Write your **student number** in the boxes above. **Letter**

Mathematical Methods ½

Examination 1 (Tech-Free)

Question and Answer Book

VCE Examination (Term 1 Mock) – April 2025

- Reading time is **15 minutes**.
- Writing time is **1 hour**.

Materials Supplied

- Question and Answer Book of 13 pages.

Instructions

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents

Student's Full Name: ______________________

Student's Email: ______________________

Tutor's Name: ______________________

Marks (Tutor Only): __________

CONTOUREDUCATION 2025

Section A

Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.

Question 1 (3 marks)

Given that $(x-2)$ is a factor of the polynomial $p(x) = 12x^3 - 19x^2 - 13x + 6$, solve the equation $12x^4 - 19x^3 - 13x^2 + 6x = 0$.

$12x^4 - 19x^3 - 13x^2 + 6x = 0$

$x(12x^3 - 19x^2 - 13x + 6) = 0$

$\because (x-2)$ is a factor

(1M) $\therefore x(x-2)(12x^2+5x-3) = 0$

$$
\begin{array}{r}
12x^2 + 5x - 3 \\
(x-2)\,\overline{)\,12x^3 - 19x^2 - 13x + 6} \\
-(12x^3 - 24x^2) \\
\hline
5x^2 - 13x \\
-(5x^2 - 10x) \\
\hline
-3x + 6 \\
-3x + 6 \\
\hline
0
\end{array}
$$

$3x \quad -1$

$4x \quad 3$

(1M) $\therefore x(x-2)(3x-1)(4x+3) = 0 \Rightarrow \therefore x = 0, 2, \frac{1}{3}, -\frac{3}{4}$ (1A)

1M Learning Objective [1.5.1] Identify the properties of Polynomials and solve Long Division.

1M Learning Objective [1.3.1] Find factorised form of quadratics.

1M Learning Objective [1.5.3] Find factored form of polynomials.

Do not write in this area.

Question 2 (3 marks)

Consider the following set of simultaneous equations:

$y = mx + c$

$$-2x + ky = 4 \quad \Rightarrow \therefore y = \frac{2}{k}x + \frac{4}{k}$$

$$(1 - k)x + y = 2 \quad \Rightarrow \therefore y = (k-1)x + 2$$

Where k is real constant.

Find the value of k, such that the set of simultaneous equations has infinitely many solutions.

infinite solns $\Rightarrow$ Same gradient & Same y-intercept:

$m_1 = m_2$:	$c_1 = c_2$:
$\frac{-2}{k} = \frac{1-k}{1}$	$\frac{4}{k} = \frac{2}{1}$ (1M)
(1M) $k - k^2 = -2$	$\therefore k = 2$
$k^2 - k - 2 = 0$	

$(k-2)(k+1) = 0$

$\therefore k = 2$ OR -1 $\Rightarrow$ $\therefore k = 2$ (1A)

Learning Objective [1.1.5] Find the unknown value for systems of linear equations.

Do not write in this area.

CONTOUREDUCATION 2025

Question 3 (3 marks)

The quadratic curves with equations $y = k(2x^2 + 1)$ and $y = x^2 - 2x$, where k is a constant, intersect each other **exactly once**. Determine the possible values of k.

$\hookrightarrow \therefore k(2x^2+1) = x^2 - 2x$ (1M)

$2kx^2 + k = x^2 - 2x$

$\underbrace{(2k-1)}_{a}x^2 + \underbrace{2}_{b}x + \underbrace{k}_{c} = 0$

1 soln : $\Delta = 0$

$b^2 - 4ac = 0$

$\therefore 2^2 - 4(2k-1)(k) = 0$ (1M)

$4 - 8k^2 + 4k = 0$

$8k^2 - 4k - 4 = 0$

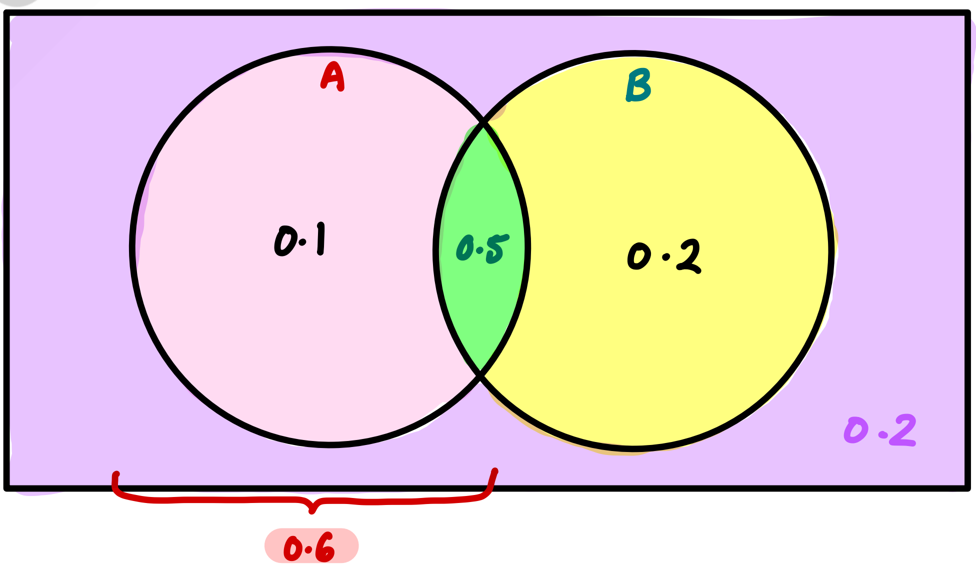
$2k^2 - k - 1 = 0$

$(2k+1)(k-1) = 0$

$\therefore k = -\frac{1}{2}$ OR 1 (1A)

Learning Objective [1.3.2] Find solutions and number of solutions to quadratic equations.

Venn Diagram:



Do not write in this area.

CONTOUREDUCATION 2025

Question 4 (5 marks)

A and B are events such that $\Pr(A) = 0.6$, $\Pr(A' \cap B) = 0.2$ and $\Pr(A \cap B') = 0.1$.

a. Find:

i. $\Pr(B)$. 1 mark

$\Pr(B) = 0.5 + 0.2 = 0.7$ // (1A)

ii. $\Pr(A \cap B)$. 1 mark

$\Pr(A \cap B) = 0.5$ // (1A)

iii. $\Pr(A \cup B')$. 1 mark

$\Pr(A \cup B') = 1 - 0.2 = 0.8$ // (1A)

iv. $\Pr(A' \mid B')$. 1 mark

$\Pr(A' \mid B') = \dfrac{\Pr(A' \cap B')}{\Pr(B')} = \dfrac{0.2}{1 - 0.7} = \dfrac{2}{3}$ // (1A)

CONTOUR EDUCATION 2025

Do not write in this area.

b. **Hence,** explain whether events A and B are mutually exclusive. 1 mark

$Pr(A \cap B) = 0$

(1A) $Pr(A \cap B) = 0.5 \neq 0$

$\therefore$ Events A and B are NOT mutually exclusive

3M Learning Objective [3.1.2] Venn Diagrams and Karnaugh Tables.

1M Learning Objective [3.1.4] Tree Diagram and Conditional Probability.

1M Learning Objective [3.1.3] Independent and Mutually Exclusive Events.

CONTOUREDUCATION 2025

Do not write in this area.

Question 5 (6 marks)

A group of five co-workers go to a live comedy show. There are three members from Team A and two members from Team B.

ORDER matters!

a. If all five co-workers must sit in a row, how many possible seating arrangements exist? 1 mark

5	4	3	2	1

$\therefore$ No. of arrangements $= 5! = 120$ (1A)

b. If the three Team A members must sit together, how many possible seating arrangements exist? 1 mark

G

P_1	P_2	P_3

$\Rightarrow 3!$

G	P_4	P_5

$3!$

$\therefore$ No. of arrangements $= 3! \cdot 3! = 36$ (1A)

On one occasion, only four seats remain for the comedy show that the co-workers wish to attend.

c. If the three Team A members and one Team B member attend, how many possible combinations of co-workers exist? 1 mark

total

$\therefore$ No. of combinations $= {}^3C_3 \cdot {}^2C_1 = 2$ (1A)

attending

d. If at least two Team A members attend, how many possible combinations of co-workers exist? 2 marks

No. of Combinations w/ ≥ 2 Team A members = No. of Combinations w/ 2 Team A members + No. of Combinations w/ 3 Team A members

$= {}^3C_2 \; {}^2C_2 + 2$ (1M)

$= 3 \cdot 1 + 2 = 5$ (1A)

Do not write in this area.

CONTOUR EDUCATION 2025

e. If the group is selected randomly, what is the probability that it consists of exactly two members from Team A and two members from Team B? 1 mark

$$\Pr(\text{2 Team A's}) = \frac{\text{No. (ways w/ 2 from Team A)}}{\text{No. (Total ways)}} = \frac{3}{\underbrace{{}^5C_4}_{\text{attending}}} \quad (1M)$$

total

$$= \frac{3}{5} \quad (1A)$$

1M Learning Objective [3.4.1] Applying pascals triangle and symmetrical properties of Combinations.

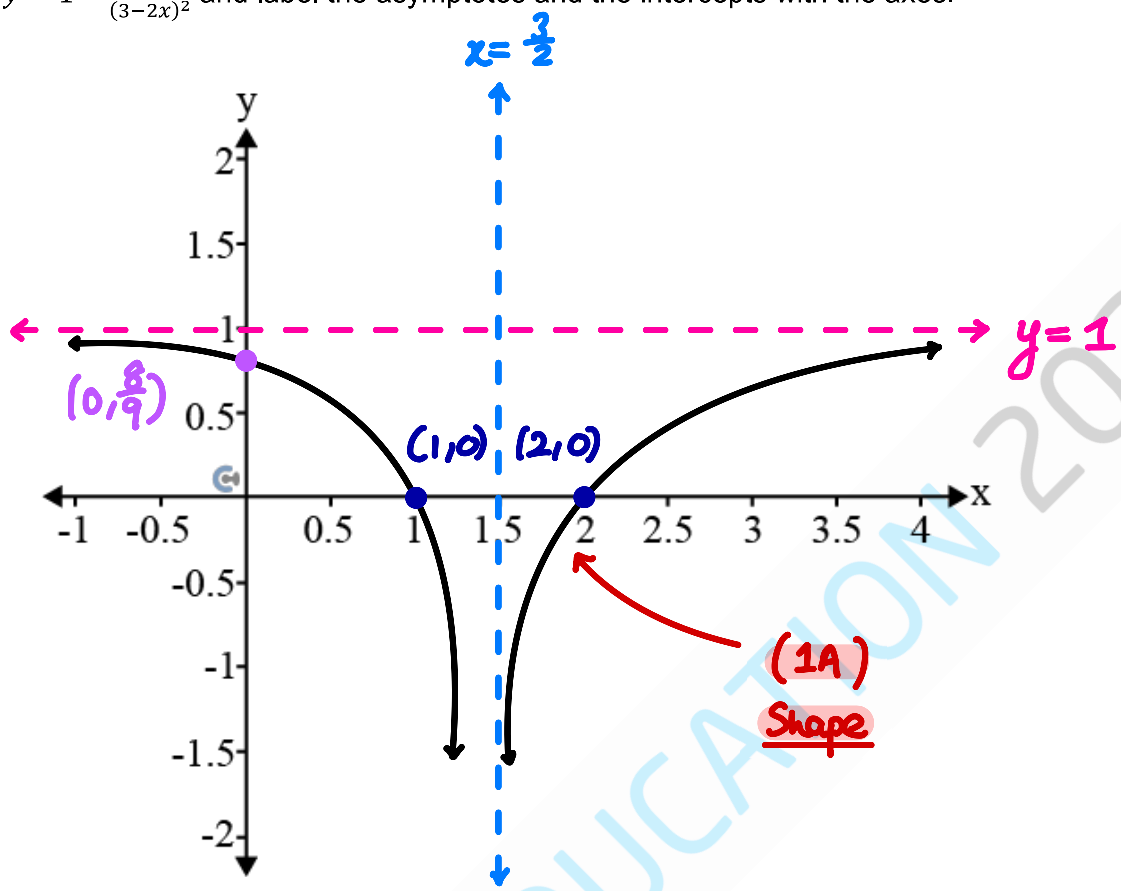
4M Learning Objective [3.4.2] - Finding selections of any size.

CONTOUREDUCATION 2025

Do not write in this area.

Question 6 (3 marks)

Sketch $y = 1 - \frac{1}{(3-2x)^2}$ and label the asymptotes and the intercepts with the axes.



x-int : y = 0

$1 - \frac{1}{(3-2x)^2} = 0$

$1 = \frac{1}{(3-2x)^2}$

$(3-2x)^2 = 1$

$3-2x = 1$ OR $3-2x = -1$

$2x = 2$ OR $2x = 4$

$\therefore x = 1$ OR $x = 2$

(1A) intercepts

y-int : x = 0

$y = 1 - \frac{1}{(3-2(0))^2}$

$= 1 - \frac{1}{3^2}$

$= 1 - \frac{1}{9}$

$\therefore y = \frac{8}{9}$

$y = 1 - \underbrace{\frac{1}{(3-2x)^2}}_{=0}$

x-asymptote :

$\therefore 3-2x = 0$

$\therefore x = \frac{3}{2}$

y-asymptote :

$\therefore y = 1$

(1A) asymptotes

Do not write in this area.

CONTOUR EDUCATION 2025

Question 7 (7 marks)

The graph of $f(x) = -\sqrt{9(x-2)} + 4$ undergoes the following transformations in the order given by:

1. Translate 3 units right (1A)
OR 2. Dilate by factor 4 from the y-axis (1A)

$$g(x) = f\left(\frac{x}{4} - 3\right)$$

$$h(x) = -g(x) + 5$$

TRANSFORMED

a. Describe a set of transformations given by $g(x) = f\left(\frac{x}{4} - 3\right)$ that maps the graph of f to the graph of g. — 2 marks

$f(x) \Rightarrow f\left(\frac{x'}{4} - 3\right)$

$\therefore x = \frac{x'}{4} - 3$

$x' = 4(x+3)$

$x' = 4x + 12 \Rightarrow$

1. Dilation by factor 4 from the y-axis (1A)
2. Translation of 12 units right (1A)

b. Describe the transformations given by $h(x) = -g(x) + 5$ that maps the graph of g to the graph of h. — 2 marks

$h(x) = -g(x) + 5$

$\therefore$
1. Reflection in the x-axis (1A)
2. Translation of 5 units up (1A)

4M Learning Objective [2.4.3] Find transformations from transformed function (Reverse Engineering).

2M Learning Objective [2.4.2] Find transformed functions.

c. Show that the image function is given by: — 3 marks

$$h(x) = \frac{3}{2}\sqrt{x - 20} + 1$$

$f(x) = -\sqrt{9(x-2)} + 4$

$g(x) = f\left(\frac{x}{4} - 3\right) = -\sqrt{9\left(\frac{x}{4} - 3 - 2\right)} + 4$

$= -\sqrt{\frac{9x}{4} - 45} + 4$ (1A)

$h(x) = -g(x) + 5$

$= -\left(-\sqrt{\frac{9x}{4} - 45} + 4\right) + 5$ (1A)

$h(x) = \sqrt{\frac{9x}{4} - 45} + 1$ (1A) $= \sqrt{\frac{1}{4}(9x - 180)}$

$= \sqrt{\frac{9}{4}(x - 20)}$

$\therefore h(x) = \frac{3}{2}\sqrt{x - 20} + 1$

CONTOUR EDUCATION 2025

Do not write in this area.

Question 8 (2 marks) Domain

Consider the function $f: \mathbb{R} \to \mathbb{R}, f(x) = 2x^2 - 3$. State the domain and range of f, and state why f does not have an inverse function, justifying your answer. Also, state a possible maximal domain so f **does** have an inverse function.

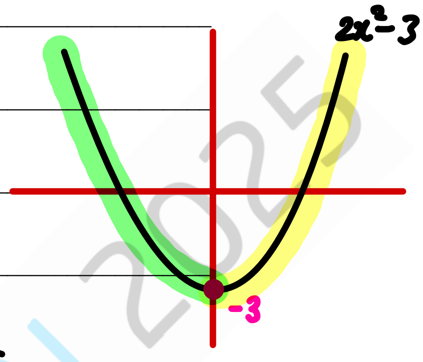
(1A)

Dom f : $x \in \mathbb{R}$

Ran f : $f(x) \in [-3, \infty)$

In order for the inverse to be a function the original function MUST be 1:1.

However, $f(x)$ is a many:1 function so there is NO inverse function.



Possible domains so inverse function exists include:

$x \leq 0$ OR $x \geq 0$ (1A)

1M Learning Objective [2.2.1] Find domain and range of functions.

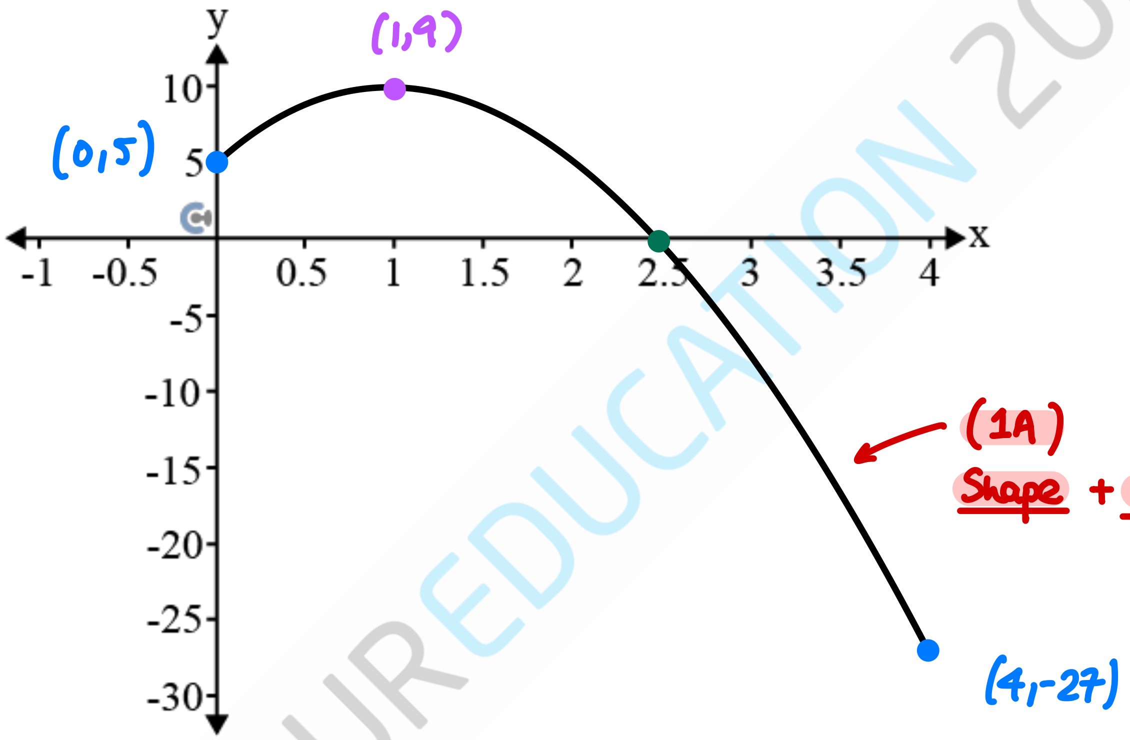
1M Learning Objective [2.3.1] Restrict domain such that the inverse function exists.

Do not write in this area.

Question 9 (4 marks) 4M Learning Objective [1.4.2] Apply Quadratics to Model a scenario.

An object is thrown by a 5 metre tall giant from the top of a high-rise building. The trajectory can be represented by the equation $H = 8x - 4x^2 + 5, x \in [0,4]$, where H is the vertical distance from the top of the building and x is the horizontal distance from the building (both distances measured in metres).

Sketch the path of the object, labelling endpoints and turning point and calculate the maximum and minimum heights of the object.



(1A) Shape + Domain

Endpoints:

(1A) $H(0) = 5\text{m}$

$H(4) = 32 - 64 + 5$

$= -27\text{m}$

X-int : $y = 0$

$8x - 4x^2 + 5 = 0$

$4x^2 - 8x - 5 = 0$

$(2x - 5)(2x + 1) = 0$

$\therefore x = \frac{5}{2}$ or $x = -\frac{1}{2}$

reject as $x \in [0,4]$

Turning Point:

$x = \frac{-b}{2a} = \frac{-8}{2(-4)}$

$= 1$

$\therefore H(1) = 8 - 4 + 5 = 9\text{m}$

$\therefore$ local max at $(1,9)$ (1A)

$\therefore$ Max height $= 9\text{m}$ & Min height $= -27\text{m}$ (1A)

Do not write in this area.

CONTOUREDUCATION 2025

Question 10 (4 marks) 4M Learning Objective [2.2.1] Find domain and range of functions.

Consider the function $f: D \to \mathbb{R}, f(x) = \frac{\sqrt{10-2x}}{x^2-4x}$.

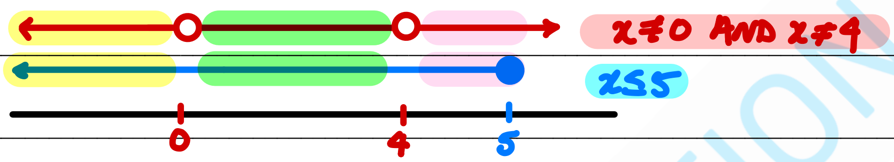
a. Find the maximal domain D of the function. Express your answer using interval notation. 3 marks

$\frac{\sqrt{10-2x}}{x^2-4x}$ $\therefore 10-2x \geq 0 \Rightarrow \therefore x \leq 5$ (1M)

$\therefore x^2-4x \neq 0$ (1M)

$x(x-4) \neq 0$

$\therefore x \neq 0$ AND $x \neq 4$

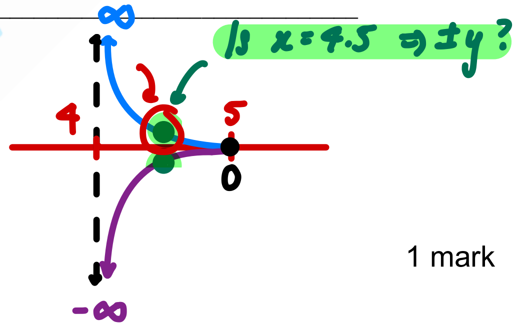


$\therefore x \leq 5$ AND $x \neq 0$ AND $x \neq 4$

(1A) $\therefore D = (-\infty, 0) \cup (0, 4) \cup (4, 5]$

OR $D = (-\infty, 5] \setminus \{0, 4\}$

Consider $g: (4,5) \to \mathbb{R}, g(x) = \frac{\sqrt{10-2x}}{x^2-4x}$.



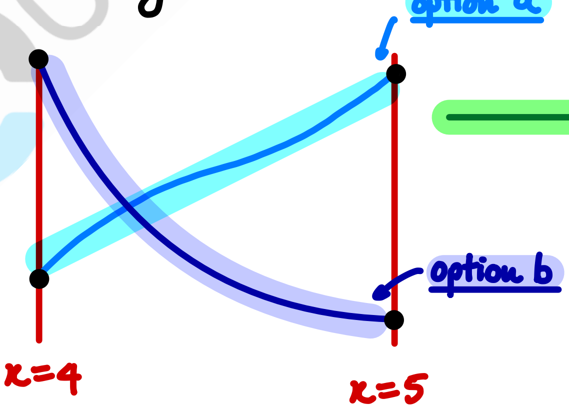
b. Given that g is a one-to-one function, state the range of g. 1 mark

$g(4.5) = \frac{\sqrt{1}}{4.5^2-4\times4.5} = \frac{+}{+} = +$

$4.5\times4.5 - 4\times4.5 > 0$ As $4.5 > 4$

$\therefore$ Ran $g \in (0, \infty)$ // (1A)

Since g is 1:1 :



⟶ Max AND min MUST occur at ENDPOINTS.

$g(5) = \frac{\sqrt{0}}{5} = 0$ //

$g(4) = \frac{\sqrt{2}}{0}$ = undefined //

Do not write in this area.

CONTOUR EDUCATION 2025