CONTOUR EDUCATION

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VCE Mathematical Methods ½
Transformations ~~Exam Skills~~ [0.10]
Workshop

Error Logbook:

Mistake/Misconception #1 Don't know		Mistake/Misconception #2 Algebra / Calculation	
Question #:	Page #:	Question #:	Page #:
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Mistake/Misconception #3 Not reading / Details		**Mistake/Misconception #4** Time management	
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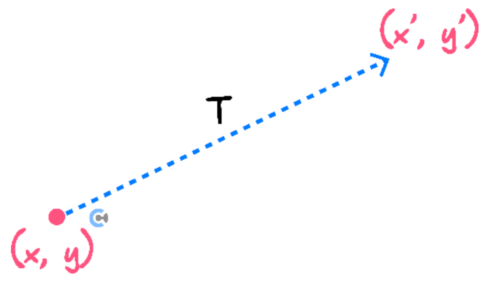
Section A: Recap

Image and Pre-Image



- The original coordinate is called the pre-image.
- The transformed coordinate is called the image.

Pre-Image: (x, y)

Image: (x', y')

Dilation

Dilation by a factor a from the x-axis: $y' = ay$

Dilation by a factor b from the y-axis: $x' = bx$

Reflection

Reflection in the x-axis: $y' = -y$

Reflection in the y-axis: $x' = -x$

Space for Personal Notes

Translation

same

Translation by c units in the positive direction of the x-axis: $x' = x + c$

Translation by d units in the positive direction of the y-axis: $y' = y + d$

The Order of Transformation

Dr.T

Order = BODMAS Order

No.

Transformation of Functions

- The aim is to get rid of the old variables, x and y, and have the new variables, x' and y', instead.

$$y = f(x) \rightarrow y' = f(x')$$

- Steps:
  1. Transform the points.
  2. Make x and y the subjects.
  3. Substitute them into the function.

Reverse Engineering

- **Steps**:
  1. Add the dashes (') back to the transformed function.
  2. Make $f(\ )$ the subject. ← main operation
  3. Equate the LHS of the original and transformed functions to the RHS of the original and transformed functions.
     y-region x-region
  4. Make x' and y' the subjects and interpret the transformations.

↳ Expanded ⇒ Dr.T

Section B: Warmup (10 Marks)

INSTRUCTION: 10 Marks. 10 Minutes Writing. → Early? Section F

Question 1 (4 marks)

a. Find the image of (x', y') after applying the following transformations to (x, y). (2 marks)

- Dilation by factor 3 from the x-axis. $y' = 3y$
- Dilation by factor 2 from the y-axis. $x' = 2x$
- Reflection in the y-axis. $x' = -2x$
- Translation 4 units to the right and 6 units down.

$x' = -2x + 4$ $\quad$ $y' = 3y - 6$

$(-2x + 4, 3y - 6)$

b. Describe a sequence of transformations that produce the same image as those described in **part a.**, but where all translations occur before reflections and dilations. (2 marks)

→ factorise

$x' = -2x + 4 = -2(x - 2)$

$y' = 3y - 6 = 3(y - 2)$

(x)
T 2 left
R in y-axis
D by factor 2 from y-axis

(y)
T 2 down
~~R~~
D by factor 3 from x-axis

Question 2 (3 marks)

Apply the following list of transformations to the function $f(x) = x^2$. Express your answer in turning point form.

Step 1 $y = x^2$

- Dilation by factor 3 from the x-axis. $y' = 3y$
- Reflection in the y-axis. $x' = -x$
- Translation 5 units to the left. $x' = -x - 5$
- Dilation by a factor of 2 from the y-axis. $x' = 2(-x-5)$

Step 2

$y = \frac{y'}{3}$ $\quad \frac{x'}{2} = -x - 5$

$x = -\frac{x'}{2} - 5$

Step 3

$y = x^2$

$\frac{y'}{3} = \left(-\frac{x'}{2} - 5\right)^2$

$\frac{y'}{3} = \left(-\frac{1}{2}(x'+10)\right)^2$

$\frac{y'}{3} = \frac{1}{4}(x'+10)^2$

$y' = \frac{3}{4}(x'+10)^2$

$y = \frac{3}{4}(x+10)^2$

Question 3 (3 marks)

Find a sequence of transformations that map $g(x) = x^2$ to $h(x) = 4\left(\frac{x+2}{3}\right)^2 + 6$.

$y = x^2 \quad y' = 4\left(\frac{x'+2}{3}\right)^2 + 6$

$y = x^2 \quad \frac{y'-6}{4} = \left(\frac{x'+2}{3}\right)^2$

$y = \frac{y'-6}{4} \Rightarrow 4y = y' - 6 \Rightarrow y' = 4y + 6$

$x = \frac{x'+2}{3} \Rightarrow x' + 2 = 3x \Rightarrow x' = 3x - 2$

x:
D factor 3 from y-axis
R
T 2 left

y:
D factor 4 from x-axis
R
T 6 up

Section C: Exam 1 Questions (16 Marks)

INSTRUCTION: 16 Marks. 25 Minutes Writing. Extension: 17 Minutes Writing.

Question 4 (3 marks)

For the function $f(x) = (x-2)^2 + 3$, the function f is dilated by a factor of 3 from the y-axis, translated 2 units in the positive x-direction and then is reflected in the y-axis to produce the function g. Find the rule for $g(x)$.

Step 1 points

1. $x' = 3x$
2. $x' = 3x + 2$
3. $x' = -(3x+2)$

$y' = y$

Step 2 Rearrange

$-x' = 3x + 2$

$-x' - 2 = 3x$

$x = \frac{-x' - 2}{3}$

Step 3 sub

$y' = \left(\frac{-x'-2}{3} - 2\right)^2 + 3$

Space for Personal Notes

$y' = \left(-\frac{x'}{3} - \frac{2}{3} - 2\right)^2 + 3$

$y' = \left(-\frac{x'}{3} - \frac{8}{3}\right)^2 + 3$

$y = \left(-\frac{x}{3} - \frac{8}{3}\right)^2 + 3$

$y = \left(\frac{x}{3} + \frac{8}{3}\right)^2 + 3$

Question 5 (6 marks)

a. Find the rule for the image of the graph of $y = -x^2$ under the following sequence of transformations: (3 marks)

- Reflection in the y-axis. $x' = -x$
- Dilation of factor $\frac{1}{2}$ from the y-axis. $x' = -\frac{1}{2}x$
- Translation 2 units in the positive direction of the x-axis and 3 units in the negative direction of the y-axis.
- Reflection in the x-axis.

$x' = -\frac{1}{2}x + 2$ $\quad$ $y' = y - 3$

$y' = -(y-3)$

Step 2

$-y' = y - 3$

$y = -y' + 3$

$x' - 2 = -\frac{1}{2}x$

$-2(x'-2) = x$

Step 3

$y = -x^2$

$-y' + 3 = -(-2(x'-2))^2$

$-y' + 3 = -4(x'-2)^2$

$-y' = -4(x'-2)^2 - 3$

$y' = 4(x'-2)^2 + 3$

b. Find a sequence of transformations that takes the graph of $y = x^4$ to the graph of $y' = 4 - 5(2(x' - 3))^4$. (3 marks)

$y = x^4$ $\quad$ $\frac{y'-4}{-5} = (2(x'-3))^4$

$y = \frac{y'-4}{-5}$

$-5y = y' - 4$

$y' = -5y + 4$

$x = 2(x'-3)$

$\frac{x}{2} = x' - 3$

$x' = \frac{x}{2} + 3$

D factor 5 from x-axis

R in x-axis

T 4 up

D factor $\frac{1}{2}$ from y-axis

~~R~~

T 3 right

Question 6 (4 marks)

A transformation $T: R^2 \to R^2$ has the rule:

$x' =$ $y' =$

$$T(x, y) = \left(-2x - 3, 2y + \frac{1}{2}\right)$$

transformed

Find the image of the curve with the equation $y = \frac{1}{2}\sqrt{-x-4} - \frac{1}{4}$ under this transformation.

Step 2 Rearrange

$x' = -2x - 3$

$x' + 3 = -2x$

$x = \frac{x'+3}{-2}$

$y' = 2y + \frac{1}{2}$

$2y = y' - \frac{1}{2}$

$y = \frac{1}{2}y' - \frac{1}{4}$

Step 3 Sub

$\frac{1}{2}y' - \frac{1}{4} = \frac{1}{2}\sqrt{-\left(-\frac{x'+3}{2}\right) - 4} - \frac{1}{4}$

$\frac{1}{2}y' - \frac{1}{4} = \frac{1}{2}\sqrt{\frac{x'}{2} + \frac{3}{2} - 4} - \frac{1}{4}$

$\frac{1}{2}y' = \frac{1}{2}\sqrt{\frac{x'}{2} - \frac{5}{2}}$

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$y' = \sqrt{\frac{x'}{2} - \frac{5}{2}}$

$y = \sqrt{\frac{x}{2} - \frac{5}{2}}$

Question 7 (3 marks)

Find a sequence of transformations that takes the graph of $y = \frac{1}{4}\left(\frac{3-x}{2}\right)^3 - 4$ to the graph of $y = x^3$.

pre

image

$4(y+4) = \left(\frac{3-x}{2}\right)^3$ $\quad y' = x'^3$

$4(y+4) = y'$ $\quad \frac{3-x}{2} = x'$

$y' = 4y + 16$ $\quad x' = \frac{3}{2} - \frac{1}{2}x$

D 4 from x-axis

~~R~~

T 16 up

D $\frac{1}{2}$ from y

R in y-axis

T $\frac{3}{2}$ right

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Section D: Tech Active Exam Skills

Calculator Tip: Finding Transformed Functions

- Save the function as $f(x)$.
- Substitute the x and y in terms of x' and y'.
- Solve for y!
- Can also apply the transformations directly to $f(x)$. Must make sure you interpret the transformations correctly, or you can easily make a mistake doing this.

Example: Apply the following transformations to $y = 2\sqrt{3x+6}$.

Dilation by a factor $\frac{1}{2}$ from the x-axis.

Dilation by a factor 3 from the y-axis.

Reflection in the y-axis.

Translation of 3 units right.

Translation of 4 units down.

```
In[22]:= f[x_] := 2 √(3 x + 6)

In[26]:= 1/2 f[-1/3 (x - 3)] - 4

Out[26]= -4 + √(9 - x)

In[27]:= Solve[(y + 4)/(1/2) == 2 √(3 * (-1/3 (x - 3)) + 6), y]

Out[27]= {{y → -4 + √(9 - x)}}
```

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Calculator Tip: Mathematica UDF

- ApplyTransformList[]

ApplyTransformList[*f[x]* , *{x, y}*, *list of transforms*]
Applies the list of transforms to f[x] in the chronological order.

```
ApplyTransformList[x^2, {x, y}, {x - 1, 2 x, y + 3}]
```

$$4 + x + \frac{x^2}{4}$$

```
ApplyTransformInvList[f[x], {x, y}, {x - 1, 2 x, y + 3}]
```

```
-3 + f[2 (-1 + x)]
```

```
ApplyTransformInvList[Sin[x], {x, y}, {x - π / 2, 2 y, y - 1 }]
```

$$\text{Sin}\left[\frac{x}{2}\right]^2$$

- ApplyTransformInvList[]

ApplyTransformInvList[*f[x]* , *{x, y}*, *list of transforms*]
Applies the list of transforms to f[x] in reverse order and as the inverse to the transforms of *ApplyTransformList.*

In[∘]:=
```
ApplyTransformInvList[x^2, {x, y}, {x - 1, 2 * x, y + 3}]
```
Out[∘]=
$$1 - 8 x + 4 x^2$$

In[∘]:=
```
ApplyTransformInvList[f[x], {x, y}, {x - 1, 2 * x, y + 3}]
```
Out[∘]=
```
-3 + f[2 (-1 + x)]
```

In[∘]:=
```
ApplyTransformInvList[2 * Cos[x] - 1, {x, y}, {x - Pi / 2, 2 * y, y - 1 }]
```
Out[∘]=
```
Sin[x]
```

Calculator Tip: TI UDF

- transform()

Transform a Function

$$transform\left(\sin(x),x,\left\{x-\frac{\pi}{2},2\cdot y,y-1\right\}\right)$$

- Translation $\frac{\pi}{2}$ units along the neg. x−dir.

$\cos(x)$

- Dilation by factor of 2 from the x−axis

$2\cdot\cos(x)$

- Translation −1 unit along the neg. y−dir.

$2\cdot\cos(x)-1$

Overview:

Apply any sequence of transformations to a function. The program will display the transformed function after each step.

Input:

```
transform(<function>, <variable>,
      <list of transformations>)
```

Other notes:

- The list of transformations can either be presented in a (horizontal or vertical) matrix of expressions or a list of expressions

- transform_inv()

Invert a Transformation

$$transform\_inv\left(x^2,x,\{x-1,2\cdot x,y+3\}\right)$$

- Inverted Transformations:

$\left\{y-3,\frac{x}{2},x+1\right\}$

- Translation −3 units along the neg. y−dir.

x^2-3

- Dilation by factor of $\frac{1}{2}$ from the y−axis

$4\cdot x^2-3$

- Translation 1 unit along the pos. x−dir.

$4\cdot x^2-8\cdot x+1$

Overview:

Find the preimage of a function under a list of transformations. The program will display the list of inverted transformations and the transformed function after each step.

Input:

```
transform_inv(<function>, <variable>,
      <list of transformations>)
```

Other notes:

- The list of transformations can either be presented in a row or column matrix, or a list of expressions

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Section E: Exam 2 Questions (24 Marks)

INSTRUCTION: 24 Marks. 36 Minutes Writing. Extension: 24 Minutes Writing.

Question 8 (1 mark)

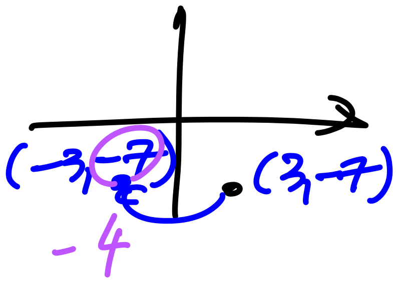
What are the coordinates of the image of the point $(3, -7)$ under a reflection in the y-axis, followed by a translation of 3 units in the positive direction of the y-axis?

A. $(-3,7)$

B. $(3,-4)$

C. $(-3,-10)$

D. $(-3,-4)$



Question 9 (1 mark)

A curve has an equation $y = f(x)$. The following transformations are applied to the curve in the given order:

- A reflection in the x-axis. $y' = -y \Rightarrow y = -y'$
- A dilation of factor 3 from the y-axis. $x' = 3x$
- A translation of 3 units in the negative direction of the x-axis. $x' = 3x - 3$

$\Rightarrow \frac{x'+3}{3} = x$

The equation of the resulting curve is:

A. $y = 3f(-x) + 3$

B. $y = f(3x - 3) + 3$

C. $y = -3f(x - 3)$

D. $y = -f\left(\frac{3+x}{3}\right)$

$y = f(x)$

$-y' = f\left(\frac{x+3}{3}\right)$

$y' = -f\left(\frac{x+3}{3}\right)$

$y = -f\left(\frac{x+3}{3}\right)$

Space for Personal Notes

Question 10 (1 mark)

If $f(x) = \begin{cases} -3x + 5 \text{ if } x \geq -2 \\ x + 5 \text{ if } x < -2 \end{cases}$ and $f(x)$ is translated 2 units in the positive direction of x-axis, $f(0)$ is equal to:

$x = -2$ Right

$x = 0$

$x = -2$

$6 + 5 = 11$

A. -5

B. 3

C. 11

D. 5

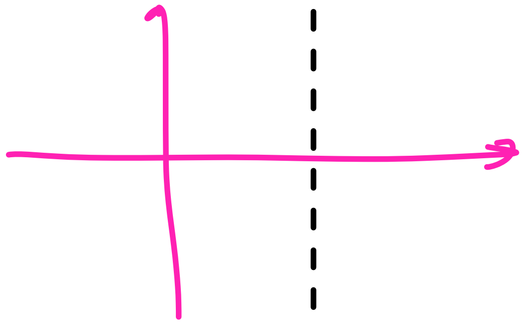
Question 11 (1 mark)

The equation of the vertical asymptote of $y = \frac{3x-2}{4-x}$ is $x = 4$. This function then undergoes a series of transformations:

S

- Dilation by a factor of $\frac{2}{27}$ from the x-axis.
- Reflection in the x-axis.
- Translation 12 units in the negative direction of the y-axis.

down



State the new equation of the vertical asymptote.

A. $x = -4$

B. $x = 4$

C. $x = -12$

D. $x = \frac{2}{27}$

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Question 12 (1 mark)

$y = \sqrt{x-2}$

Consider the following functions:

DRT

$$f(x) = \sqrt{x-2}$$

$$g(x) = 2\sqrt{3x-4} + 2$$

$\frac{y'-2}{2} = \sqrt{3x'-4}$

State the set of transformations required to transform from $f(x)$ to $g(x)$.

$y = \frac{y'-2}{2}$ $\quad x - 2 = 3x' - 4$

$y' = 2y + 2$ $\quad x' = \frac{x+2}{3}$

A. Dilation by a factor of 3 from the x-axis, translation 2 up, translation 2 right and dilation by a factor of $\frac{1}{3}$ from the y-axis.

B. Dilation by a factor of $\frac{1}{3}$ from the x-axis, translation 2 up, translation 2 right and dilation by a factor of 2 from the y-axis.

C. Dilation by a factor of 2 from the x-axis, translation 2 up, translation 2 right and dilation by a factor of $\frac{1}{3}$ from the y-axis.

D. Dilation by a factor of 3 from the x-axis, translation 2 up, translation 2 right and dilation by a factor of 2 from the y-axis.

(0,3)

TRD

Question 13 (1 mark)

For the function $f: [0, \infty) \to R$, where $f(x) = x^2 + 3$, the function is dilated by a factor of 2 from the x-axis and is then translated 4 units in the positive direction of the y-axis. The range and rule of the transformed function are:

$x' = x$ $\quad y' = 2y + 4$

$x = x'$ $\quad y = \frac{y'-4}{2}$

A. Ran: $[0, \infty)$, where $f(x) = \left(\frac{x}{2}\right)^2 + 7$

B. Ran: $[10, \infty)$, where $f(x) = 2x^2 + 10$ ✓

C. Ran: $[2, \infty)$, where $f(x) = \left(\frac{x}{2}\right)^2 + 10$

D. Ran: $[12, \infty)$, where $f(x) = 2x^2 + 7$

$\frac{y'-4}{2} = x'^2 + 3$

Space for Personal Notes

old
Ran = $[3, \infty)$
$\Rightarrow [6, \infty)$
$\Rightarrow [10, \infty)$

$y' = 2x'^2 + 6 + 4$

$y = 2x^2 + 10$

Question 14 (1 mark)

Consider the following function and the following sequence of transformations:

$$g: R \to R, g(x) = -x^2 + 4$$

- A translation of 3 units in the negative direction of the x-axis. $x' = x - 3$ $\Rightarrow x = x' + 3$
- A translation of 2 units in the negative direction of the y-axis. $y' = y - 2$
- A dilation from the x-axis by a factor of $\frac{3}{2}$. $y' = \frac{3}{2}(y-2)$

$\hookrightarrow y - 2 = \frac{2}{3}y'$

$y = \frac{2}{3}y' + 2$

$\frac{2}{3}y' + 2 = -(x'+3)^2 + 4$

$\frac{2}{3}y' = -(x'+3)^2 + 2$

$y' = -\frac{3}{2}(x'+3)^2 + 3$

What is the image of g?

A. $4(x-3)^2 - 2$

B. $-\frac{3}{2}(x+3)^2 + 3$

C. $-\frac{3}{2}(x+3)^2 + 2$

D. $-\left(\frac{x}{3} + 4\right)^2 - 7$

Question 15 (1 mark)

Consider the following function and the sequence of transformations:

$$g: R\backslash\{-1\} \to R, g(x) = -\frac{1}{(x+1)^2} + 1$$

- ~~A translation of 3 units in the negative direction of the y-axis.~~
- ~~A dilation by a factor of $\frac{1}{2}$ from the x-axis.~~
- A dilation by a factor of 4 from the y-axis.

$R\backslash\{-1\}$

$\downarrow$ 4 from y

$R\backslash\{-4\}$

What will be the domain of the image of g? x

A. $R\backslash\{4\}$

B. $R\backslash\left\{-\frac{1}{2}\right\}$

C. $R\backslash\{-4\}$

D. $(-1, \infty)$

Question 16 (7 marks)

The graph of the function with the rule $y = -\sqrt{x}$ is transformed under the following ordered list of transformations:

- A translation of 2 units right on the x-axis. $x' = x + 2$
- A translation of 2 unit up on the y-axis. $y' = y + 2 \Rightarrow y = y' - 2$
- A reflection in the y-axis. $x' = -(x+2)$
- A dilation of factor $\frac{1}{2}$ away from the y-axis. $x' = -\frac{1}{2}(x+2)$

a. Write down the equation of the rule of the transformed function. (4 marks)

$-2x' = x + 2$

$-2x' - 2 = x$

$x = -2x' - 2$

$y = -\sqrt{x}$

$y' - 2 = -\sqrt{-2x' - 2}$

$y' = -\sqrt{-2x' - 2} + 2$

$y = -\sqrt{-2x - 2} + 2$

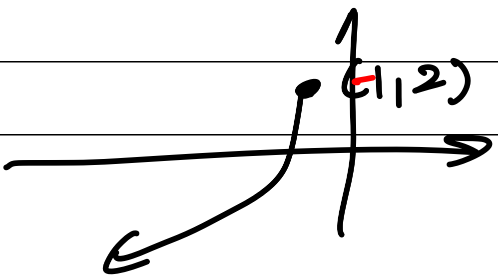
b. Hence, state the domain and range of the transformed function. (2 marks)

in ≥ 0 $\quad -2x - 2 = 0$

$2x = -2$

$x = -1$

start $(-1, 2)$



dom $= (-\infty, -1]$

ran $= (-\infty, 2]$

c. To where does the point $\left(2, -\sqrt{2}\right)$ on the original graph get transformed? (1 mark)

$(4, -\sqrt{2})$

$(4, -\sqrt{2}+2)$

$(-4, -\sqrt{2}+2)$

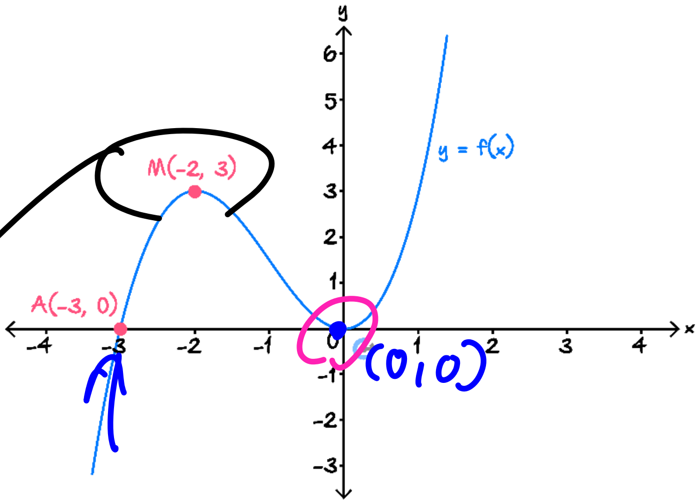
Space for Personal Notes $(-2, -\sqrt{2}+2)$

Question 17 (9 marks)

Eco is an avid wildlife photographer. On his recent trip to the Amazon rainforest, he managed to set up a go-pro on a selfie stick and took a birds-eye view picture of one of the world's largest anacondas.

He has modelled the anaconda via the function $y = f(x)$, shown in the diagram below.

The curve meets the x-axis at the origin and the point $A(-3, 0)$ and has a local maximum at $M(-2, 3)$.



a. Given that f is a cubic function, show that $f(x) = \frac{3}{4}x^2(x+3)$. (2 marks)

$y = a(x+3)(x-0)^2$

$y = a(x+3)x^2$

$(-2, 3)$

$3 = a(-2+3)(-2)^2$

$3 = a(1)(4)$

$3 = 4a$

$a = \frac{3}{4}$

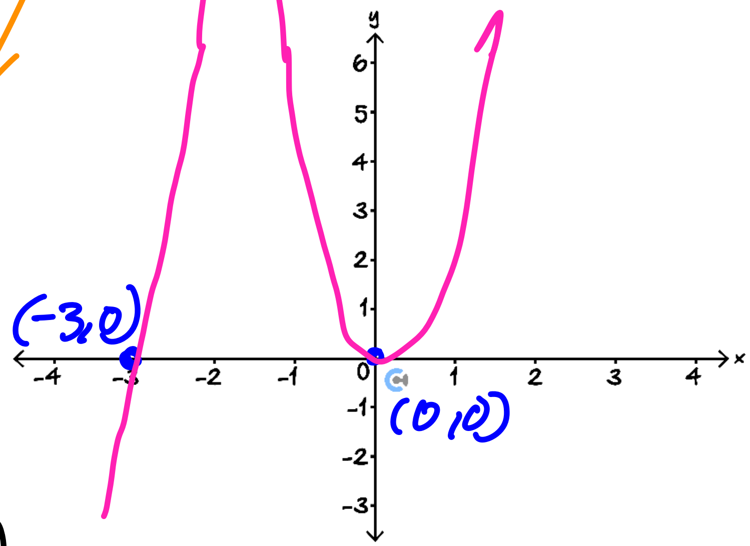
$y = \frac{3}{4}x^2(x+3)$

$(-2, 12)$

b. When the anaconda tries to mark its territory, it spreads its head and tail out. If this movement of the anaconda is a dilation by a factor of 4 from the x-axis, write the equation of the transformed graph in the form $y = cf(x)$ and sketch the transformed graph. (2 marks)

$y' = 4y$

$y = \frac{y'}{4}$



$y = f(x)$

$\frac{y'}{4} = f(x')$

$y = 4f(x)$

c. Which points are not affected by the transformation described above? (1 mark)

↳ x-ints

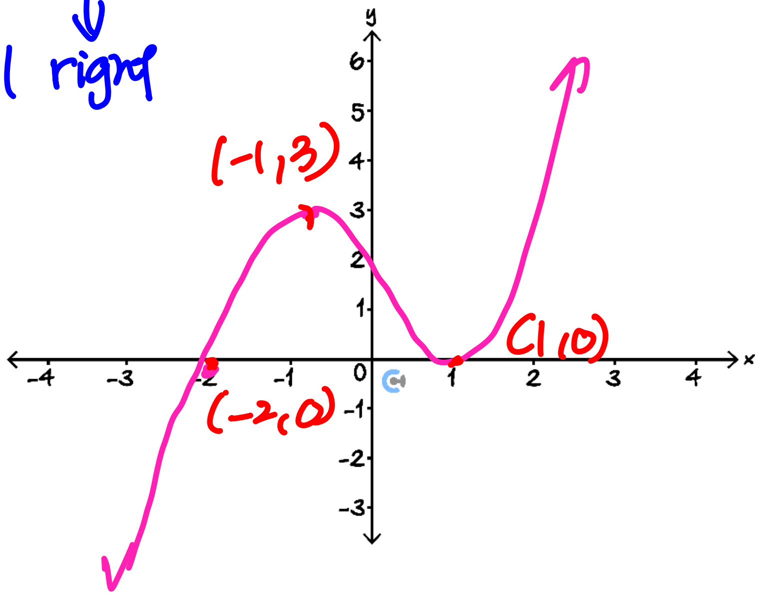
$x =$ c − image

d. Sketch the graph of $y = f(x - 1)$, and state what this transformation represents geometrically. (2 marks)

1 right

$x = x' - 1$

$x' = x + 1$



e. Suppose that Leo's go-pro accidentally took the mirror image of the snake, but he wants to reproduce the original image.

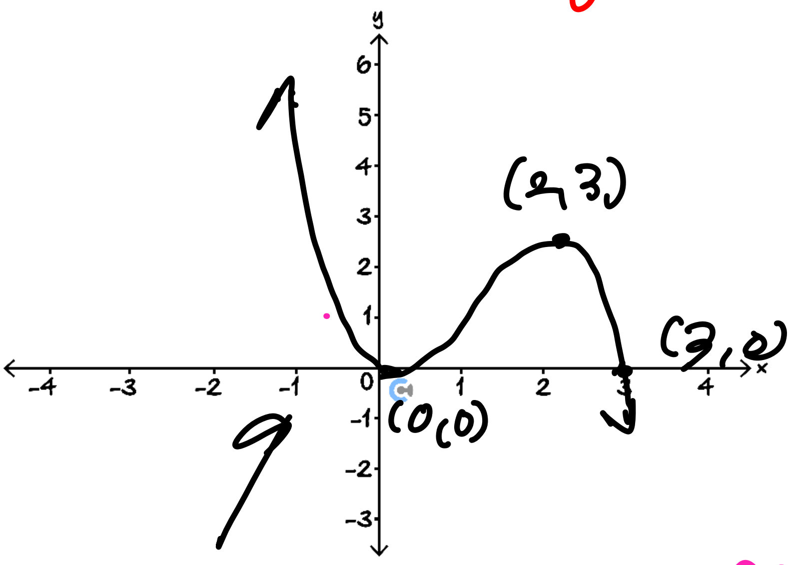
If he wants to reflect the image across the vertical axis, write down the equation of the transformed function and sketch it. (2 marks)

↳ Reflect y-axis

$x \mapsto -x$

$x = -x'$

$y' = f(-x')$



$y = f(-x)$

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Section F: Extension Exam 1 (15 Marks)

INSTRUCTION: 15 Marks. 18 Minutes Writing.

Question 18 (4 marks)

The transformation $T: R^2 \to R^2$ is defined by $T(x, y) = (-3x - 4, y + 2)$.
The images of the curve $y = (x - 2)^2 + 4$ under the transformation T has equation $y = ax^2 + bx + c$.
Find the values of a, b and c.

$$y = \frac{x^2}{9} + \frac{20x}{9} + \frac{154}{9}$$

$$a = \frac{1}{9}, b = \frac{20}{9}, c = \frac{154}{9}$$

```
In[177]:= f[x_] := (x - 2)^2 + 4
In[178]:= f[(x + 4)/(-3)] + 2 // Expand
Out[178]= 154/9 + 20 x/9 + x^2/9
```

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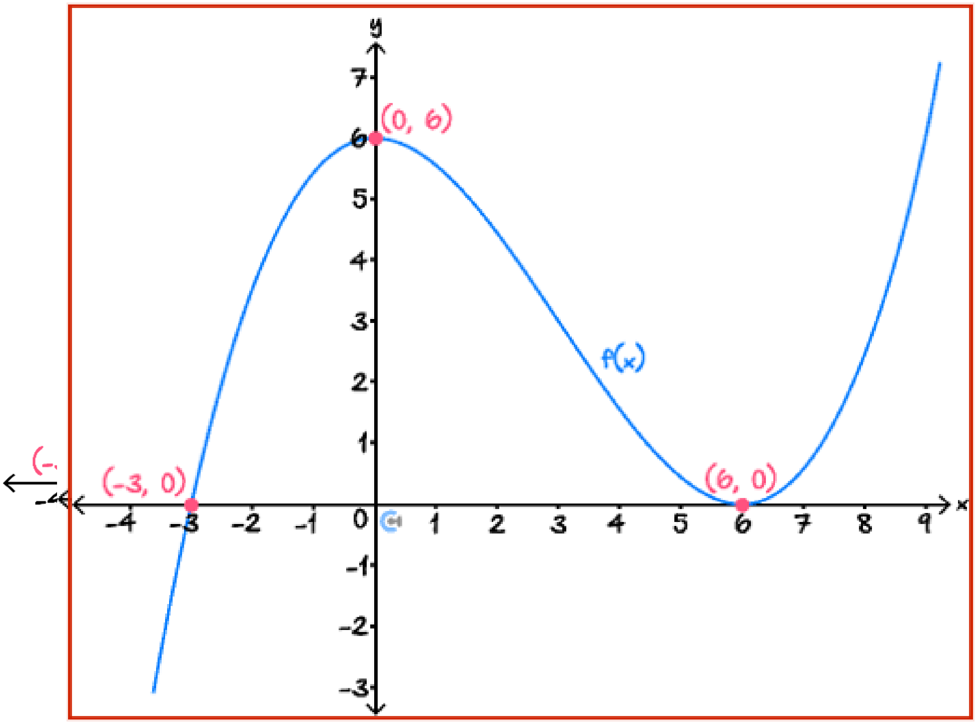
Question 19 (5 marks)

Consider the graph of $f(x)$ shown below.

a. Sketch the graph of $y = \frac{2}{3}f(3x+3)$ alongside the graph of $y = f(x)$, on the axes below. Label all axes, intercepts and turning points with coordinates. (3 marks)



b. It is known that $f(x)$ is a cubic function. Hence, or otherwise, determine the rule for the funct $g: R \to R, g(x) = \frac{2}{3}f(3x+3)$. (2 marks)

If our sketch in **part a.** was accurate, we can see that $g(x) = a(x+2)(x-1)^2$ and using a reference coordinate, we find that $a = 1$.
Thus $g(x) = (x+2)(x-1)^2$.

Question 20 (6 marks)

Let $f(x) = \sqrt{kx - 2} + 1$, where $k \in R^+$.

a. State the maximal domain of f, expressing your answer in terms of k where appropriate. (1 mark)

$\left[\frac{2}{k}, \infty\right)$

b. Find a sequence of transformations that map $f(x)$ to $g(x) = \sqrt{x+1}$. (2 marks)

- A dilation by factor k from the y-axis.
- A translation 3 units to the right.
- A translation 1 unit down.

c. Determine the values of k, for which f and f^{-1}, intersect twice. (3 marks)

$f(x)$ is an increasing function so intersections with its inverse must occur along the line $y = x$.

Solve $f(x) = x$.

$$x = 1 + \frac{k}{2} \pm \frac{1}{2}\sqrt{k^2 + 4k - 8}$$

For two solutions we require $k^2 + 4k - 8 > 0 \Rightarrow k > 2\sqrt{3} - 2$ (recall we also have $k > 0$).

The last time they had two solutions was when f and f^{-1}, have the same starting point. Note that this starting point is on the line $y = x$, and f starts at $\left(\frac{2}{k}, 1\right)$, therefore $k = 2$.

Two solutions for $2\sqrt{3} - 2 < k \leq 2$.

We could also use brute force algebra to find $k = 2$ by solving;

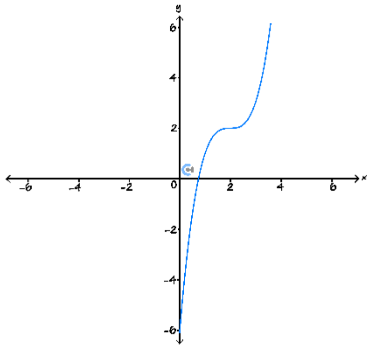
$$1 + \frac{k}{2} - \frac{1}{2}\sqrt{k^2 + 4k - 8} = \frac{2}{k} \Rightarrow k = 2$$

Section G: Extension Exam 2 (13 Marks)

INSTRUCTION: 13 Marks. 18 Minutes Writing.

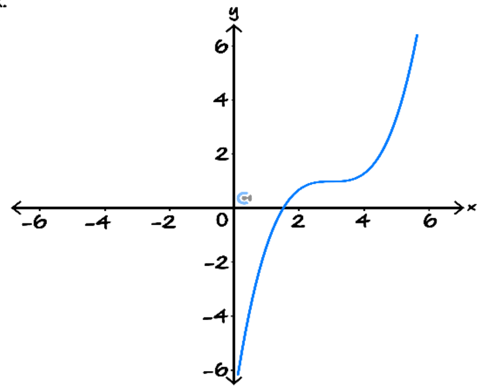
Question 21 (1 mark)

The graph whose equation is $y = f(x)$ is shown below:

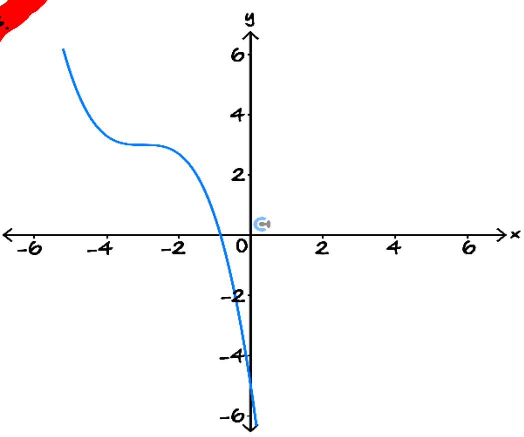


The graph whose equation is $y = 1 + f\left(-\frac{2x}{3}\right)$ is:

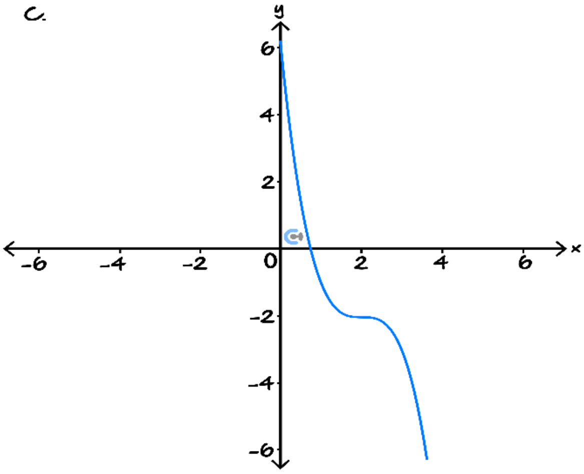
A.



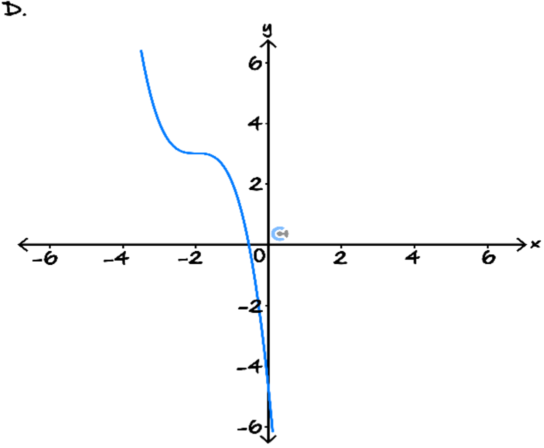
B.



C.



D.



Question 22 (1 mark)

Which of the following transformation functions $T : R^2 \to R^2$, could map the parabola $y = x^2$, to a parabola with a turning point at $(-2, 5)$:

A. $T(x, y) = (x + 2, y + 5)$

B. $T(x, y) = (x - 2, y - 5)$

C. $T(x, y) = (6x - 2, 3y + 5)$

D. $T(x, y) = (3x - 6, y + 5)$

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Question 23 (11 marks)

Consider the functions:

$$f : \mathbb{R} \to \mathbb{R}, f(x) = x(x-2)^2(x+1)$$
$$g : \mathbb{R} \to \mathbb{R}, g(x) = x^4 + 11x^3 + 42x^2 + 64x + 32$$

a.

i. Determine the x-intercepts of g. (1 mark)

i. Determine the x-intercepts of g. (1 mark)

Solution: $g(x) = (x+4)^2(x+2)(x+1)$
So x-intercepts at $(-4,0)$, $(-2,0)$ and $(-1,0)$

ii. Find a sequence of transformations that map f to g. (2 marks)

Solution: Graphing both functions will help us see that $g(x) = f(-(x+2))$.
Thus the transfromations are a reflection in the y-axis followed by a translation 2 units to the left.

b. Consider the hybrid function:

$$h(x) = \begin{cases} f(x), & x \geq -1 \\ p(x), & x < -1 \end{cases}$$

The function h has the property $h(x-1) = h\big(-(x+1)\big)$.

Determine the rule for $p(x)$. (2 marks)

Solution: The property can be interpreted as h being symmetric about the line $x = -1$.
Visually we see that $p(x) = g(x) = (x+4)^2(x+2)(x+1)$

Consider the following transformations:

$$T:\ \mathbb{R}^2 \to \mathbb{R}^2, T(x,y) = (2x-3, 3y+1)$$
$$S:\ \mathbb{R}^2 \to \mathbb{R}^2, S(x,y) = (-x+2, 2y-2)$$

c.

i. Find the rule for the image of f after it has undergone the transformation T followed by the transformation S. (3 marks)

Solution: Let $f_T(x)$ be the image of f under the transformation T.

$$f_T(x) = 3f\left(\frac{1}{2}(x+3)\right) + 1 = \frac{1}{16}\left(3x^4 + 18x^3 - 66x + 61\right)$$

Now we apply the transformation S to f_T.

$$\begin{aligned} f_{TS}(x) = 2f_T(2-x) - 2 &= \frac{3x^4}{8} - \frac{21x^3}{4} + \frac{45x^2}{2} - \frac{123x}{4} + \frac{105}{8} \\ &= \frac{3}{8}(x-7)(x-5)(x-1)^2 \end{aligned}$$

ii. The transformation T is applied to the function g. The transformation:

$$U:\ \mathbb{R}^2 \to \mathbb{R}^2, U(x,y) = a(x+b), c(y+d))$$

Maps the image of g under T, to the function f.

Find the values of a, b, c and d. (3 marks)

Solution: We must undo the transformation T and then undo the transformation from f to g.

- A translation 3 units right
- A translation 1 unit down
- A dilation by factor $\frac{1}{2}$ from the y-axis
- A dilation by factor $\frac{1}{3}$ from the x-axis

this undos the transformation T. $(x', y') = \left(\frac{1}{2}(x+3), \frac{1}{3}(y-1)\right)$. Then

- A translation 2 units to the right
- A reflection in the y-axis.

This gives

$$U(x,y) = \left(-\frac{1}{2}(x+7), \frac{1}{3}(y-1)\right)$$

thus $a = -\frac{1}{2}, b = 7, c = \frac{1}{3}$ and $d = -1$.

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