CONTOUR EDUCATION

Website: contoureducation.com.au | **Phone:** 1800 888 300
Email: hello@contoureducation.com.au

VCE Mathematical Methods ½
Linear & Coordinate Geometry [0.1]
Workshop Solutions

Section A: Recap

Linear equations

Definition

- **Definition:** Equations where the highest power of a variable is 1.
  - **Gradient-intercept form:**

$$y = mx + c$$

$$\text{Where } m = \text{gradient} = \frac{\text{rise}}{\text{run}} = \boxed{\frac{y_2 - y_1}{x_2 - x_1}}$$

$$\text{and } c = \boxed{y\text{-intercept}}$$

- No singular solution for a linear equation in two variables.
  - All pairs of coordinates (x, y) that satisfy the equation lie on a **line.** (Hence, linear equations.)

Sub-Section: Inequality

Inequalities rule

Definition

$$x > \frac{b}{a}, \text{ where } a < 0$$

- Multiplying both sides by a negative number __reverses__ the inequality sign.

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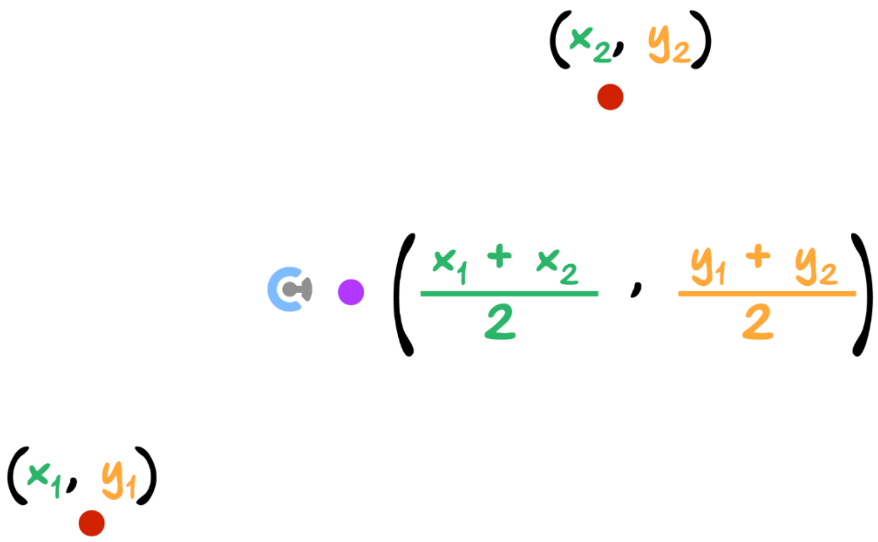
Sub-Section: Midpoint

Midpoint

Definition



- **Definition:** The midpoint, M, of two points A and B is the point halfway between A and B.

$$M(x_m, y_m) = \left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2} \right)$$

- The midpoint can be found by taking the ___average___ of the x-coordinate and y-coordinate of the two points.

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Sub-Section: Distance Between Two Points

Distance between two points

- **Definition:** The distance between two points (x_1, x_2) and (y_1, y_2) can be found using Pythagoras' theorem:

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

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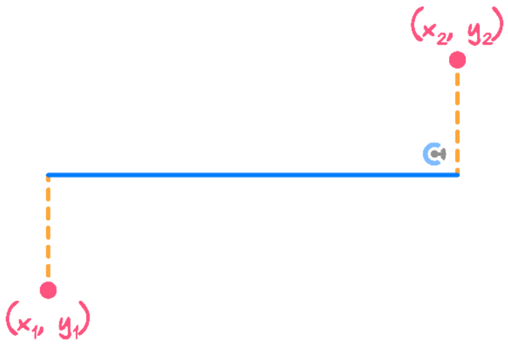
Sub-Section: Vertical Distance Vs Horizontal Distance

Horizontal distance

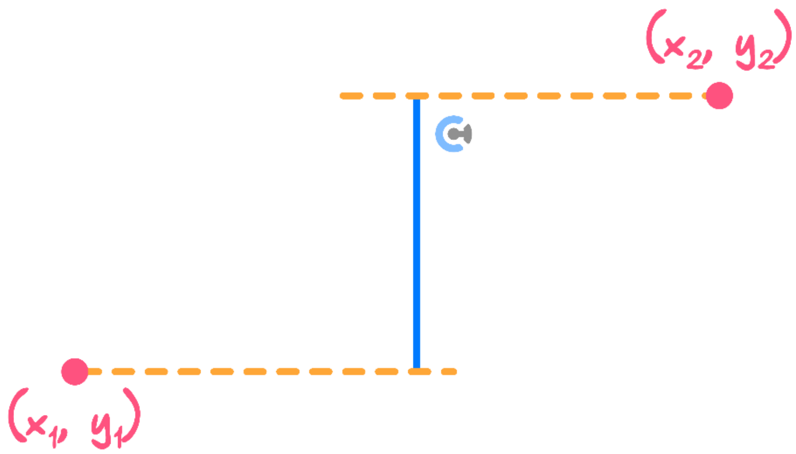


Horizontal Distance $= x_2 - x_1$ where, $x_2 > x_1$.

- Find the difference between their x-values.

What about vertical distance then?

Vertical distance



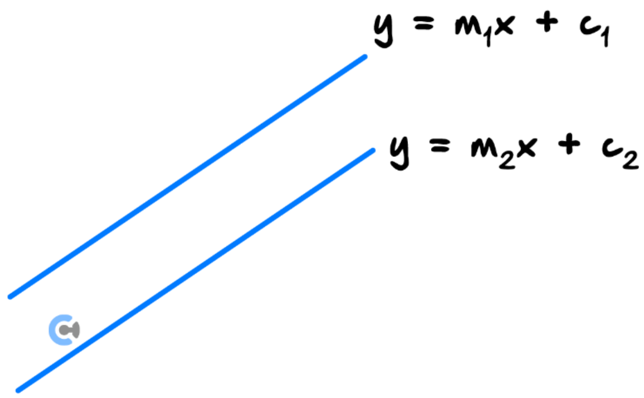
Vertical Distance $= y_2 - y_1$ where, $y_2 > y_1$.

- Find the difference between their y-values.

Sub-Section: Parallel and Perpendicular Lines

Definition

Parallel lines

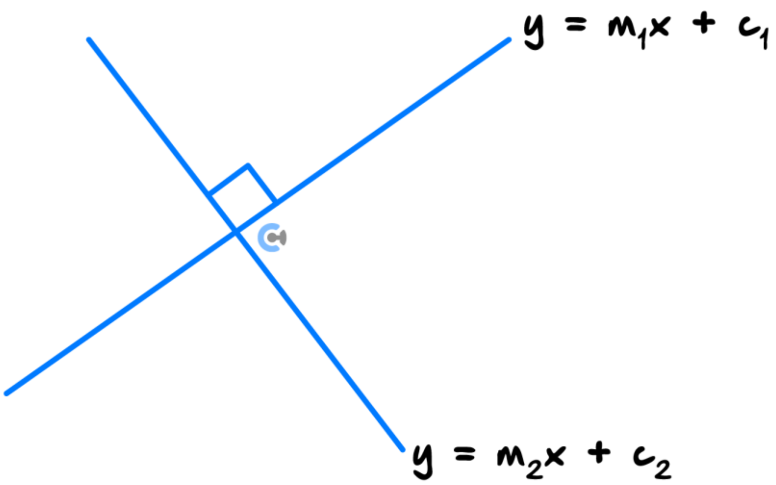


- Parallel lines have the ___same___ gradient.

$$m_1 = m_2$$

Definition

Perpendicular lines



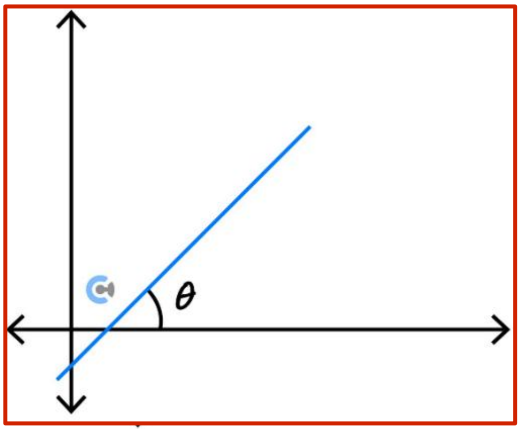
- A line that is perpendicular to another line has a gradient, which is the ___negative reciprocal___ of the gradient of the first line.

$$m_{\perp} = -\frac{1}{m}$$

Sub-Section: Angle Between a Line and the x-axis

How do we find the angle between a line and the x-axis?

The angle between a line and the x-axis



- The angle between a line and the positive **direction of the x-axis** (anticlockwise) is given by:

$$\tan(\theta) = m$$

NOTE: Angles from the x-axis measured anticlockwise = positive angles.

- Don't worry about it too much, it's just convention! (More on this in circular functions.)

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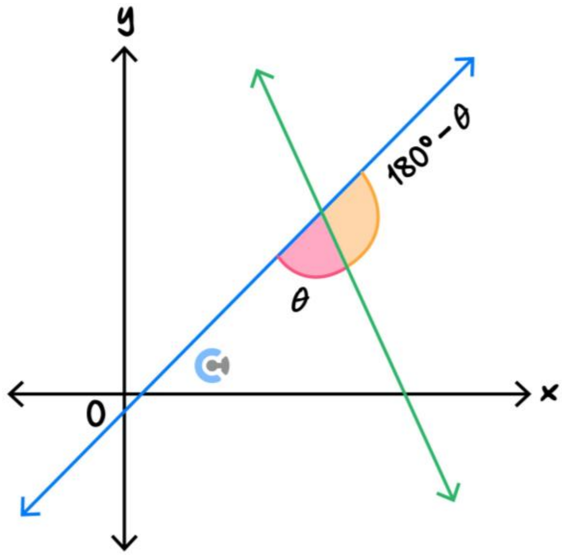
Sub-Section: Angle Between the Two Lines

Slightly more complicated now!
How about an angle between two lines?

The acute angle between two lines



$$\theta = |\tan^{-1}(m_1) - \tan^{-1}(m_2)|$$

- **Alternatively:**

$$\tan(\theta) = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

For your understanding, note that this formula is derived from the tan compound angle formula covered in SM12.

NOTE: $|x|$ just takes the positive value of x.

TIP: Make sure your CAS is in degrees.

Sub-Section: Finding Simultaneous Equations for Two Variables

Definition

Simultaneous linear equations

- Elimination method:
  - Add or subtract one equation from the other in order to __eliminate__ one of the variables. Then have an equation in one variable that can be solved easily.
- Substitution method:
  - Make one of the variables the subject (generally x or y) and ___substitute___ that value into the other equation.

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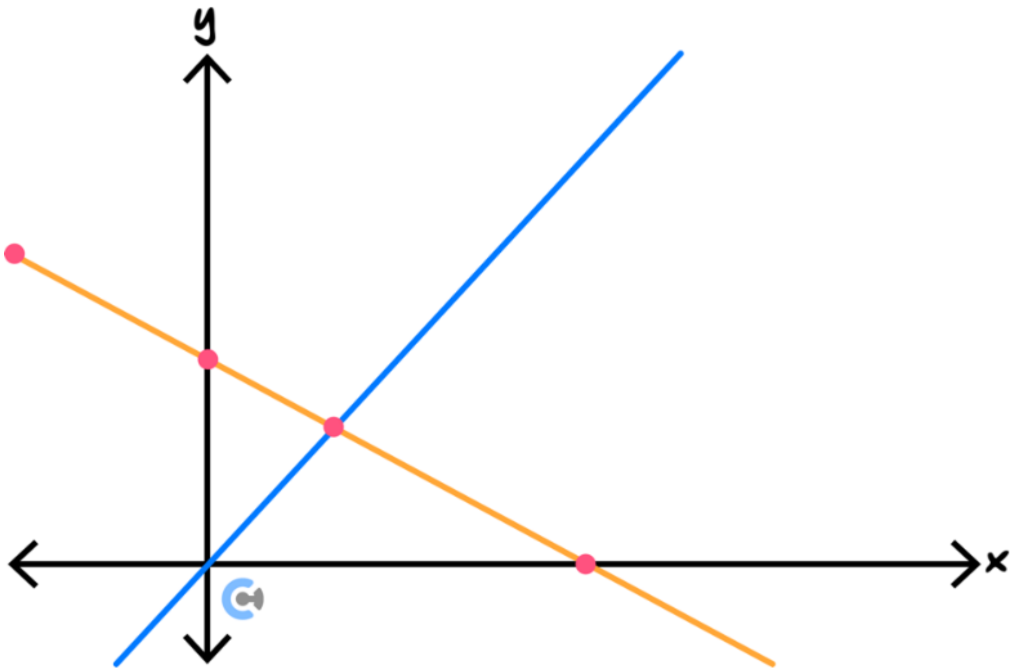
Sub-Section: Number of Solutions for Two Variables

What does the geometry look like for each number of solutions?

Exploration: Geometry of the number of solutions between linear graphs

- Unique solution:

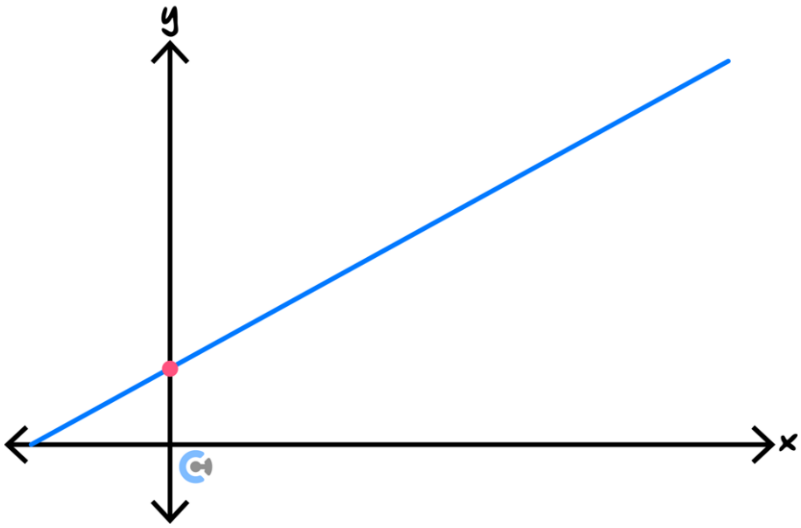
$$m_1 \neq m_2$$



They just need to have __ different gradients __.

- **Infinite solutions:**

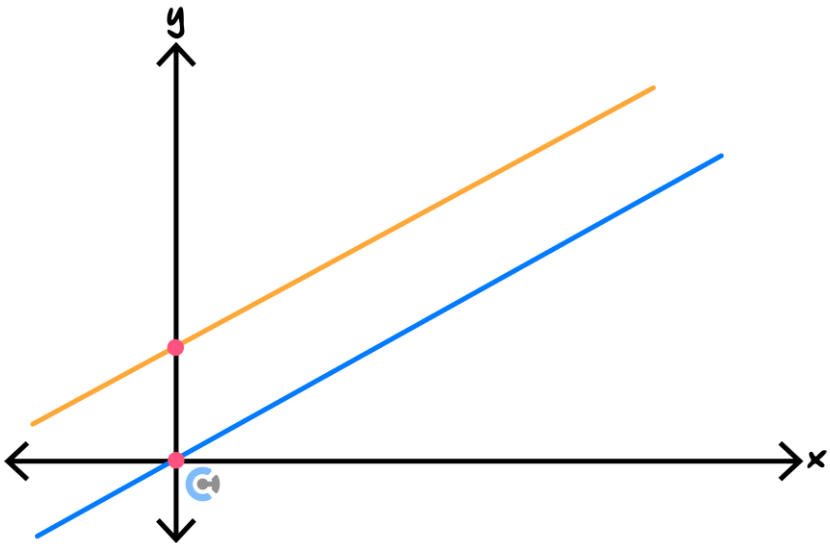
$$m_1 = m_2 \text{ and } c_1 = c_2$$



- They just need to have the same gradient and the same c-values.
- In other words, they have to be the exact same line.

- **No solutions:**

$$m_1 = m_2 \text{ and } c_1 \neq c_2$$



- They need to have the same gradient but different $+c$.
- They have to be two different parallel lines.

<u>**General solutions of simultaneous linear equations**</u>

- Two linear equations are either:
  - The same line is expressed in a different form. In this case, they have ___ infinite ___ solutions.
  - Unique lines which are parallel. In this case, they have ____ no ____ solutions.
  - Unique lines which are not parallel. In this case, they have ___ exactly one ___ solution.

TIP: It's a good idea to substitute your answer back into the equations to see if the criteria are met for each part.

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Section B: Warmup

INSTRUCTION: 5 Minutes Writing.

Question 1

a. Find the horizontal distance between the points (2,6) and (5,6).

3

b. Find the equation of the line parallel to $y = 2x - 5$ that goes through the midpoint of (1,2) and (5,4).

```
In[2]:= Solve[y - 3 == 2 (x - 3), y]
Out[2]= {{y → -3 + 2 x}}
```

c. Find the distance between the points (1,3) and (5,7).

In[1]:= $\sqrt{4\text{^}2 + 4\text{^}2}$

Out[1]= $4\sqrt{2}$

d. Determine the value of k for which the equations:

$$3kx - 2y = k$$

$$6x - 4y = 6$$

Have no solutions.

$k = 1$

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Section C: Exam 1 (23 Marks)

INSTRUCTION: 23 Marks. 30 Minutes Writing.

Question 2 (2 marks)

Solve the following simultaneous linear equations for x and y.

$$\begin{aligned} 5x + 3y &= 41 \\ -2x - 3y &= -20 \\ 4x + 6y &= 40 \end{aligned}$$

Solution: Equations 2 and 3 are the same line. So we just need to solve

$$\begin{aligned} 5x + 3y &= 41 \\ 2x + 3y &= 20 \end{aligned}$$

$$\implies 3x = 21$$

$$\implies x = 7 \text{ and } y = 2.$$

Question 3 (4 mark)

a. Point B $(2,1)$ is the midpoint of A $(-1,-1)$ and point C. Find the coordinates of C. (2 marks)

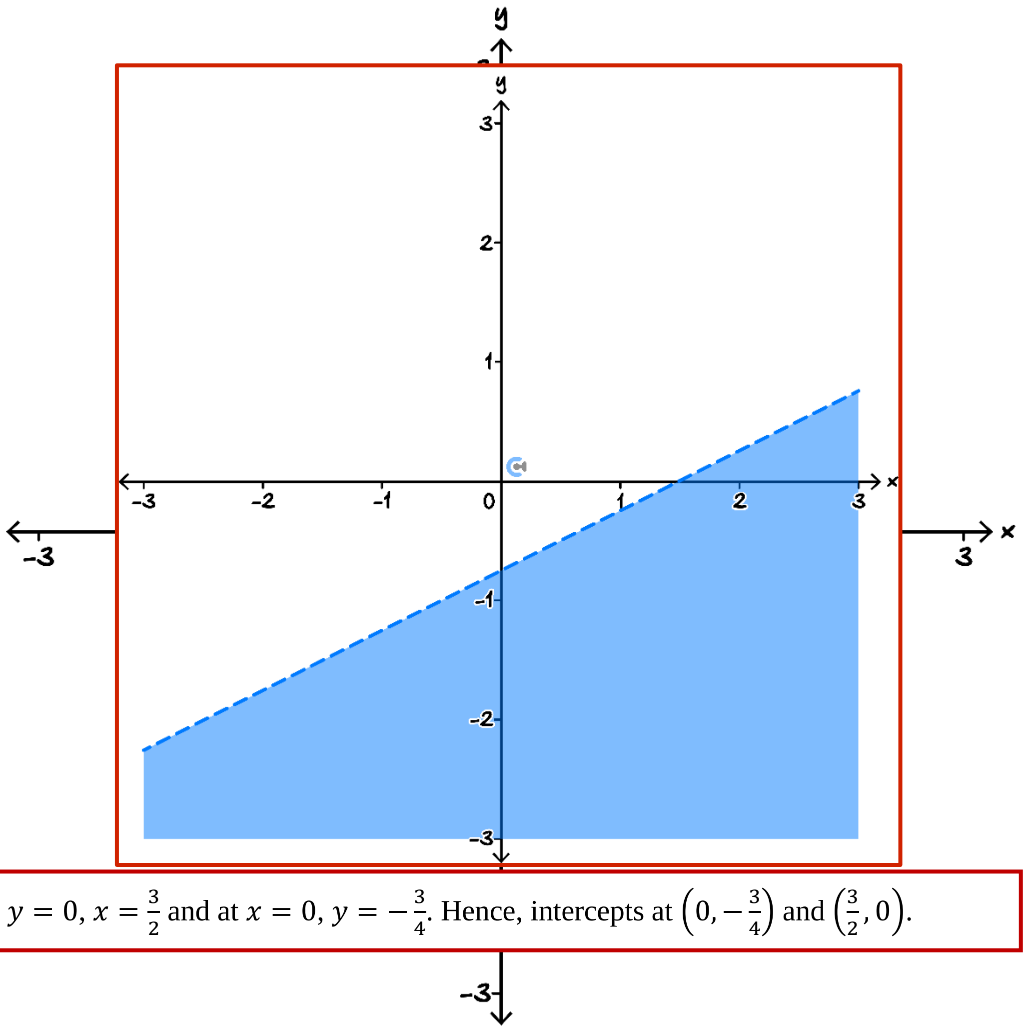
(5,3)

b. Find in the form $y = ax + b$ the equation of the line that makes an angle of $45°$ anticlockwise from the positive direction of the x-axis and passes through B. (Hint: $\tan 45° = 1$) (2 marks)

Gradient $= a = \tan 45° = 1$
Equation of the line $y = x - 1$

Question 4 (3 marks)

Sketch the following inequality: $2x > 4y + 3$



At $y = 0$, $x = \frac{3}{2}$ and at $x = 0$, $y = -\frac{3}{4}$. Hence, intercepts at $\left(0, -\frac{3}{4}\right)$ and $\left(\frac{3}{2}, 0\right)$.

Question 5 (6 marks)

Sam's home is at the point $(-1, -2)$. He wants to walk straight from his home to a river bank which is given by the linear line $3x + 4y - 12 = 0$. The units are kilometres.

a. Find the equation of the straight-line path taken by Sam if he walked the least distance. (2 marks)

Solution: We want the equation of a line perpendicular to $3x + 4y - 12 = 0$ that passes through $(-1, -2)$.
$m_1 = -\frac{3}{4} \implies m_2 = \frac{4}{3}$.
Therefore, the line has equation

$$y + 2 = \frac{4}{3}(x+1)$$
$$y = \frac{4}{3}x - \frac{2}{3}$$

Equivalently, $4x - 3y - 2 = 0$

b. Find the coordinate of the point on the riverbank where Sam reaches. (2 marks)

Solution: Sub in $y = \frac{4}{3}x - \frac{2}{3}$ into $3x + 4y - 12 = 0$

$$3x + \frac{16}{3}x - \frac{8}{3} = 12$$
$$9x + 16x - 8 = 36$$
$$25x = 44x = \frac{44}{25}$$

$$y = \frac{4}{3} \times \frac{44}{25} - \frac{50}{75} = \frac{176 - 50}{75} = \frac{126}{75} = \frac{42}{25}$$

Therefore, coordinates are $\left(\frac{44}{25}, \frac{42}{25}\right)$

c. Calculate the distance travelled by Sam. It is given that $\sqrt{69^2 + 92^2} = 115$. (2 marks)

$$d = \sqrt{\left(1 + \frac{44}{25}\right)^2 + \left(2 + \frac{42}{25}\right)^2}$$
$$= \sqrt{\left(\frac{69}{25}\right)^2 + \left(\frac{92}{25}\right)^2}$$
$$= \frac{1}{25}\sqrt{69^2 + 92^2}$$
$$= \frac{115}{25}$$
$$= \frac{23}{5}$$

Question 6 (2 marks)

The distance between two stations is 320 km. Two trains start simultaneously from different stations and travel on parallel tracks towards each other.

If the speed of one of them is greater than the other by 10 km/hr and the distance between the two trains after 2 hours of their start is 20 km, find the speed of each train.

Solution: Let one train have the speed x kmh^{-1}
then the other train has speed $(x+10)$ kmh^{-1}. After 2 hours we have the equation

$$\begin{aligned} 320 - 2x - 2(x+10) &= 20 \\ 320 - 4x - 20 - 20 &= 0 \\ 4x &= 280 \\ x &= 70 \end{aligned}$$

Therefore the trains have speeds 70 kmh^{-1} and 80 kmh^{-1}

Question 7 (3 marks)

Jeff is creating two chemical solutions, Solution A and B, by mixing two key ingredients:
Chemical X and Chemical Y. Each litre of Solution A requires 3 grams of Chemical X and 2 grams of Chemical Y, while each litre of Solution B requires 2 grams of Chemical X and 5 grams of Chemical Y.

Jeff has 38 grams of Chemical X and 49 grams of Chemical Y available. How many litres of each solution should the lab prepare to use up all the chemicals?

Solution: Let $x =$ numbers of litres of solution A
Let $y =$ number of litres of solution B, then we have

$$\begin{aligned} 3x + 2y &= 38 \\ 2x + 5y &= 49 \end{aligned}$$

Solving yields, $x = \dfrac{92}{11}$ and $y = \dfrac{71}{11}$

Question 8 (3 marks)

Determine the value of k so that the following linear equations have no solution.

$$(3k + 1)x + 3y - 5 = 0$$

$$(k^2 + 3)x + (k - 2)y - 5 = 0$$

Solution: The lines will have the same gradient if

$$\begin{aligned}\frac{3k+1}{3} &= \frac{k^2+3}{k-2}\\(3k+1)(k-2) &= 3(k^2+3)\\3k^2-5k-2 &= 3k^2+9\\-5k &= 11\\k &= -\frac{11}{5}\end{aligned}$$

The lines will have the same y-intercept if

$$\begin{aligned}\frac{5}{3} &= \frac{5}{k-2}\\k &= 5\end{aligned}$$

Therefore, the linear equations will have no solution if $k = -\frac{11}{5}$.

Let's take a BREAK (standard stream)!

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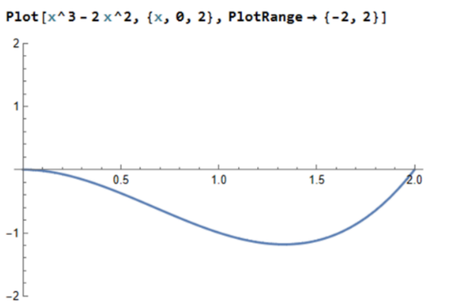
Section D: Tech Active Exam Skills

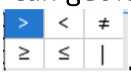
INSTRUCTION: 5 Minutes Writing.

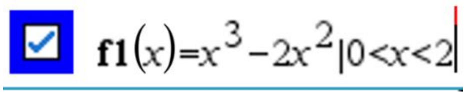
Calculator Commands: Graphing

Mathematica

- Plot [function, {x, $x$min, $x$max}. Plot Range → {$y$min, $y$max}]
- Plot Range is optional but makes the scale appropriate for the question.

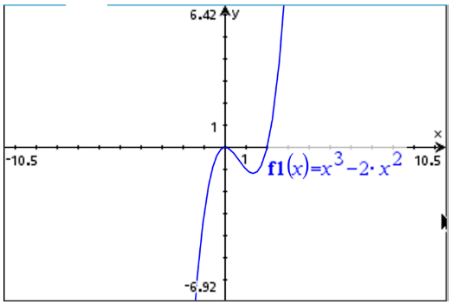


- Menu → 6 (Analyse) to find min/max x and y-intercepts.
- Restrict domain to $0 < x < 2$ use the bar can get it from ctrl+ = .

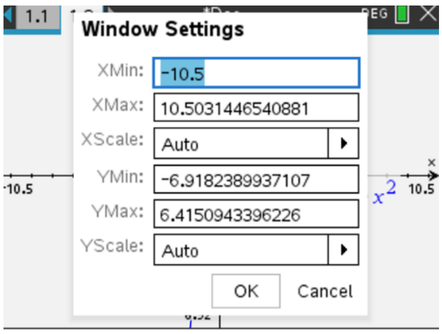


TI-Nspire

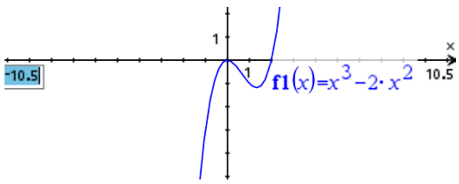
- Open a graph page and plot your function.



- Zoom settings: Menu → 4 (window/zoom) → 1 enter your x and y ranges.

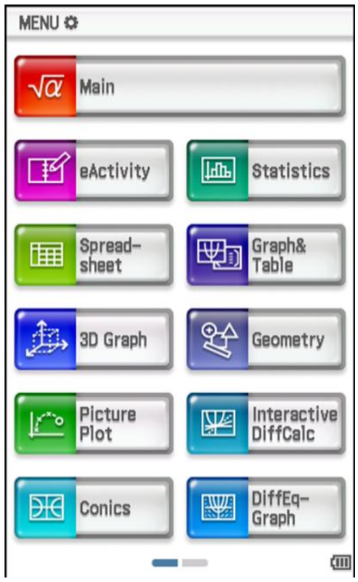


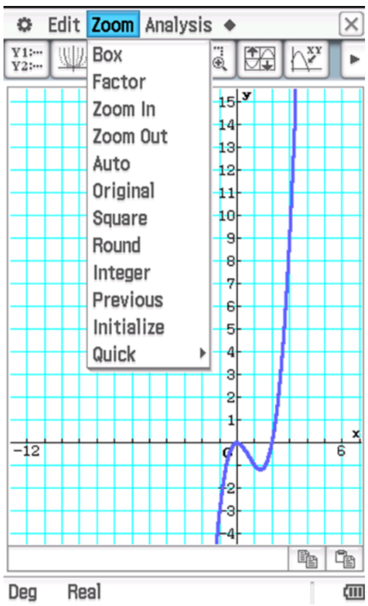
- Can also click the axis numbers on the graph and alter them directly.



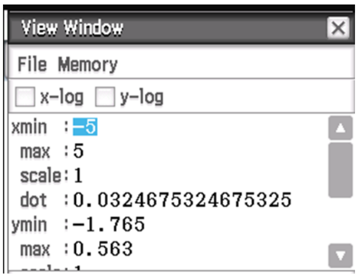
Casio Classpad

- Click Graph & Table, and enter the function.





- Analysis → G- Solve to find intercepts.
- Use this button to set the view window.



- Use | to restrict domain → Find it in Math 3.

$y1 = x^3 - 2 \cdot x^2 \mid 0 < x < 2$

Calculator Commands: Solving Equations

TI-Nspire

- Menu → 3 → 1

$\text{solve}(x^2 - 4 \cdot x - 9 = 0, x)$

$x = -(\sqrt{13} - 2) \text{ or } x = \sqrt{13} + 2$

Casio Classpad

- Action → Advanced → Solve

$\text{solve}(x\char`^2 - 4x - 9 = 0, x)$

$\{x = -\sqrt{13} + 2, x = \sqrt{13} + 2\}$

In[122]:= Solve[x^2 - 4 x - 9 == 0, x]

Out[122]= $\{\{x \to 2 - \sqrt{13}\}, \{x \to 2 + \sqrt{13}\}\}$

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Calculator Commands: Simultaneous Equations

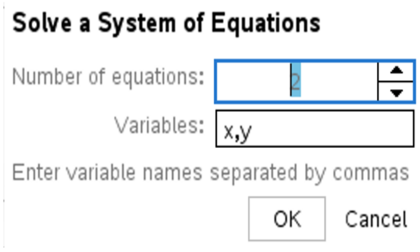
Mathematica

- Just do && between.
- Solve[equation&&equation , {var1, var2}]

```
In[128]:= Solve[2 x - 3 y == 16 && x + y == 3, {x, y}]
Out[128]= {{x → 5, y → -2}}
```

TI-Nspire

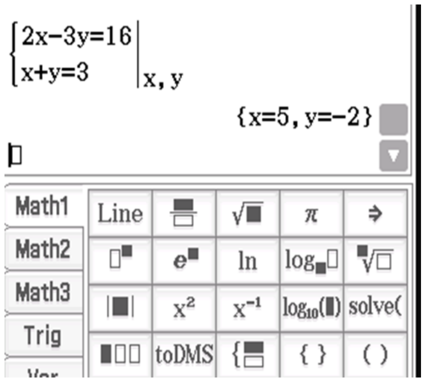
- Menu 3 7 1



$\text{solve}\left(\begin{cases}2\cdot x-3\cdot y=16\\ x+y=3\end{cases},\{x,y\}\right)$ $x=5$ and $y=-2$

Casio Classpad

- Math1 → Click highlighted box→ Enter equations and variables you are solving for:



Question 9 Tech-Active.

Solve the equations $2x + 7y = 16$ and $5x + 3y = 20$ for x and y.

```
In[2]:= Solve[2 x + 7 y == 16 && 5 x + 3 y == 20, {x, y}]
Out[2]= {{x → 92/29, y → 40/29}}
```

Space for Personal Notes

Calculator Commands: Finding the Angle between a Line and x-axis

Mathematica

In[124]:= ArcTan[2] / Degree // N

Out[124]= 63.4349

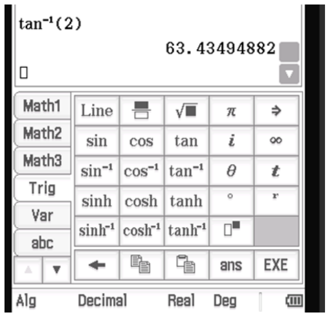
TI-Nspire

- Trig button. Check that you are in degrees.

$\tan^{-1}(2)$ 63.4349

Casio Classpad

- Keyboard →Trig. Change to decimals and degrees.



Calculator Commands: Finding the Angle between Two Lines

Mathematica

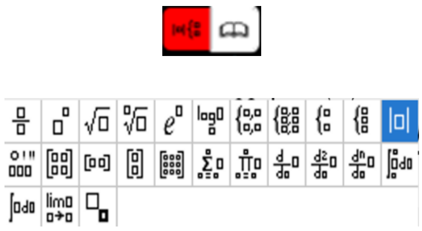
- Use the Abs[] function.

In[126]:= Abs[ArcTan[2] - ArcTan[1]] / Degree // N

Out[126]= 18.4349

TI-Nspire

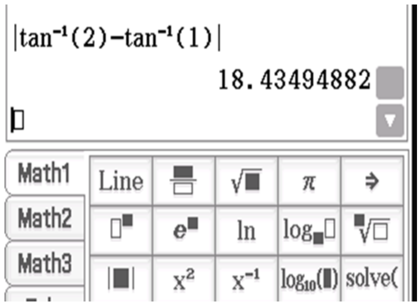
- Find the modulus sign.



$|\tan^{-1}(2)-\tan^{-1}(1)|$ 18.4349

Casio Classpad

- Modulus sign under Math1.



Space for Personal Notes

Question 10 Tech-Active.

Find the obtuse angle, correct to 3 decimal places, between the lines $y = -2x - 9$ and $y = x + 5$.

```
In[222]:= Abs[ArcTan[-2] - ArcTan[1]] / Degree // N
Out[222]= 108.435
```

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Section E: Exam 2 (27 Marks)

INSTRUCTION: 27 Marks. 34 Minutes Writing.

Question 11 (1 mark)

The linear function $f(x) = 3x - 2$ has a maximum value of 3 and minimum value of -5.

The function can only take x values in the range:

A. $1 \leq x \leq 7$

B. $1 < x \leq 7$

C. $-1 \leq x \leq \frac{5}{3}$

D. $1 \leq x \leq \frac{4}{3}$

Question 12 (1 mark)

The gradient of the line that is the perpendicular bisector of the points $\left(\frac{7}{2}, -4\right)$ and $\left(\frac{5}{2}, 3\right)$:

A. -7

B. 7

C. $\frac{1}{7}$

D. $\frac{7}{2}$

Space for Personal Notes

Question 13 (1 mark)

The simultaneous linear equations:

$$mx + 12y = 24$$

$$3x + my = m$$

Have no solution for:

A. $m = 6$ or $m = -6$

B. $m = 12$ or $m = 3$

C. $m \neq -6$ and $m \neq 6$

D. $m = 2$ or $m = 1$

Question 14 (1 mark)

In a cinema, adult tickets cost \$10 each while child tickets cost \$6. For a certain film, there were 125 people in the cinema, having paid in total \$878.

Find how many adults and how many children were watching this film:

A. 15 children and 67 adults.

B. 32 children and 90 adults.

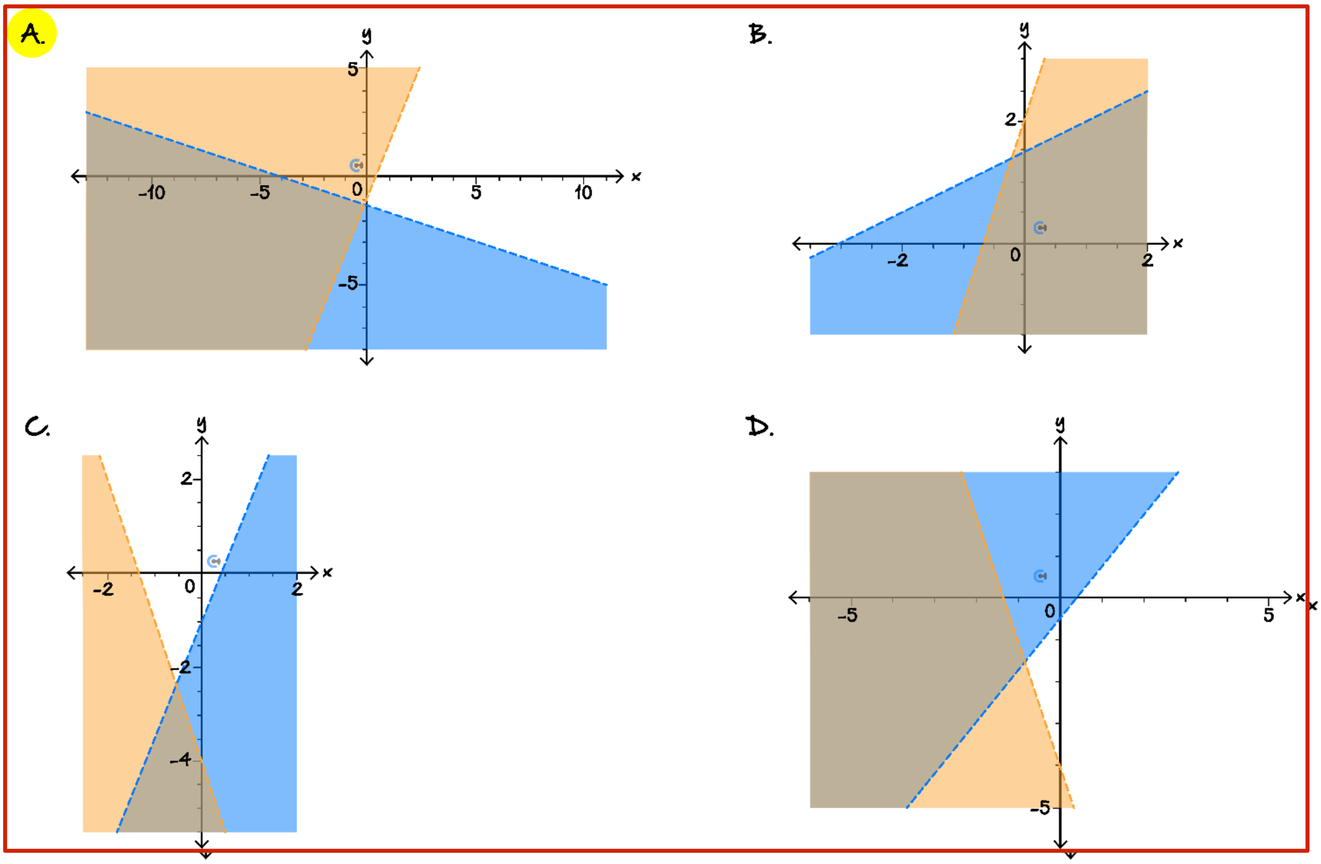
C. 45 children and 16 adults.

D. 93 children and 32 adults.

Space for Personal Notes

Question 15 (1 mark)

Find the graph that represents the 2 inequalities $3y + x < -4$ and $2.5x - 1 < y$.



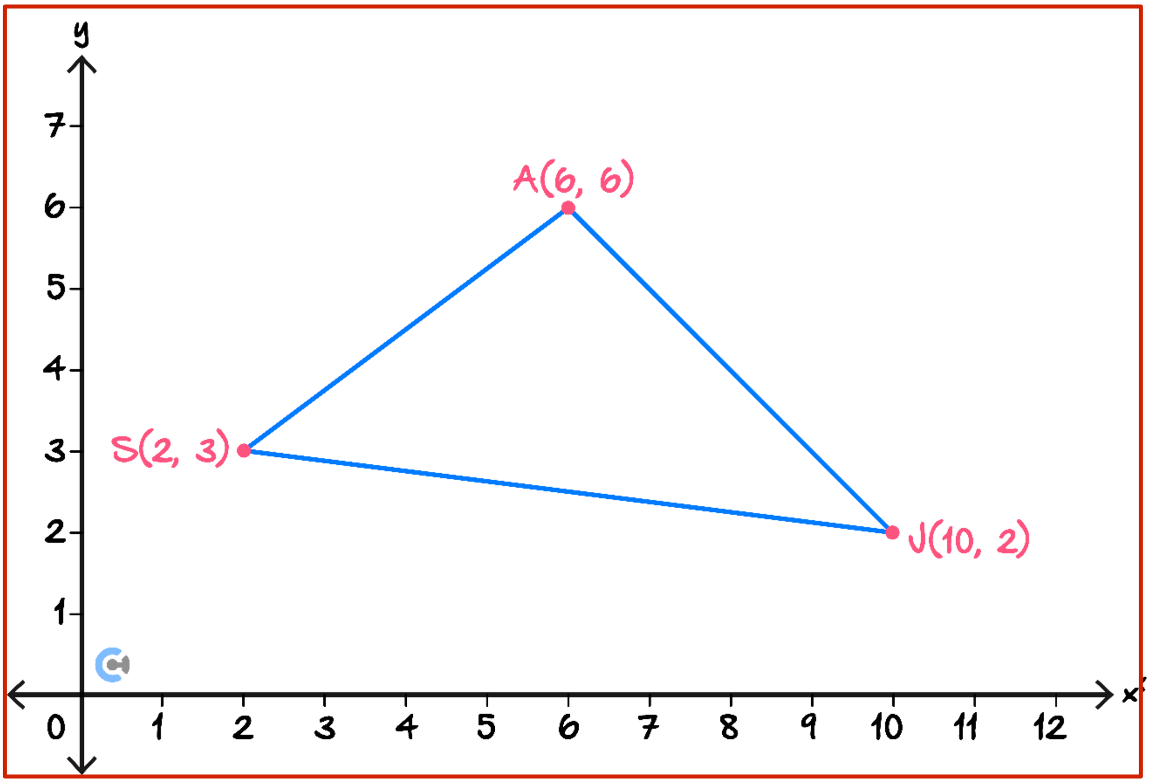
Space for Personal Notes

Question 16 (10 marks)

Alex, Jacob and Sam are playing a game on the beach. The play zone is in the shape of a triangle and each player starts by standing at a vertex. These vertices are $A(6,6), J(10,2)$ and $S(2,3)$.

Measurements are in metres.

a. Sketch the play zone border on the axes below. Label all vertices with their coordinates. (1 mark)



b. Find the equation of the line segment SJ in terms of x and y. (2 marks)

Solution: $m = \dfrac{3-2}{2-10} = -\dfrac{1}{8}$. Therefore,

$$y - 3 = -\frac{1}{8}(x-2)$$
$$y = -\frac{1}{8}x + \frac{13}{4}$$

c. Find the equation of the line perpendicular to SJ that goes through A. (2 marks)

$m = 8$ and through $(6, 6)$.

$$y - 6 = 8(x - 6)$$
$$y = 8x - 42$$

d. Hence, find the area of the play zone. (2 marks)

Solution: $8x - 42 = -\frac{1}{8}x + \frac{13}{4} \implies x = \frac{362}{65}$. Therefore intersection $\left(\frac{362}{65}, \frac{166}{65}\right)$

Triangle height length $= \sqrt{\left(6 - \frac{362}{65}\right)^2 + \left(6 - \frac{166}{65}\right)^2} = \frac{28}{\sqrt{65}}$

Triangle base length $= |SJ| = \sqrt{8^2 + 1^2} = \sqrt{65}$

Area play zone $= \frac{1}{2} \times \frac{28}{\sqrt{65}} \times \sqrt{65} = 14 \text{ m}^2$

e. Find the angles $\angle ASJ, \angle AJS$ and angle $\angle JAS$ of the triangle. Give all angles in degrees correct to two decimal places. (3 marks)

Solution: Gradient $SA = \frac{3}{4}$ and gradient $AJ = -1$

$\angle ASJ = \left|\arctan\left(\frac{3}{4}\right) - \arctan\left(-\frac{1}{8}\right)\right| =$ 43.99

$\angle AJS = \left|\arctan(-1) - \arctan\left(-\frac{1}{8}\right)\right| = 37.87°$

$\angle JAS = 180 - \left|\arctan(-1) - \arctan\left(-\frac{3}{4}\right)\right| = 98.13°$

Question 17 (12 marks)

The coordinates of three points on the Cartesian plane are given by $P(-19, 24)$, $Q(-23, -37)$ and $R(38, -41)$.

a. Find the coordinates of A the midpoint of PQ. (1 mark)

$$A\left(-21, -\frac{13}{2}\right)$$

b. Show that $\angle PQR = 90°$. (2 marks)

- Gradient of PQ

In[127]:= $\frac{-37-24}{-23+19}$

Out[127]= $\frac{61}{4}$

- Gradient of QR

In[128]:= $\frac{-41+37}{38+23}$

Out[128]= $-\frac{4}{61}$

c. Given that $PQRS$ is a rectangle, find the coordinates of the point S. (3 marks)

- Let S be the point (a,b)
- Gradient for SR

$\frac{b+41}{a-38}$

- Gradient of SP

$\frac{b-24}{a+19}$

- Using side lengths PQ is the same as RS

```
32]:= Solve[
  {(b + 41)/(a - 38) == -1/((b-24)/(a+19)), EuclideanDistance[p, q] == EuclideanDistance[{a, b}, r]},
  {a, b}]
```

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

```
32]= {{a → -23, b → -37}, {a → 42, b → 20}}
```

- S: (42,20) as (-23,-37) is point Q

d. Find the coordinates of B, the midpoint of the diagonal PR. (1 mark)

$$\left(\frac{19}{2}, -\frac{17}{2}\right)$$

e. Find the equation of the line that connects AB. (2 marks)

$$y = -\frac{961}{122} - \frac{4x}{61}$$

f. Find the perimeter of the rectangle $PQRS$. (2 marks)

```
In[155]:= EuclideanDistance[s, r]
Out[155]= √3737
In[156]:= EuclideanDistance[p, q]
Out[156]= √3737
4 * √3737
```

g. Find the area of the rectangle $PQRS$. (1 mark)

```
In[157]:= √3737 * √3737
Out[157]= 3737
```

Let's take a BREAK (Extension Stream)!

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Section F: Extension Exam 1 (16 Marks)

INSTRUCTION: 16 Marks. 20 Minutes Writing.

Question 18 (4 marks)

Consider the system of linear equations:

$$(k+1)x + 5y = 0$$

$$3x + (k-1)y = k$$

a. Find the value(s) of k for which the system of equations will have no solution. (3 marks)

Solution: Equate the gradients.

$$\frac{k+1}{5} = \frac{3}{k-1}$$
$$k^2 - 1 = 15$$
$$k^2 = 16$$
$$k = -4, 4$$

The y-intercepts for the two equations are 0 and k so they are never equal. Therefore no solution when $k = -4, 4$

b. Find the value(s) of k for which the system of equations will have a unique solution. (1 mark)

Solution: Unique solution when $k \in \mathbb{R} \setminus \{-4, 4\}$

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Question 19 (5 marks)

Consider the line L with equation $y = 2x - 1$ and the point $A(6, 1)$.

a. Find the equation of the line perpendicular to L passing through the point A. (2 marks)

$m = -\frac{1}{2}$ and through $(6, 1)$. Therefore,

$$y = -\frac{1}{2}x + 4$$

b. Use this perpendicular line to find the coordinates of the point B, which is the reflection of A in the line L. (2 marks)

Solution: $2x - 1 = -\frac{1}{2}x + 4 \implies x = 2$. Lines intersect at $(2, 3)$
$(2, 3)$ is the midpoint of A and B. Therefore, $B(-2, 5)$

c. Point A can also be mapped to point B if it is reflected in the line $x = p$ and then reflected in the line $y = q$. Find the values of p and q. (1 mark)

$p = 2$ and $q = 3$

Question 20 (4 marks)

Consider the quadratic function $f(x) = 2x^2 - 4x + 5$.

a. The point P has a horizontal distance of 2 units from two different points on $f(x)$ and a vertical distance of 0 units from the same two points. Find the coordinates of the point P. (2 marks)

Solution: $f(x) = 2(x-1)^2 + 4$. P must lie on the axis of symmetry $\implies a = 1$. Then $f(3) = f(-1) = 12$ and so $P(1, 12)$.

b. Find the possible value(s) of c such that the vertical distance between f and $(c, 2)$ when $x = c$ is 4. (2 marks)

We solve

$$\begin{aligned} 2(c-1)^2 + 4 - 2 &= 4 \\ 2(c-1)^2 &= 2 \\ (c-1)^2 &= 1 \\ c &= 0, 2 \end{aligned}$$

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Question 21 (3 marks)

The point $P(a, b)$ where a and b are positive real numbers, lies on the line $y + 2x - 6 = 0$.

Find the minimum distance between the point P and the origin, without using calculus.

Solution: Min distance on perpendicular line.

Perpendicular line has equation $y = \frac{1}{2}x$

$\frac{1}{2}x + 2x = 6 \implies \frac{5}{2}x = 6 \implies x = \frac{12}{5}$.

So intersection point at $\left(\frac{12}{5}, \frac{6}{5}\right)$. Therefore min distance is

$$
\begin{aligned}
d &= \sqrt{\left(\frac{12}{5}\right)^2 + \left(\frac{6}{5}\right)^2} \\
&= \frac{\sqrt{180}}{5} \\
&= \frac{\sqrt{36} \times \sqrt{5}}{5} \\
&= \frac{6\sqrt{5}}{5}.
\end{aligned}
$$

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Section G: Extension Exam 2 (20 Marks)

INSTRUCTION: 20 Marks. 24 Minutes Writing.

Question 22 (1 mark)

George and Lucy are preparing for a Mathematics exam by doing the same set of practice papers. They both have one practice paper left to do and their mean scores are identical.

Lucy scores 47% on her last paper and her mean score drops to 69%. George scored 83% on his last paper and his mean score rises to 72%. Determine the number of practice papers in the set.

A. 36

B. 10

C. 42

D. 12

```
In[74]:= Solve[{(47 + (n - 1) a)/n == 69, (83 + (n - 1) a)/n == 72}, {a, n}]
Out[74]= {{a → 71, n → 12}}
```

Question 23 (1 mark)

The two lines $px + qy + r = 0$ and $p^2x + q^2y + r^2 = 0$ are perpendicular when:

A. $p = \pm q$

B. $p^2 + q^2 = r^2$

C. $p + q + r = 0$

D. $p^3 + q^3 = 0$

Question 24 (1 mark)

Two simultaneous linear equations are $4x - 6y = 2k$ and $mx + 6y = 10$. Which of the following statements is **false**?

A. If $m \neq -4$ and $k \in \mathbb{R}$, there is a unique solution.

B. If $m = -4$ and $k = -5$, then there is an infinite number of solutions.

C. If $m = -4$ and $k \neq -5$, then there is more than one solution.

D. If $m = -4$ and $k \in \mathbb{R}$, there is no unique solution.

Question 25 (1 mark)

The minimum distance between the origin and a point on the line $y = 4 - x$ is:

A. 2

B. $2\sqrt{2}$

C. $\sqrt{6}$

D. $\sqrt{10}$

Question 26 (1 mark)

The obtuse angle formed by the lines $y = 3x + 5$ and $y = mx + 3$ is $135°$. The possible value(s) of m are:

A. 2

B. $-2, \frac{1}{2}$

C. $1, -1$

D. -1

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Question 27 (15 marks)

Suppose that Amy is standing at the point A (2,6) and Sachin is standing at the point B (−3,2).

a. Show that the distance between Amy and Sachin is $\sqrt{41}$. (1 mark)

$$d = \sqrt{(2+3)^2 + (6-2)^2} = \sqrt{5^2 + 4^2} = \sqrt{25+16} = \sqrt{41}$$

b. Find the equation of the line segment AB in the form $ax + by + c = 0$, for integers a, b, c. (1 mark)

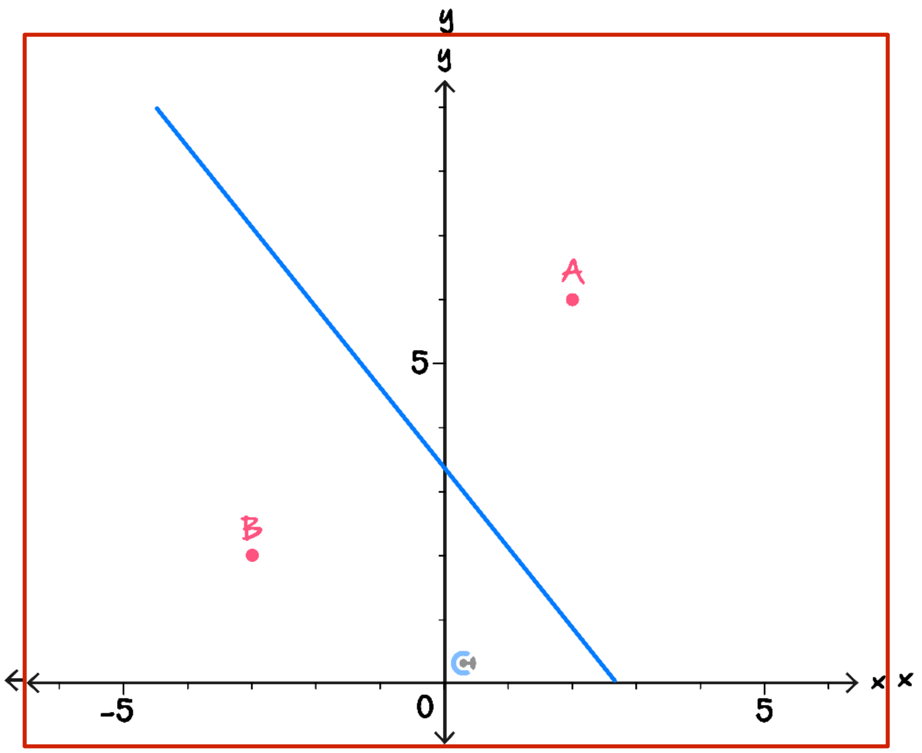
$$4x - 5y + 22 = 0$$

c. Find the perpendicular bisector of the points A and B. (2 marks)

Solution: Want equation of the line with gradient $-\frac{5}{4}$ through the midpoint of AB $\left(-\frac{1}{2}, 4\right)$

$$y - 4 = -\frac{5}{4}\left(x + \frac{1}{2}\right)$$
$$y = -\frac{5}{4}x + \frac{27}{8}$$

d. Sketch this perpendicular bisector along with the points A and B. (2 marks)



e. Give an example of a point that is equidistant to both A and B. (1 mark)

Any point on the line that is a perpendicular bisector of A and B.

f. Explain why there are infinitely many points with this property from **part e.** (1 mark)

Because there are infinitely many points on the perpendicular bisector of A and B.

Also, suppose that strict social distancing measures have been enforced so that no person is allowed within a $\frac{41}{8}$ radius of any other person.

g. Michael stands at a point P to talk to Amy and Stuart stands at a point Q to talk to Sachin. Point Q has only positive coordinates.

i. Find the coordinates of P and Q such everyone is as close as possible to each other while still meeting social distancing requirements. (4 marks)

Solution: Points P and Q must be located on the perpendicular bisector and they are both a distance of $\frac{41}{8}$ from points A and B. The distance between point A and a point $\left(x, -\frac{5}{4}x + \frac{27}{8}\right)$ on the perpendicular bisector is given by

$$d = \sqrt{(x-2)^2 + \left(\frac{27}{8} - \frac{5}{4}x - 6\right)^2}$$

Now we use CAS to solve $d = \frac{41}{8} \implies x = -3, 2$.
Sub $x = -3$ into $y = \frac{27}{8} - \frac{5}{4}x \implies y = \frac{57}{8}$
Sub $x = 2$ into $y = \frac{27}{8} - \frac{5}{4}x \implies y = \frac{7}{8}$
Therefore we have coordinates

$$P\left(-3, \frac{57}{8}\right) \quad \text{and} \quad Q\left(2, \frac{7}{8}\right)$$

ii. Find the angle $\angle BQA$ in degrees correct to two decimal places. (2 marks)

Solution: Many different ways that work.

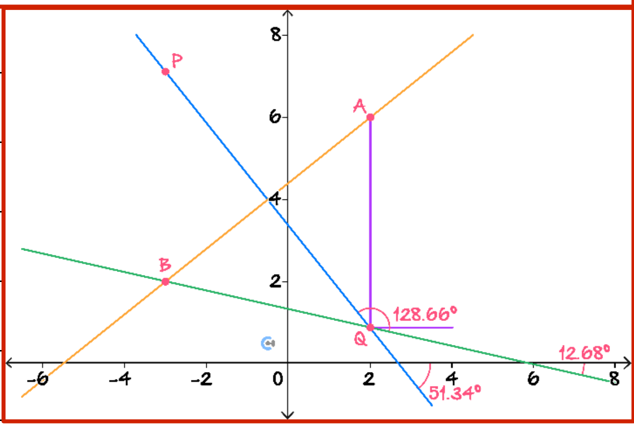
The line BQ has equation $y = \frac{53}{40} - \frac{9}{40}x$

$\tan^{-1}\left(-\frac{9}{40}\right) = -12.68^\circ$

$\angle BQA = 90 - 12.68 = 77.32^\circ$.

OR $\tan^{-1}\left(-\frac{5}{4}\right) = -51.34^\circ$

$\angle BQA = 2 \times (180 - 51.34 - 90) = 77.32^\circ$.



iii. Hence, find the angles $\angle QBA$ and $\angle QAB$ in degrees correct to two decimal places. (1 mark)

Solution: The triangle BQA is isosceles.

Therefore $\angle QBA = \angle QAB = \frac{180 - 77.32}{2} = 51.34^\circ$

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