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Write your **student number** in the boxes above.

Letter

Mathematical Methods $\frac{1}{2}$

Examination 2 (Tech-Active)

Question and Answer Book

VCE Examination (Term 1 Mock) – April 2025

- Reading time is **15 minutes**.
- Writing time is **2 hours**.

Materials Supplied

- Question and Answer Book of 21 pages.
- Multiple-Choice Answer Sheet.

Instructions

- Follow the instructions on your Multiple-Choice Answer Sheet.
- At the end of the examination, place your Multiple-Choice Answer Sheet inside the front cover of this book.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents

	pages
Section A (2 questions, 20 marks)	2–7
Section B (7 questions, 60 marks)	8–21

Student's Full Name: _____

Student's Email: _____

Tutor's Name: _____

Marks (Tutor Only): _____

Section A

Instructions

- Answer **all** questions in pencil on the Multiple-Choice Answer Sheet.
- Choose the response that is **correct** or that **best answers** the question.
- A correct answer scores 1; an incorrect answer scores 0.
- Marks will **not** be deducted for incorrect answers.
- No marks will be given if more than one answer is completed for any question.
- Unless otherwise indicated, the diagrams in this book are not drawn to scale.

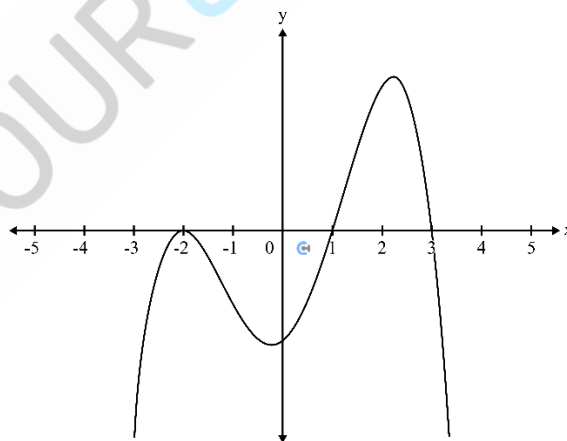
Question 1

The gradient of a line perpendicular to the line which passes through $(-2, 0)$ and $(0, -4)$ is:

- A. $\frac{1}{2}$
- B. -2
- C. $-\frac{1}{2}$
- D. 2

Question 2

The diagram below shows part of the graph of a polynomial function.



A possible rule for this function is:

- A. $y = (x + 2)^2(x - 1)(x - 3)$
- B. $y = (x + 2)^2(x - 1)(3 - x)$
- C. $y = -(x - 2)^2(x - 1)(3 - x)$
- D. $y = -(x + 2)(x - 1)(x - 3)$

Question 3

The largest value of a such that the function $f: (-\infty, a] \rightarrow \mathbb{R}, f(x) = x^2 + 3x - 10$ is not many-to-one is equal to

- A. -5
- B. -1.5
- C. 0
- D. 2

Question 4

The graph of $y = \frac{1}{(x+3)^2} + 4$ has a horizontal asymptote with the equation:

- A. $y = 4$
- B. $y = 3$
- C. $x = -2$
- D. $x = -3$

Question 5

If $f(x-1) = x^2 - 2x + 3$, then $f(x)$ is equal to:

- A. $x^2 - 2$
- B. $x^2 + 2$
- C. $x^2 - 2x + 2$
- D. $x^2 - 4x + 6$

Question 6

The graph of a function f is obtained from the graph of the function g with the rule $g(x) = \sqrt{2x-5}$ via a reflection in the x -axis followed by a dilation from the y -axis by a factor of $\frac{1}{2}$.

Which one of the following is the rule for the function f ?

- A. $f(x) = \sqrt{5-4x}$
- B. $f(x) = -\sqrt{x-5}$
- C. $f(x) = \sqrt{x+5}$
- D. $f(x) = -\sqrt{4x-5}$

Question 7

The point $A(3, 2)$ lies on the graph of the function f . A transformation maps the graph of f to the graph of g , where $g(x) = \frac{1}{2}f(x - 1)$. The same transformation maps point A to point P .

The coordinates of the point P are:

- A. $(2, 1)$
- B. $(2, 4)$
- C. $(4, 1)$
- D. $(4, 2)$

Question 8

The domain and the range for the relation with the equation $2 - y = \frac{3}{(x-1)^2}$ are given by:

- A. $\{x: x \in \mathbb{R} \setminus \{1\}\}$ and $\{y: y > 2\}$.
- B. $\{x: x \in \mathbb{R} \setminus \{1\}\}$ and $\{y: y < 2\}$.
- C. $\{x: x \in \mathbb{R} \setminus \{1\}\}$ and $\{y: y < -2\}$.
- D. $\{x: x \in \mathbb{R} \setminus \{1\}\}$ and $\{y: y \in \mathbb{R} \setminus \{2\}\}$.

Question 9

The graph of a cubic function of the form $y = a(x + h)^3 + k$ has a stationary point of inflection at $(-2, 2)$ and intercepts the y -axis at 10. The equation of the function is:

- A. $y = (x - 2)^3 + 2$
- B. $y = 2(x + 2)^3 + 10$
- C. $y = (x + 2)^3 + 10$
- D. $y = (x + 2)^3 + 2$

Question 10

A function has the rule, $f(x) = 2x^{\frac{1}{3}} + 8, x \in \mathbb{R}$. The rule for the inverse function is:

- A. $f^{-1}(x) = \frac{2}{x} - 8$
- B. $f^{-1}(x) = \frac{1}{2x^3 - 8}$
- C. $f^{-1}(x) = \left(\frac{x-8}{2}\right)^3$
- D. $f^{-1}(x) = 2x^3 - 8$

Question 11

Which of the following functions is not one-to-one?

- A. $f(x) = 25 - x^2, x < 0$
- B. $f(x) = \frac{1}{x^2} + 25$
- C. $f(x) = 4\sqrt{x}$
- D. $f(x) = -\frac{5}{x}$

Question 12

If $f(x) = \begin{cases} -3x + 5 & x \geq -2 \\ x + 5 & x < -2 \end{cases}$ then the range of f is:

- A. $(-\infty, 1]$
- B. $(-\infty, 11)$
- C. $(-\infty, 11]$
- D. $(-5, \infty) \cup (-\infty, 5]$

Question 13

If $4(x - 1)^2 = a(x + 1)^2 + bx$ for all real values of x , the values of a and b are:

- A. $a = 4, b = 16$
- B. $a = 4, b = -16$
- C. $a = 4, b = 4$
- D. $a = 4, b = -6$

Question 14

The angle, in degrees, between the lines $y = 2x + \frac{5}{3}$ and $y = 3x - \frac{10}{27}$ is closest to:

- A. 11.31
- B. 0.20
- C. 8.13
- D. 0.14

Question 15

Let $p(x) = x^3 - ax + 1$ where $a \in \mathbb{R}$. If the remainder of $p(x)$ when divided by $ax + 8$ is 1, then the value of a is:

- A. 3
- B. 4
- C. 8
- D. -2

Question 16

Three-letter words are to be made by arranging the letters of the word **METHODS**. What is the probability that a randomly chosen arrangement begins with a vowel?

- A. $\frac{2}{7}$
- B. $\frac{3}{10}$
- C. $\frac{1}{3}$
- D. $\frac{4}{11}$

Question 17

Suppose that for three events A , B and C , $\Pr(A | B) = 1$ and $\Pr(C | B) = \Pr(C)$. Which of the following is true? **HINT:** $X \subseteq Y$ means X is part of Y .

- A. $\Pr(A) = 1$.
- B. $C \subseteq B$.
- C. B and C are independent events and $B \subseteq A$.
- D. B and C are not independent events and $B \subseteq A$.

The following information applies to the two questions that follow:

The probability that Zoe bakes cookies on Friday is 0.55. If she bakes on Friday, the probability that she bakes again on Saturday is 0.75. If she does not bake on Friday, the probability that she bakes on Saturday is 0.25.

Question 18

The probability that Zoe bakes on **both** Friday and Saturday is closest to

- A. 0.41
- B. 0.20
- C. 0.15
- D. 0.55

Question 19

The probability that Zoe bakes on **Saturday** is closest to

- A. 0.57
- B. 0.48
- C. 0.53
- D. 0.34

Question 20

If the solutions of $x^2 + bx + 6 = 0$ are integers, the possible values of b are:

- A. 5 and 7.
- B. -5 and -7 .
- C. ± 5 and ± 7 .
- D. -5 and 7.

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Section B

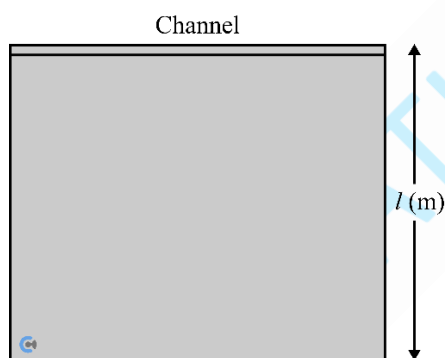
Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.

Question 1 (5 marks)

A piece of fencing 960 m long will be used to enclose three sides of a rectangular field. The fourth side has a straight channel along it.

Let l be the length of the field as shown. Let A be the area of the field.



All measurements are in metres.

- a. Express A as a function of l .

2 marks

- b. What is a suitable domain of the function A ?

1 mark

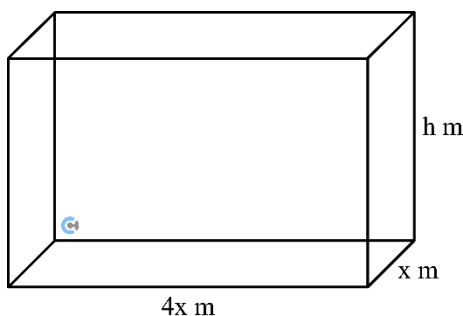
- c. Determine the range of A . Also state the value of l for which the maximum area occurs. 2 marks

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Question 2 (11 marks)

A cuboid (rectangular prism) has dimensions x metres, h metres and $4x$ metres as shown on the diagram. The cuboid is made of 240 m of wire.



- a. Find h in terms of x .

2 marks

- b. Find the volume, $V \text{ m}^3$, of the cuboid in terms of x .

1 mark

- c. Find V when $x = 11$.

1 mark

- d. Find the possible values of x for the cuboid to exist.

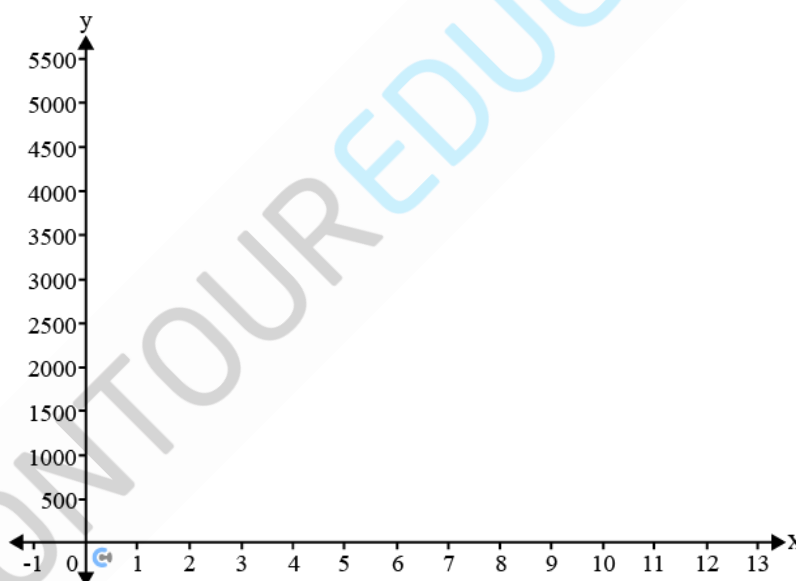
2 marks

- e. Find the possible values of x when $V = 1620$, correct to two decimal places.

1 mark

- f. Sketch the graph for the volume of the cuboid on the axes below. Label the endpoints with coordinates.

2 marks



- g. Hence, state the maximum volume of the cuboid and the value of x for which it occurs.

2 marks

Question 3 (8 marks)

The table shows the results of an experiment in which the air resistance (R) of a new car was measured for different speeds (S).

Speed (km/hr)	Resistance (kN)
0	0
5	1
10	5
15	10
20	18
25	28
30	41
35	55
40	72
45	91
50	113
55	136
60	162
65	190
70	221
75	253

- a. Using a linear model, $R = aS + b$ (where a and b are constants) and the results for speeds 0 km/hr and 50 km/hr , find appropriate values for a and b .

2 marks

- b.** Using a quadratic model, $R = cS^2 + d$ (where c and d are constants) and the results for speeds 0 km/hr and 50 km/hr , find appropriate values for c and d . 2 marks

- c.** Using a cubic model, $R = eS^3 + f$ (where e and f are constants) and the results for speeds 0 km/hr and 50 km/hr , find appropriate values for e and f . 2 marks

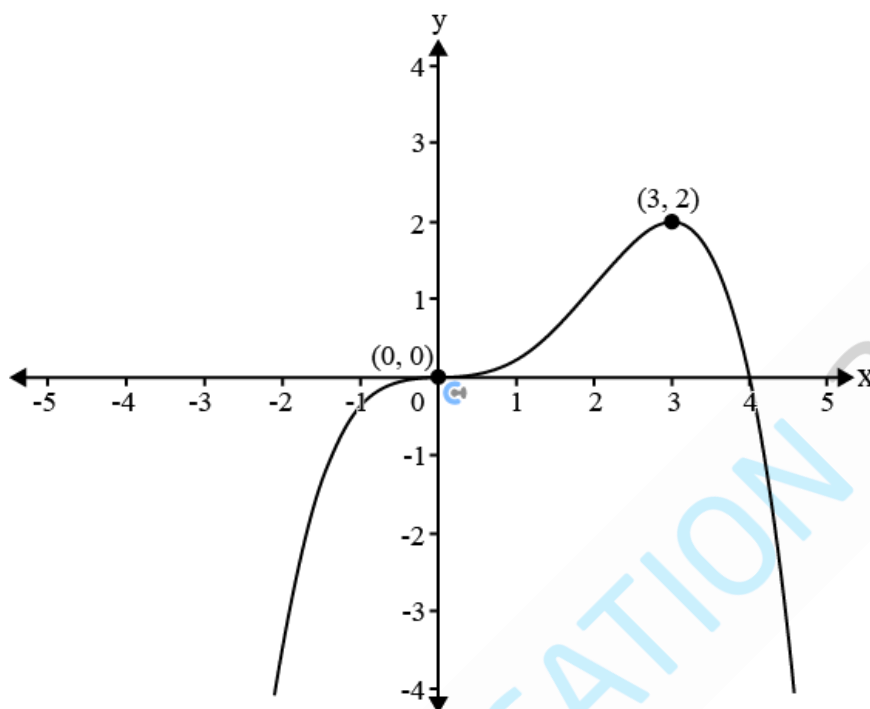
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- d. Using points (25, 28) and (75, 253), evaluate each of the three models. Which of the models fits the data best? 2 marks

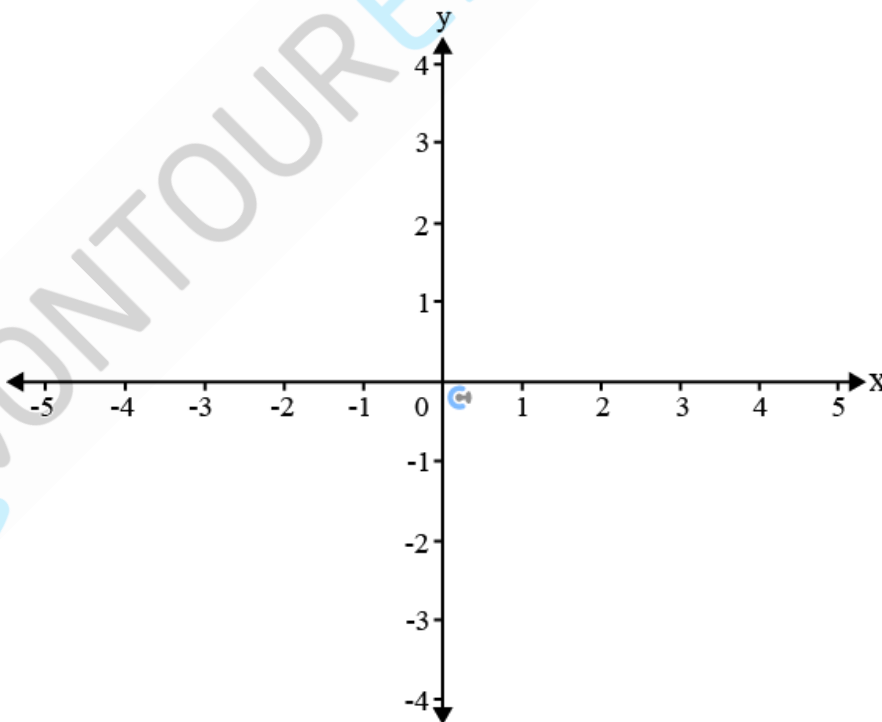
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Question 4 (9 marks)

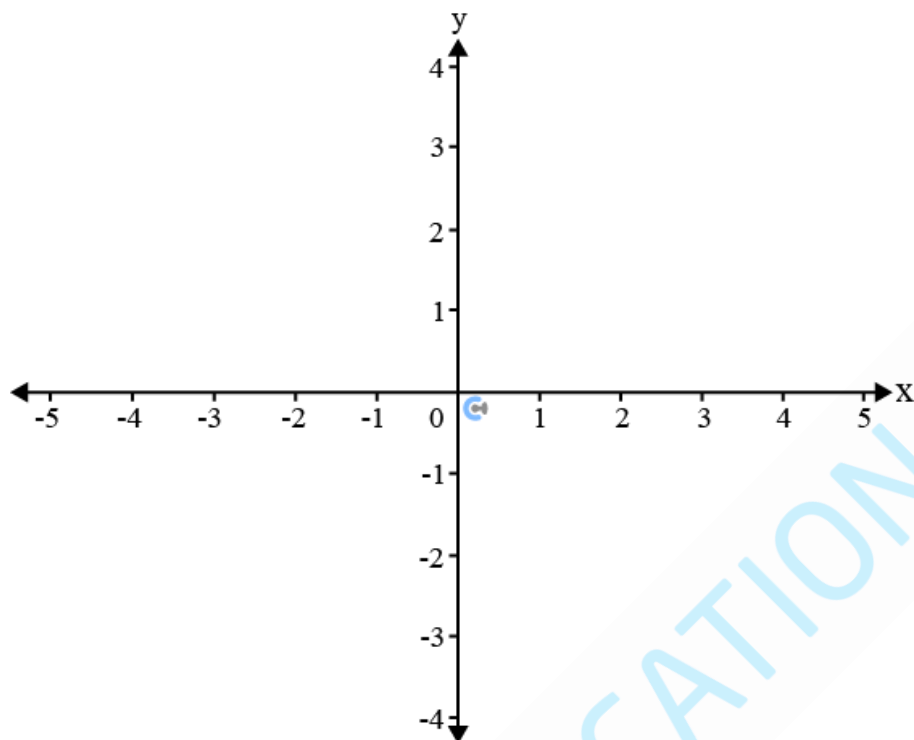
The diagram shows the graph of $y = f(x)$ which passes through $(0,0)$.



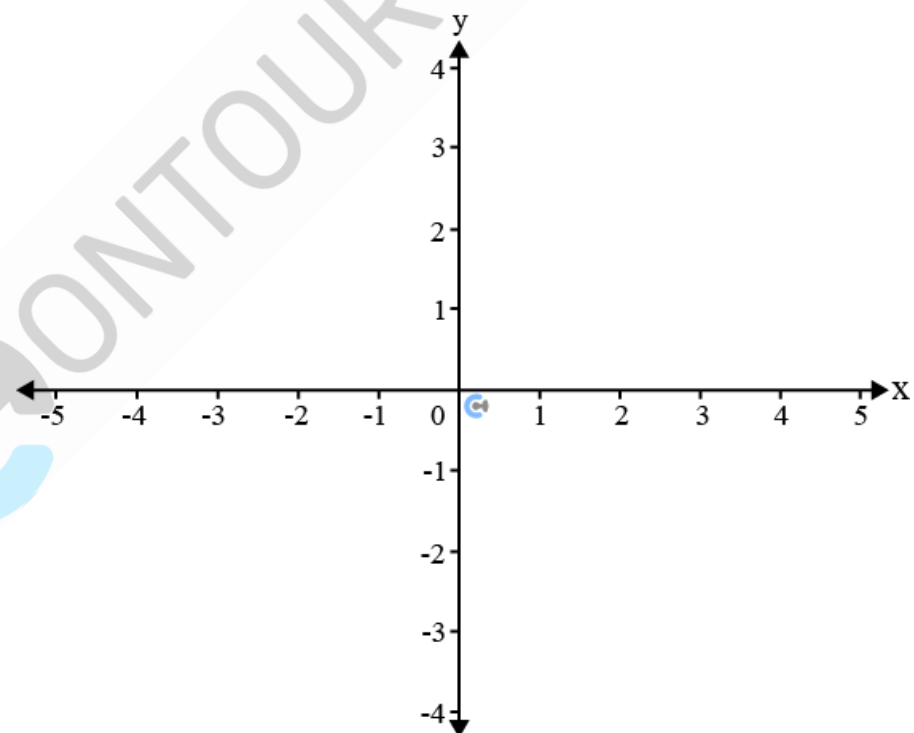
- a. Sketch the graph of $y = f(x + 1) - 1$, Label the coordinates of the image of the point $(0,0)$ and the point $(3,2)$ under this transformation. 3 marks



- b. Sketch the graph of $y = f(2x)$. Label the coordinates of the image of the point $(0,0)$ and the point $(3,2)$ under this transformation. 3 marks



- c. Sketch the graph of $y = -f(-x)$. Label the coordinates of the image of the point $(0,0)$ and the point $(3,2)$ under this transformation. 3 marks



Question 5 (10 marks)

The revenue (in thousands of dollars) from the sale of x thousand items is given by $R(x) = 6(2x^2 + 10x + 3)$ and the manufacturing cost (in thousands of dollars) of x thousand items is $C(x) = x(6x^2 - x + 1)$.

- a. State the degree of $R(x)$ and $C(x)$.

1 mark

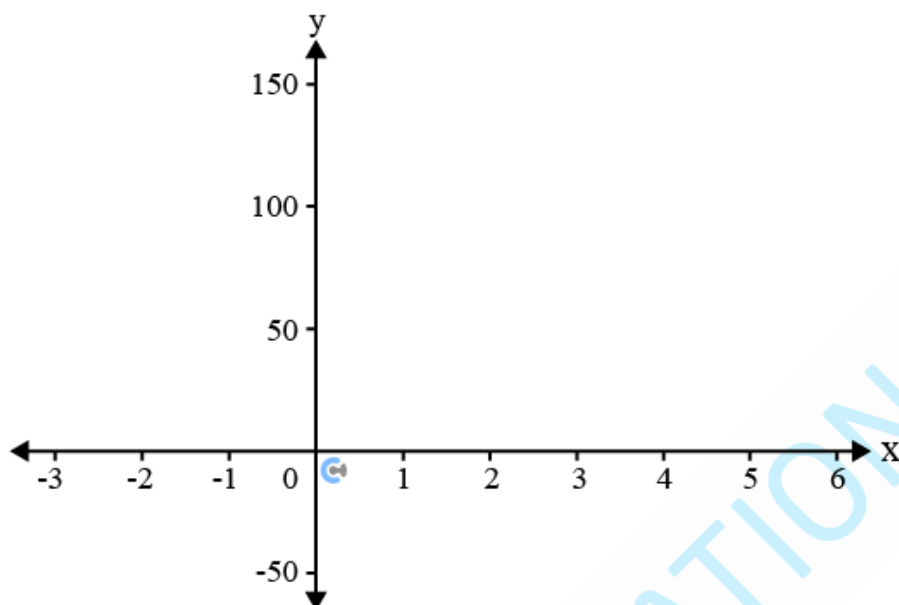
- b. Calculate the revenue and the cost of 1000 items sold and explain whether a profit is made.

2 marks

- c. Show that the profit (in thousand dollars) from the sale of x thousand items is given by $P(x) = -6x^3 + 13x^2 + 59x + 18$.

2 marks

- d. Given the graph of $y = -6x^3 + 13x^2 + 59x + 18$ cuts the x -axis at $x = -2$, sketch the graph of $y = P(x)$ for appropriate values of x . Label all axial intercepts and turning points correct to one decimal place. 3 marks



- e. If a loss occurs when the number of items manufactured is d , state the smallest value of d . 2 marks

Question 6 (11 marks)

Ravi has two jars, each containing blue marbles and red marbles.

a. The first jar contains 8 blue marbles and 4 red marbles.

- i. If Ravi picks one marble at random from this jar, what is the probability it is blue? 1 mark

- ii. If he picks two marbles from this jar without replacement, what is the probability they are both blue? 2 marks

- iii. If he picks two marbles from this jar without replacement, what is the probability that the second marble is blue? 2 marks

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- iv. What is the probability that the first marble was red given that the second marble is blue? 2 marks

- b. The second jar also contains 8 blue marbles, but Ravi does not know how many red marbles it holds. Suppose the second jar contains a total of m marbles.

- i. Ravi picks two marbles at random from the second jar without replacement. Write an expression in terms of m for the probability that both marbles are blue. 2 marks

- ii. If the probability of selecting two blue marbles from the second jar is $\frac{4}{15}$, how many red marbles are in the second jar? 2 marks

Question 7 (6 marks)

A music festival organiser needs to assemble a four-member performance panel from a pool of n male performers and m female performers.

You may leave your answers in terms of nC_r or nP_r .

- a. How many different panels can be formed? Give your answers in terms of m and n . 1 mark

- b. How many panels consist of exactly two male performers and two female performers? 1 mark
Give your answers in terms of m and n .

- c. If there are twice as many male performers as female performers, and the number of gender-balanced panels is less than 720, what are the possible values of n ? 4 marks
