Write your **student number** in the boxes above. **Letter**

Mathematical Methods ½

Examination 1 (Tech-Free)

Question and Answer Book - SOLUTIONS

VCE Examination (Term 1 Mock) – April 2025

- Reading time is **15 minutes**.
- Writing time is **1 hour**.

Materials Supplied

- Question and Answer Book of 13 pages.

Instructions

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents

Student's Full Name: ______________________

Student's Email: ______________________

Tutor's Name: ______________________

Marks (Tutor Only): __________

Section A

Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.

Question 1 (3 marks)

Given that $(x - 2)$ is a factor of the polynomial $p(x) = 12x^3 - 19x^2 - 13x + 6$, solve the equation $12x^4 - 19x^3 - 13x^2 + 6x = 0$. **D2**

1M Learning Objective [1.5.1] Identify the properties of Polynomials and solve Long Division.

1M Learning Objective [1.3.1] Find factorised form of quadratics.

1M Learning Objective [1.5.3] Find factored form of polynomials.

We have $f(x) = 12x^4 - 19x^3 - 13x^2 + 6x = xp(x)$.
$p(x) = (x - 2)(12x^2 + 5x - 3)$. (1M, long division or compare coefficients to factor)
Solve $12x^2 + 5x - 3 = 0 \implies (3x - 1)(4x + 3) = 0 \implies x = \frac{1}{3}, -\frac{3}{4}$. (1M).
Thus, all solutions to $f(x) = 0$ are $x = -\frac{3}{4}, 0, \frac{1}{3}, 2$. (1A)

Question 2 (3 marks)

Learning Objective [1.1.5] Find the unknown value for systems of linear equations.

Consider the following set of simultaneous equations:

D3

$$-2x + ky = 4$$
$$(1-k)x + y = 2$$

Where k is real constant.

Find the value of k, such that the set of simultaneous equations has infinitely many solutions.

Gradients are equal if $\frac{-2}{k} = 1 - k \implies k - k^2 = -2 \implies k^2 - k - 2 = 0$. (1M for a quadratic)

$k^2 - k - 2 = 0 \implies (k-2)(k+1) = 0 \implies k = 2, -1$. (1M)

Equate y-intercepts: $\frac{4}{k} = 2 \implies k = 2$.

Thus infinite solutions if $k = 2$. (1A)

Question 3 (3 marks) Learning Objective [1.3.2] Find solutions and number of solutions to quadratic equations.

The quadratic curves with equations $y = k(2x^2 + 1)$ and $y = x^2 - 2x$, where k is a constant, intersect each other **exactly once**. Determine the possible values of k. **D2**

$y = k(2x^2+1)$, $y = x^2 - 2x$ $\Rightarrow$ look for intersections

$\Rightarrow k(2x^2+1) = x^2 - 2x$

$\Rightarrow 2kx^2 + k = x^2 - 2x$

$\Rightarrow 2kx^2 - x^2 + 2x + k = 0$

$\Rightarrow (2k-1)x^2 + 2x + k = 0$

If they touch, we must have repeated roots

$b^2 - 4ac = 0$

Hence $2^2 - 4 \times (2k-1) \times k = 0$

$\Rightarrow 4 - 4k(2k-1) = 0$

$\Rightarrow 1 - k(2k-1) = 0$

$\Rightarrow 1 - 2k^2 + k = 0$

$\Rightarrow 0 = 2k^2 - k - 1$

$\Rightarrow (2k+1)(k-1) = 0$

$k = 1, -\frac{1}{2}$

- 1M – Correctly equating two curves into one single quadratic equation
- 1M – Correct use of discriminant in terms of k
- 1M – Correct answers

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Question 4 (5 marks) D1

A and B are events such that $\Pr(A) = 0.6$, $\Pr(A' \cap B) = 0.2$ and $\Pr(A \cap B') = 0.1$.

a. Find:

3M Learning Objective [3.1.2] Venn Diagrams and Karnaugh Tables.

1M Learning Objective [3.1.4] Tree Diagram and Conditional Probability.

1M Learning Objective [3.1.3] Independent and Mutually Exclusive Events.

i. $\Pr(B)$. 1 mark

D1

$$\Pr(B) = 0.7$$

3M Learning Objective [3.1.2] Venn Diagrams and Karnaugh Tables.

1M Learning Objective [3.1.4] Tree Diagram and Conditional Probability.

1M Learning Objective [3.1.3] Independent and Mutually Exclusive Events.

ii. $\Pr(A \cap B)$. 1 mark

D1

$$\Pr(A \cap B) = 0.5$$

3M Learning Objective [3.1.2] Venn Diagrams and Karnaugh Tables.

1M Learning Objective [3.1.4] Tree Diagram and Conditional Probability.

1M Learning Objective [3.1.3] Independent and Mutually Exclusive Events.

iii. $\Pr(A \cup B')$. 1 mark

D1

$$\Pr(A \cup B') = 0.6 + 0.2 = 0.8 \text{ (1A)}.$$

3M Learning Objective [3.1.2] Venn Diagrams and Karnaugh Tables.

1M Learning Objective [3.1.4] Tree Diagram and Conditional Probability.

1M Learning Objective [3.1.3] Independent and Mutually Exclusive Events.

iv. $\Pr(A' \mid B')$. 1 mark

D1

$$\Pr(A' \mid B') = \frac{\Pr(A' \cap B')}{\Pr(B')} = \frac{0.2}{0.3} = \frac{2}{3}$$

Do not write in this area.

b. **Hence,** explain whether events A and B are mutually exclusive. **D1** 1 mark

3M Learning Objective [3.1.2] Venn Diagrams and Karnaugh Tables.

1M Learning Objective [3.1.4] Tree Diagram and Conditional Probability.

1M Learning Objective [3.1.3] Independent and Mutually Exclusive Events.

Events A and B are not mutually exclusive as $\Pr(A \cap B) = 0.5 \neq 0$

Do not write in this area.

Question 5 (6 marks)

A group of five co-workers go to a live comedy show. There are three members from Team A and two members from Team B.

1M Learning Objective [3.4.1] Applying pascals triangle and symmetrical properties of Combinations.

a. If all five co-workers must sit in a row, how many possible seating arrangements exist? 1 mark

4M Learning Objective [3.4.2] - Finding selections of any size.

D1

$5! = 120$ 1A

b. If the three Team A members must sit together, how many possible seating arrangements exist? 1 mark

1M Learning Objective [3.4.1] Applying pascals triangle and symmetrical properties of Combinations.

D1

4M Learning Objective [3.4.2] - Finding selections of any size.

$3!\,3! = 36$ 1A

On one occasion, only four seats remain for the comedy show that the co-workers wish to attend.

c. If the three Team A members and one Team B member attend, how many possible combinations of co-workers exist? 1 mark

D1

1M Learning Objective [3.4.1] Applying pascals triangle and symmetrical properties of Combinations.

4M Learning Objective [3.4.2] - Finding selections of any size.

$\binom{3}{3}\binom{2}{1} = 2$ 1A

d. If at least two Team A members attend, how many possible combinations of co-workers exist? 2 marks

1M Learning Objective [3.4.1] Applying pascals triangle and symmetrical properties of Combinations.

D1

4M Learning Objective [3.4.2] - Finding selections of any size.

1M ${}^3C_3 \times {}^2C_1 + {}^3C_2 \times {}^2C_2 = 2 + 3$
$= 5$ 1A

e. If the group is selected randomly, what is the probability that it consists of exactly two members from Team A and two members from Team B? **D1** 1 mark

1M Learning Objective [3.4.1] Applying pascals triangle and symmetrical properties of Combinations.

4M Learning Objective [3.4.2] - Finding selections of any size.

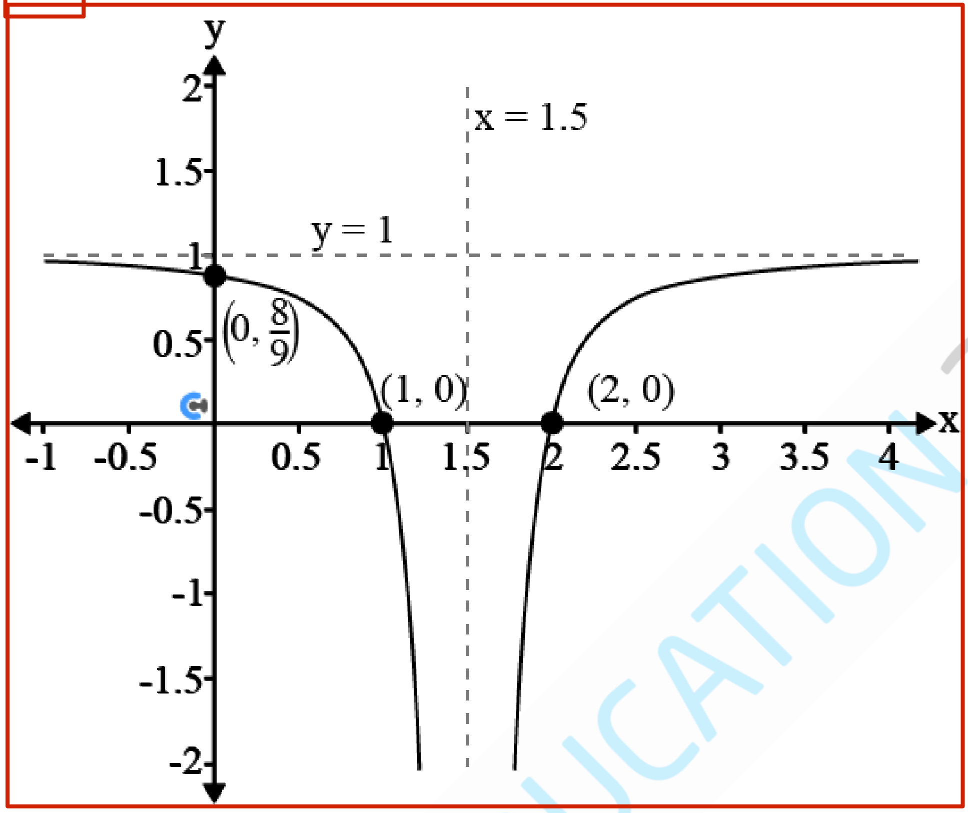
$$\frac{\text{Ways to choose 2 from Team A and 2 from Team B}}{\text{Total ways to choose any 4 members}} = \frac{\binom{3}{2}\binom{2}{2}}{\binom{5}{4}} \ \textbf{1}\boldsymbol{M}$$

$$= \frac{3 * 1}{5} = \frac{3}{5} \ \textbf{1}\boldsymbol{A}$$

Question 6 (3 marks) 3M Learning Objective [2.1.2] Sketch and find the rule of Truncus Functions.

Sketch $y = 1 - \frac{1}{(3-2x)^2}$ and label the asymptotes and the intercepts with the axes.

D2



1A shape
1A asymptotes
1A intercepts

$3 - 2x = 0 \implies x = 1.5$ is an asymptote. Horizontal asymptote $y = 1$.

When $x = 0$ $y = 1 - \frac{1}{3^2} = \frac{8}{9}$.

$1 - \frac{1}{(3-2x)^2} = 0 \implies (3-2x)^2 = 1$

$3 - 2x = \pm 1 \implies x = 1, 2.$

1A for shape, 1A asymptotes and 1A intercepts.

Question 7 (7 marks)

The graph of $f(x) = -\sqrt{9(x-2)} + 4$ undergoes the following transformations in the order given by:

$$g(x) = f\left(\frac{x}{4} - 3\right)$$

$$h(x) = -g(x) + 5$$

a. Describe a set of transformations given by $g(x) = f\left(\frac{x}{4} - 3\right)$ that maps the graph of f to the graph of g. 2 marks

D2

2M Learning Objective [2.4.2] Find transformed functions.

4M Learning Objective [2.4.3] Find transformations from transformed function (Reverse Engineering).

A dilation by factor 4 from the y-axis (1A) followed by a translation 12 units right (1A).
OR
A translation 3 units right (1A) followed by a dilation by factor 4 from the y-axis (1A).

b. Describe the transformations given by $h(x) = -g(x) + 5$ that maps the graph of g to the graph of h. 2 marks

D2

4M Learning Objective [2.4.3] Find transformations from transformed function (Reverse Engineering).

2M Learning Objective [2.4.2] Find transformed functions.

A reflection in the x-axis **(1A)** followed by a translation of 5 units up **(1A)**

c. Show that the image function is given by: 3 marks

D2

$$h(x) = \frac{3}{2}\sqrt{x - 20} + 1$$

4M Learning Objective [2.4.3] Find transformations from transformed function (Reverse Engineering).

2M Learning Objective [2.4.2] Find transformed functions.

$g(x) = -\sqrt{9\left(\frac{x}{4} - 3 - 2\right)} + 4 = -\sqrt{9\left(\frac{x}{4} - 5\right)} + 4.$ (1M)

$h(x) = \sqrt{9\left(\frac{x}{4} - 5\right)} - 4 + 5 = \sqrt{9\left(\frac{x}{4} - 5\right)} + 1.$ (1M)

Now factoring we get

$$h(x) = 3\sqrt{\frac{x}{4} - 5} + 1 = 3\sqrt{\frac{1}{4}(x - 20)} + 1$$
$$= \frac{3}{2}\sqrt{x - 20} + 1 \quad \text{(1M)}$$

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Question 8 (2 marks) **D2**

Consider the function $f: \mathbb{R} \to \mathbb{R}, f(x) = 2x^2 - 3$. State the domain and range of f, and state why f does not have an inverse function, justifying your answer. Also, state a possible maximal domain so f **does** have an inverse function.

1M Learning Objective [2.2.1] Find domain and range of functions.

1M Learning Objective [2.3.1] Restrict domain such that the inverse function exists.

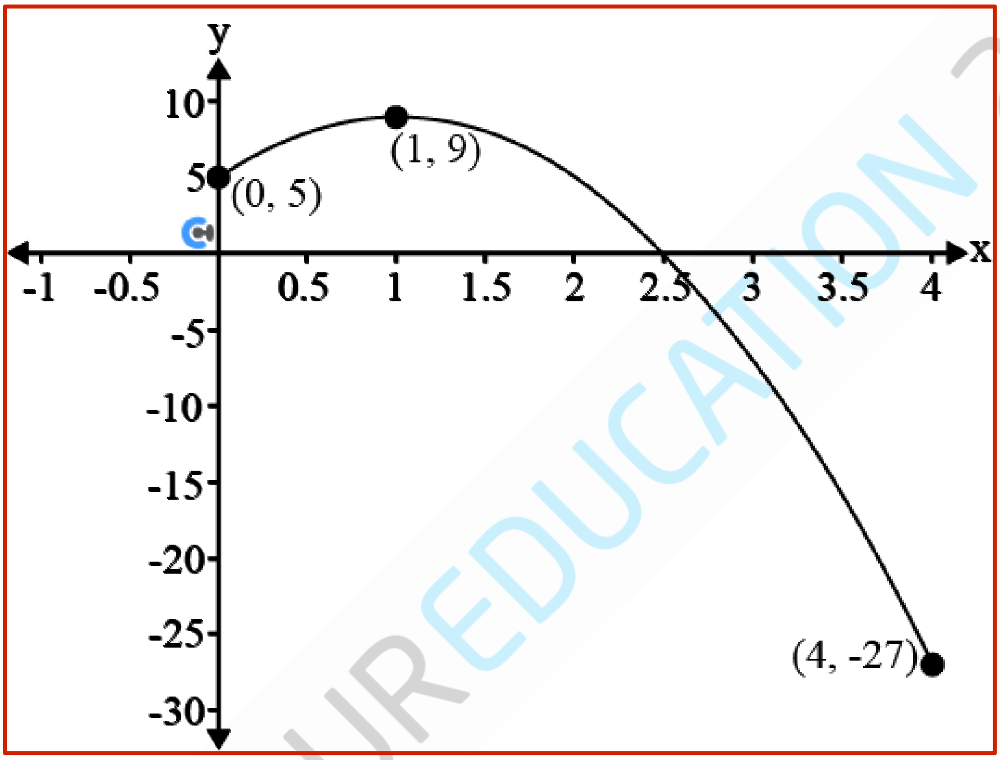
Many-to-one function (or not one-to-one) since the function has two x-values for every y-value (except when x=0)
Domain *R*
Range $[-3, \infty)$
1A for all 3 above with justification

Inverse function when domain of f restricted to $[0, \infty)$ or $(-\infty, 0]$ **1A**

Question 9 (4 marks) 4M Learning Objective [1.4.2] Apply Quadratics to Model a scenario.

An object is thrown by a 5 metre tall giant from the top of a high-rise building. The trajectory can be represented by the equation $H = 8x - 4x^2 + 5, x \in [0,4]$, where H is the vertical distance from the top of the building and x is the horizontal distance from the building (both distances measured in D2 s).

Sketch the path of the object, labelling endpoints and turning point and calculate the maximum and minimum heights of the object.



$$H = 8x - 4x^2 + 5,\ x \in [0, 4]$$
$$= {}^{-}4\left(x^2 - 2x - \frac{5}{4}\right)$$
$$= {}^{-}4\left(x^2 - 2x + 1 - 1 - \frac{5}{4}\right)$$
$$= {}^{-}4\left[(x-1)^2 - \frac{9}{4}\right]$$
$$H = {}^{-}4(x-1)^2 + 9$$

The maximum turning point is (1, 9).
Therefore the object reaches a maximum height of 9 m.
At $x = 0$, $H = 5$ m
At $x = 4$, $H = {}^{-}27$ m
Therefore the object has a minimum height of 27 m below the top of the building.

Question 10 (4 marks)

Consider the function $f: D \to \mathbb{R}, f(x) = \frac{\sqrt{10-2x}}{x^2-4x}$.

a. Find the maximal domain D of the function. Express your answer using interval notation. 3 marks

4M Learning Objective [2.2.1] Find domain and range of functions. **D2**

$10 - 2x \geq 0$ **1M**
$10 \geq 2x$
$x \leq 5$

$x^2 - 4x \neq 0$ **1M**
$x(x-4) \neq 0$
$x \neq 0, x \neq 4$

Thus, $D = (-\infty, 0) \cup (0,4) \cup (4,5]$ **1A must use interval notation**

Consider $g: (4,5) \to \mathbb{R}, g(x) = \frac{\sqrt{10-2x}}{x^2-4x}$.

b. Given that g is a one-to-one function, state the range of g. 1 mark

D1 4M Learning Objective [2.2.1] Find domain and range of functions.

$\text{ran } g = (0, \infty)$. (1A)