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VCE Mathematical Methods ½
Linear & Coordinate Geometry [0.1]
Workshop



Section A: Recap

Linear equations

Definition

- Definition: Equations where the highest power of a variable is 1.
 - Gradient-intercept form:

$$y = mx + c$$

Where
$$m = \text{gradient} = \frac{\text{rise}}{\text{run}} =$$

and
$$c =$$

- No singular solution for a linear equation in two variables.
 - \bullet All pairs of coordinates (x, y) that satisfy the equation lie on a **line**. (Hence, linear equations.)

Sub-Section: Inequality



Inequalities rule



$$x > \frac{b}{a}$$
, where $a < 0$

Multiplying both sides by a negative number _____ the inequality sign.



Sub-Section: Midpoint



Midpoint



$$\bullet \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

 (x_2, y_2)

Definition: The midpoint, M, of two points A and B is the point halfway between A and B.

$$M(x_m, y_m) = \left(\right.$$

The midpoint can be found by taking the _____ of the x-coordinate and y-coordinate of the two points.





Sub-Section: Distance Between Two Points



Distance between two points



Definition: The distance between two points (x_1, x_2) and (y_1, y_2) can be found using Pythagoras' theorem:

Distance = ____

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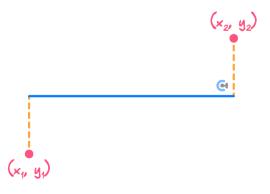


Sub-Section: Vertical Distance Vs Horizontal Distance



Horizontal distance





Horizontal Distance = $x_2 - x_1$ where, ______.

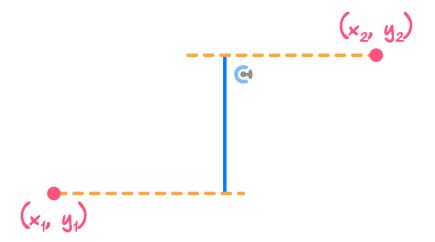
Find the difference between their *x*-values.

What about vertical distance then?



Vertical distance





Vertical Distance = $y_2 - y_1$ where, $y_2 > y_1$.

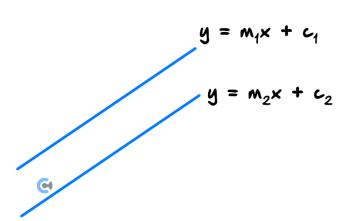
Find the difference between their y-values.



Sub-Section: Parallel and Perpendicular Lines



Parallel lines

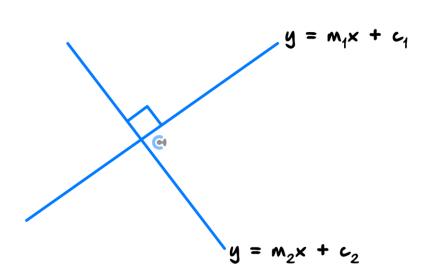


Parallel lines have the _____ gradient.

$$m_1 = m_2$$

Definition

Perpendicular lines



A line that is perpendicular to another line has a gradient, which is the ______ of the gradient of the first line.

$$m_{\perp}=-rac{1}{m}$$



<u>Sub-Section</u>: Angle Between a Line and the x-axis

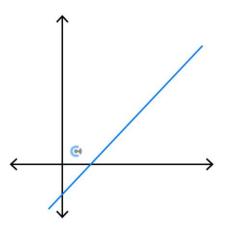


How do we find the angle between a line and the x-axis?



The angle between a line and the x-axis





The angle between a line and the ______ direction of the x-axis (anticlockwise) is given by:

$$tan(\theta) = m$$

NOTE: Angles from the x-axis measured anticlockwise = _____ angles.



Don't worry about it too much, it's just convention! (More on this in circular functions.)



Sub-Section: Angle Between the Two Lines

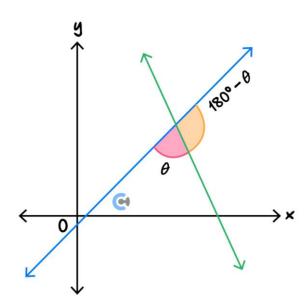


Slightly more complicated now! How about an angle between two lines?



The acute angle between two lines





$$\theta =$$

Alternatively:

$$tan(\theta) =$$

• For your understanding, note that this formula is derived from the tan compound angle formula covered in SM12.

NOTE: |x| just takes the positive value of x.



TIP: Make sure your CAS is in degrees.







Sub-Section: Finding Simultaneous Equations for Two Variables

Definition

Simultaneous linear equations

- Elimination method:
 - Add or subtract one equation from the other in order to ______ one of the variables. Then have an equation in one variable that can be solved easily.
- Substitution method:
 - Make one of the variables the subject (generally x or y) and _____ that value into the other equation.

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Sub-Section: Number of Solutions for Two Variables



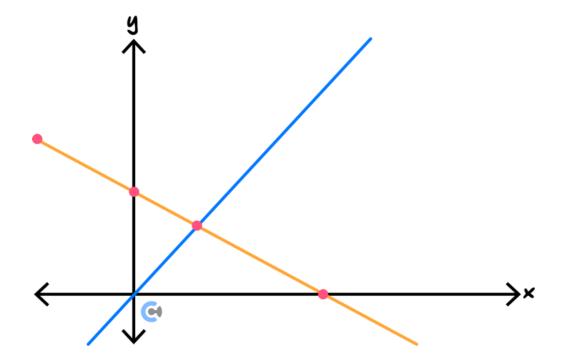
What does the geometry look like for each number of solutions?



Exploration: Geometry of the number of solutions between linear graphs

Unique solution:

$$m_1 \neq m_2$$

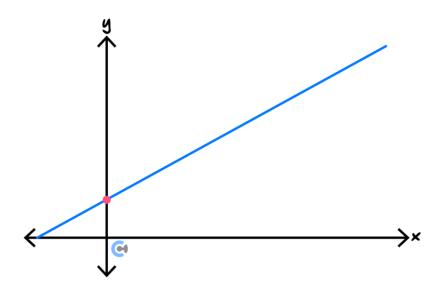


They just need to have ______



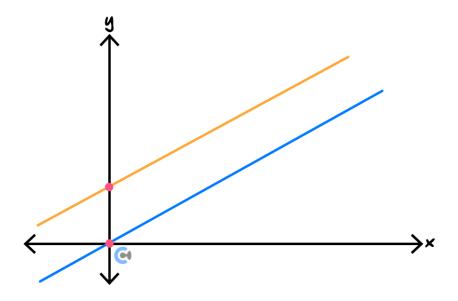
Infinite solutions:

$$m_1=m_2$$
 and $c_1=c_2$



- They just need to have the same _____ and the same _____.
- In other words, they have to be the ______.
- No solutions:

$$m_1=m_2$$
 and $c_1 \neq c_2$



- lacktriangledown They need to have the ______ but _____+c.
- They have to be two different _____ lines.



General solutions of simultaneous linear equations



- Two linear equations are either:
 - The same line is expressed in a different form. In this case, they have _____ solutions.
 - Unique lines which are parallel. In this case, they have _____ solutions.
 - Unique lines which are not parallel. In this case, they have ______ solution.

TIP: It's a good idea to substitute your answer back into the equations to see if the criteria are met for each part.



Section B: Warmup

INSTRUCTION: 5 Minutes Writing.



Question 1					
a.	Find the horizontal distance between the points (2,6) and (5,6).				
ĺ					
b.	Find the equation of the line parallel to $y = 2x - 5$ that goes through the midpoint of (1,2) and (5,4).				

c.	Find the distance between the points (1,3) and (5,7).
d.	Determine the value of k for which the equations:
	3kx - 2y = k
	6x - 4y = 6 Have no solutions.
Sp	pace for Personal Notes



Section C: Exam 1 (23 Marks)

INSTRUCTION: 23 Marks. 30 Minutes Writing.



Question 2 (2 marks)

Solve the following simultaneous linear equations for x and y.

$$5x + 3y = 41$$

$$-2x - 3y = -20$$

$$4x + 6y = 40$$

 $\textbf{Question 3} \; (4 \; mark)$

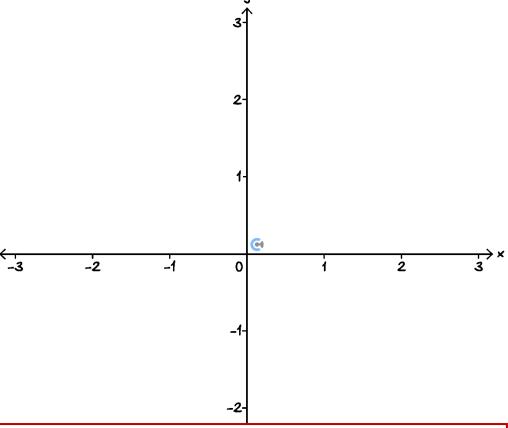
a. Point B (2,1) is the midpoint of A $(-1,-1)$ and point C. Find the coordinates of C. (2 marks)					



b. Find in the form y = ax + b the equation of the line that makes an angle of 45° anticlockwise from the positive direction of the *x*-axis and passes through *B*. (Hint: $\tan 45^\circ = 1$) (2 marks)

Question 4 (3 marks)

Sketch the following inequality: 2x > 4y + 3

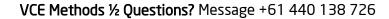


At y = 0, $x = \frac{3}{2}$ and at x = 0, $y = -\frac{3}{4}$. Hence, intercepts at $\left(0, -\frac{3}{4}\right)$ and $\left(\frac{3}{2}, 0\right)$.

-3-



Question 5 (6 marks)				
	n's home is at the point $(-1, -2)$. He wants to walk straight from his home to a river bank which is given by linear line $3x + 4y - 12 = 0$. The units are kilometres.			
a.	Find the equation of the straight-line path taken by Sam if he walked the least distance. (2 marks)			
b.	Find the coordinate of the point on the riverbank where Sam reaches. (2 marks)			
υ.				
c.	Calculate the distance travelled by Sam. It is given that $\sqrt{69^2 + 92^2} = 115$. (2 marks)			





Question 6 (2 marks)
The distance between two stations is 320 km. Two trains start simultaneously from different stations and travel on parallel tracks towards each other.
If the speed of one of them is greater than the other by $10 km/hr$ and the distance between the two trains after 2 hours of their start is $20 km$, find the speed of each train.
Question 7 (3 marks)
Jeff is creating two chemical solutions, Solution <i>A</i> and <i>B</i> , by mixing two key ingredients: Chemical <i>X</i> and Chemical <i>Y</i> . Each litre of Solution <i>A</i> requires 3 grams of Chemical <i>X</i> and 2 grams of Chemical <i>Y</i> , while each litre of Solution <i>B</i> requires 2 grams of Chemical <i>X</i> and 5 grams of Chemical <i>Y</i> .
Jeff has 38 grams of Chemical <i>X</i> and 49 grams of Chemical <i>Y</i> available. How many litres of each solution should the lab prepare to use up all the chemicals?



Question 8	(3 marks)		

Determine the value of k so that the following linear equations have no solution.

$$(3k+1)x + 3y - 5 = 0$$

$$(k^2 + 3)x + (k - 2)y - 5 = 0$$

Let's take a BREAK (standard stream)!

R



Section D: Tech Active Exam Skills

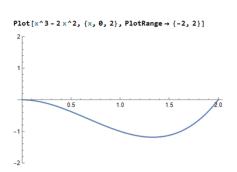
INSTRUCTION: 5 Minutes Writing.



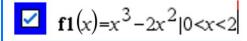
Calculator Commands: Graphing

Mathematica

- Plot [function, {x, xmin, xmax}.
 Plot Range → {ymin, ymax}]
- Plot Range is optional but makes the scale appropriate for the question.

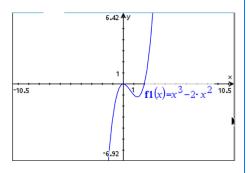


- Menu → 6 (Analyse) to find min/max x and y-intercepts.
- Restrict domain to 0 < x < 2 use the bar can get it from ctrl+ = $\begin{vmatrix} x & y & y \\ y & y & z \end{vmatrix}$

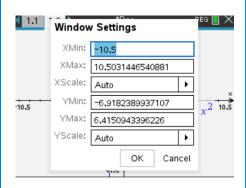


➤ TI-Nspire

Open a graph page and plot your function.



Zoom settings: Menu $\rightarrow 4$ (window/zoom) $\rightarrow 1$ enter your x and y ranges.



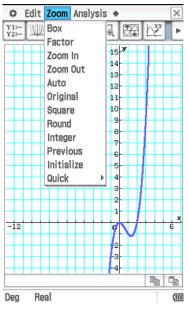
Can also click the axis numbers on the graph and alter them directly.



Casio Classpad

Click Graph & Table, and enter the function.





- Analysis → G- Solve to find intercepts.
- Use this button to set the view window.



- Ge Use | to restrict domain → Find it in Math 3.
- $Vy1=_{X}3-_{2}\cdot_{X}2|_{0\leq x\leq 2}$

Calculator Commands: Solving Equations



- TI-Nspire

solve
$$(x^2-4\cdot x-9=0,x)$$

 $x=-(\sqrt{13}-2) \text{ or } x=\sqrt{13}+2$

- Casio Classpad
 - Action → Advanced → Solve

solve
$$(x^2-4x-9=0, x)$$

 $\{x=-\sqrt{13}+2, x=\sqrt{13}+2\}$

$$\begin{split} & \text{In[122]:= Solve[x^2 - 4 x - 9 == 0, x]} \\ & \text{Out[122]=} \; \left\{ \left\{ x \to 2 - \sqrt{13} \; \right\}, \; \left\{ x \to 2 + \sqrt{13} \; \right\} \right\} \end{split}$$



Calculator Commands: Simultaneous Equations

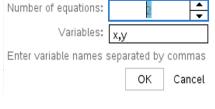
(4)

- Mathematica
 - Just do && between.
 - Solve[equation&&equation , {var1, var2}]

In[128]:= Solve[2 x - 3 y = 16 && x + y = 3, {x, y}]
Out[128]= { $x \to 5, y \to -2$ }

- TI-Nspire
 - Menu 3 7 1

Solve a System of Equations



solve
$$\begin{cases} 2 \cdot x - 3 \cdot y = 16 \\ x + y = 3 \end{cases}$$
, $\{x, y\}$ $x = 5$ and $y = -2$

Casio Classpad

Math1 → Click highlighted box→ Enter equations and variables you are solving for:

Question 9 Tech-Active.

Solve the equations 2x + 7y = 16 and 5x + 3y = 20 for x and y.



<u>Calculator Commands:</u> Finding the Angle between a Line and x-axis



Mathematica

In[124]:= ArcTan[2] / Degree // N
Out[124]= 63.4349

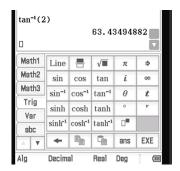
TI-Nspire

Firig button. Check that you are in degrees.

tan⁻¹(2) 63.4349

Casio Classpad

G Keyboard →Trig. Change to decimals and degrees.



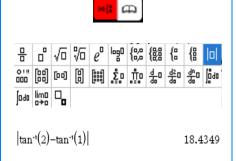
Calculator Commands: Finding the Angle between Two Lines

CAS GI

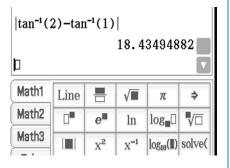
- Mathematica
 - Use the Abs[] function.

In[126]:= Abs[ArcTan[2] - ArcTan[1]] / Degree // N
Out[126]:= 18.4349

- TI-Nspire
 - Find the modulus sign.



- Casio Classpad
 - Modulus sign under Math1.





Question 10 Tech-Active.	
Find the obtuse angle, correct to 3 decimal places, between the lines $y = -2x - 9$ and $y = x + 5$.	

Space for Personal Notes	



Section E: Exam 2 (27 Marks)

INSTRUCTION: 27 Marks. 34 Minutes Writing.



Question 11 (1 mark)

The linear function f(x) = 3x - 2 has a maximum value of 3 and minimum value of -5.

The function can only take x values in the range:

- **A.** $1 \le x \le 7$
- **B.** $1 < x \le 7$
- C. $-1 \le x \le \frac{5}{3}$
- **D.** $1 \le x \le \frac{4}{3}$

Question 12 (1 mark)

The gradient of the line that is the perpendicular bisector of the points $(\frac{7}{2}, -4)$ and $(\frac{5}{2}, 3)$:

- **A.** -7
- **B.** 7
- C. $\frac{1}{7}$
- **D.** $\frac{7}{2}$



Question 13 (1 mark)

The simultaneous linear equations:

$$mx + 12y = 24$$

$$3x + my = m$$

Have no solution for:

- **A.** m = 6 or m = -6
- **B.** m = 12 or m = 3
- C. $m \neq -6$ and $m \neq 6$
- **D.** m = 2 or m = 1

Question 14 (1 mark)

In a cinema, adult tickets cost \$10 each while child tickets cost \$6. For a certain film, there were 125 people in the cinema, having paid in total \$878.

Find how many adults and how many children were watching this film:

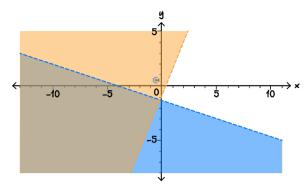
- **A.** 15 children and 67 adults.
- **B.** 32 children and 90 adults.
- C. 45 children and 16 adults.
- **D.** 93 children and 32 adults.



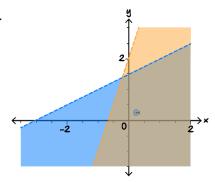
Question 15 (1 mark)

Find the graph that represents the 2 inequalities 3y + x < -4 and 2.5x - 1 < y.

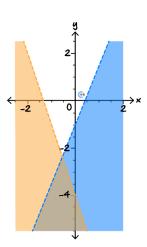
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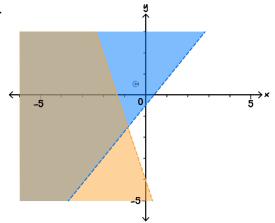
₿.



C.



D.



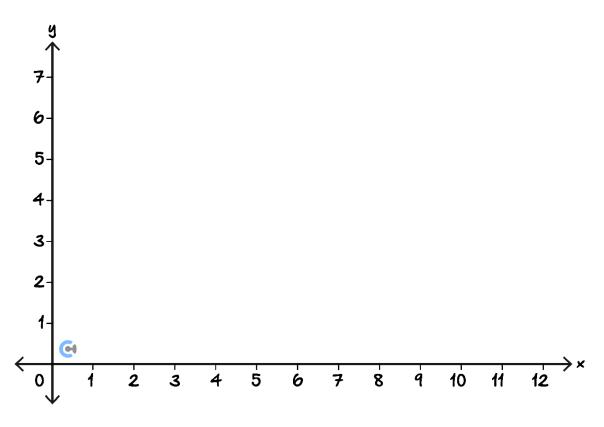


Question 16 (10 marks)

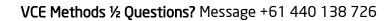
Alex, Jacob and Sam are playing a game on the beach. The play zone is in the shape of a triangle and each player starts by standing at a vertex. These vertices are A(6,6), J(10,2) and S(2,3).

Measurements are in metres.

a. Sketch the play zone border on the axes below. Label all vertices with their coordinates. (1 mark)



b. Find the equation of the line segment SJ in terms of x and y. (2 marks)





c.	Find the equation of the line perpendicular to SJ that goes through A . (2 marks)
d.	Hence, find the area of the play zone. (2 marks)
e.	Find the angles $\angle ASJ$, $\angle AJS$ and angle $\angle JAS$ of the triangle. Give all angles in degrees correct to two decimal places. (3 marks)

CONTOUREDUCATION

Question 17 (12 marks)

The coordinates of three points on the Cartesian plane are given by P(-19, 24), Q(-23, -37) and R(38, -41).

a. Find the coordinates of A the midpoint of PQ. (1 mark)

 $A\left(-21,-\frac{13}{2}\right)$

b. Show that $\angle PQR = 90^{\circ}$. (2 marks)

In[127]:= $\frac{-37 - 24}{-23 + 19}$ Out[127]= $\frac{61}{4}$

■ Gradient of QR

 $\ln[128] = \frac{-41 + 37}{38 + 23}$ $\int_{\text{Dut}[128] = -\frac{4}{3}}$

c. Given that *PQRS* is a rectangle, find the coordinates of the point *S*. (3 marks)

S: (42,20) as (-23,-37) is point Q

d. Find the coordinates of B, the midpoint of the diagonal PR. (1 mark)

 $(\frac{19}{2}, -\frac{1}{2})$

e. Find the equation of the line that connects AB. (2 marks)

 $y = -\frac{961}{122} - \frac{4x}{61}$

f. Find the perimeter of the rectangle *PQRS*. (2 marks)

In[155]:= EuclideanDistance[s, r] $Out[155] = \sqrt{3737}$ In[156]:= EuclideanDistance[p, q] $Out[156] = \sqrt{3737}$ $4 * \sqrt{3737}$

g. Find the area of the rectangle *PQRS*. (1 mark)

t[157]:= $\sqrt{3737} * \sqrt{3737}$

Let's take a BREAK (Extension Stream)!



Section F: Extension Exam 1 (16 Marks)

INSTRUCTION: 16 Marks. 20 Minutes Writing.



Ouestion	18	(4	marks)	١
Oueshon	10	١4	marks	,

Consider the system of linear equations:

$$(k+1)x + 5y = 0$$

3x + (k - 1)y = k and the value(s) of k for which the system of equations will have no solution. (3 marks)
nd the value(s) of k for which the system of equations will have a unique solution. (1 mark)
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Qu	estion 19 (5 marks)
Co	nsider the line L with equation $y = 2x - 1$ and the point $A(6, 1)$.
a.	Find the equation of the line perpendicular to L passing through the point A . (2 marks)
b.	Use this perpendicular line to find the coordinates of the point B , which is the reflection of A in the line L . (2 marks)
c.	Point A can also be mapped to point B if it is reflected in the line $x = p$ and then reflected in the line $y = q$. Find the values of p and q. (1 mark)



Question 20 (4 marks)			
Consider the quadratic function $f(x) = 2x^2 - 4x + 5$.			
a. The point P has a horizontal distance of 2 units from two different points on $f(x)$ and a vertical distance of 0 units from the same two points. Find the coordinates of the point P . (2 marks))		
·			
b. Find the possible value(s) of c such that the vertical distance between f and $(c, 2)$ when $x = c$ is 4. (2 marks))		
Space for Personal Notes			



Question 21 (3 marks)	
The point $P(a, b)$ where a and b are positive real numbers, lies on the line $y + 2x - 6 = 0$.	
Find the minimum distance between the point P and the origin, without using calculus.	
Space for Personal Notes	



Section G: Extension Exam 2 (20 Marks)

INSTRUCTION: 20 Marks. 24 Minutes Writing.



Question 22 (1 mark)

George and Lucy are preparing for a Mathematics exam by doing the same set of practice papers. They both have one practice paper left to do and their mean scores are identical.

Lucy scores 47% on her last paper and her mean score drops to 69%. George scored 83% on his last paper and his mean score rises to 72%. Determine the number of practice papers in the set.

- **A.** 36
- **B.** 10
- **C.** 42
- **D.** 12

Question 23 (1 mark)

The two lines px + qy + r = 0 and $p^2x + q^2y + r^2 = 0$ are perpendicular when:

- A. $p = \pm q$
- **B.** $p^2 + q^2 = r^2$
- **C.** p + q + r = 0
- **D.** $p^3 + q^3 = 0$

Question 24 (1 mark)

Two simultaneous linear equations are 4x - 6y = 2k and mx + 6y = 10. Which of the following statements is **false**?

- **A.** If $m \neq -4$ and $k \in \mathbb{R}$, there is a unique solution.
- **B.** If m = -4 and k = -5, then there is an infinite number of solutions.
- C. If m = -4 and $k \neq -5$, then there is more than one solution.
- **D.** If m = -4 and $k \in \mathbb{R}$, there is no unique solution.

Question 25 (1 mark)

The minimum distance between the origin and a point on the line y = 4 - x is:

- **A.** 2
- **B.** $2\sqrt{2}$
- C. $\sqrt{6}$
- **D.** $\sqrt{10}$

Question 26 (1 mark)

The obtuse angle formed by the lines y = 3x + 5 and y = mx + 3 is 135°. The possible value(s) of m are:

- **A.** 2
- **B.** $-2,\frac{1}{2}$
- C. 1, -1
- **D.** −1



Question 27 (15 marks)

Suppose that Amy is standing at the point A (2,6) and Sachin is standing at the point B (-3,2).

a. Show that the distance between Amy and Sachin is $\sqrt{41}$. (1 mark)

b. Find the equation of the line segment AB in the form ax + by + c = 0, for integers a, b, c. (1 mark)

4x - 5y + 22 = 0

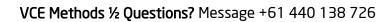
c. Find the perpendicular bisector of the points A and B. (2 marks)

d. Sketch this perpendicular bisector along with the points A and B. (2 marks)

5--5 ×



GIV	ive an example of a point that is equidistant to both A and B . (1 mark)				
			Any point on the line that is a perpendicular bisector of A and B.		
Ex	splain why there are infinitely many points with this property from part e. (1 mark)				
		Beca	ause there are infinitely many points on the perpendicular bisector of A and B.		
			trict social distancing measures have been enforced so that no person is allowed within a $\frac{41}{8}$ person.		
			at a point P to talk to Amy and Stuart stands at a point Q to talk to Sachin. Point Q has only ates.		
i.			ordinates of P and Q such everyone is as close as possible to each other while still meeting acing requirements. (4 marks)		
	o, sius	o, suppose ius of any of Michael st positive co	Beca o, suppose that status of any other p Michael stands a positive coordination. Find the coordination		





ii.	Find the angle $\angle BQA$ in degrees correct to two decimal places. (2 marks)			
				
				
iii.	Hence, find the angles $\angle QBA$ and $\angle QAB$ in degrees correct to two decimal places. (1 mark)			
	- <u></u> -			
pace	for Personal Notes			





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VCE Mathematical Methods ½

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