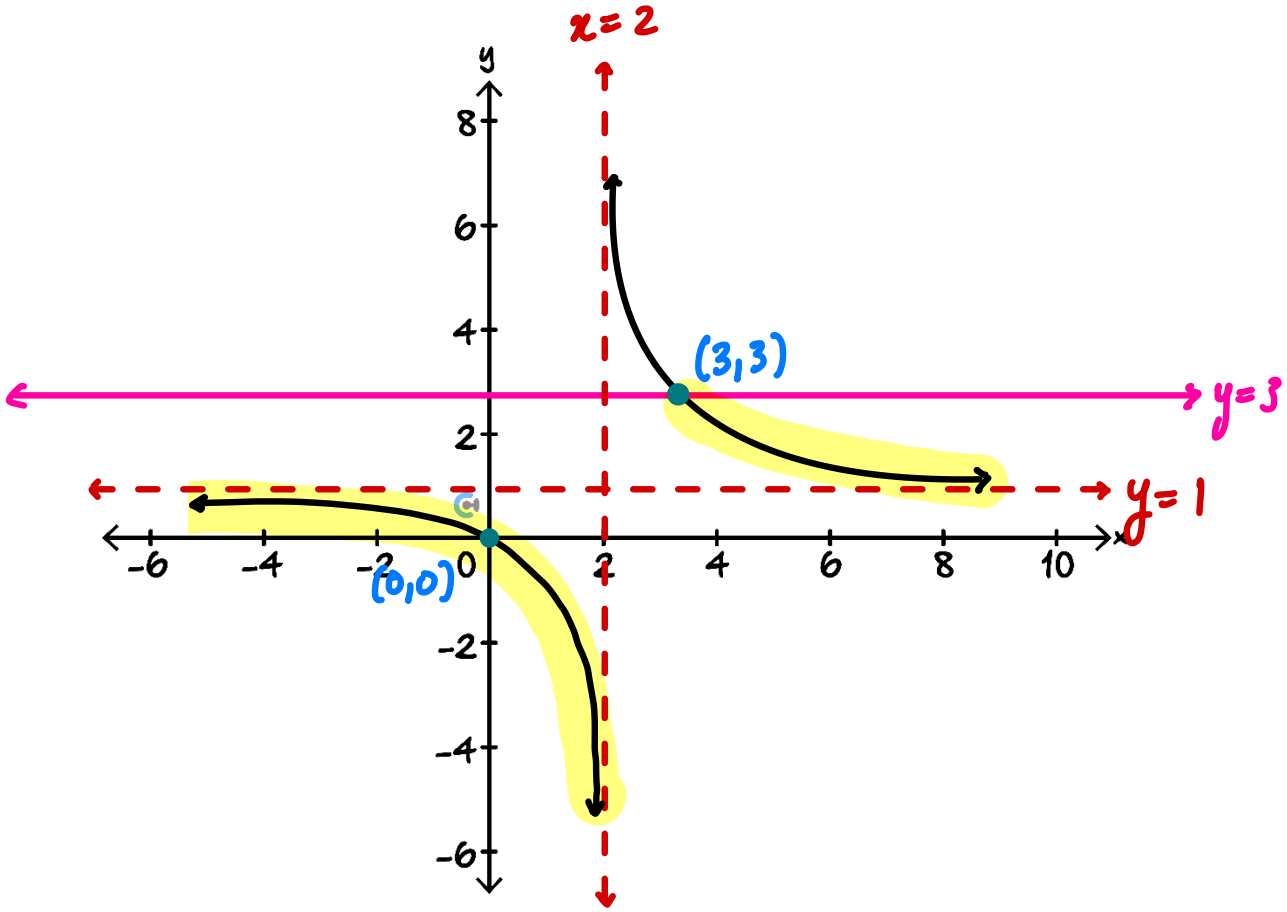
Question 1 (5 marks)

Let $f: R\backslash\{2\} \to R, f(x) = \frac{x+6}{x-2}$. → $\frac{x-2+8}{x-2} = \frac{x-2}{x-2} + \frac{8}{x-2}$

a. Express f in the form $a + \frac{b}{x-2}$, stating the values of a and b. (1 mark) **[2.1.1]**

$f(x) = 1 + \frac{8}{x-2}$ $\Rightarrow \therefore a = 1$ & $b = 8$

b. Sketch the graph of $y = 1 + \frac{2}{x-2}$ on the axes below. Label asymptotes with their equations and axis intercepts with their coordinates. (3 marks) **[2.1.1]**



c. Find the values of x for which $1 + \frac{2}{x-2} < 3$. (1 mark)

$\frac{2}{x-2} < 2 \Rightarrow$

Case 1: $x-2>0$

$2 < 2(x-2)$

$x-2>1$

$\therefore x>3$

Case 2: $x-1<0$

$2(x-2)<2$

$x-2<0 \Rightarrow x<2$

$\therefore$

Question 2 (5 marks)

The function $f(x) = a\sqrt{x-h}+k$, where a, h and k are non-zero integers, has an x-intercept at $(-2,0)$ and has an endpoint at $(-6,4)$.

a. Determine the values of a, h and k. (2 marks) **[2.1.3]**

$\because h=-6$ & $k=4$

$\therefore y = a\sqrt{x+6}+4$

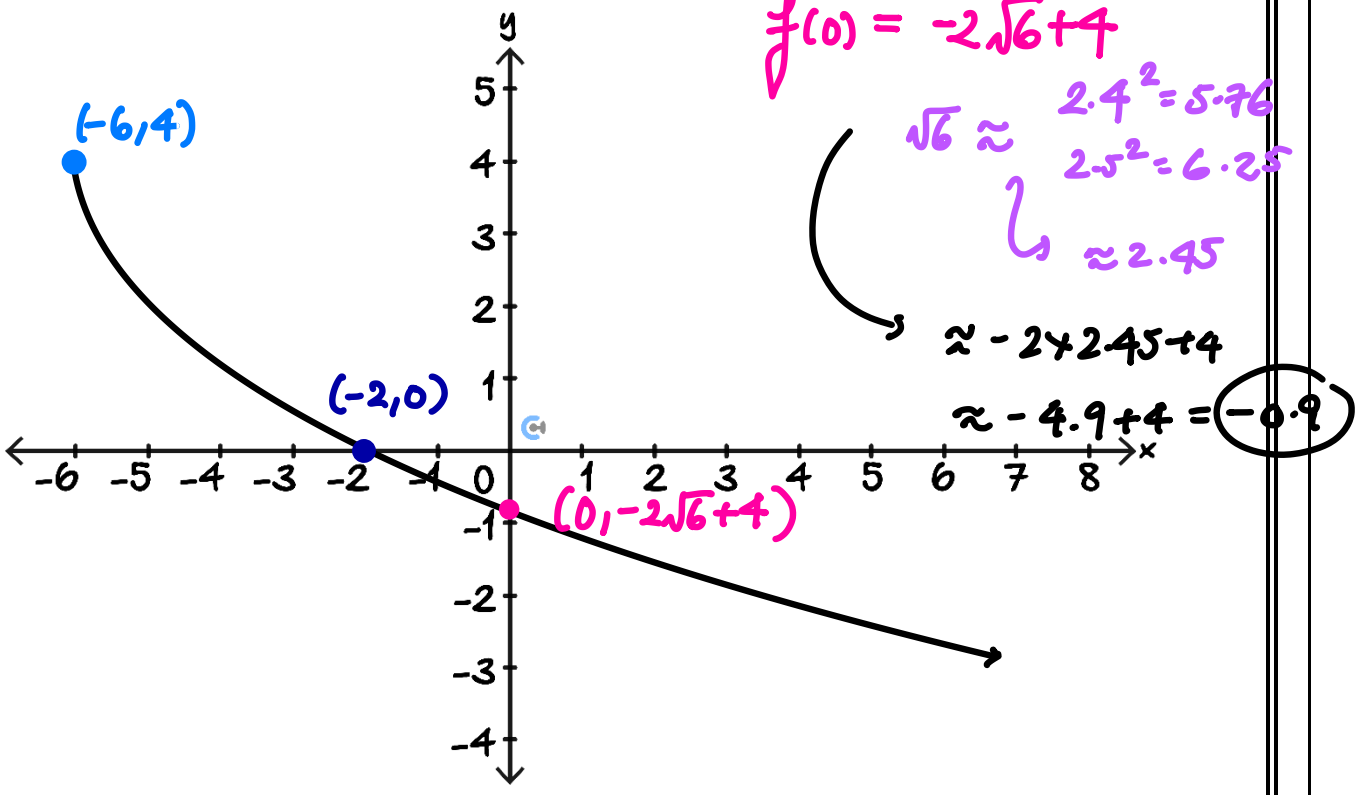
Sub$(-2,0)$:

$0 = a\sqrt{4}+4$

$2a=-4$

$\therefore a=-2$

b. Sketch the graph $y = f(x)$ on the axis below, labelling all key features. (2 marks) **[2.1.3]**



$f(0) = -2\sqrt{6}+4$

$\sqrt{6} \approx$ $2.4^2 = 5.76$, $2.5^2 = 6.25$, ≈ 2.45

$\approx -2\times2.45+4$

$\approx -4.9+4 = -0.9$

c. The image of f under a transformation T passes through the origin. Given that T is a singular translation, describe a possible transformation T.
(1 mark) **[2.4.1]**

2 right OR $2\sqrt{6}-4$ up

Question 3 (3 marks) **[2.4.2]**

Consider the following function:

$$f(x) = (x-2)^2$$

Apply the following transformations below to the function above.

Dilation by a factor of $\frac{1}{2}$ from the y-axis $\left(\frac{1}{2}x, y\right)$

Dilation by a factor of 3 from the x-axis $\left(\frac{1}{2}x, 3y\right)$

Translation by 4 units in the negative direction of the x-axis $\left(\frac{1}{2}x-4, 3y\right)$

Translation by 1 unit in the positive direction of the y-axis $\left(\frac{1}{2}x-4, 3y+1\right)$

Reflection in the y-axis $\left(-\left(\frac{1}{2}x-4\right), 3y+1\right)$

$x' = -\frac{1}{2}x+4$ $\quad y' = 3y+1$

$x = -2(x'-4)$

$y = (x-2)^2$

$\Downarrow$

$\therefore y' = 3(-2(x'-4)-2)^2+1$

$\therefore f(x) = 3(-2x+6)^2+1$ //

Question 4 (3 marks) **[2.4.3] [2.5.1]**

Consider the following functions:

$$f(x) = \sqrt{x+5}$$

$$g(x) = -\frac{1}{2}\sqrt{5-2x}+1$$

Find the set of transformations that map $f(x)$ to $g(x)$ in DRT order.

$x+5 = 5-2x'$

$2x' = -x$

$x' = -\frac{1}{2}x$ $\Rightarrow$

$y' = -\frac{1}{2}y + 1$

1. Dil $\frac{1}{2}$ from y
2. Dil $\frac{1}{2}$ from x
3. Reflection in y
4. Reflection in x
5. 1 up

Space for Personal Notes

Question 5 (11 marks)

Consider the function $f: [0, a] \to R, f(x) = 6x - x^2$.

a. Show that the largest value of a such that the inverse function f^{-1} exists is $a = 3$. (1 mark)

$f(x) = -(x^2 - 6x)$

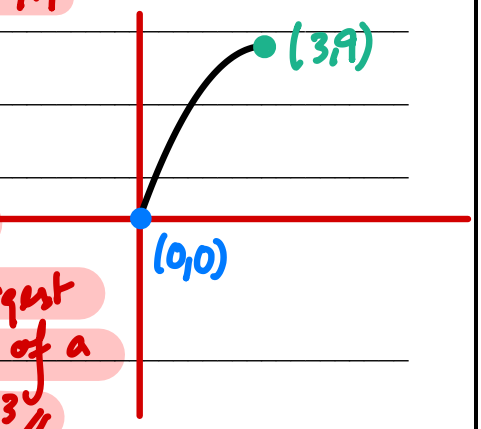
$= -((x-3)^2 - 9)$

$= -(x-3)^2 + 9$

TP: (3,9)

∴ If $a > 3$, f is no longer 1:1

f must be 1:1



↳ ∴ Largest value of a is 3 //

b. State the domain and range of the inverse of f. (2 marks)

$\text{Dom } f^{-1} = \text{Ran } f = [0, 9]$ //

$\text{Ran } f^{-1} = \text{Dom } f = [0, 3]$ //

c. Determine the rule of the inverse function f^{-1}. (2 marks)

Let $y = f(x)$:

Swap x & y:

$x = -(y-3)^2 + 9$

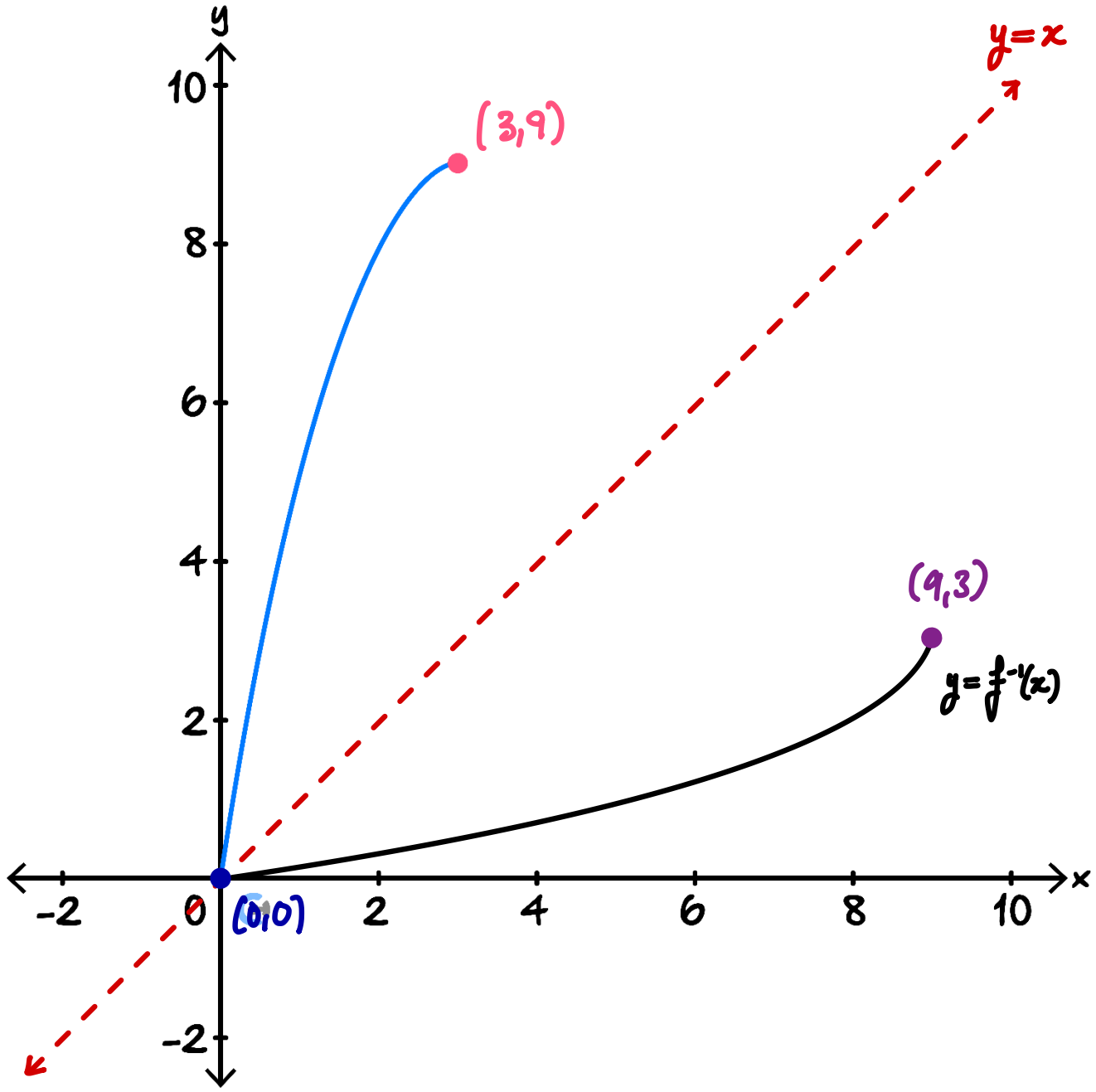
$(y-3)^2 = 9 - x$

$y - 3 = \pm\sqrt{9-x}$

$y = 3 \pm \sqrt{9-x}$

↳ $\therefore f^{-1}(x) = 3 - \sqrt{9-x}$

d. The graph of f is shown on the graph below. On the same set of axes, sketch accurately the graph of the inverse of f. (2 marks)



e. Find an intersection point between f and f^{-1}. (1 mark)

∴ IP: (0,0)

f. **Extension.** Consider the function $g: [0, 3] \to R, g(x) = f(x) - k$, where $k > 0$.
Find all values of k such that g and g^{-1} have two points of intersection. (3 marks) **[2.5.4]**

∴ let $g(x) = x$:

$6x - x^2 - k = x$

∴ $x^2 - 5x + k = 0$

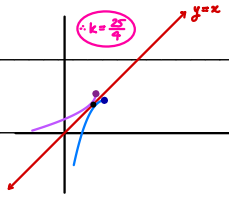
Since 2 solns:

∴ $\Delta = (-5)^2 - 4(1)(k) > 0$

$25 - 4(1)(k) > 0$

$4k < 25$

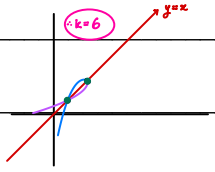
∴ $k < \frac{25}{4}$



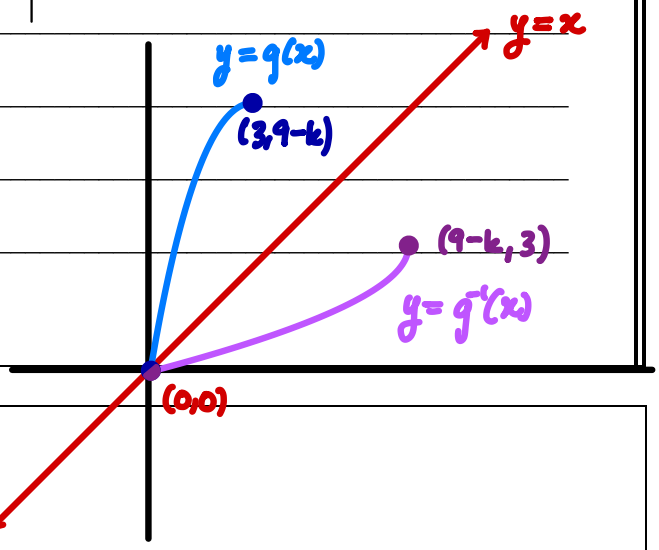
Solve for EP intersection:

∴ $9 - k = 3$

∴ $k = 6$



↳ ∴ $6 \leq k < \frac{25}{4}$



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For 2 intersections :

we know $g(x)$ and $y=x$ intersect twice when the endpoint lies on $y=x$.

↳ ∴ $9 - k = 3$

Section C: Tech Active Exam Skills

Calculator Commands: Using Sliders/Manipulate on CAS

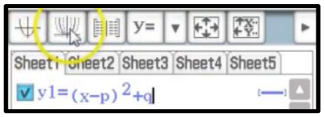
Mathematica

```
Manipulate[Plot[function, {x, xmin, xmax}],
    {unknown, lowerbound, upperbound}]
```

- **NOTE:** The function **must** be typed out instead of using its saved name.

TI-Nspire

Casio Classpad



Calculator Commands: Finding maximal domain

Mathematica

```
FunctionDomain[func, x]
```

TI-Nspire

- Type up domain (or find it under the book button).

$$\text{domain}(\text{func},x)$$

Casio Classpad

- Sketch the function and analyse.

Space for Personal Notes

Defining Hybrid Functions on CAS

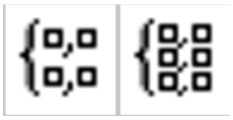
Mathematica

- Piecewise

$$\text{Piecewise}[\{\{val_1, cond_1\}, \{val_2, cond_2\}, \ldots\}]$$

- represents a piecewise function with values val_i in the regions defined by the conditions $cond_i$.

TI-Nspire



$$\begin{cases} func\,1, dom\,1 \\ func\,2, dom\,2 \end{cases}$$

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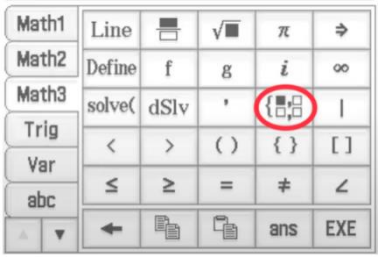

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Calculator Tip: Finding Transformed Functions

- Save the function as $f(x)$.
- Substitute the x and y in terms of x' and y'.
- Solve for y!
- Can also apply the transformations directly to $f(x)$. Must make sure you interpret the transformations correctly or you can easily make a mistake doing this.
- **Example:** Apply the following transformations to $y = 2\sqrt{3x + 6}$.

Dilation by a factor $\frac{1}{2}$ from the x-axis.

Dilation by a factor 3 from the y-axis.

Reflection in the y-axis.

Translation of 3 units right.

Translation of 4 units down.

In[22]:= $f[x\_] := 2\sqrt{3x + 6}$

In[26]:= $1/2\, f[-1/3\ (x - 3)] - 4$

Out[26]= $-4 + \sqrt{9 - x}$

In[27]:= $\text{Solve}\left[\frac{y + 4}{1/2} == 2\sqrt{3 * (-1/3\ (x - 3)) + 6}, y\right]$

Out[27]= $\{\{y \to -4 + \sqrt{9 - x}\}\}$

Space for Personal Notes

Section D: Exam 2 Questions (34 Marks)

INSTRUCTION:

- Regular: 34 Marks. 5 Minutes Reading. 45 Minutes Writing.
- Extension: 34 Marks. 5 Minutes Reading. 34 Minutes Writing.

Question 6 (1 mark)

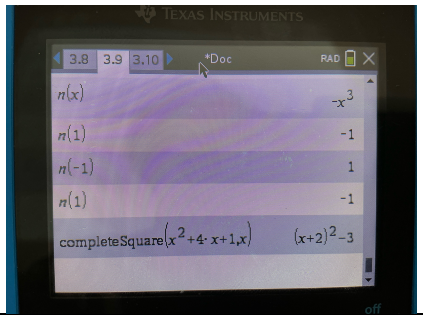
The largest value of a for which $f : (-\infty, a] \to \mathbb{R}, f(x) = x^2 + 4x + 1$ has an inverse function is: **[2.3.1]**

A. -2

B. 2

C. -3

D. 1



Question 7 (1 mark)

Which of the following relations is many to many? **[2.1.5]**

A. $y = \sqrt{4 - x^2}$

B. $y^2 = x$

C. $x^2y - y^2x^2 = -1$

D. $y = 2x^2 + 3x - 1$

Space for Personal Notes

Question 8 (1 mark)

Consider the function $f(x) = x^3$. Transformations S and T are described in the options below. Select the option where transformations S and T give the same image of f. **[2.5.5]**

A. S: A dilation by factor 2 from the x-axis. T: A dilation by factor 8 from the y-axis.

B. S: A dilation by factor 8 from the x-axis. T: A dilation by factor 2 from the y-axis.

C. S: A dilation by factor 8 from the x-axis. T: A dilation by factor $\frac{1}{2}$ from the y-axis.

D. ~~S: A dilation by factor 8 from the x-axis. T: A dilation by factor 2 from the y-axis.~~

Question 9 (1 mark)

Consider the function $f : [1, \infty) \to R, f(x) = (x - 1)^2 + 3$. Let $f^{-1}(x) = g(x)$, and let $h(x) = f(x - k)$, where k is a positive real constant. Then it is true that: **[2.5.4]**

A. $h^{-1}(x) = g(x - k) + k$

B. $h^{-1}(x) = g(x) + k$

C. $h^{-1}(x) = g(x - k)$

D. $h^{-1}(x) = g(x) - k$

Question 10 (1 mark)

Consider the hybrid function:

$$f(x) = \begin{cases} -3x, & -2 \leq x < 1 \\ x^2 - 6x + 4, & 2 < x < 6 \end{cases}$$

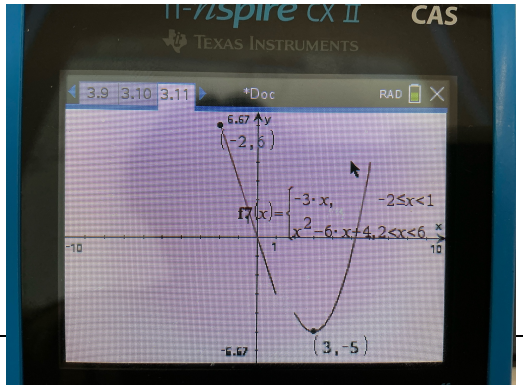
The domain and range of f respectively are: **[2.2.2]**

A. $[-2, 1) \cup (2, 6)$ and $[-5, 4]$

B. $[-2, 1) \cup (2, 6)$ and $[-3, 6]$

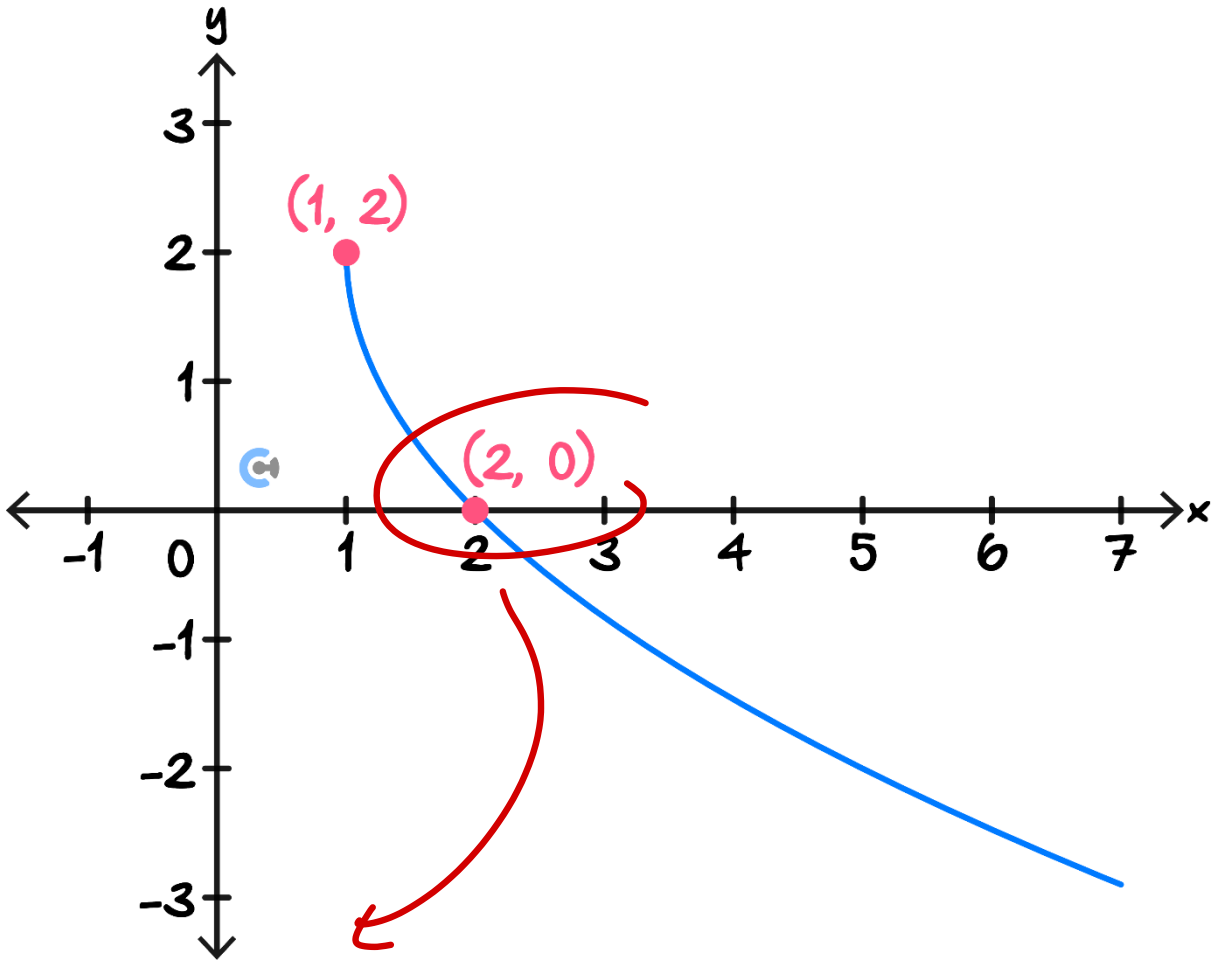
C. $[-2, 1) \cup (2, 6)$ and $[-5, 6]$

D. $[-2, 1) \cup [2, 6]$ and $[-6, 5]$



Question 11 (1 mark)

The rule for the graph shown below could be:



A. $y = -4\sqrt{x-1} + 2$

B. $y = -\sqrt{4x-4} + 2$

C. $y = \sqrt{4x-4} + 2$

D. $y = 4\sqrt{x-1} + 2$

Space for Personal Notes

Question 12 (1 mark)

$\frac{1}{3}(x-2) = x'$ OR $x' = \frac{1}{3}x - \frac{2}{3}$

Consider the functions f and g, where $g(x) = \frac{1}{2} f\left(\frac{1}{3}(x-2)\right)$.

$y = \frac{1}{2}y'$

$\therefore y' = 2y$

A sequence of transformations that maps g to f is: **[2.5.2]**

A. A dilation by factor $\frac{1}{2}$ from the x-axis followed by a dilation by factor 3 from the y-axis and a translation 2 units to the right.

B. A translation 2 units to the left followed by a dilation by factor $\frac{1}{3}$ from the y-axis and a dilation by factor 2 from the x-axis.

C. A dilation by factor $\frac{1}{3}$ from the y-axis and a dilation by factor 2 from the x-axis followed by a translation 2 units to the right.

D. A dilation by factor $\frac{1}{2}$ from the x-axis followed by a dilation by factor $\frac{1}{3}$ from the y-axis and a translation 2 units to the left.

Question 13 (18 marks)

Consider the function $f(x) = \frac{8}{(x-1)^2} - 2$.

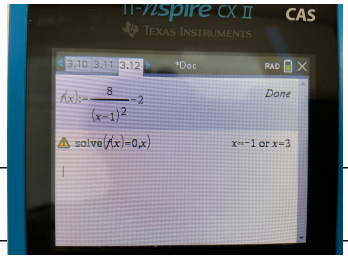
a. State the maximal domain and range of f. (2 marks) **[2.2.1]**

Dom f : $x \in \mathbb{R} \setminus \{1\}$

Ran f : $f(x) \in (-2, \infty)$

b.

i. State the coordinates for the x-intercepts of f. (1 mark)



$x = -1$ OR 3

↳ $\therefore (-1, 0)$ & $(3, 0)$

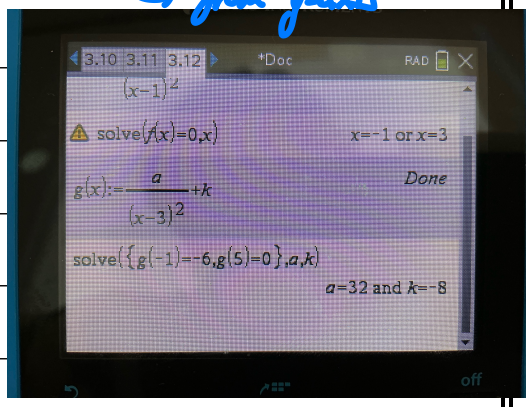
ii. The function f undergoes a dilation by factor b from the y-axis. Determine the value of b if the image of f has x-axis intercepts of $(-2, 0)$ and $(6, 0)$. (1 mark) **[2.5.3]**

Dil 2 from y ⇒ $\therefore b = 2$

iii. **Extension.** The function g has a vertical asymptote at $x = 3$ and include the points $(-1, -6)$ and $(5, 0)$.

Determine a sequence of transformations, with translations first, and no dilation from the x-axis, that maps the graph of f to the graph of g. (4 marks) **[2.1.2] [2.4.3] [2.5.5]**

↳ from y-axis



$\because g(x) = \frac{a}{(x-3)^2} + k$

$\therefore g(x) = \frac{32}{(x-3)^2} + 8$

$y = \frac{8}{(x-1)^2} - 2 \Rightarrow y' = \frac{32}{(x'-3)^2} + 8$

$y = \frac{1}{\frac{1}{8}(x-1)^2} - 2 \Rightarrow y' = \frac{1}{\frac{1}{32}(x'-3)^2} + 8$

$y = \frac{1}{\left(\frac{1}{2\sqrt{2}}(x-1)\right)^2} - 2 \Rightarrow y' = \frac{1}{\left(\frac{1}{4\sqrt{2}}(x'-3)\right)^2} + 8$

$\frac{1}{2\sqrt{2}}(x-1) = \frac{1}{4\sqrt{2}}(x'-3)$

$2x - 2 = x' - 3$

$x' = 2(x + \frac{1}{2})$

1. $\frac{1}{2}$ right
2. 10 up
3. Dil 2 from y

Now, consider the family of function $h: (-\infty, 2) \to \mathbb{R}, h(x) = \frac{a}{(x-2)^2} - 2$, where $a > 0$.

c. Find the domain and range for the inverse function h^{-1}. (2 marks) **[2.2.3]**

Dom h^{-1} = Ran h = $(-2, \infty)$

Ran h^{-1} = Dom h = $(-\infty, 2)$

d. Find the rule for $h^{-1}(x)$. (2 marks) **[2.2.3]**

let $y = h(x)$:

Swap x & y:

$x = \frac{a}{(y-2)^2} - 2$

$x + 2 = \frac{a}{(y-2)^2}$

$(y-2)^2 = \frac{a}{x+2}$

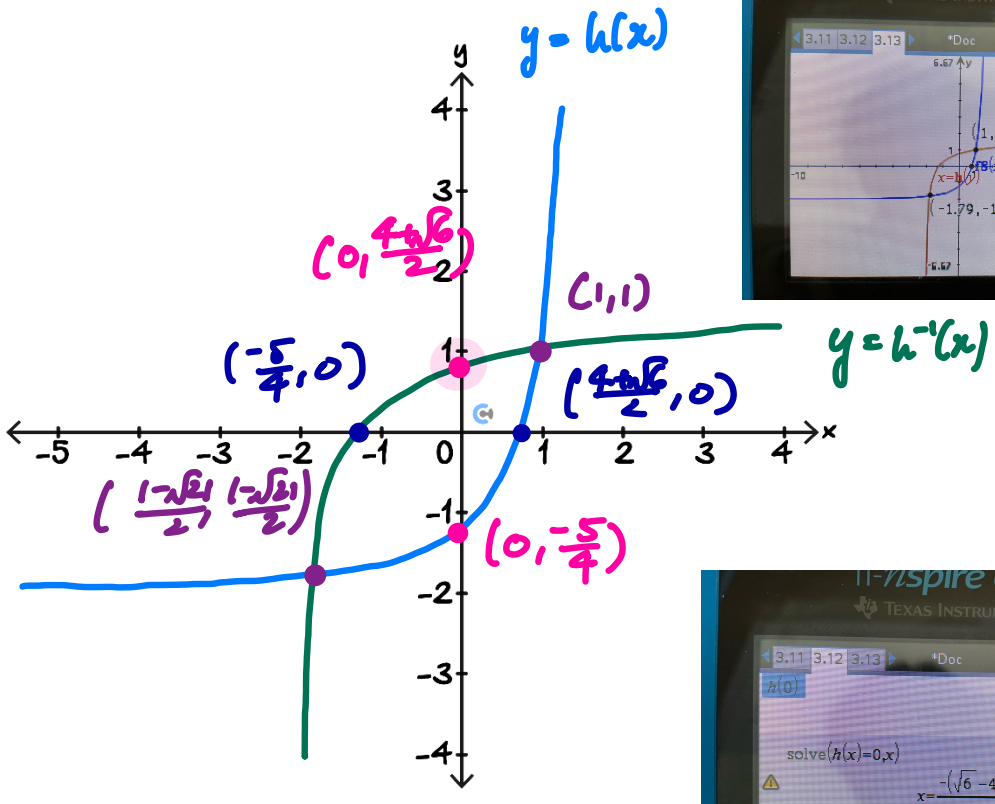
$y - 2 = \pm\sqrt{\frac{a}{x+2}}$

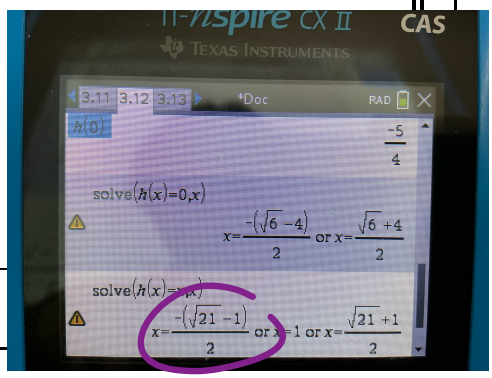
$y = 2 \pm \sqrt{\frac{a}{x+2}}$

$\therefore h^{-1}(x) = 2 - \sqrt{\frac{a}{x+2}}$

Gas ran $h^{-1} \in (-\infty, 2)$

e. Sketch the graphs of h and h^{-1}, where $a = 3$, on the axes below. Label all axes intercepts and points of intersection ~~with correct to 2 decimal places~~. (3 marks) **[2.1.2]**



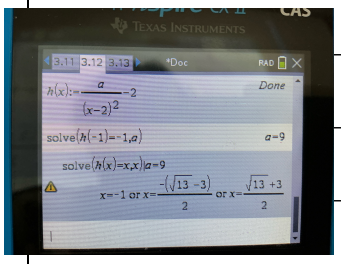


f. It is known that the graphs of h and h^{-1} intersect at $(-1,-1)$. Find the value of a and the other points of intersection. (2 marks)

$\because h(-1) = -1$:

$\therefore a = 9$ $\Rightarrow$ Solve $h(x) = x$:

$\therefore x = -1$ OR $\frac{3-\sqrt{13}}{2}$ OR $\frac{3+\sqrt{13}}{2}$

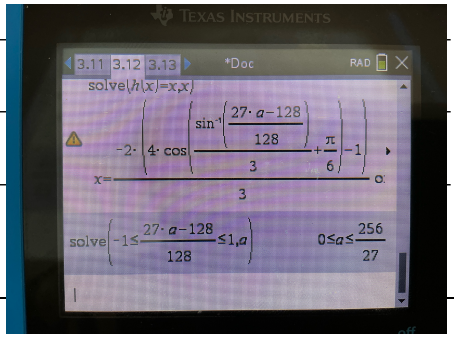


Other IP: $\left(\frac{3-\sqrt{13}}{2}, \frac{3-\sqrt{13}}{2}\right)$

Reject as Dom $h \in (-2,\infty)$

g. Find the range of values of a such that h and h^{-1} do not intersect each other. Give your answer correct to one decimal place.

TIP: Use sliders. **Extension.** Bonus: Can you find the exact range? (1 mark) **[2.3.3]**



a cannot be < 0

a cannot be $\in [0, \frac{256}{27}]$

$\therefore a > \frac{256}{27}$

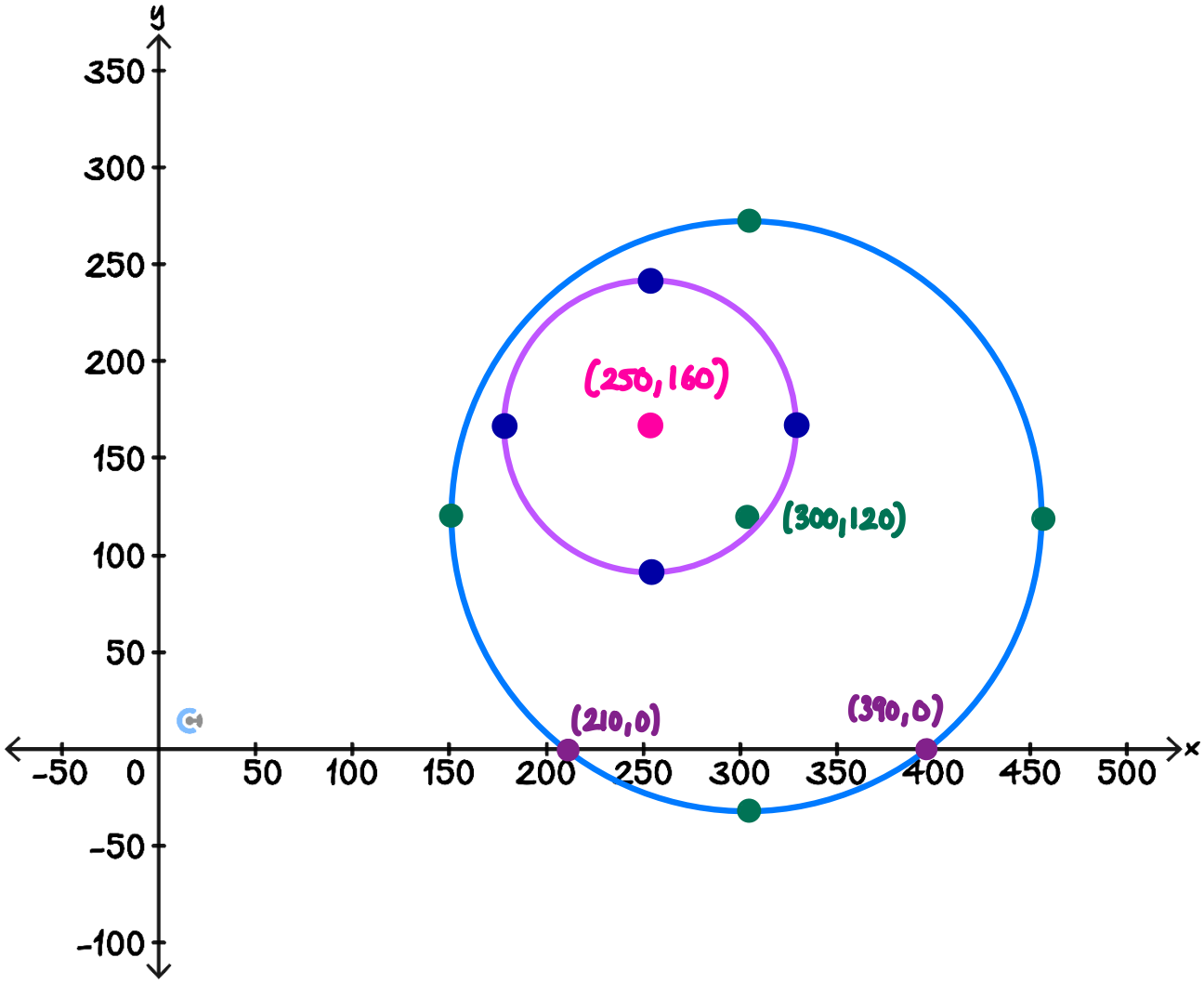
Question 14 (9 marks)

A futuristic floating athletics arena is designed in the shape of a perfect circle with a radius of 150 metres. The centre of the arena is located at $(300, 120)$ in a coordinate system where x represents the horizontal distance east and y represents the vertical distance north.

a. Find the equation for the boundary of the athletics arena. (1 mark) **[2.1.4]**

$\therefore (x-300)^2 + (y-120)^2 = 150^2$ //

b. Sketch the boundary, labelling all axial intercepts. (2 marks) **[2.1.4]**



To prevent athletes from falling off the floating arena, a protective energy shield extends 10 metres inside the arena boundary, forming an inner circular safety zone.

c. Write down the equation for the top half of the energy shield's boundary. (2 marks) **[2.1.4]**

$(x-300)^2 + (y-120)^2 = 140^2$

$\Downarrow$

$\therefore y = +\sqrt{140^2-(x-300)^2} + 120$ //

A mysterious teleportation vortex appears at the point $(250, 160)$ in the arena. Any athlete who gets within 80 metres of this vortex is randomly transported to another point within the vortex's area of influence.

d. State whether anyone will be transported outside the arena by the vortex. (1 mark)

$(x-250)^2+(y-160)^2 = 80^2$

$\therefore$ No!

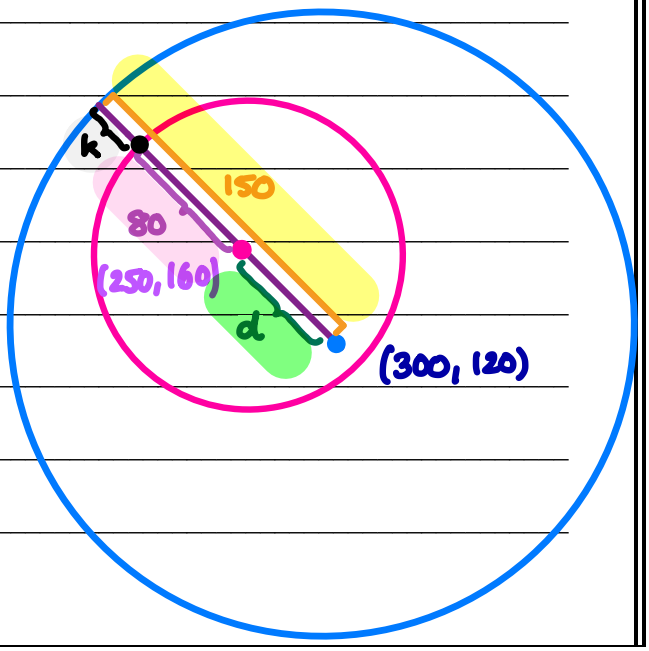
e. **Extension.** While scientists try to find a way to close the vortex, it is decided to extend the arena's energy shield so that anyone transported by the vortex remains inside the energy shield.

Find the largest distance, k metres, that the energy shield can be away from the edge of the arena, such that the vortex cannot transport anyone past the energy shield. Give your answer correct to two decimal places. (3 marks) **[2.1.4] [2.3.3]**

$\therefore d = \sqrt{(300-250)^2+(120-160)^2} \approx 64.03\text{m}$

$\because d + 80 + k = 150$

$\hookrightarrow \therefore k = 70 - d \approx 5.97\text{m}$ //



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