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Write your **student number** in the boxes above.

**Letter**

# Mathematical Methods $\frac{1}{2}$

## Examination 2 (Tech-Active)

### Question and Answer Book - **SOLUTIONS**

VCE Examination (Term 1 Mock) – April 2025

- Reading time is **15 minutes**.
- Writing time is **2 hours**.

#### Materials Supplied

- Question and Answer Book of 21 pages.
- Multiple-Choice Answer Sheet.

#### Instructions

- Follow the instructions on your Multiple-Choice Answer Sheet.
- At the end of the examination, place your Multiple-Choice Answer Sheet inside the front cover of this book.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

#### Contents

	pages
<b>Section A</b> (2 questions, 20 marks)	2–7
<b>Section B</b> (7 questions, 60 marks)	8–21

**Student's Full Name:** \_\_\_\_\_

**Student's Email:** \_\_\_\_\_

**Tutor's Name:** \_\_\_\_\_

**Marks (Tutor Only):** \_\_\_\_\_

## Section A

### Instructions

- Answer **all** questions in pencil on the Multiple-Choice Answer Sheet.
- Choose the response that is **correct** or that **best answers** the question.
- A correct answer scores 1; an incorrect answer scores 0.
- Marks will **not** be deducted for incorrect answers.
- No marks will be given if more than one answer is completed for any question.
- Unless otherwise indicated, the diagrams in this book are not drawn to scale.

**Question 1** Learning Objective [1.1.3] Find parallel and perpendicular lines.

The gradient of a line perpendicular to the line which passes through  $(-2, 0)$  and  $(0, -4)$  is:

A.  $\frac{1}{2}$

B.  $-2$

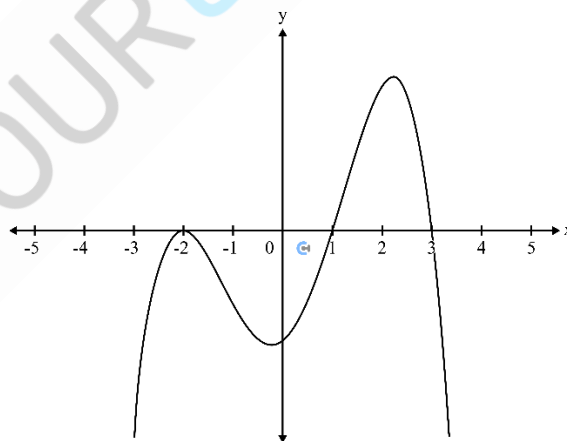
C.  $-\frac{1}{2}$

D.  $2$

D1

**Question 2** Learning Objective [2.3.2] Figure out possible rule of a graph.

The diagram below shows part of the graph of a polynomial function.



A possible rule for this function is:

A.  $y = (x + 2)^2(x - 1)(x - 3)$

B.  $y = (x + 2)^2(x - 1)(3 - x)$

C.  $y = -(x - 2)^2(x - 1)(3 - x)$

D.  $y = -(x + 2)(x - 1)(x - 3)$

D1

**Question 3** Learning Objective [2.3.1] Restrict domain such that the inverse function exists.

**D1**

The largest value of  $a$  such that the function  $f: (-\infty, a] \rightarrow \mathbb{R}, f(x) = x^2 + 3x - 10$  is not many-to-one is equal to

A.  $-5$

B.  $-1.5$

C.  $0$

D.  $2$

**Question 4** Learning Objective [2.2.1] Find domain and range of functions.

The graph of  $y = \frac{1}{(x+3)^2} + 4$  has a horizontal asymptote with the equation:

A.  $y = 4$

B.  $y = 3$

C.  $x = -2$

D.  $x = -3$

**D1**

**Question 5** Learning Objective [2.5.2] Find opposite transformations.

If  $f(x-1) = x^2 - 2x + 3$ , then  $f(x)$  is equal to:

**D1**

A.  $x^2 - 2$

B.  $x^2 + 2$

C.  $x^2 - 2x + 2$

D.  $x^2 - 4x + 6$

**Question 6** Learning Objective [2.4.2] Find transformed functions.

The graph of a function  $f$  is obtained from the graph of the function  $g$  with the rule  $g(x) = \sqrt{2x-5}$  via a reflection in the  $x$ -axis followed by a dilation from the  $y$ -axis by a factor of  $\frac{1}{2}$ .

**D1**

Which one of the following is the rule for the function  $f$ ?

A.  $f(x) = \sqrt{5-4x}$

B.  $f(x) = -\sqrt{x-5}$

C.  $f(x) = \sqrt{x+5}$

D.  $f(x) = -\sqrt{4x-5}$

**Question 7** Learning Objective [2.5.3] Apply transformations of functions to find its domain, range, transformed points.

The point  $A(3, 2)$  lies on the graph of the function  $f$ . A transformation maps the graph of  $f$  to the graph of  $g$ , where  $g(x) = \frac{1}{2}f(x - 1)$ . The same transformation maps point  $A$  to point  $P$ .

The coordinates of the point  $P$  are:

**D1**

- A.  $(2, 1)$
- B.  $(2, 4)$
- C.  $(4, 1)$
- D.  $(4, 2)$

**Question 8** Learning Objective [2.2.1] Find domain and range of functions.

The domain and the range for the relation with the equation  $2 - y = \frac{3}{(x-1)^2}$  are given by:

- A.  $\{x: x \in \mathbb{R} \setminus \{1\}\}$  and  $\{y: y > 2\}$ .
- B.  $\{x: x \in \mathbb{R} \setminus \{1\}\}$  and  $\{y: y < 2\}$ .
- C.  $\{x: x \in \mathbb{R} \setminus \{1\}\}$  and  $\{y: y < -2\}$ .
- D.  $\{x: x \in \mathbb{R} \setminus \{1\}\}$  and  $\{y: y \in \mathbb{R} \setminus \{2\}\}$ .

**D1**

**Question 9** Learning Objective [1.5.3] Find factored form of polynomials.

The graph of a cubic function of the form  $y = a(x + h)^3 + k$  has a stationary point of inflection at  $(-2, 2)$  and intercepts the  $y$ -axis at 10. The equation of the function is:

**D1**

- A.  $y = (x - 2)^3 + 2$
- B.  $y = 2(x + 2)^3 + 10$
- C.  $y = (x + 2)^3 + 10$
- D.  $y = (x + 2)^3 + 2$

**Question 10** Learning Objective [2.2.3] Find the rule, domain, range and intersections between Inverse Functions.

A function has the rule,  $f(x) = 2x^{\frac{1}{3}} + 8, x \in \mathbb{R}$ . The rule for the inverse function is:

A.  $f^{-1}(x) = \frac{2}{x} - 8$

B.  $f^{-1}(x) = \frac{1}{2x^3 - 8}$

C.  $f^{-1}(x) = \left(\frac{x-8}{2}\right)^3$

D.  $f^{-1}(x) = 2x^3 - 8$

**D1**

**Question 11** Learning Objective [2.3.1] Restrict domain such that the inverse function exists.

Which of the following functions is not one-to-one?

A.  $f(x) = 25 - x^2, x < 0$

D1

B.  $f(x) = \frac{1}{x^2} + 25$

C.  $f(x) = 4\sqrt{x}$

D.  $f(x) = -\frac{5}{x}$

**Question 12** Learning Objective [2.2.2] Sketch and find the domain and range of Hybrid Functions.

If  $f(x) = \begin{cases} -3x + 5 & x \geq -2 \\ x + 5 & x < -2 \end{cases}$  then the range of  $f$  is:

D1

A.  $(-\infty, 1]$

B.  $(-\infty, 11)$

C.  $(-\infty, 11]$

D.  $(-5, \infty) \cup (-\infty, 5]$

**Question 13** Learning Objective [1.5.1] Identify the properties of Polynomials and solve Long Division.

If  $4(x - 1)^2 = a(x + 1)^2 + bx$  for all real values of  $x$ , the values of  $a$  and  $b$  are:

A.  $a = 4, b = 16$

B.  $a = 4, b = -16$

C.  $a = 4, b = 4$

D.  $a = 4, b = -6$

D1

**Question 14** Learning Objective [1.1.4] Find angle between a line and x axis or two lines.

The angle, in degrees, between the lines  $y = 2x + \frac{5}{3}$  and  $y = 3x - \frac{10}{27}$  is closest to:

D1

A. 11.31

B. 0.20

C. 8.13

D. 0.14

$$|\tan^{-1}(2) - \tan^{-1}(3)| \quad 8.130102$$

**Question 15** Learning Objective [1.5.2] Apply Remainder and Factor Theorem to find reminders and factors.

Let  $p(x) = x^3 - ax + 1$  where  $a \in \mathbb{R}$ . If the remainder of  $p(x)$  when divided by  $ax + 8$  is 1, then the value of  $a$  is:

D1

A. 3

B. 4

C. 8

D. -2

$x^3 - a \cdot x + 1 \rightarrow p(x)$	Done
$\Delta \text{ solve } \left( p\left(\frac{-8}{a}\right) = 1, a \right)$	$a = 4$

**Question 16** Learning Objective [3.4.2] Finding selections of any size.

Three-letter words are to be made by arranging the letters of the word **METHODS**. What is the probability that a randomly chosen arrangement begins with a vowel?

**D1**

A.  $\frac{2}{7}$

B.  $\frac{3}{10}$

C.  $\frac{1}{3}$

D.  $\frac{4}{11}$

Start with E or O

$$\frac{1 \times 6 \times 5 + 1 \times 6 \times 5}{7 \times 6 \times 5} = \frac{2}{7}$$

**Question 17** Learning Objective [3.1.4] Tree Diagram and Conditional Probability.

Suppose that for three events  $A$ ,  $B$  and  $C$ ,  $\Pr(A | B) = 1$  and  $\Pr(C | B) = \Pr(C)$ . Which of the following is true? **HINT:**  $X \subseteq Y$  means  $X$  is part of  $Y$ .

**D1**

A.  $\Pr(A) = 1$ .

B.  $C \subseteq B$ .

C.  $B$  and  $C$  are independent events and  $B \subseteq A$ .

D.  $B$  and  $C$  are not independent events and  $B \subseteq A$ .

The following information applies to the two questions that follow:

The probability that Zoe bakes cookies on Friday is 0.55. If she bakes on Friday, the probability that she bakes again on Saturday is 0.75. If she does not bake on Friday, the probability that she bakes on Saturday is 0.25.

**Question 18** Learning Objective [3.1.4] Tree Diagram and Conditional Probability.

**D1**

The probability that Zoe bakes on **both** Friday and Saturday is closest to

A. 0.41

B. 0.20

C. 0.15

D. 0.55

$$\Pr(F \cap S) = \Pr(S | F) \times \Pr(F) = 0.75 \times 0.55 = 0.4125 \approx 0.41.$$

**Question 19** Learning Objective [3.1.4] Tree Diagram and Conditional Probability.

The probability that Zoe bakes on **Saturday** is closest to

**D1**

A. 0.57

B. 0.48

C. 0.53

D. 0.34

$$(0.75)(0.55) + (0.25)(0.45) = 0.4125 + 0.1125 = 0.525 \approx 0.53.$$

**Question 20** Learning Objective [1.3.2] Find solutions and number of solutions to quadratic equations.

If the solutions of  $x^2 + bx + 6 = 0$  are integers, the possible values of  $b$  are:

- A. 5 and 7.
- B.  $-5$  and  $-7$ .
- C.  $\pm 5$  and  $\pm 7$ .
- D.  $-5$  and 7.

D1

Do not write in this area.

## Section B

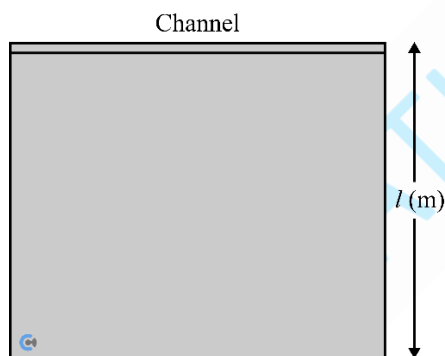
### Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.

### Question 1 (5 marks)

A piece of fencing 960 m long will be used to enclose three sides of a rectangular field. The fourth side has a straight channel along it.

Let  $l$  be the length of the field as shown. Let  $A$  be the area of the field.



All measurements are in metres.

a. Express  $A$  as a function of  $l$ .

2 marks

**D2**

Learning Objective [1.4.2] Apply Quadratics to Model a scenario.

$$960 = 2l + \text{width} \Rightarrow \text{width} = 960 - 2l \quad \mathbf{1M}$$

$$A = l \times \text{width}$$

$$A(l) = l(960 - 2l). \quad \mathbf{1A}$$

b. What is a suitable domain of the function  $A$ ?

1 mark

**D1**

Learning Objective [1.4.2] Apply Quadratics to Model a scenario.

$$0 < l < 480. \quad \mathbf{1A}$$



- c. Determine the range of  $A$ . Also state the value of  $l$  for which the maximum area occurs. 2 marks

D2

Learning Objective [1.4.2] Apply Quadratics to Model a scenario.

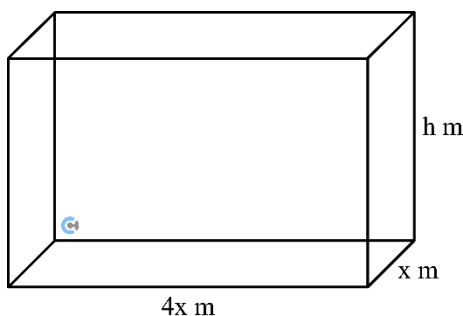
$$\text{Max length} = -\frac{b}{2a} = -\frac{960}{2(-2)} = 240. \mathbf{1M}$$

$$A(240) = 240 \times (960 - 480) = 240 \times 480 = 115200$$

$$0 < A \leq 115200. \mathbf{1A}$$

**Question 2** (11 marks)

A cuboid (rectangular prism) has dimensions  $x$  metres,  $h$  metres and  $4x$  metres as shown on the diagram. The cuboid is made of 240 m of wire.



- a. Find  $h$  in terms of  $x$ . Learning Objective [1.5.4] Graph factored and unfactored polynomials.

2 marks

D1

$$\text{solve}(4 \cdot 4 \cdot x + 4 \cdot x + 4 \cdot h = 240, h) \quad h = 60 - 5 \cdot x$$

$$h = 60 - 5x$$

1M equation that needs to be solved  
1A for answer

- b. Find the volume,  $V \text{ m}^3$ , of the cuboid in terms of  $x$ .

1 mark

D1

Learning Objective [1.5.4] Graph factored and unfactored polynomials.

$$V = x \cdot 4 \cdot x \cdot h \mid h = 60 - 5 \cdot x$$

$$V = -20 \cdot x^2 \cdot (x - 12)$$

$$V = 4x^2(60 - 5x)$$

1A

- c. Find  $V$  when  $x = 11$ . Learning Objective [1.5.4] Graph factored and unfactored polynomials.

1 mark

D1

$$V = -20 \cdot x^2 \cdot (x - 12) \mid x = 11$$

$$V = 2420$$

$$2420 \text{ m}^3$$

1A

- d. Find the possible values of
- $x$
- for the cuboid to exist.

2 marks

D2

Learning Objective [1.5.4] Graph factored and unfactored polynomials.

1M

$$\text{solve } (x > 0 \text{ and } 60 - 5 \cdot x > 0 \text{ and } -20 \cdot x^2 \cdot (x - 12) > 0, x)$$

$$0 < x < 12$$

1A

- e. Find the possible values of
- $x$
- when
- $V = 1620$
- , correct to two decimal places.

D1

1 mark

Learning Objective [1.5.4] Graph factored and unfactored polynomials.

$$\text{solve } (1620 = -20 \cdot x^2 \cdot (x - 12), x) | x > 0$$

$$x = 3 \text{ or } x = \frac{3 \cdot (\sqrt{21} + 3)}{2}$$

$$x = 3 \text{ or } x = \frac{3 \cdot (\sqrt{21} + 3)}{2}$$

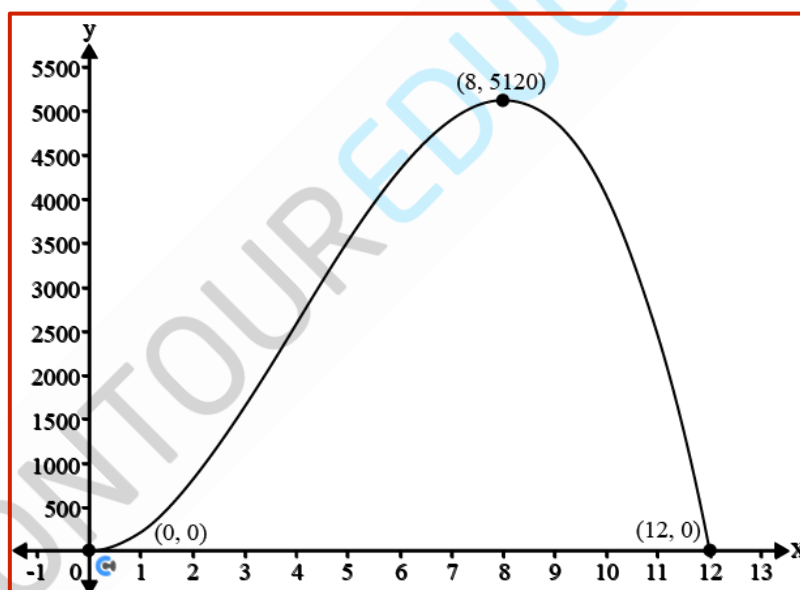
1A

$$x = 3. \text{ or } x = 11.373864$$

- f. Sketch the graph for the volume of the cuboid on the axes below. Label the endpoints with coordinates.

2 marks

D2



1A

1A

$$5120 \text{ m}^3 \text{ when } x = 8 \text{ m}$$

- g. Hence, state the maximum volume

2 marks

1M shape, 1M endpoints labelled with open circles

D1

$$\text{Max volume of } 5120 \text{ m}^3 \text{ (1A) when } x = 8 \text{ metres (1A).}$$

Question 3 (8 marks)

The table shows the results of an experiment in which the air resistance ( $R$ ) of a new car was measured for different speeds ( $S$ ).

Speed (km/hr)	Resistance (kN)
0	0
5	1
10	5
15	10
20	18
25	28
30	41
35	55
40	72
45	91
50	113
55	136
60	162
65	190
70	221
75	253

- a. Using a linear model,  $R = aS + b$  (where  $a$  and  $b$  are constants) and the results for speeds 0 km/hr and 50 km/hr, find appropriate values for  $a$  and  $b$ . 2 marks

Learning Objective [1.1.1] Solve and Graph Linear Equations and Inequalities. D2

Using the data points (0,0) & (50,113)  
(0,0) gives  $b = 0$ . 1A  
At (50,113):  
 $R = aS$   
 $113 = a(50)$   
 $a = \frac{113}{50}$  1A  
 $= 2.26$

- b. Using a quadratic model,  $R = cS^2 + d$  (where  $c$  and  $d$  are constants) and the results for speeds 0 km/hr and 50 km/hr, find appropriate values for  $c$  and  $d$ . **D2** 2 marks

Learning Objective [1.4.3] Apply family of functions to find an unknown of a function.

$(0,0)$ gives $d = 0$ . At $(50,113)$ : $R = cS^2$ $113 = c(50)^2$ Solving gives: $c = \frac{113}{50^2} = \frac{113}{2500} = 0.0452$	1A	1A
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- c. Using a cubic model,  $R = eS^3 + f$  (where  $e$  and  $f$  are constants) and the results for speeds 0 km/hr and 50 km/hr, find appropriate values for  $e$  and  $f$ . 2 marks

**D2**

Learning Objective [2.3.2] Figure out possible rule of a graph.

$R = eS^3 + f$ $(0,0)$ gives $f = 0$ . At $(50,113)$ : $113 = e(50)^3$ $e = \frac{113}{50^3}$ $= \frac{113}{125000}$ $= 0.000904$	1A	1A
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- d. Using points (25, 28) and (75, 253), evaluate each of the three models. Which of the models fits the data best? D2 2 marks

The three models are:

Linear:  $R = 2.26S$

Quadratic:  $R = 0.0452S^2$

Cubic:  $R = 0.000904S^3$

Linear:  $R = 2.26S$

$S = 25, R = 2.26(25) = 56.5$ , error  $28 - 56.5 = -28.5$

$S = 75, R = 2.26(75) = 169.5$ , error  $253 - 169.5 = 83.5$

Quadratic:  $R = 0.0452S^2$

$S = 25, R = 0.0452(25^2) = 28.25$ , error  $28 - 28.25 = -0.25$

$S = 75, R = 0.0452(75^2) = 254.25$ , error  $253 - 254.25 = -1.25$

Cubic:  $R = 0.000904S^3$

$S = 25, R = 0.000904(25^3) = 14.125$ , error  $28 - 14.125 = 13.875$

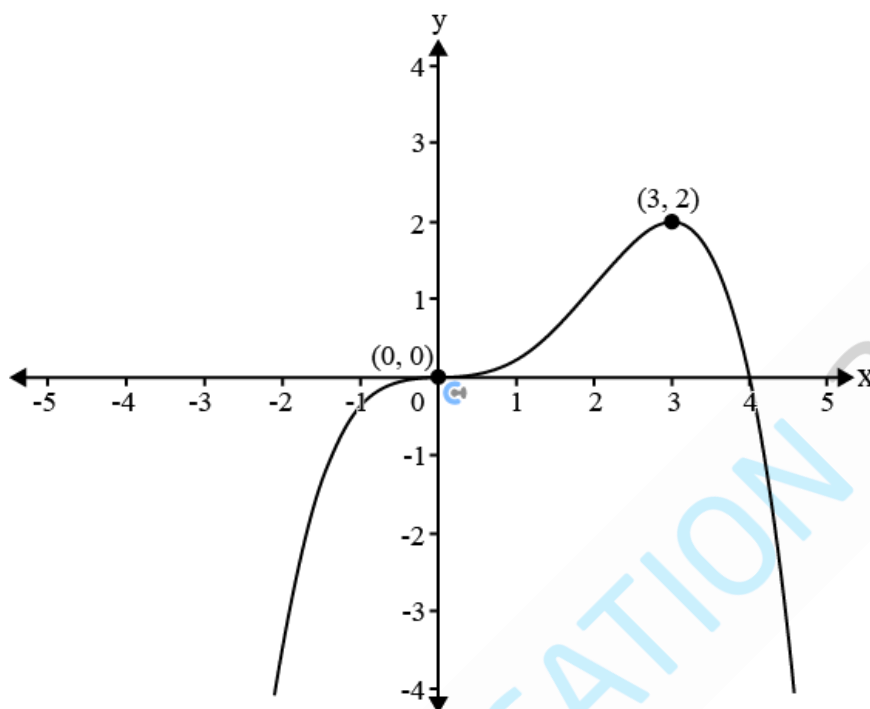
$S = 75, R = 0.000904(75^3) = 381.375$ , error  $253 - 381.375 = -128.375$

The quadratic model is by far the best. 1A

1M subbing in the points into a model

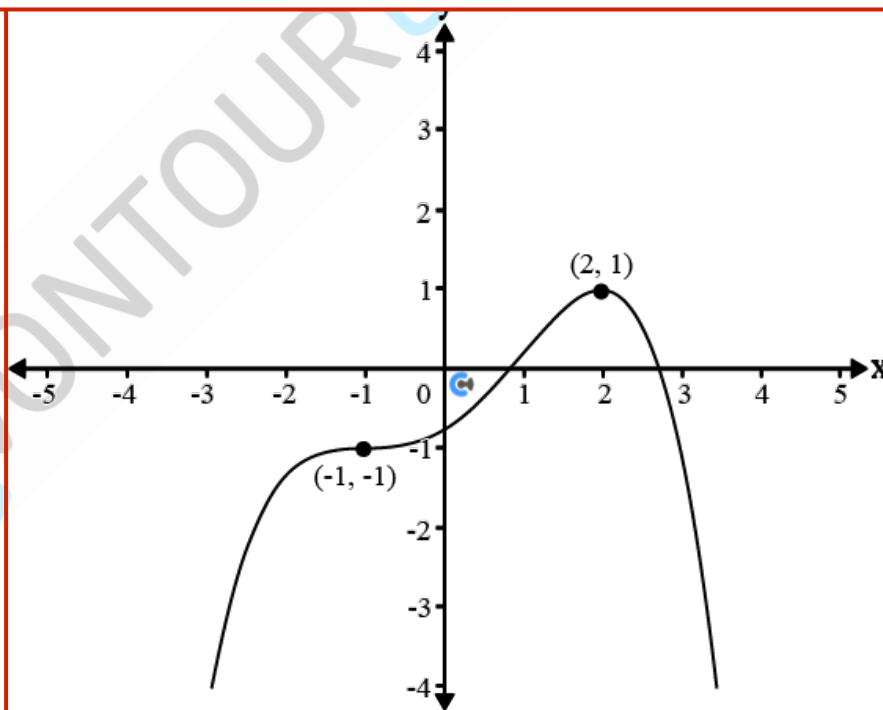
**Question 4** (9 marks)

The diagram shows the graph of  $y = f(x)$  which passes through  $(0,0)$ .



- a. Sketch the graph of  $y = f(x + 1) - 1$ , Label the coordinates of the image of the point  $(0,0)$  and the point  $(3,2)$  under this transform **D2** 3 marks

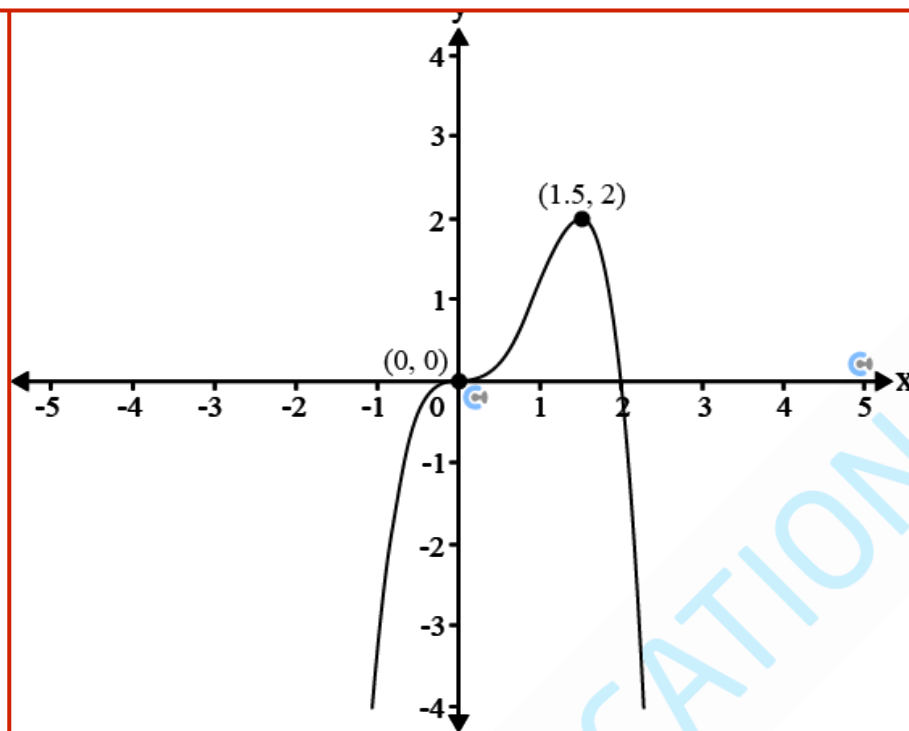
Learning Objective [2.5.3] Apply transformations of functions to find its domain, range, transformed points.



1A General shape  
1A Coordinates  
1A Correctly shifted

- b. Sketch the graph of  $y = f(2x)$ . Label the coordinates of the image of the point  $(0,0)$  and the point  $(3,2)$  under this transformation. **D2** 3 marks

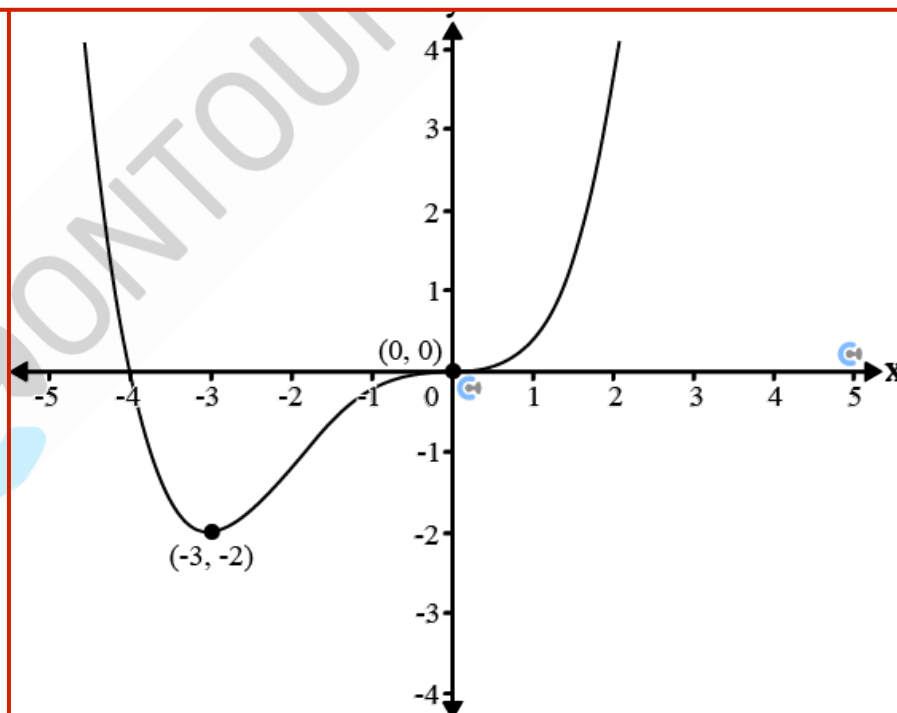
Learning Objective [2.5.3] Apply transformations of functions to find its domain, range, transformed points.



1A Compressed graph  
1A Coordinates  
1A General Shape

- c. Sketch the graph of  $y = -f(-x)$ . Label the coordinates of the image of the point  $(0,0)$  and the point  $(3,2)$  under this transformation. **D2** 3 marks

Learning Objective [2.5.3] Apply transformations of functions to find its domain, range, transformed points.



1A Compressed graph  
1A Coordinates  
1A General Shape

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**Question 5** (10 marks)

The revenue (in thousands of dollars) from the sale of  $x$  thousand items is given by  $R(x) = 6(2x^2 + 10x + 3)$  and the manufacturing cost (in thousands of dollars) of  $x$  thousand items is  $C(x) = x(6x^2 - x + 1)$ .

- a. State the degree of  $R(x)$  and  $C(x)$ .

1 mark

**D2** Learning Objective [1.5.1] Identify the properties of Polynomials and solve Long Division.

Revenue:  $R(x) = 6(2x^2 + 10x + 3)$ ; Cost  $C(x) = x(6x^2 - x + 1)$

a  $R(x)$  is a degree 2 polynomial and  $C(x)$  is a degree 3 polynomial.

**1A**

- b. Calculate the revenue and the cost of 1000 items sold and explain whether a profit is made.

2 marks

**D2**

If 1000 items are sold then  $x = 1$ .

$$R(1) = 6(2 + 10 + 3) \quad \text{and} \quad C(1) = 1(6 - 1 + 1)$$

$$= 90 \quad \text{1M for calculations} \quad = 6$$

The revenue is \$90 while the cost is \$6 so a profit of \$84 is made.

**1A**

- c. Show that the profit (in thousand dollars) from the sale of  $x$  thousand items is given by  $P(x) = -6x^3 + 13x^2 + 59x + 18$ .

2 marks

**D2**

The profit is Revenue – Cost.

$$\therefore P(x) = R(x) - C(x)$$

$$\text{1M} \quad = 6(2x^2 + 10x + 3) - x(6x^2 - x + 1)$$

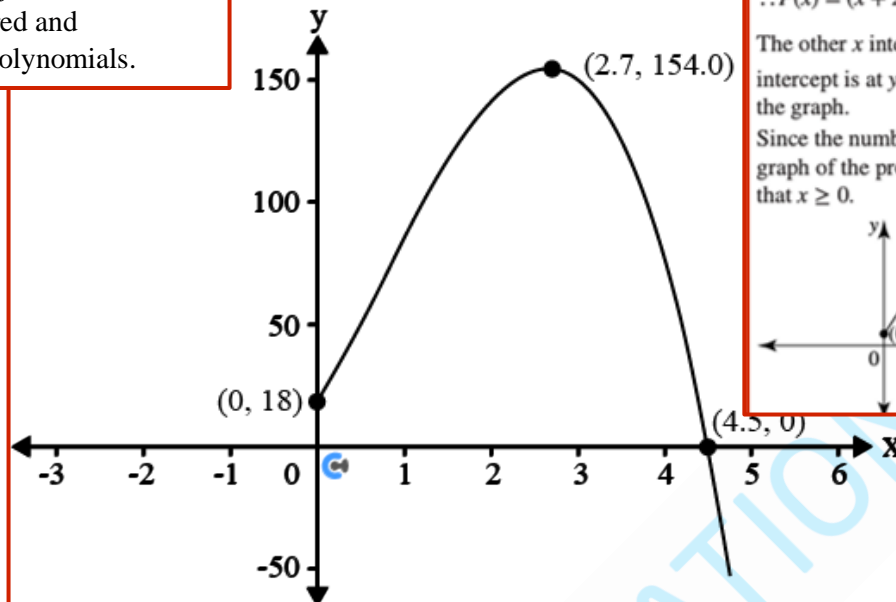
$$\text{1M} \quad = 12x^2 + 60x + 18 - 6x^3 + x^2 - x$$

$$\therefore P(x) = -6x^3 + 13x^2 + 59x + 18$$

- d. Given the graph of  $y = -6x^3 + 13x^2 + 59x + 18$  cuts the  $x$ -axis at  $x = -2$ , sketch the graph of  $y = P(x)$  for appropriate values of  $x$ . Label all axial intercepts and turning points correct to one decimal place. 3 marks

D2

Learning Objective [1.5.4]  
Graph factored and unfactored polynomials.



$x$  intercept at  $x = -2 \Rightarrow (x + 2)$  is a factor

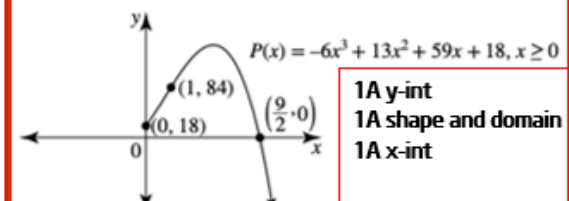
$$\therefore P(x) = -6x^3 + 13x^2 + 59x + 18$$

$$= (x + 2)(-6x^2 + 25x + 9)$$

$$\therefore P(x) = (x + 2)(-2x + 9)(3x + 1)$$

The other  $x$  intercepts are at  $x = \frac{9}{2}$ ,  $x = -\frac{1}{3}$  and the  $y$  intercept is at  $y = 18$ . From part b the point  $(1, 84)$  lies on the graph.

Since the number of items sold cannot be negative, the graph of the profit can only be drawn with the restriction that  $x \geq 0$ .



1A y-int  
1A shape and domain  
1A x-int

- e. If a loss occurs when the number of items manufactured is  $d$ , state the smallest value of  $d$ . 2 marks

D2

From the graph it can be seen that a Loss occurs for  $x > \frac{9}{2}$ .  
 If  $x = \frac{9}{2} = 4.5$ , then 4500 items are sold. The number of items must be a whole number so the least number manufactured which results in a Loss is 4501. The least value of  $d$  is 4501.

1M

1A

Do not write in this area.

**Question 6** (11 marks)

Ravi has two jars, each containing blue marbles and red marbles.

a. The first jar contains 8 blue marbles and 4 red marbles.

i. If Ravi picks one marble at random from this jar, what is the probability it is blue?

1 mark

**D1**

Learning Objective [3.1.1] Sample Space, Uncertainty and Equally Likely Events.

$$\Pr(\text{blue}) = \frac{8}{12} = \frac{2}{3}, \mathbf{1A}$$

ii. If he picks two marbles from this jar without replacement, what is the probability they are both blue?

2 marks

**D1**

Learning Objective [3.1.4] Tree Diagram and Conditional Probability.

$$\Pr(\text{both blue}) = \frac{8}{12} \times \frac{7}{11} \mathbf{1M} = \frac{56}{132} = \frac{14}{33}, \mathbf{1A}$$

iii. If he picks two marbles from this jar without replacement, what is the probability that the second marble is blue?

2 marks

**D1**

Learning Objective [3.1.4] Tree Diagram and Conditional Probability.

$$\begin{aligned} \Pr(\text{2nd is blue}) &= \Pr(\text{1st blue})\Pr(\text{2nd blue} \mid \text{1st blue}) + \Pr(\text{1st red})\Pr(\text{2nd blue} \mid \text{1st red}). \\ &= \frac{8}{12} \times \frac{7}{11} + \frac{4}{12} \times \frac{8}{11} \mathbf{1M} = \frac{2}{3}, \mathbf{1A} \end{aligned}$$

- iv. What is the probability that the first marble was red given that the second marble is blue? 2 marks

Learning Objective [3.1.4] Tree Diagram and Conditional Probability.

D1

$$\begin{aligned}\Pr(1\text{st red} \mid 2\text{nd blue}) &= \frac{\Pr(1\text{st red and } 2\text{nd blue})}{\Pr(2\text{nd blue})} \\ \Pr(1\text{st red and } 2\text{nd blue}) &= \frac{4}{12} \times \frac{8}{11} = \frac{32}{132} \\ \Pr(2\text{nd blue}) &= \frac{2}{3} \\ \Pr(1\text{st red} \mid 2\text{nd blue}) &= \frac{\frac{32}{132}}{\frac{2}{3}} \mathbf{1M} = \frac{4}{11} \cdot \mathbf{1A}\end{aligned}$$

- b. The second jar also contains 8 blue marbles, but Ravi does not know how many red marbles it holds. Suppose the second jar contains a total of  $m$  marbles.

- i. Ravi picks two marbles at random from the second jar without replacement. Write an expression in terms of  $m$  for the probability that both marbles are blue. 2 marks

Learning Objective [3.1.4] Tree Diagram and Conditional Probability.

D1

$$\Pr(\text{both blue}) = \frac{8}{m} \times \frac{7}{m-1} \mathbf{1M} = \frac{56}{m(m-1)} \cdot \mathbf{1A}$$

- ii. If the probability of selecting two blue marbles from the second jar is  $\frac{4}{15}$ , how many red marbles are in the second jar? 2 marks

Learning Objective [3.1.4] Tree Diagram and Conditional Probability.

D1

$$\begin{aligned}\frac{8}{m} \times \frac{7}{m-1} &= \frac{4}{15} \Rightarrow m = 15 \text{ or } m = -14 \\ \text{As } m > 0, m &= 15 \mathbf{1M} \\ \text{Red marbles} &= 15 - 8 = 7 \mathbf{1A}\end{aligned}$$

**Question 7** (6 marks)

A music festival organiser needs to assemble a four-member performance panel from a pool of  $n$  male performers and  $m$  female performers.

You may leave your answers in terms of  ${}^nC_r$  or  ${}^nP_r$ .

- a. How many different panels can be formed? Give your answers in terms of  $m$  and  $n$ . 1 mark

Learning Objective [3.4.2] Finding selections of any size.

D1

$${}^{n+m}C_4$$

1A

- b. How many panels consist of exactly two male performers and two female performers? 1 mark

D1

Give your answers in terms of  $m$  and  $n$ .

Learning Objective [3.4.2] Finding selections of any size.

$${}^nC_2 \times {}^mC_2$$

1A

- c. If there are twice as many male performers as female performers, and the number of gender-balanced panels is less than 720, what are the possible values of  $n$ ? 4 marks

D3

Learning Objective [3.3.2] - Find number of permutations and combinations with restrictions/composite.

$$\begin{aligned}
 n &= 2m \\
 {}^{2m}C_2 \times {}^mC_2 &< 720 \\
 \frac{(2m)!}{2!(2m-2)!} \times \frac{m!}{2!(m-2)!} &< 720 \quad 1M \\
 \frac{(2m)!}{(2m-2)!} \times \frac{m!}{(m-2)!} &< 2880 \\
 \text{Note } \frac{(2m)!}{(2m-2)!} &= (2m)(2m-1), \frac{m!}{(m-2)!} = m(m-1). \\
 (2m)(2m-1)m(m-1) &< 2880. \quad 1M \\
 -4.82 < m &< 5.57 \quad 1M \\
 -9.64 < 2m &< 11.14 \\
 \text{As } n \text{ is a natural number, hence} \\
 n &= 2, 4, 6, 8, 10. \quad 1A
 \end{aligned}$$